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Research Article

Some Inequalities of Simpson Type for h-Convex Functions via Fractional Integrals

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We establish some inequalities of Simpson type involving Riemann-Liouville fractional integrals for mappings whose first derivatives are h-convex.

1. Introduction

The following inequality is well known in the literature as Simpson's inequality.

Theorem 1. *Let $f : [a,b] \to R$ be four times continuously differentiable mapping on (a,b) and $\|f^{(4)}\|_\infty = \sup_{x\in(a,b)}|f^{(4)}(x)| < \infty$.*

Then, the following inequality holds:

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\frac{a+b}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{1}{2880}\left\|f^{(4)}\right\|_\infty (b-a)^4. \tag{1}$$

In [1], Dragomir et al. proved the following inequality.

Theorem 2. *Suppose $f : [a,b] \to R$ is a differentiable mapping whose derivative is continuous on (a,b) and $f' \in L_1[a,b]$. Then the following inequality holds:*

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\frac{a+b}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{b-a}{3}\left\|f'\right\|_1, \tag{2}$$

where $\|f'\|_1 = \int_a^b |f'(x)|dx$.

In [2], Sarikaya et al. obtained inequalities for differentiable convex mappings. The main inequality is as follows.

Theorem 3. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a,b \in I^\circ$ with $a < b$. If $|f'|^q$ is convex on $[a,b]$, $q > 1$, then the following inequality holds:*

$$\begin{aligned}\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]-\frac{1}{b-a}\int_a^b f(x)\,dx\right| &\le \frac{b-a}{12}\left(\frac{1+2^{p+1}}{3(p+1)}\right)^{1/p} \\ &\cdot\left\{\left(\frac{3\left|f'(b)\right|^q+\left|f'(a)\right|^q}{4}\right)^{1/q}\right. \\ &\left.+\left(\frac{\left|f'(b)\right|^q+3\left|f'(a)\right|^q}{4}\right)^{1/q}\right\},\end{aligned} \tag{3}$$

where $1/p + 1/q = 1$.

In [3], Sarikaya et al. obtained the following inequality for s-convex function.

Theorem 4. *Let $f : I \subset [0,\infty] \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a,b \in I^\circ$ with $a < b$. If $|f'|^q$ is s-convex on $[a,b]$, for some fixed $s \in [0,1]$ and $q > 1$, then the following inequality holds:*

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]-\frac{1}{b-a}\int_a^b f(x)\,dx\right|\\&\quad\le\frac{(b-a)}{12}\left(\frac{1+2^{p+1}}{3(p+1)}\right)^{1/p}\\&\quad\cdot\left\{\left(\frac{\left|f'(b)\right|^q+\left|f'((a+b)/2)\right|^q}{s+1}\right)^{1/q}\right.\\&\quad\left.+\left(\frac{\left|f'(a)\right|^q+\left|f'((a+b)/2)\right|^q}{s+1}\right)^{1/q}\right\},\end{aligned}\tag{4}$$

where $1/p + 1/q = 1$.

For recent refinements, counterparts, generalizations, and inequalities of Simpson type, see [1–7].

In 2007, Varošanec in [8] introduced a large class of functions, the so-called h-convex functions. This class contains several well-known classes of functions such as nonnegative convex functions and s-convex functions. This class is defined in the following way: a function $f : I \to R$, $\emptyset \neq I \subset R$ being an interval, is called h-convex if

$$f(tx+(1-t)y)\le h(t)f(x)+h(1-t)f(y)\tag{5}$$

holds for all $x, y \in I$, $t \in (0,1)$, where $h : J \to R$, $h \not\equiv 0$, and J is an interval, $(0,1) \subseteq J$.

The aim of this paper is to establish inequalities of Simpson type for h-convex mappings via fractional integrals which are defined in the following way: left-sided and right-sided Riemann-Liouville fractional integrals of the order $\alpha > 0$ are defined by

$$\begin{aligned}I_{a^+}^{\alpha}f(x)&=\frac{1}{\Gamma(\alpha)}\int_a^x(x-t)^{\alpha-1}f(t)\,dt\qquad(0\le a<x\le b),\\I_{b^-}^{\alpha}f(x)&=\frac{1}{\Gamma(\alpha)}\int_x^b(t-x)^{\alpha-1}f(t)\,dt\qquad(0\le a\le x<b),\end{aligned}\tag{6}$$

where $\Gamma(\alpha) = \int_0^\infty e^{-u}u^{\alpha-1}du$ is the gamma function. Here is $I_{a^+}^{\circ}f(x) = I_{b^-}^{\circ}f(x) = f(x)$.

2. Main Results

To prove our main results, we consider the following lemma.

Lemma 5. *Let $f : I \subset R \to R$ be an absolutely continuous mapping on I° such that $f' \in L_1[a,b]$, where $a,b \in I^\circ$ with $a < b$. Then the following inequality holds:*

$$\begin{aligned}&\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\\&\quad-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\\&\quad=\frac{b-a}{2}\left[\int_0^1\left(\frac{t^{\alpha}}{2}-\frac{1}{3}\right)f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)dt\right.\\&\quad\left.+\int_0^1\left(\frac{1}{3}-\frac{t^{\alpha}}{2}\right)f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)dt\right].\end{aligned}\tag{7}$$

Proof. By integration by parts and by the change of the variables, we have

$$\begin{aligned}&\int_0^1\left(\frac{t^{\alpha}}{2}-\frac{1}{3}\right)f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)dt\\&\quad=\frac{2}{b-a}\left(\frac{t^{\alpha}}{2}-\frac{1}{3}\right)f\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\Bigg|_0^1\\&\quad-\frac{\alpha}{b-a}\int_0^1t^{\alpha-1}f\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)dt\\&\quad=\frac{1}{3(b-a)}f(b)+\frac{2}{3(b-a)}f\left(\frac{a+b}{2}\right)\\&\quad-\frac{2^{\alpha}\cdot\alpha}{(b-a)^{\alpha+1}}\int_{(a+b)/2}^b\left(u-\frac{a+b}{2}\right)^{\alpha-1}f(u)\,du\\&\quad=\frac{1}{3(b-a)}f(b)+\frac{2}{3(b-a)}f\left(\frac{a+b}{2}\right)\\&\quad-\frac{2^{\alpha}\Gamma(\alpha+1)}{(b-a)^{\alpha+1}}I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right).\end{aligned}\tag{8}$$

Similarly, we have

$$\begin{aligned}&\int_0^1\left(\frac{1}{3}-\frac{t^{\alpha}}{2}\right)f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)dt\\&\quad=\frac{2}{a-b}\left(\frac{1}{3}-\frac{t^{\alpha}}{2}\right)f\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\Bigg|_0^1\\&\quad+\frac{\alpha}{a-b}\int_0^1t^{\alpha-1}f\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)dt\\&\quad=\frac{1}{3(b-a)}f(a)+\frac{2}{3(b-a)}f\left(\frac{a+b}{2}\right)\\&\quad+\frac{2^{\alpha}\cdot\alpha}{(b-a)^{\alpha+1}}\int_{(a+b)/2}^a\left(\frac{a+b}{2}-u\right)^{\alpha-1}f(u)\,du\\&\quad=\frac{1}{3(b-a)}f(a)+\frac{2}{3(b-a)}f\left(\frac{a+b}{2}\right)\\&\quad-\frac{2^{\alpha}\Gamma(\alpha+1)}{(b-a)^{\alpha+1}}I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right).\end{aligned}\tag{9}$$

From (8) and (9), we get (7). This completes the proof. □

The following theorems give a new result of Simpson's inequality for h-convex functions via fractional integrals.

Theorem 6. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a,b \in I^\circ$ with $a < b$. If $|f'|$ is h-convex on $[a,b]$, then the following inequality holds:*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+I_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right| \le \frac{b-a}{3}\left[\left|f'(a)\right|+\left|f'(b)\right|\right]\int_0^1 h(t)\,dt. \tag{10}$$

Proof. From Lemma 5 and since $|f'|$ is h-convex on $[a,b]$, we get

$$\begin{aligned}
&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+I_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right| \\
&\le \frac{b-a}{2}\int_0^1\left[\left|\frac{t^\alpha}{2}-\frac{1}{3}\right|\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right| + \left|\frac{1}{3}-\frac{t^\alpha}{2}\right|\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\right]dt \le \frac{b-a}{2} \\
&\cdot\int_0^1\left|\frac{t^\alpha}{2}-\frac{1}{3}\right|\left[h\left(\frac{1+t}{2}\right)\left|f'(b)\right| + h\left(\frac{1-t}{2}\right)\left|f'(a)\right| + h\left(\frac{1+t}{2}\right)\left|f'(a)\right| + h\left(\frac{1-t}{2}\right)\left|f'(b)\right|\right]dt = \frac{b-a}{2}\left[\left|f'(a)\right| + \left|f'(b)\right|\right]\int_0^1\left|\frac{t^\alpha}{2}-\frac{1}{3}\right|\left[h\left(\frac{1+t}{2}\right)+h\left(\frac{1-t}{2}\right)\right]dt \\
&\le \frac{b-a}{3}\left[\left|f'(a)\right|+\left|f'(b)\right|\right]\int_0^1 h(t)\,dt,
\end{aligned} \tag{11}$$

where we used the fact that $|t^\alpha/2-1/3| \le 1/3$ for all $x \in [0,1]$.

The proof is completed. □

Corollary 7. *If in Theorem 6 one takes $h(t) = t$ then inequality (10) reduces to the following inequality for the convex function:*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+I_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right| \le \frac{b-a}{6}\left[\left|f'(a)\right|+\left|f'(b)\right|\right]. \tag{12}$$

Corollary 8. *If in Theorem 6 one takes $h(t) = t^s$ then inequality (10) reduces to the following inequality for the s-convex function:*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+I_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right| \le \frac{b-a}{3(s+1)}\left[\left|f'(a)\right|+\left|f'(b)\right|\right]. \tag{13}$$

Corollary 9. *If in Theorem 6 one takes $h(t) = t$ and $\alpha = 1$ then from the proof of Theorem 6 it follows that the following inequality holds:*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{5(b-a)}{72}\left[\left|f'(a)\right| + \left|f'(b)\right|\right]. \tag{14}$$

Theorem 10. *Let $f : I \subset R \to R$ be a differentiable mapping on I° such that $f' \in L_1[a,b]$, where $a,b \in I^\circ$ with $a < b$. If $|f'|^q$ is h-convex on $[a,b]$ and $q > 1$, then the following inequality holds:*

$$\begin{aligned}
&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+I_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right| \\
&\le \frac{b-a}{6}\left\{\left[\left|f'(b)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt + \left|f'(a)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt\right]^{1/q} + \left[\left|f'(a)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt + \left|f'(b)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt\right]^{1/q}\right\}.
\end{aligned} \tag{15}$$

Proof. From Lemma 5 and the Hőlder inequality, we have

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\\&\quad\left.-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\right|\\&\quad\le\frac{b-a}{2}\left\{\left(\int_0^1\left|\frac{t^{\alpha}}{2}-\frac{1}{3}\right|^p dt\right)^{1/p}\right.\\&\quad\cdot\left(\int_0^1\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|^q dt\right)^{1/q}\\&\quad+\left(\int_0^1\left|\frac{1}{3}-\frac{t^{\alpha}}{2}\right|^p dt\right)^{1/p}\\&\quad\left.\cdot\left(\int_0^1\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q dt\right)^{1/q}\right\},\end{aligned}\tag{16}$$

where $1/p+1/q=1$.

Because $|f'|^q$ is h-convex, we have

$$\begin{aligned}&\int_0^1\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|^q dt\\&\quad\le\left|f'(b)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt\\&\qquad+\left|f'(a)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt,\\&\int_0^1\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q dt\\&\quad\le\left|f'(a)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt\\&\qquad+\left|f'(b)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt.\end{aligned}\tag{17}$$

Using the fact that $|t^{\alpha}/2-1/3|=|1/3-t^{\alpha}/2|\le 1/3$ for all $t\in[0,1]$ and using the last two inequalities in (16) we obtain (15).

This completes the proof of the theorem. □

Corollary 11. *If in Theorem 10 one takes $h(t)=t$ then inequality (15) reduces to the following inequality for the convex function:*

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\\&\quad\left.-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\right|\\&\quad\le\frac{b-a}{2}\left[\left(\frac{3}{4}\left|f'(b)\right|^q+\frac{1}{4}\left|f'(a)\right|^q\right)^{1/q}\right.\\&\quad\left.+\left(\frac{1}{4}\left|f'(b)\right|^q+\frac{3}{4}\left|f'(a)\right|^q\right)^{1/q}\right].\end{aligned}\tag{18}$$

Corollary 12. *If in Theorem 10 one takes $h(t)=t^s$ then inequality (15) reduces to the following inequality for the s-convex function:*

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\\&\quad\left.-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\right|\\&\quad\le\frac{b-a}{2}\left(\frac{2}{s+1}\right)^{1/q}\\&\quad\cdot\left\{\left(\left|f'(b)\right|^q\left(1-\frac{1}{2^{s+1}}\right)+\left|f'(a)\right|^q\frac{1}{2^{s+1}}\right)^{1/q}\right.\\&\quad\left.+\left(\left|f'(b)\right|^q\frac{1}{2^{s+1}}+\left|f'(a)\right|^q\left(1-\frac{1}{2^{s+1}}\right)\right)^{1/q}\right\}.\end{aligned}\tag{19}$$

Theorem 13. *Let $f:I\subset R\to R$ be a differentiable mapping on I° such that $f'\in L_1[a,b]$, where $a,b\in I^{\circ}$ with $a<b$. If $|f'|^q$ is h-convex on $[a,b]$ and $q\ge 1$, then the following inequality holds:*

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\\&\quad\left.-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\right|\\&\quad\le\frac{b-a}{2}\left\{\left(\left|f'(b)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt\right.\right.\\&\quad\left.+\left|f'(a)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt\right)^{1/q}\\&\quad+\left(\left|f'(a)\right|^q\int_0^1 h\left(\frac{1+t}{2}\right)dt\right.\\&\quad\left.\left.+\left|f'(b)\right|^q\int_0^1 h\left(\frac{1-t}{2}\right)dt\right)^{1/q}\right\}.\end{aligned}\tag{20}$$

Proof. From Lemma 5 and the power mean inequality, we have that the following inequality holds:

$$\begin{aligned}&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\\&\quad\left.-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}^{\alpha}f\left(\frac{a+b}{2}\right)+I_{b^-}^{\alpha}f\left(\frac{a+b}{2}\right)\right]\right|\\&\quad\le\frac{b-a}{2}\left(\int_0^1\left|\frac{t^{\alpha}}{2}-\frac{1}{3}\right|dt\right)^{1-1/q}\\&\quad\cdot\left[\left(\int_0^1\left|\frac{t^{\alpha}}{2}-\frac{1}{3}\right|\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|^q dt\right)^{1/q}\right.\\&\quad+\left(\int_0^1\left|\frac{1}{3}-\frac{t^{\alpha}}{2}\right|dt\right.\\&\quad\left.\left.\cdot\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q dt\right)^{1/q}\right].\end{aligned}\tag{21}$$

By the h-convexity of $|f'|^q$ and using the fact that $|t^\alpha/2 - 1/3| = |1/3 - t^\alpha/2| \leq 1/3$ for all $t \in [0,1]$, we have

$$\begin{aligned}
&\int_0^1 \left|\frac{t^\alpha}{2} - \frac{1}{3}\right| \left|f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right)\right|^q dt \\
&\quad \leq \frac{1}{3}\left(\left|f'(b)\right|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right. \\
&\quad \left. + \left|f'(a)\right|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt\right), \\
&\int_0^1 \left|\frac{1}{3} - \frac{t^\alpha}{2}\right| \left|f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)\right|^q dt \\
&\quad \leq \frac{1}{3}\left(\left|f'(a)\right|^q \int_0^1 h\left(\frac{1+t}{2}\right) dt \right. \\
&\quad \left. + \left|f'(b)\right|^q \int_0^1 h\left(\frac{1-t}{2}\right) dt\right).
\end{aligned} \tag{22}$$

Using the last two inequalities in (21) we obtain (20). This completes the proof. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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