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Research Article

Three Different Methods for New Soliton Solutions of the Generalized NLS Equation

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Three different methods are applied to construct new types of solutions of nonlinear evolution equations. First, the Csch method is used to carry out the solutions; then the Extended Tanh-Coth method and the modified simple equation method are used to obtain the soliton solutions. The effectiveness of these methods is demonstrated by applications to the RKL model, the generalized derivative NLS equation. The solitary wave solutions and trigonometric function solutions are obtained. The obtained solutions are very useful in the nonlinear pulse propagation through optical fibers.

1. Introduction

Partial differential equations describe various nonlinear phenomena in natural and applied sciences such as fluid dynamics, plasma physics, solid state physics, optical fibers, acoustics, biology, and mathematical finance. Partial differential equations which arise in real-world physical problems are often too complicated to be solved exactly. It is of significant importance to solve nonlinear partial differential equations (NLPDEs) from both theoretical and practical points of view. The analysis of some physical phenomena is investigated by the exact solutions of nonlinear evolution equations (NLEEs) [1–9].

In this paper, the third-order generalized NLS equation is studied, which is proposed by Radhakrishnan, Kundu, and Lakshmanan (RKL) [10]. The normalized RKL model can be written as

$$iq_t + q_{xx} + 2|q|^2 q + i\alpha q_{xxx} + i\beta\left(|q|^2 q\right)_x + i\gamma\left(|q|^4 q\right)_x + \delta|q|^4 q = 0. \tag{1}$$

Equation (1) describes the propagation of femtosecond optical pulses, $q(x,t)$ represents normalized complex slowly varying amplitude of the pulse envelope, and α, β, γ, and δ are real constants. Some solitary wave solutions and combined Jacobian elliptic function solution were constructed by different methods [3, 4].

The Csch method is used to carry out the solutions. Then, the Extended Tanh-Coth method and the modified simple equation method are used to obtain the soliton solutions of this equation.

2. Traveling Wave Solution

Consider the nonlinear partial differential equation in the form

$$F\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xxx}, \ldots\right) = 0, \tag{2}$$

where $u(x,t)$ is a traveling wave solution of nonlinear partial differential equation (2). We use the transformations,

$$u(x,t) = f(\xi), \tag{3}$$

where $\xi = kx - \lambda t$. This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\cdot) = -\lambda \frac{d}{d\xi}(\cdot),$$
$$\frac{\partial}{\partial x}(\cdot) = k \frac{d}{d\xi}(\cdot). \tag{4}$$

Using (4) to transfer the nonlinear partial differential equation (2) to nonlinear ordinary differential equation,

$$Q\left(f, f', f'', f''', \ldots\right) = 0. \tag{5}$$

The ordinary differential equation (5) is then integrated as long as all terms contain derivatives, where we neglect the integration constants.

3. The Generalized NLS Equation (RKL)

In this section, the generalized third-order NLS equation (RKL) (1) is chosen to illustrate the effectiveness of three methods.

The solution of (1) may be supposed as

$$q(x,t) = e^{i\theta} u(\xi), \quad \text{where } \theta = k_1 x + k_2 t,\ \xi = kx - \lambda t. \tag{6}$$

Substituting (6) into (1) and by defining the derivatives,

$$\begin{aligned} q_t &= \left[-\lambda u'(\xi) + i k_2 u(\xi)\right] e^{i\theta}, \\ q_x &= \left[k u'(\xi) + i k_1 u(\xi)\right] e^{i\theta}, \\ q_{xx} &= \left[k^2 u'' + 2ikk_1 u' - {k_1}^2 u\right] e^{i\theta}, \\ q_{xxx} &= \left[k^3 u''' + 2ik^2 k_1 u'' - k{k_1}^2 u' + ik_1 k^2 u'' \right. \\ &\quad \left. - 2k{k_1}^2 u' - i{k_1}^3 u\right] e^{i\theta}; \end{aligned} \tag{7}$$

then decomposing (1) into real and imaginary parts yields a pair of relations which represented nonlinear ordinary differential equations. The real part is

$$\begin{aligned} &k^2\left[1 - 3\alpha k_1\right] u'' + \left[\alpha {k_1}^3 - {k_1}^2 - k_2\right] u \\ &\quad + \left[2 - 3k_1\beta\right] u^3 + \left[\delta - 5k_1\gamma\right] u^5 = 0, \end{aligned} \tag{8}$$

while the imaginary part is

$$\begin{aligned} &\alpha k^3 u''' + \left[2kk_1 - \lambda - 3\alpha k{k_1}^2\right] u' + 3\beta k u^2 u' \\ &\quad + 5k\gamma u^4 u' = 0. \end{aligned} \tag{9}$$

Integrating (9) once and setting the integration constant to zero, we obtain

$$\alpha k^3 u'' + \left[2kk_1 - \lambda - 3\alpha k{k_1}^2\right] u + \beta k u^3 + k\gamma u^5 = 0. \tag{10}$$

Equations (8) and (10) will be equivalent, provided that

$$\begin{aligned} \frac{k^2\left[1 - 3\alpha k_1\right]}{\alpha k^3} &= \frac{\left[\alpha {k_1}^3 - {k_1}^2 - k_2\right]}{\left[2kk_1 - \lambda - 3\alpha k{k_1}^2\right]} = \frac{\left[2 - 3\beta k_1\right]}{\beta k} \\ &= \frac{\left[\delta - 5\gamma k_1\right]}{k\gamma}, \end{aligned} \tag{11}$$

from which we get the parametric constraints

$$\begin{aligned} \delta &= \left(\frac{1}{\alpha} + 2k_1\right)\gamma, \\ \beta &= 2\alpha, \\ \alpha &= \frac{-\left[kk_2 - 8k{k_1}^2 + 3\lambda k_1\right] \mp \sqrt{\left[kk_2 - 8k{k_1}^2 + 3\lambda k_1\right]^2 + 32k{k_1}^3\left(\lambda - 2kk_1\right)}}{16k{k_1}^3}; \end{aligned} \tag{12}$$

multiplying both sides of (10) by u' and integrating with respect to ξ with zero constant, we get

$$\begin{aligned} &u'^2 + \frac{\beta\left[\alpha {k_1}^3 - {k_1}^2 - k_2\right]}{\alpha k^2} u^2 + \frac{\beta}{2\alpha k^2} u^4 + \frac{\gamma}{3\alpha k^2} u^6 \\ &\quad = 0; \end{aligned} \tag{13}$$

assume that

$$\begin{aligned} c_2 &= \frac{\beta\left[\alpha {k_1}^3 - {k_1}^2 - k_2\right]}{\alpha k^2}, \\ c_4 &= \frac{\beta}{2\alpha k^2}, \\ c_6 &= \frac{\gamma}{3\alpha k^2}. \end{aligned} \tag{14}$$

Then

$$u'^2 + c_2u^2 + c_4u^4 + c_6u^6 = 0. \tag{15}$$

4. Methodology

In this section we will apply three different methods to solve (15). These methods are Csch method, Extended Tanh-Coth method, and the modified simple equation method (MSEM).

4.1. Csch Function Method. The solution of many nonlinear equations can be expressed in the form [11]

$$u(\xi) = A\,\mathrm{csch}^{\tau}(\mu\xi) \tag{16}$$

and their derivative

$$\begin{aligned} u'(\xi) &= -A\tau\mu\,\mathrm{csch}^{\tau}(\mu\xi)\cdot\coth(\mu\xi), \\ u''(\xi) &= A\tau\mu^2\left[(\tau+1)\,\mathrm{csch}^{\tau+2}(\mu\xi) + \tau\,\mathrm{csch}^{\tau}(\mu\xi)\right], \end{aligned} \tag{17}$$

where A, μ, and τ are parameters to be determined and μ and λ are the wave number and the wave speed, respectively. We substitute (16)-(17) into the reduced equation (15); we get

$$\begin{aligned} &A^2\tau^2\mu^2\mathrm{csch}^{2\tau}(\mu\xi) + A^2\tau^2\mu^2\mathrm{csch}^{2\tau+2}(\mu\xi) \\ &\quad + c_2A^2\mathrm{csch}^{2\tau}(\mu\xi) + c_4A^4\mathrm{csch}^{4\tau}(\mu\xi) \\ &\quad + c_6A^6\mathrm{csch}^{6\tau}(\mu\xi) = 0. \end{aligned} \tag{18}$$

Balance the terms of the Csch functions to find τ

$$2\tau + 2 = 6\tau, \quad \text{Then } \tau = \frac{1}{2}. \tag{19}$$

We next collect all terms in (18) with the same power in $\mathrm{csch}^k(\mu\xi)$ and set their coefficients to zero to get a system of algebraic equations among the unknowns A, μ, and τ and solve the subsequent system

$$\begin{aligned} A^2\frac{1}{4}\mu^2 + c_2A^2 &= 0, \\ A^2\frac{1}{4}\mu^2 + c_6A^6 &= 0. \end{aligned} \tag{20}$$

Solving the system of equations in (20), we get

$$\begin{aligned} \mu &= 2i\sqrt{c_2} = \frac{2i}{k}\sqrt{\frac{\beta}{\alpha}\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]}, \\ A &= \mp\sqrt[4]{\frac{c_2}{c_6}} = \mp\sqrt[4]{\frac{3\beta}{\gamma}\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]}; \end{aligned} \tag{21}$$

then

$$u(\xi) = \mp\sqrt[4]{\frac{3\beta}{\gamma}\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]}\sqrt{\mathrm{csch}(\mu\xi)} \tag{22}$$

therefore

$$q(x,t) = \mp\sqrt[4]{\frac{3\beta}{\gamma}\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]}e^{i[k_1x+k_2t]}\sqrt{\mathrm{csch}\left(\frac{2i}{k}\sqrt{\frac{\beta}{\alpha}\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]}(kx-\lambda t)\right)}; \tag{23}$$

Figure 1 represents the solitary wave in (23) for $k = k_1 = k_2 = 1$, $\lambda = -1$, $\gamma = -1$, $\delta = -10/3$, $\beta = 3/2$, $\alpha = 3/4$, and then $q(x,t) = 1.54\sqrt{\mathrm{csch}(3.1622(x+t))}$.

4.2. Tanh-Coth Method. The key step is to introduce the ansatz, the new independent variable [12, 13]

$$Y = \tanh(\xi) \tag{24}$$

that leads to the change of variables:

$$\frac{dU}{d\xi} = \left(1-Y^2\right)\frac{dU}{dY}, \tag{25}$$

$$\frac{d^2U}{d\xi^2} = -2Y\left(1-Y^2\right)\frac{dU}{dY} + \left(1-Y^2\right)^2\frac{d^2U}{dY^2}, \tag{26}$$

$$\begin{aligned} \frac{d^3U}{d\xi^3} &= 2\left(1-Y^2\right)\left(3Y^2-1\right)\frac{dU}{dY} \\ &\quad - 6Y\left(1-Y^2\right)^2\frac{d^2U}{dY^2} + \left(1-Y^2\right)^3\frac{d^3U}{dY^3}. \end{aligned} \tag{27}$$

Assume

$$V(\xi) = u^2(\xi). \tag{28}$$

Equation (15) can be written as

$$V'^2 + 4c_2V^2 + 4c_4V^3 + 4c_6V^4 = 0. \tag{29}$$

The next step is that the solution of (29) is expressed in the form

$$V(\xi) = \sum_{i=0}^{m} a_iY^i + \sum_{i=1}^{m} b_iY^{-i}, \tag{30}$$

where the parameter m can be found by balancing the highest-order linear term with the nonlinear terms in (29).

We balance V^4 with $(dV/dY)^2$, to obtain $4m = (m+1)^2$; then $m = 1$. The Tanh-Coth method admits the use of the finite expansion for

$$\begin{aligned} V &= a_0 + a_1Y + b_1Y^{-1}, \\ V' &= a_1 - b_1Y^{-2}. \end{aligned} \tag{31}$$

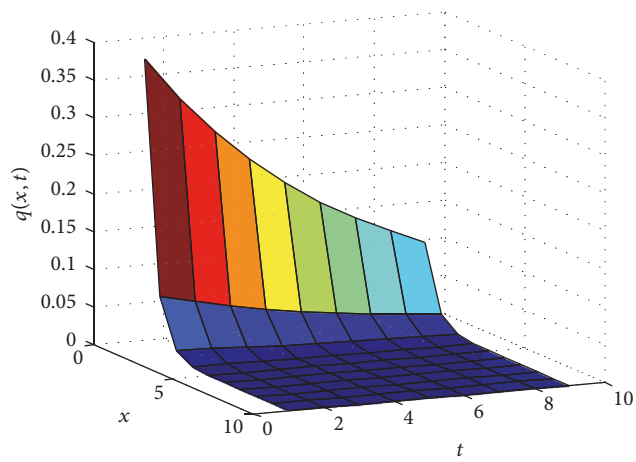

FIGURE 1: The solitary wave in (23) for $0 \le t \le 1,\ 0 \le x \le 10$.

$\lambda, k, a_0, a_1, b_1$ are to be determined. Substituting (31) into (25) and then into (29) will yield a set of algebraic equations because all coefficients of Y have to vanish

$$\begin{aligned}&\left(1-Y^2\right)^2\left(a_1-b_1Y^{-2}\right)^2+4c_2\left(\left[a_0+a_1Y\right]+b_1Y^{-1}\right)^2\\&\quad+4c_4\left(a_0+a_1Y+b_1Y^{-1}\right)^3\\&\quad+4c_6\left(a_0+a_1Y+b_1Y^{-1}\right)^4=0.\end{aligned}\tag{32}$$

Equation (32) can be written as

$$\begin{aligned}&\left(a_1{}^2\left(1-2Y^2+Y^4\right)-2a_1b_1\left(Y^{-2}-2+Y^2\right)\right.\\&\quad\left.+b_1{}^2\left(Y^{-4}-2Y^{-2}+1\right)\right)+4c_2\left(a_0{}^2+2a_1b_1\right.\\&\quad\left.+2a_0a_1Y+a_1{}^2Y^2+2a_0b_1Y^{-1}+b_1{}^2Y^{-2}\right)+4c_4\left(a_0{}^3\right.\\&\quad+3a_0{}^2a_1Y+3a_0a_1{}^2Y^2+a_1{}^3Y^3+3\left(a_0{}^2Y^{-1}\right.\\&\quad\left.+2a_0a_1+a_1{}^2Y\right)b_1+3\left(a_0Y^{-2}+a_1Y^{-1}\right)b_1{}^2\\&\quad\left.+b_1{}^3Y^{-3}\right)+4c_6\left(\left[a_0{}^4+4a_0{}^3a_1Y+6a_0{}^2a_1{}^2Y^2\right.\right.\\&\quad\left.+4a_0a_1{}^3Y^3+a_1{}^4Y^4\right]+4\left[a_0{}^3Y^{-1}+3a_0{}^2a_1\right.\\&\quad\left.+3a_0a_1{}^2Y+a_1{}^3Y^2\right]b_1+6\left[a_0{}^2Y^{-2}+2a_0a_1Y^{-1}\right.\\&\quad\left.\left.+a_1{}^2\right]b_1{}^2+4\left[a_0Y^{-3}+a_1Y^{-2}\right]b_1{}^3+b_1{}^4Y^{-4}\right)=0.\end{aligned}\tag{33}$$

Equating expressions at Y^i, $(i=-4,-3,-2,-1,0,1,2,3,4)$ to zero, we have the following system of equations:

$$\begin{aligned}&\text{Coefficients of } Y^{-4}\text{: } \left[1+4c_6b_1{}^2\right]b_1{}^2=0\\&\text{Coefficients of } Y^{-3}\text{: } \left\{c_4+4c_6a_0\right\}b_1{}^3=0\\&\text{Coefficients of } Y^{-2}\text{: } -2a_1b_1-2b_1{}^2+4c_2b_1{}^2\\&\quad+12c_4a_0b_1{}^2+4c_6\left(6a_0{}^2+4a_1b_1\right)b_1{}^2=0\\&\text{Coefficients of } Y^{-1}\left\{8c_2a_0+12c_4\left(a_0{}^2+a_1b_1\right)\right.\\&\quad\left.+4c_6\left(4a_0{}^3+12a_0a_1b_1\right)\right\}b_1=0\\&\text{Coefficients of } Y^{0}\text{: } a_1{}^2+4a_1b_1+b_1{}^2+4c_2a_0{}^2\\&\quad+4c_4\left(a_0{}^3+6a_0a_1b_1\right)+4c_6\left(a_0{}^4+12a_0{}^2a_1b_1\right.\\&\quad\left.+6a_1{}^2b_1{}^2\right)=0\\&\text{Coefficients of } Y\text{: } 8c_2a_0a_1+12c_4\left(a_0{}^2+a_1b_1\right)a_1\\&\quad+16c_6a_0a_1\left(a_0{}^2+3a_1b_1\right)=0\\&\text{Coefficients of } Y^{2}\text{: } -2a_1{}^2-2a_1b_1+4c_2a_1{}^2\\&\quad+12c_4a_0a_1{}^2+4c_6a_1{}^2\left(6a_0{}^2+4a_1b_1\right)=0\\&\text{Coefficients of } Y^{3}\text{: } \left(c_4+4c_6a_0\right)a_1{}^3=0\\&\text{Coefficients of } Y^{4}\text{: } \left(1+4c_6a_1{}^2\right)a_1{}^2=0.\end{aligned}\tag{34}$$

Solving the system of equations (34), we get

$$\begin{aligned}b_1&=\mp\frac{i}{2\sqrt{c_6}},\\a_0&=-\frac{1}{2\sqrt{c_6}},\\a_1&=0,\\c_2&=1,\\c_4&=2\sqrt{c_6}.\end{aligned}\tag{35}$$

Substitute for c_6 from (14), and then

$$\begin{aligned}b_1&=\mp ik^2,\\a_0&=-k^2,\\a_1&=0,\\c_4&=\frac{1}{k}\sqrt{\frac{1}{k}},\\k&=\sqrt{2\left[\alpha k_1{}^3-k_1{}^2-k_2\right]},\\\gamma&=\frac{3\alpha}{4k^2},\\\delta&=\frac{3\left(1+2\alpha k_1\right)}{4k^2};\end{aligned}\tag{36}$$

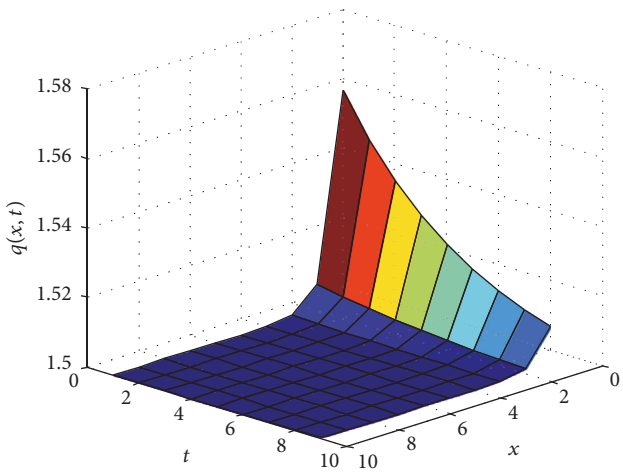

FIGURE 2: The solitary wave in (39) for $0 \le t \le 1,\ 0 \le x \le 10$.

therefore

$$V(\xi) = k^2\left[-1 \mp i\coth(\xi)\right],$$
$$u(\xi) = k\sqrt{-1 \mp i\coth(\xi)}. \tag{37}$$

Then

$$q(x,t) = e^{i[k_1x+k_2t]}k\sqrt{-1 \mp i\coth(kx-\lambda t)} \tag{38}$$

for $k = \sqrt{1/2}$, $k_1 = k_2 = 1$, $\lambda = 3k$, $\alpha = 1/4$, and then

$$q(x,t) = \left(\frac{1}{\sqrt{2}}\right)\sqrt{-1 + i\coth\left\{\left(\frac{1}{\sqrt{2}}\right)(x-3t)\right\}}. \tag{39}$$

Figure 2 represents the solitary wave in (39).

4.3. The Modified Simple Equation Method. We look for solutions of (29) in the form [14]

$$V = A_0 + A_1\frac{\psi_\xi}{\psi},$$
$$V_\xi = A_1\left(\frac{\psi_{\xi\xi}}{\psi} - \frac{\psi_\xi^2}{\psi^2}\right). \tag{40}$$

Then (29) can be written as

$$A_1^2\left(\frac{\psi_{\xi\xi}}{\psi} - \frac{\psi_\xi^2}{\psi^2}\right)^2 + 4c_2\left(A_0 + A_1\frac{\psi_\xi}{\psi}\right)^2 + 4c_4\left(A_0 + A_1\frac{\psi_\xi}{\psi}\right)^3 + 4c_6\left(A_0 + A_1\frac{\psi_\xi}{\psi}\right)^4 = 0. \tag{41}$$

Then (41) can be written as

$$\begin{aligned} &A_1^2\left[\frac{\psi_{\xi\xi}^2}{\psi^2} - 2\psi_{\xi\xi}\frac{\psi_\xi^2}{\psi^3} + \frac{\psi_\xi^4}{\psi^4}\right] + 4c_2\left[A_0^2 + 2A_0A_1\frac{\psi_\xi}{\psi} + A_1^2\frac{\psi_\xi^2}{\psi^2}\right] + 4c_4\left[A_0^3 + 3A_0^2A_1\frac{\psi_\xi}{\psi} + 3A_0A_1^2\frac{\psi_\xi^2}{\psi^2} + A_1^3\frac{\psi_\xi^3}{\psi^3}\right] \\ &+ 4c_6\left[A_0^4 + 4A_0^3A_1\frac{\psi_\xi}{\psi} + 6A_0^2A_1^2\frac{\psi_\xi^2}{\psi^2} + 4A_0A_1^3\frac{\psi_\xi^3}{\psi^3} + A_1^4\frac{\psi_\xi^4}{\psi^4}\right] = 0. \end{aligned} \tag{42}$$

Equating expressions in (42) at ψ^{-1}, ψ^{-2}, ψ^{-3}, and ψ^{-4} to zero, we have the following system of equations:

$$\begin{aligned} \left[1 + 4c_6A_1^2\right]A_1^2 &= 0, \\ \left[\psi_{\xi\xi} - 2A_1\left\{c_4 + 4c_6A_0\right\}\psi_\xi\right]A_1^2 &= 0, \\ \left[\psi_{\xi\xi}^2 + 4\left\{c_2 + 3c_4A_0 + 6c_6A_0^2\right\}\psi_\xi^2\right]A_1^2 &= 0, \\ \left\{2c_2 + 3c_4A_0 + 4c_6A_0^2\right\}A_0A_1 &= 0. \end{aligned} \tag{43}$$

Solving the system of equations in (43),

$$A_1 = \mp i\frac{k}{2}\sqrt{\frac{3\alpha}{\gamma}}. \tag{44}$$

Family 1

$$\begin{aligned} A_0 &= \frac{-\beta + \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1^3 - k_1^2 - k_2\right]/3}}{4\gamma/3}, \\ \psi_1(\xi) &= \varepsilon_{11} + \varepsilon_{12}e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0^2}\xi}, \\ q_1(x,t) &= e^{i[k_1x+k_2t]}\left\{A_0 - k\varepsilon_{12}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0^2\right)}{\gamma}} \cdot \frac{e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0^2}(kx-\lambda t)}}{\varepsilon_{11} + \varepsilon_{12}e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0^2}(kx-\lambda t)}}\right\}^{1/2}. \end{aligned} \tag{45}$$

Family 2

$$A_0 = \frac{-\beta - \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{4\gamma/3},$$
$$\psi_2(\xi) = \varepsilon_{21} + \varepsilon_{22} e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_2(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - k\varepsilon_{22}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{21}+\varepsilon_{22}e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{46}$$

Family 3

$$A_0 = \frac{-3\beta + \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_3(\xi) = \varepsilon_{31} + \varepsilon_{32} e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_3(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - k\varepsilon_{32}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{31}+\varepsilon_{32}e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{47}$$

Family 4

$$A_0 = \frac{-3\beta - \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_4(\xi) = \varepsilon_{41} + \varepsilon_{42} e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_4(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - k\varepsilon_{42}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{41}+\varepsilon_{42}e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{48}$$

Family 5

$$A_0 = \frac{-\beta + \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{4\gamma/3},$$
$$\psi_5(\xi) = \varepsilon_{51} + \varepsilon_{52} e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_5(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 + k\varepsilon_{52}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{51}+\varepsilon_{52}e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{49}$$

Family 6

$$A_0 = \frac{-\beta - \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{4\gamma/3},$$
$$\psi_6(\xi) = \varepsilon_{61} + \varepsilon_{62} e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_6(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 + k\varepsilon_{62}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{61}+\varepsilon_{62}e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{50}$$

Family 7

$$A_0 = \frac{-3\beta + \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_7(\xi) = \varepsilon_{71} + \varepsilon_{72} e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_7(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 + k\varepsilon_{72}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{71}+\varepsilon_{72}e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{51}$$

Family 8

$$A_0 = \frac{-3\beta - \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_8(\xi) = \varepsilon_{81} + \varepsilon_{82} e^{2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}\xi},$$
$$q_8(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 + k\varepsilon_{82}\sqrt{\frac{3\alpha\left(c_2+3c_4A_0+6c_6A_0{}^2\right)}{\gamma}} \cdot \frac{e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}{\varepsilon_{81}+\varepsilon_{82}e^{-2i\sqrt{c_2+3c_4A_0+6c_6A_0{}^2}(kx-\lambda t)}}\right\}^{1/2}. \tag{52}$$

Family 9

$$A_0 = \frac{-\beta + \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{4\gamma/3},$$
$$\psi_9(\xi) = \varepsilon_{91} + \varepsilon_{92} e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0]\xi},$$
$$q_9(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - \frac{3k^2\alpha}{\gamma}\left[c_4+2c_6A_0\right]^2 \cdot \frac{\varepsilon_{92}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}{\varepsilon_{91}+\varepsilon_{92}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}\right\}^{1/2}. \tag{53}$$

Family 10

$$A_0 = \frac{-\beta - \sqrt{\beta^2 - 16\gamma\beta\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{4\gamma/3},$$
$$\psi_{10}(\xi) = \varepsilon_{101} + \varepsilon_{102} e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0]\xi},$$
$$q_{10}(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - \frac{3k^2\alpha}{\gamma}\left[c_4+2c_6A_0\right]^2 \cdot \frac{\varepsilon_{102}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}{\varepsilon_{101}+\varepsilon_{102}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}\right\}^{1/2}. \tag{54}$$

Family 11

$$A_0 = \frac{-3\beta + \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_{11}(\xi) = \varepsilon_{111} + \varepsilon_{112} e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0]\xi},$$
$$q_{11}(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - \frac{3k^2\alpha}{\gamma}\left[c_4+2c_6A_0\right]^2 \cdot \frac{\varepsilon_{112}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}{\varepsilon_{111}+\varepsilon_{112}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}\right\}^{1/2}. \tag{55}$$

Family 12

$$A_0 = \frac{-3\beta - \sqrt{9\beta^2 - 128\beta\gamma\left[\alpha k_1{}^3 - k_1{}^2 - k_2\right]/3}}{16\gamma/3},$$
$$\psi_{12}(\xi) = \varepsilon_{121} + \varepsilon_{122} e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0]\xi},$$
$$q_{12}(x,t) = e^{i[k_1x+k_2t]}\left\{A_0 - \frac{3k^2\alpha}{\gamma}\left[c_4+2c_6A_0\right]^2 \cdot \frac{\varepsilon_{122}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}{\varepsilon_{121}+\varepsilon_{122}e^{ik\sqrt{3\alpha/\gamma}[c_4+2c_6A_0](kx-\lambda t)}}\right\}^{1/2}. \tag{56}$$

5. Conclusion

In this paper, series of new traveling wave solutions have been obtained. The Csch method and the Extended Tanh-Coth method and modified simple equation method are used to carry out the integration of the generalized NLS equation, which is RKL. These methods can be also applied to solve other types of the generalized nonlinear evolution equations with complex coefficients. The solitary waves in Figures 1 and 2 obtained by the Csch and Tanh-Coth methods, respectively, are identical in form and behavior. The obtained solutions are very useful and may be important to explain some physical phenomena and find applications in the nonlinear pulse propagation through optical fibers.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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