

Research Article

The Approximate Solutions of Three-Dimensional Diffusion and Wave Equations within Local Fractional Derivative Operator

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We used the local fractional variational iteration transform method (LFVITM) coupled by the local fractional Laplace transform and variational iteration method to solve three-dimensional diffusion and wave equations with local fractional derivative operator. This method has Lagrange multiplier equal to minus one, which makes the calculations more easily. The obtained results show that the presented method is efficient and yields a solution in a closed form. Illustrative examples are included to demonstrate the high accuracy and fast convergence of this new method.

1. Introduction

The diffusion equation is a partial differential equation that portrays density dynamics in a material that undertakes diffusion. It is also used to describe progression demonstrating diffusive-like performance, for example, the transmission of alleles in a population genetics [1–3]. The three-dimensional diffusion equation in fractal heat transfer involving local fractional derivatives was presented as

$$\nabla^{2\alpha} \varphi(x, y, z, t) = \frac{1}{K^\alpha} \frac{\partial^\alpha \varphi(x, y, z, t)}{\partial t^\alpha}, \quad (1)$$

subject to the initial condition

$$\varphi(x, y, z, 0) = \eta(x, y, z), \quad (2)$$

where the local fractional Laplace operator is defined as follows (see [4–8]):

$$\nabla^{2\alpha} = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} + \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}. \quad (3)$$

K^α is a nondifferentiable diffusion coefficient, and $\varphi(x, y, z, t)$ is satisfied with the nondifferentiable temperature distribution, while the three-dimensional wave equation involving local fractional derivatives was presented as

$$\nabla^{2\alpha} \varphi(x, y, z, t) = \frac{1}{K^\alpha} \frac{\partial^\alpha \varphi(x, y, z, t)}{\partial t^{2\alpha}}, \quad (4)$$

subject to the initial conditions

$$\begin{aligned} \varphi(x, y, z, 0) &= \eta_1(x, y, z), \\ \frac{\partial^\alpha \varphi(x, y, z, 0)}{\partial t^\alpha} &= \eta_2(x, y, z). \end{aligned} \quad (5)$$

Many physical problems are governed by partial differential equations (PDEs), and the solution of these equations has been a subject of many investigators in recent years. The diffusion and wave equations have been successfully modeled for many physical and engineering phenomena such as seismic analysis, rheology, fluid flow, viscous damping, viscoelastic materials, and polymer physics [9–11].

Recently, the diffusion and wave problems were studied by several authors by using local fractional decomposition method [12–15], local fractional variational iteration [15–17], local fractional series expansion [18], local fractional functional decomposition method [19, 20], local fractional Laplace decomposition method [21], local fractional homotopy perturbation method [22], local fractional similarity solution [23], and local fractional differential transform method [24, 25]. In this paper, our aims are to present the coupling method of local fractional Laplace transform and variational iteration method, which is called the local fractional variational iteration transform method, and to use

it to solve three-dimensional diffusion and wave equations with local fractional derivative.

2. Mathematical Fundamentals

In this section, we present the basic theory of local fractional calculus and concepts of local fractional Laplace transform (see [12–15]).

Definition 1. One says that a function $f(x)$ is local fractional continuous at $x = x_0$; if it holds,

$$|f(x) - f(x_0)| < \varepsilon^\alpha, \quad 0 < \alpha \leq 1 \quad (6)$$

with $|x - x_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in R$. For $x \in (a, b)$, it is called local fractional continuous on (a, b) , denoted by $f(x) \in C_\alpha(a, b)$.

Definition 2. Setting $f(x) \in C_\alpha(a, b)$, the local fractional derivative of $f(x)$ at $x = x_0$ is defined as

$$\begin{aligned} D_x^\alpha f(x_0) &= \left. \frac{d^\alpha}{dx^\alpha} f(x) \right|_{x=x_0} = f^{(\alpha)}(x_0) \\ &= \lim_{x \rightarrow x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \end{aligned} \quad (7)$$

where $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(\alpha + 1)(f(x) - f(x_0))$.

Definition 3. Let one denote a partition of the interval $[a, b]$ as (t_j, t_{j+1}) , $j = 0, \dots, N - 1$, and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \dots\}$. The local fractional integral of $f(x)$ in the interval $[a, b]$ is given by

$$\begin{aligned} {}_a I_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t) (dt)^\alpha \\ &= \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)^\alpha. \end{aligned} \quad (8)$$

Definition 4. Let $(1/\Gamma(1 + \alpha)) \int_0^\infty |f(x)|(dx)^\alpha < k < \infty$. The Yang-Laplace transform of $f(x)$ is given by

$$\begin{aligned} L_\alpha \{f(x)\} &= f_s^{L,\alpha}(s) \\ &= \frac{1}{\Gamma(1 + \alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x) (dx)^\alpha, \end{aligned} \quad (9)$$

$0 < \alpha \leq 1,$

where the latter integral converges and $s^\alpha \in R^\alpha$.

Definition 5. The inverse formula of the Yang-Laplace transforms of $f(x)$ is given by

$$\begin{aligned} L_\alpha^{-1} \{f_s^{L,\alpha}(s)\} &= f(x) \\ &= \frac{1}{(2\pi)^\alpha} \int_{\beta-i\omega}^{\beta+i\omega} E_\alpha(s^\alpha x^\alpha) f_s^{L,\alpha}(s) (ds)^\alpha, \end{aligned} \quad (10)$$

$0 < \alpha \leq 1,$

where $s^\alpha = \beta^\alpha + i^\alpha \omega^\alpha$; fractal imaginary unit is i^α , and $\text{Re}(s) = \beta > 0$.

The properties for local fractional Laplace transform used in the paper are given as

$$\begin{aligned} L_\alpha \{af(x) + bg(x)\} &= af_s^{L,\alpha}(s) + bg_s^{L,\alpha}(s), \\ L_\alpha \{E_\alpha(c^\alpha x^\alpha) f(x)\} &= f_s^{L,\alpha}(s - c), \\ L_\alpha \{f^{(k\alpha)}(x)\} &= s^{k\alpha} f_s^{L,\alpha}(s) - s^{(k-1)\alpha} f(0) \\ &\quad - s^{(k-2)\alpha} f^{(\alpha)}(0) - \dots \\ &\quad - f^{((k-1)\alpha)}(0), \\ L_\alpha \{E_\alpha(a^\alpha x^\alpha)\} &= \frac{1}{s^\alpha - a^\alpha}, \\ L_\alpha \{\sin_\alpha(a^\alpha x^\alpha)\} &= \frac{a^\alpha}{s^{2\alpha} + a^{2\alpha}}, \\ L_\alpha \{x^{k\alpha}\} &= \frac{\Gamma(1 + k\alpha)}{s^{(k+1)\alpha}}. \end{aligned} \quad (11)$$

3. LFVITM for Three-Dimensional Diffusion Problems

We first rewrite problem (1) in the local fractional operator form

$$\begin{aligned} L_t^{(\alpha)} \varphi(x, y, z, t) &= K^\alpha (L_{xx}^{(2\alpha)} \varphi(x, y, z, t) \\ &\quad + L_{yy}^{(2\alpha)} \varphi(x, y, z, t) + L_{zz}^{(2\alpha)} \varphi(x, y, z, t)), \end{aligned} \quad (12)$$

where the local fractional differential operators $L_t^{(\alpha)}$, $L_{xx}^{(2\alpha)}$, $L_{yy}^{(2\alpha)}$, and $L_{zz}^{(2\alpha)}$ are defined by

$$\begin{aligned} L_t^{(\alpha)}(\cdot) &= \frac{\partial^\alpha}{\partial t^\alpha}(\cdot), \\ L_{xx}^{(2\alpha)}(\cdot) &= \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}(\cdot), \\ L_{yy}^{(2\alpha)}(\cdot) &= \frac{\partial^{2\alpha}}{\partial y^{2\alpha}}(\cdot), \\ L_{zz}^{(2\alpha)}(\cdot) &= \frac{\partial^{2\alpha}}{\partial z^{2\alpha}}(\cdot). \end{aligned} \quad (13)$$

Adopting the local fractional Laplace transform (denoted in this paper by L_α) to both sides of (12) and using the initial condition leads to

$$\begin{aligned} L_\alpha \{\varphi(x, y, z, t)\} &= \frac{1}{s^\alpha} \varphi(x, y, z) + \frac{1}{s^\alpha} \\ &\quad \cdot L_\alpha \{K^\alpha (L_{xx}^{(2\alpha)} \varphi(x, y, z, t) + L_{yy}^{(2\alpha)} \varphi(x, y, z, t) \\ &\quad + L_{zz}^{(2\alpha)} \varphi(x, y, z, t))\}. \end{aligned} \quad (14)$$

Operating with the inverse of local fractional Laplace transform on both sides of (14) gives

$$\begin{aligned} \varphi(x, y, z, t) &= \eta(x, y, z) \\ &+ \mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi + L_{yy}^{(2\alpha)} \varphi + L_{zz}^{(2\alpha)} \varphi \} \right). \end{aligned} \tag{15}$$

Deriving both sides of (15) with respect to t , we have

$$\begin{aligned} L_t^{(\alpha)} \varphi(x, y, z, t) &= L_t^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi + L_{yy}^{(2\alpha)} \varphi + L_{zz}^{(2\alpha)} \varphi \} \right) \right]. \end{aligned} \tag{16}$$

By the correction function of the irrational method

$$\varphi_{n+1}(x, y, z, t) = \varphi_n(x, y, z, t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_n - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi_n + L_{yy}^{(2\alpha)} \varphi_n + L_{zz}^{(2\alpha)} \varphi_n \} \right) \right] \right) (d\tau)^\alpha, \tag{17}$$

finally, the solution $\varphi(x, y, z, t)$ is given by

$$\varphi(x, y, z, t) = \lim_{n \rightarrow \infty} \varphi_n(x, y, z, t). \tag{18}$$

$$\begin{aligned} \varphi(x, y, z, 0) &= E_\alpha(x^\alpha + y^\alpha + z^\alpha), \\ K &= 1, \end{aligned} \tag{19}$$

We now consider the initial conditions of (2); namely,

we have

$$\begin{aligned} \varphi_0(x, y, z, t) &= E_\alpha(x^\alpha + y^\alpha + z^\alpha), \\ \varphi_{n+1}(x, y, z, t) &= \varphi_n(x, y, z, t) \\ &- \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_n(\tau) - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{1^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi_n(\tau) + L_{yy}^{(2\alpha)} \varphi_n(\tau) + L_{zz}^{(2\alpha)} \varphi_n(\tau) \} \right) \right] \right) (d\tau)^\alpha. \end{aligned} \tag{20}$$

Consequently, we obtain

$$\begin{aligned} \varphi_0(x, y, z, t) &= E_\alpha(x^\alpha + y^\alpha + z^\alpha), \\ \varphi_1(x, y, z, t) &= \varphi_0(x, y, z, t) \\ &- \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_0(\tau) - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{1^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi_0(\tau) + L_{yy}^{(2\alpha)} \varphi_0(\tau) + L_{zz}^{(2\alpha)} \varphi_0(\tau) \} \right) \right] \right) (d\tau)^\alpha \\ &= E_\alpha(x^\alpha + y^\alpha + z^\alpha) \left(1 + \frac{3t^\alpha}{\Gamma(1+\alpha)} \right), \end{aligned} \tag{21}$$

$$\begin{aligned} \varphi_2(x, y, z, t) &= \varphi_1(x, y, z, t) \\ &- \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_1(\tau) - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{1^\alpha}{s^\alpha} \mathcal{L}_\alpha \{ L_{xx}^{(2\alpha)} \varphi_1(\tau) + L_{yy}^{(2\alpha)} \varphi_1(\tau) + L_{zz}^{(2\alpha)} \varphi_1(\tau) \} \right) \right] \right) (d\tau)^\alpha \\ &= E_\alpha(x^\alpha + y^\alpha + z^\alpha) \left(1 + \frac{(3t)^\alpha}{\Gamma(1+\alpha)} + \frac{9t^{2\alpha}}{\Gamma(1+2\alpha)} \right), \end{aligned}$$

and so on.

The solution in a nondifferentiable series form

$$\begin{aligned} &\cdot \left(1 + \frac{3t^\alpha}{\Gamma(1+\alpha)} + \frac{9t^{2\alpha}}{\Gamma(1+2\alpha)} + \dots \right) \end{aligned} \tag{22}$$

$$\varphi(x, y, z, t) = E_\alpha(x^\alpha + y^\alpha + z^\alpha)$$

is readily obtained.

Therefore, the exact solution can be written as

$$\varphi(x, y, z, t) = E_\alpha(x^\alpha + y^\alpha + z^\alpha + 3t^\alpha). \quad (23)$$

4. LFVITM for Three-Dimensional Wave Problems

We first rewrite the problem (4) in the local fractional operator form

$$\begin{aligned} L_{tt}^{(\alpha)} \varphi(x, y, z, t) &= K^\alpha \left(L_{xx}^{(2\alpha)} \varphi(x, y, z, t) \right. \\ &\quad \left. + L_{yy}^{(2\alpha)} \varphi(x, y, z, t) + L_{zz}^{(2\alpha)} \varphi(x, y, z, t) \right). \end{aligned} \quad (24)$$

Applying the local fractional Laplace transform to both sides of (24) and using the initial condition leads to

$$\begin{aligned} \mathcal{L}_\alpha \{ \varphi(x, y, z, t) \} &= \frac{1}{s^\alpha} \eta_1(x, y, z) + \frac{1}{s^{2\alpha}} \eta_2(x, y, z) \\ &\quad + \frac{1}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ K^\alpha \left(L_{xx}^{(2\alpha)} \varphi(x, y, z, t) \right. \right. \\ &\quad \left. \left. + L_{yy}^{(2\alpha)} \varphi(x, y, z, t) + L_{zz}^{(2\alpha)} \varphi(x, y, z, t) \right) \right\}. \end{aligned} \quad (25)$$

Operating with the inverse of local fractional Laplace transform on both sides of (25) gives

$$\begin{aligned} \varphi(x, y, z, t) &= \eta_1(x, y, z) + \frac{t^\alpha}{\Gamma(1+\alpha)} \eta_2(x, y, z) \\ &\quad + \mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ L_{xx}^{(2\alpha)} \varphi + L_{yy}^{(2\alpha)} \varphi + L_{zz}^{(2\alpha)} \varphi \right\} \right). \end{aligned} \quad (26)$$

Deriving both sides of (26) with respect to t , we obtain

$$\begin{aligned} L_t^{(\alpha)} \varphi(x, y, z, t) &= \eta_2(x, y, z) \\ &\quad + L_t^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ L_{xx}^{(2\alpha)} \varphi + L_{yy}^{(2\alpha)} \varphi + L_{zz}^{(2\alpha)} \varphi \right\} \right) \right]. \end{aligned} \quad (27)$$

By the correction function of the irrational method,

$$\begin{aligned} \varphi_{n+1}(x, y, z, t) &= \varphi_n(x, y, z, t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_n \right. \\ &\quad \left. - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{K^\alpha}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ L_{xx}^{(2\alpha)} \varphi_n + L_{yy}^{(2\alpha)} \varphi_n + L_{zz}^{(2\alpha)} \varphi_n \right\} \right) \right] \right. \\ &\quad \left. - \eta_2(x, y, z) \right) (d\tau)^\alpha. \end{aligned} \quad (28)$$

Finally, the solution $\varphi(x, y, z, t)$ is given by

$$\varphi(x, y, z, t) = \lim_{n \rightarrow \infty} \varphi_n(x, y, z, t). \quad (29)$$

We now consider the initial conditions of (5); namely,

$$\begin{aligned} \varphi(x, y, z, 0) &= 0, \\ \frac{\partial^\alpha \varphi(x, y, z, 0)}{\partial t^\alpha} &= 3 \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha), \\ K &= 3. \end{aligned} \quad (30)$$

Starting with the zeroth approximation,

$$\begin{aligned} \varphi_0(x, y, z, t) &= \frac{3t^\alpha}{\Gamma(1+\alpha)} \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha). \end{aligned} \quad (31)$$

Substituting (31) in (28) we obtain the following successive approximations:

$$\begin{aligned} \varphi_1(x, y, z, t) &= \varphi_0(x, y, z, t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_0(\tau) \right. \\ &\quad \left. - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{3^\alpha}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ L_{xx}^{(2\alpha)} \varphi_0(\tau) + L_{yy}^{(2\alpha)} \varphi_0(\tau) + L_{zz}^{(2\alpha)} \varphi_0(\tau) \right\} \right) \right] \right. \\ &\quad \left. - 3 \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha) \right) (d\tau)^\alpha = \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \\ &\quad \cdot \sin_\alpha(z^\alpha) \left(\frac{3t^\alpha}{\Gamma(1+\alpha)} - \frac{27t^{3\alpha}}{\Gamma(1+3\alpha)} \right), \end{aligned} \quad (32)$$

$$\begin{aligned} \varphi_2(x, y, z, t) &= \varphi_1(x, y, z, t) - \frac{1}{\Gamma(1+\alpha)} \int_0^t \left(L_\tau^{(\alpha)} \varphi_1(\tau) \right. \\ &\quad \left. - L_\tau^{(\alpha)} \left[\mathcal{L}_\alpha^{-1} \left(\frac{3^\alpha}{s^{2\alpha}} \mathcal{L}_\alpha \left\{ L_{xx}^{(2\alpha)} \varphi_1(\tau) + L_{yy}^{(2\alpha)} \varphi_1(\tau) + L_{zz}^{(2\alpha)} \varphi_1(\tau) \right\} \right) \right] \right. \\ &\quad \left. - 3 \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha) \right) (d\tau)^\alpha = \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \\ &\quad \cdot \sin_\alpha(z^\alpha) \left(\frac{3t^\alpha}{\Gamma(1+\alpha)} - \frac{27t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{243t^{5\alpha}}{\Gamma(1+5\alpha)} \right), \end{aligned}$$

and so on.

The solution in a nondifferentiable series form

$$\begin{aligned} \varphi(x, y, z, t) &= \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha) \\ &\quad \cdot \left(\frac{3t^\alpha}{\Gamma(1+\alpha)} - \frac{27t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{243t^{5\alpha}}{\Gamma(1+5\alpha)} \dots \right) \end{aligned} \quad (33)$$

is readily obtained.

Therefore, the exact solution can be written as

$$\begin{aligned} \varphi(x, y, z, t) &= \sin_\alpha(x^\alpha) \sin_\alpha(y^\alpha) \sin_\alpha(z^\alpha) \sin_\alpha(3t^\alpha). \end{aligned} \quad (34)$$

5. Conclusion

In this work, we studied the local fractional variational iteration transform method to solve three-dimensional diffusion and wave equations involving local fractional derivative operator and their nondifferentiable solutions were obtained. This method can also be applied to a large class of system of partial differential equations with approximations that converges rapidly to accurate solutions.

Competing Interests

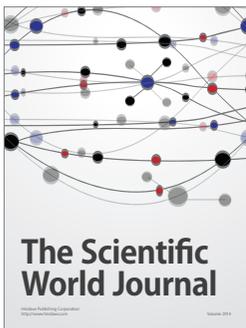
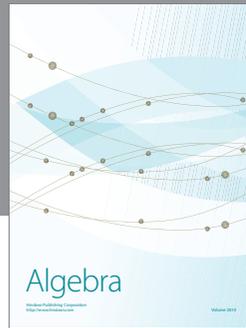
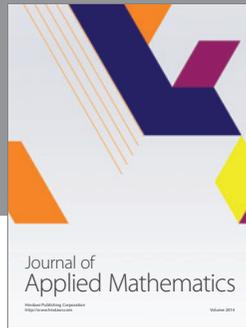
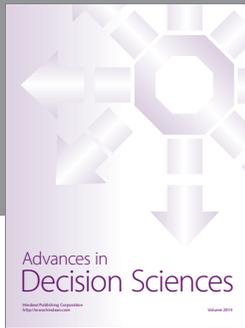
The author declares that there are no competing interests regarding this paper.

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