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Research Article

Studying Radiation and Reaction Effects on Unsteady MHD Non-Newtonian (Walter's B) Fluid in Porous Medium

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This paper describes the studied effects of thermal radiation and chemical reaction on unsteady MHD non-Newtonian (obeying Walter's B model) fluid in porous medium. The resulting problems are solved numerically. Graphical results for various interesting parameters are presented. Also the effects of the different parameters on the skin-friction and the heat fluxes are obtained and discussed numerically.

1. Introduction

Thermal convection in porous medium has attracted considerable interest during the last few decades, because it has various applications in geophysics, soil sciences, ground water hydrology, astrophysics, food processing, oceanography, limnology, engineering, and so forth. There are many elasticoviscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walter's B model elasticoviscous fluid having relevance in chemical technology and industry. However, the interest and research activities regarding the boundary layer flow of non-Newtonian fluids have increased considerably in the past few decades and it is one of the thrust areas of current research [1–7]. Of course, this is because of the industrial and engineering applications of non-Newtonian fluids as well as their interesting mathematical challenges in the form of highly nonlinear equations governing the flows.

The effect of magnetic field on thermal instability of Walter's B model elasticoviscous fluid finds importance in geophysics, particularly, in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion.

Kumar and Srivastava [8] have worked on the effects of chemical reaction on MHD flow of dusty viscoelastic (Walter's liquid model B) liquid with heat source/sink. Khan et al. [9] have studied the hydromagnetic rotating flows of an Oldroyd-B fluid in a porous medium. Khan et al. [10] have investigated the viscoelastic MHD flow and heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work. The numerical or approximate solutions for both steady and transient flows of Walters' B fluid have been studied at great length in a diverse range of geometries using a wide spectrum of computational or analytical techniques [11–13]. Effects of thermal diffusion and chemical reaction on MHD flow of dusty viscoelastic (Walter's liquid model B) fluid have been inspected by Prakash et al. [14]. Abdul Hakeem et al. [15] have found the effect of heat radiation in Walter's liquid B fluid over a stretching sheet with nonuniform heat source/sink and elastic deformation. Madhurai and Kalpana [16] have discussed thermal effect on unsteady flow of a dusty viscoelastic fluid between two parallel plates under different pressure gradients. Recently, unsteady free convection flow in Walters' B fluid and heat transfer analysis have been presented by Khan et al. [17].

In the present problem, the effects of design viscoelastic parameter, unsteadiness parameter, magnetic field parameter,

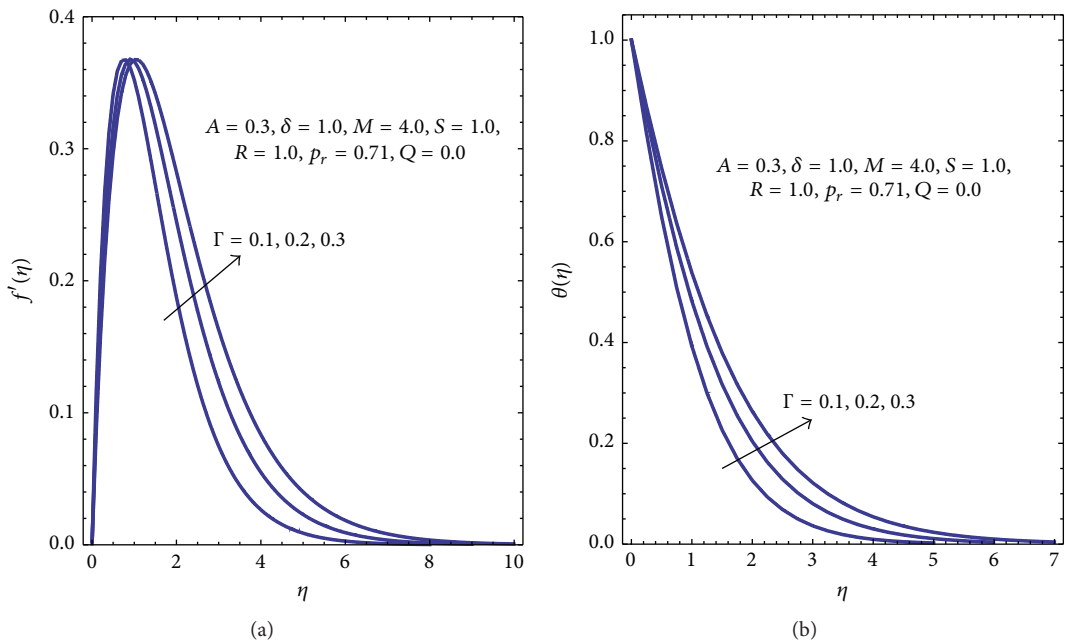

Figure 1: Effect of Γ on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

porosity parameter, thermal radiation parameter, heating source parameter, and Prandtl number parameter on account of fluid flow, heat transfer, the skin-friction, and the heat fluxes are obtained and discussed numerically.

2. Mathematical Model

Consider unsteady MHD non-Newtonian (obeying Walter's B model) fluid in porous medium in the presence of thermal radiation and chemical reaction over a vertical rigid plate at $y = 0$. The fluid is assumed to be falling down vertically over a vertical plane. The governing equations are given by the following.

The momentum equations:

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{k_1}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} + g\beta_1 (T - T_\infty) - \frac{\sigma B^2}{\rho}u - \frac{\mu}{\rho\kappa}u, \tag{1}$$

the energy equation:

$$\frac{\partial T}{\partial t} = \frac{k_2}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{Q_1}{\rho C_p}(T - T_\infty). \tag{2}$$

The initial conditions are given by

$$u(y,t) = 0, \quad T(y,t) = T_w, \quad \text{for } t < 0. \tag{3}$$

The boundary conditions for $t \geq 0$ are

$$\begin{aligned} u(0,t) &= 0, \\ u(\infty,t) &= 0, \\ u_y(\infty,t) &= 0, \\ T(0,t) &= T_w, \\ T(\infty,t) &= T_\infty. \end{aligned} \tag{4}$$

In the above equations u is the velocity fluid phase, t is the time, ρ is the densities of the fluid, g is the acceleration due to gravity, σ is the electrical conductivity, $\kappa = \kappa_0(1 - at)^4$ is the permeability of the porous medium, a magnetic field $B = B_0(1 - at)^{-1/2}$ normal to the stretching sheet is applied, T is the fluid temperature, C_p is the specific heat of the fluid at constant pressure, $\mu = \mu_0(1 - at)^3$ is the viscosity of the fluid, $k_1 = k_0(1 - at)^4$ is Walters' viscoelasticity parameter, $k_2 = k_0^*(1 - at)^3$ is the thermal conductivity of the fluid, $\beta_1 = \beta_0 b(1-at)^{-1}$ is the expansion coefficient of temperature, $Q_1 = Q_0 b(1 - at)^{-1}$ is the internal heating parameter, $\nu_0 = \mu_0/\rho$ is the kinematic viscosity of the fluid, Q_0 is the volumetric heat generation/absorption rate, $T_w = T_\infty + bT_0(1 - at)^{-1}$ is the wall temperature, a, b, k_0, k_0^*, and β_0 are positive constants, and T_∞ is the uniform temperature. Using the Rosseland approximation (Sparrow and Cess [18] and Abdel-Rahman Rashed [19]), the radiative heat flux q_r could be expressed by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ^* represents the Stefan-Boltzmann constant and k^* is the Rosseland mean absorption coefficient.

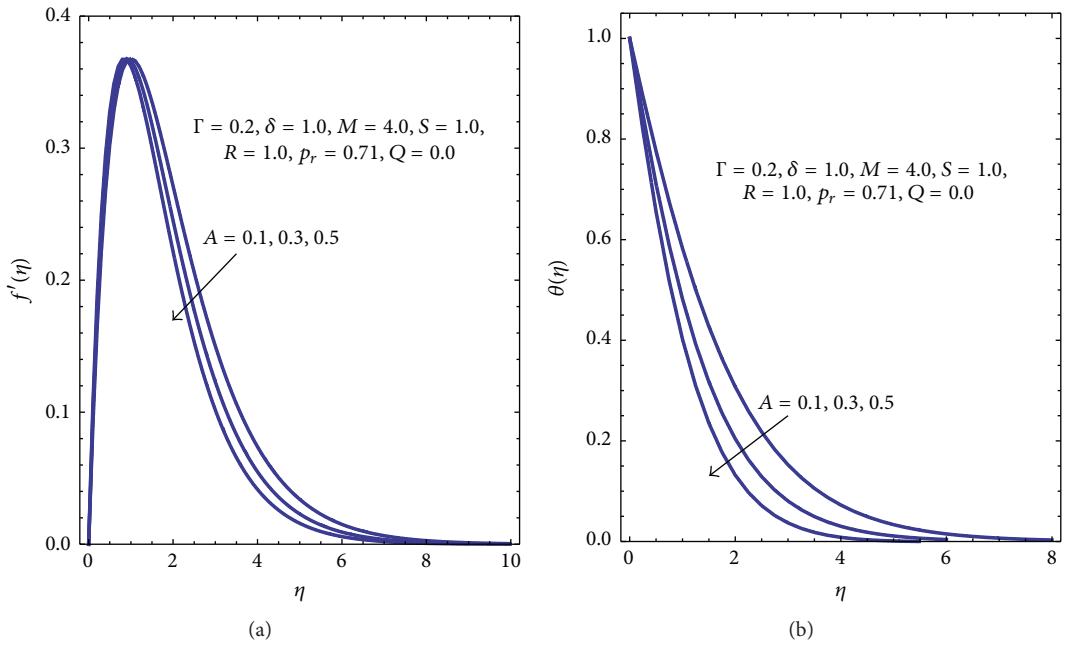

FIGURE 2: Effect of A on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

Assume that the temperature difference within the flow is sufficiently small such that T^4 could be approached as the linear function of temperature:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

The similarity variables and parameters are as

$$\begin{aligned} u(y,t) &= b(1-at)^{-1} f'(\eta), \\ \eta &= b^2(1-at)^{-2} y, \\ T(y,t) &= T_\infty + bT_0(1-at)^{-1}\theta(\eta). \end{aligned} \tag{7}$$

Substituting (5)–(7) in (1)–(4), we obtain

$$\begin{aligned} &f''' - \Gamma\left(5f''' + 2\eta f^{1v}\right) - A\left(f' + 2\eta f''\right) + \delta\theta \\ &\quad - (M+S) f' = 0, \\ &(1+R)\theta'' - Ap_r\left(\theta + 2\eta\theta'\right) + Q\theta = 0 \end{aligned} \tag{8}$$

with the boundary conditions

$$\begin{aligned} f(0) &= 0, \\ f'(0) &= 0, \\ \theta(0) &= 1, \\ f'(\infty) &= 0, \\ f''(\infty) &= 0, \\ \theta(\infty) &= 0, \end{aligned} \tag{9}$$

where $A = a/\nu_0 b^4$ is the unsteadiness parameter, $\Gamma = k_0 a/\rho\nu_0$ is the viscoelastic parameter, $\delta = g\beta_0 T_0/\nu_0 b^3$ is the temperature buoyancy parameter, $M = \sigma B_0^2/\mu_0 b^4$ is the magnetic field parameter, $S = 1/\kappa_0 b^4$ is the porosity parameter, $p_r = \mu_0 C_p/k_0^*$ is the Prandtl number, $R = 16\sigma^* T_\infty^3 (T_w - T_\infty)^3/3k^* k_0^* b^3 T_0^3$ is the radiation parameter, and $Q = Q_0/k_0^* b^4$ is the heating source parameter.

3. Skin-Friction Coefficient and Nusselt Number

A quantity of interest for the present problem is the local skin-friction coefficients C_f and the local Nusselt number N_u, which are defined as

$$\begin{aligned} C_f &= \frac{\tau_w|_{y=0}}{\mu_0 b^3}, \\ N_u &= \frac{q_w|_{y=0}}{k_0^* T_0 b^3}, \end{aligned} \tag{10}$$

where τ_w and q_w are the surface of Walter's B fluid, which are given by

$$\begin{aligned} \tau_w &= \mu\left(\frac{\partial u}{\partial y} - k_1 \frac{\partial^2 u}{\partial y \partial t}\right)\Bigg|_{y=0}, \\ q_w &= -k_2\left(\frac{\partial T}{\partial y}\right)\Bigg|_{y=0}. \end{aligned} \tag{11}$$

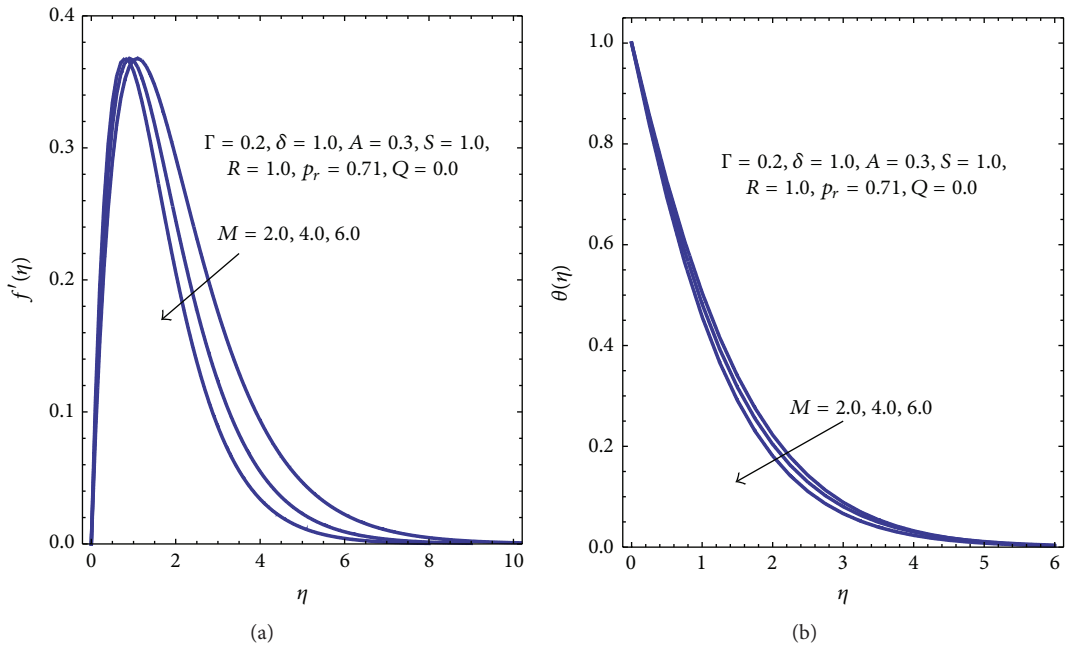

Figure 3: Effect of M on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

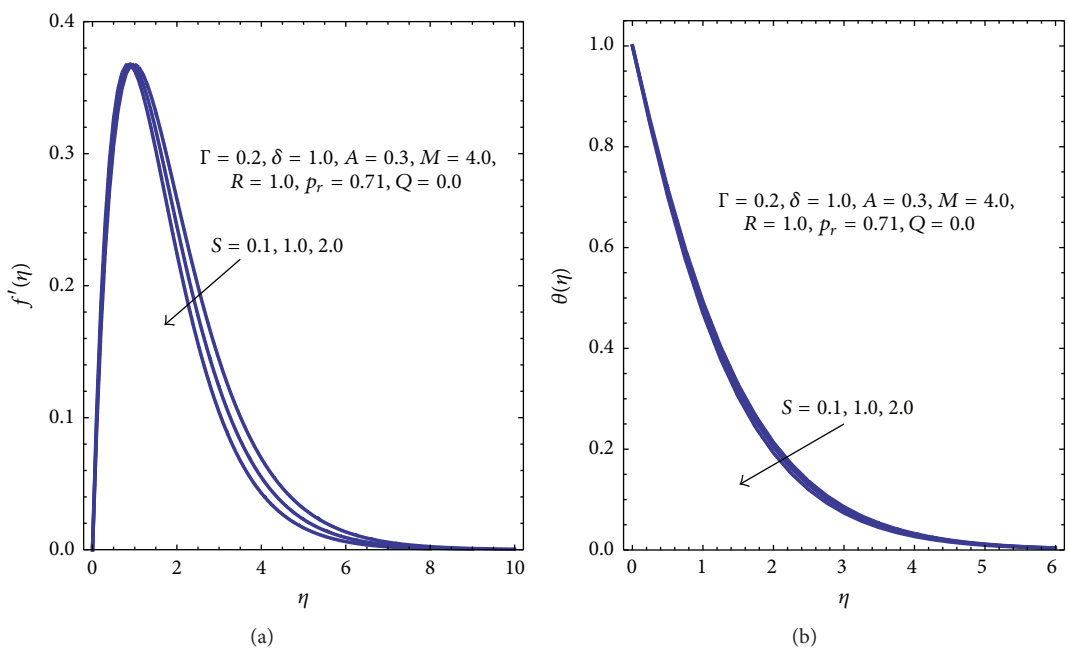

Figure 4: Effect of S on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

Using (10) and (11), we get

$$C_f = (1 - 3\Gamma) f''(0),$$
$$N_u = -\theta'(0). \tag{12}$$

4. Results and Discussion

The system of coupled nonlinear ordinary differential equations (8) subject to boundary conditions (9) is solved numerically by using the shooting technique with forth order of Runge-Kutta algorithm. In order to gain physical insight the velocity and temperature profiles have been discussed.

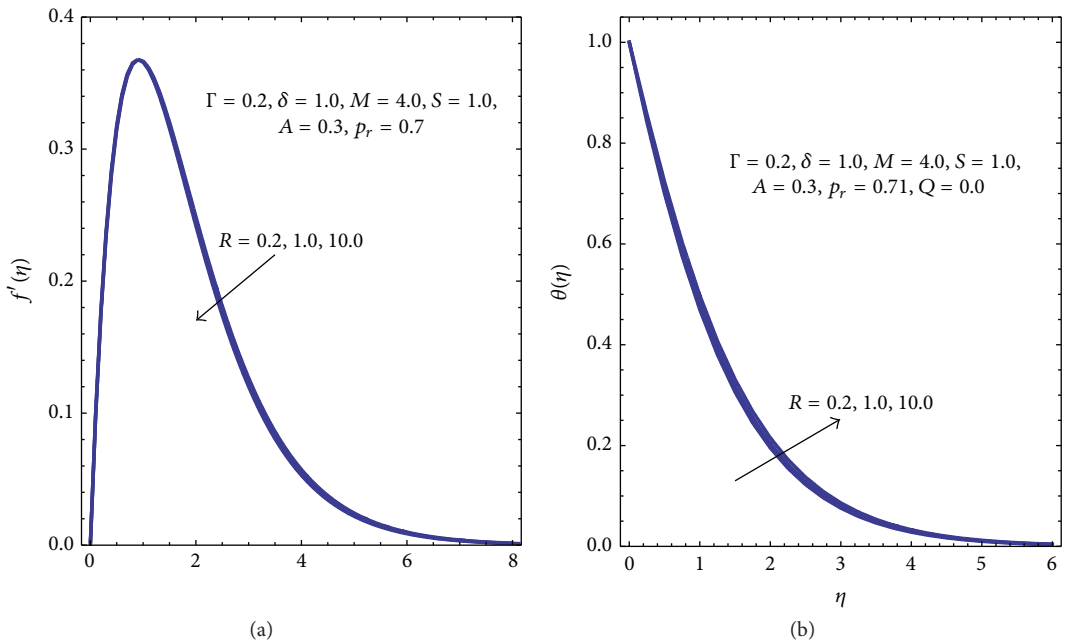

Figure 5: Effect of R on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

The velocity $f'(\eta)$ and the temperature $\theta(\eta)$ profiles increase with the increase of viscoelastic parameter Γ, shown in Figures 1(a) and 1(b).

Figures 2, 3, 4, and 6 show the effect of unsteadiness parameter, magnetic field parameter, porosity parameter, and Prandtl number on the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ profiles. We found that A, M, S, and p_r increase with decrease of the velocity and the temperature profiles.

Figures 5 and 7 confirm that the velocity profile decreases and the temperature profile increases when thermal radiation parameter and heating parameter increase.

The numerical values of the local skin-friction and the local Nusselt number are given in Table 1. For an increase in Γ, we observe that the local skin-friction coefficient decreases and the local Nusselt number increases, while, with the increase in each of A, M, S, and p_r, we observe that the local skin-friction coefficient increases. But the local Nusselt number decreases.

With an increase in each of R and Q, we observe that the local skin-friction coefficient decreases but the local Nusselt number increases.

Table 1: Numerical of the values skin-friction coefficient (C_f) and Nusselt number (N_u) with Γ, A, M, S, R, p_r, and Q.

Γ	A	M	S	R	p_r	Q	C_f	N_u
0.1	0.3	4.0	1.0	1.0	0.71	0.0	1.32385	−0.764965
0.2							1.09487	−0.623587
0.3							0.95622	−0.544546
0.2	0.1	4.0	1.0	1.0	0.71	0.0	0.99741	−0.466889
	0.3						1.09487	−0.623587
	0.5						1.18435	−0.759761
0.2	0.3	2.0	1.0	1.0	0.71	0.0	0.91739	−0.588509
		4.0					1.09487	−0.623587
		6.0					1.24735	−0.661887
0.2	0.3	4.0	0.1	1.0	0.71	0.0	1.01884	−0.607028
			1.0				1.09487	−0.623587
			2.0				1.17359	−0.642623
0.2	0.3	4.0	1.0	0.2	0.71	0.0	1.10639	−0.658349
				1.0			1.09487	−0.623587
				10.0			1.08057	−0.580003
0.2	0.3	4.0	1.0	1.0	0.71	0.0	1.09487	−0.623587
					7.0		1.23924	−1.04038
					10.0		1.30247	−1.21198
0.2	0.3	4.0	1.0	1.0	0.71	−0.5	1.13492	−0.743195
						0.0	1.09487	−0.623587
						0.5	1.05331	−0.495614

5. Conclusions

We computationally investigate the effects of thermal radiation and chemical reaction on unsteady MHD non-Newtonian (obeying Walter's B model) fluid in porous medium. The following is observed.

(1) The velocity and the temperature profiles decrease with the increase of each of the magnetic field and porosity parameters.

(2) With the increase of each of thermal radiation and heating source parameters, the velocity profile decreases while the temperature profile increases.

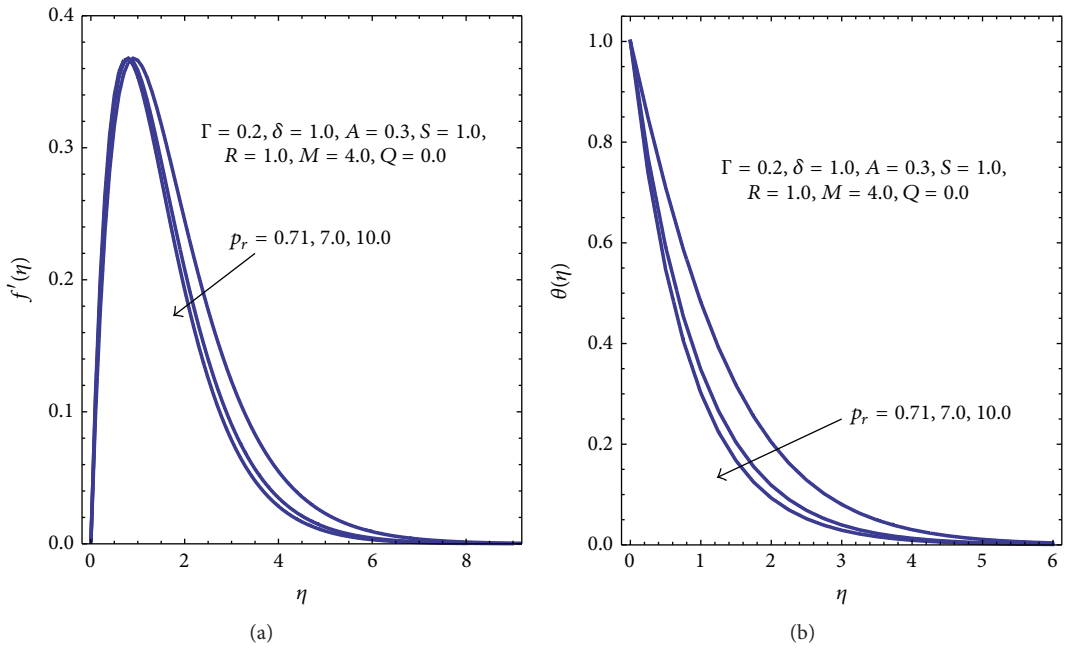

Figure 6: Effect of p_r on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

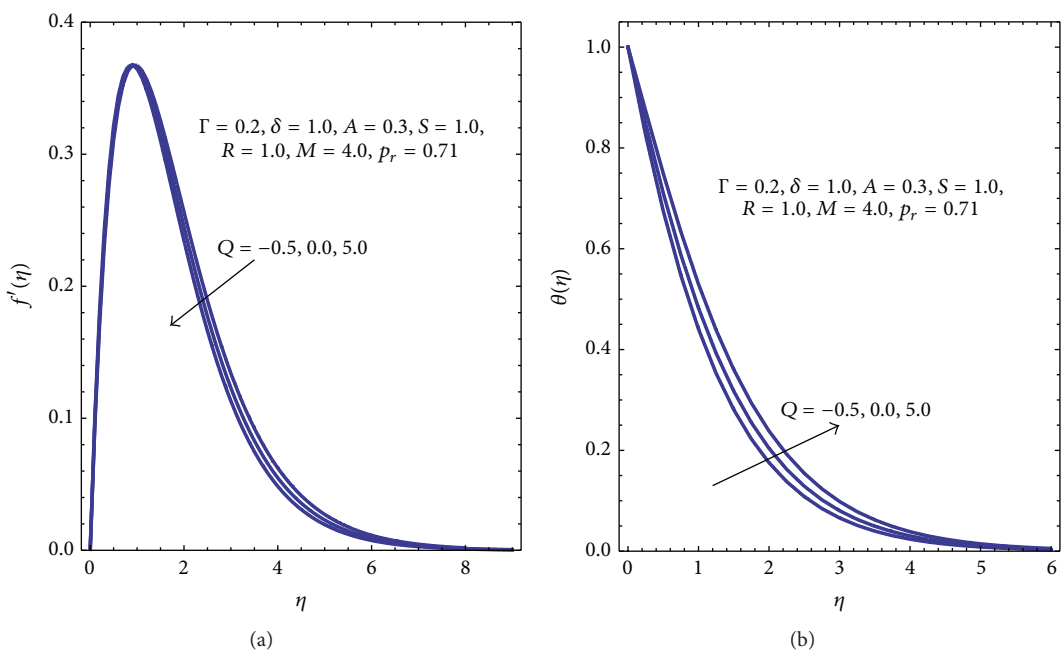

Figure 7: Effect of Q on (a) velocity profile $f'(\eta)$ and (b) temperature profile $\theta(\eta)$.

(3) For an increase of each of the magnetic field and porosity parameters, the local skin-friction coefficient increases while the local Nusselt number decreases.

(4) With the increase of each of thermal radiation and heating source parameters, the local skin-friction coefficient decreases but the local Nusselt number increases.

Competing Interests

The authors declare that they have no competing interests.

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