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Research Article

Soliton Solutions of the Coupled Schrödinger-Boussinesq Equations for Kerr Law Nonlinearity

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In this paper, the coupled Schrödinger-Boussinesq equations (SBE) will be solved by the sech, tanh, csch, and the modified simplest equation method (MSEM). We obtain exact solutions of the nonlinear for bright, dark, and singular 1-soliton solution. Kerr law nonlinearity media are studied. Results have proven that modified simple equation method does not produce the soliton solution in general case. Solutions may find practical applications and will be important for the conservation laws for dispersive optical solitons.

1. Introduction

All optical communications are being used for transcontinental and transoceanic data transfer, through long-haul optical fibers, at the present time. There are various aspects of soliton communication that still need to be addressed. One of the features is the dispersive optical solitons. In presence of higher order dispersion terms, soliton communications are sometimes a hindrance as these dispersion terms produce soliton radiation. Nonlinear evolution equations have a major role in various scientific and engineering fields, such as optical fibers. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction, and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. In recent years, exact homoclinic and heteroclinic solutions were proposed for some NEEs like nonlinear Schrödinger equation, Sine-Gordon equation, Davey-Stewartson equation, Zakharov equation, and Boussinesq equation [1–7].

In particular, the study of the coupled Schrödinger-Boussinesq equations has attracted much attention of mathematicians and physicists [8–10]. The existence of the global solution of the initial boundary problem for the equations was investigated in [8]. The existence of a periodic solution for the equations was considered in [9]. Kilicman and Abazari [10] used the (G'/G)-expansion method to construct periodic and soliton solutions for the Schrödinger-Boussinesq. The investigation of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena [9–12].

The nonlinear coupled Schrödinger-Boussinesq equation (SBE) governs the propagation of optical solitons in a dispersive optical fiber and is a very important equation in the area of theoretical and mathematical physics. This paper is going to take a look at the bright, dark, and singular soliton solutions for Kerr law nonlinearity media.

2. Governing Equations

Consider the coupled Schrödinger-Boussinesq equations (SBE). They appeared in [13] as a special case of general systems governing the stationary propagation of coupled nonlinear upper hybrid and magneto sonic waves in magnetized plasma. These equations were in the form [14]

$$\begin{gathered} i\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}+\alpha_1 E-NE=0, \\ 3\frac{\partial^2 N}{\partial t^2}-\frac{\partial^4 N}{\partial x^4}+3\frac{\partial^2}{\partial x^2}\left(N^2\right)+\alpha_2\frac{\partial^2 N}{\partial x^2}-\frac{\partial^2}{\partial x^2}\left(|E|^2\right) \\ =0, \end{gathered} \tag{1}$$

where α_1, α_2 are real constants, $E(x,t)$ is a complex function, and $N(x,t)$ is a real function. The complete integrability of (1) was studied by Chowdhury et al. [15], and N-soliton solution, homoclinic orbit solution, and rogue solution were obtained by Hu et al. [16], Dai et al. [17–19], and Mu and Qin [20].

3. The Traveling Solution

Consider the nonlinear partial differential equation in the form

$$F\left(u, u_t, u_x, u_{tt}, u_{xx}, \ldots\right) = 0, \tag{2}$$

where $u(x,t)$ is a traveling wave solution of nonlinear partial differential equation (2). We use the transformations

$$u(x,t) = f(\xi), \tag{3}$$

where $\xi = x - \rho t + \chi$. This enables us to use the following changes:

$$\begin{aligned} \frac{\partial}{\partial t}(\cdot) &= -\rho \frac{d}{d\xi}(\cdot), \\ \frac{\partial}{\partial x}(\cdot) &= \frac{d}{d\xi}(\cdot). \end{aligned} \tag{4}$$

Using (4) to transfer the nonlinear partial differential equation (2) to nonlinear ordinary differential equation,

$$Q\left(f, f', f'', f''', \ldots\right) = 0. \tag{5}$$

The ordinary differential equation (5) is then integrated as long as all terms contain derivatives, where we neglect the integration constants.

4. Hyperbolic Function Methods

The solutions of many nonlinear equations can be expressed in the following form.

4.1. Sech Function Method (Bright Soliton) [21]

$$\begin{aligned} f(\xi) &= A \operatorname{sech}^{\beta}(\mu\xi), \\ f'(\xi) &= -A\beta\mu \operatorname{sech}^{\beta}(\mu\xi) \cdot \tanh(\mu\xi), \\ f''(\xi) &= -A\beta\mu^2 \left[(\beta+1) \operatorname{sech}^{\beta+2}(\mu\xi) \right. \\ &\quad \left. - \beta \operatorname{sech}^{\beta}(\mu\xi)\right], \\ f'''(\xi) &= A\beta\mu^3 \left[(\beta+1)(\beta+2) \operatorname{sech}^{\beta+2}(\mu\xi) \right. \\ &\quad \left. - \beta^2 \operatorname{sech}^{\beta}(\mu\xi)\right] \tanh(\mu\xi). \end{aligned} \tag{6}$$

4.2. Tanh Function Method (Dark Soliton) [22]

$$\begin{aligned} f(\xi) &= A \tanh^{\beta}(\mu\xi), \quad |\xi| \le \frac{\pi}{2\mu}, \\ f'(\xi) &= A\beta\mu \left[\tanh^{\beta-1}(\mu\xi) - \tanh^{\beta+1}(\mu\xi)\right], \\ f''(\xi) &= A\beta\mu^2 \left[(\beta-1)\tanh^{\beta-2}(\mu\xi) - 2\beta\tanh^{\beta}(\mu\xi) \right. \\ &\quad \left. + (\beta+1)\tanh^{\beta+2}(\mu\xi)\right], \\ f'''(\xi) &= A\beta\mu^3 \left[(\beta-1)(\beta-2)\tanh^{\beta-3}(\mu\xi) \right. \\ &\quad - \left\{(\beta-1)(\beta-2) + 2\beta\right\} \tanh^{\beta-1}(\mu\xi) \\ &\quad + \left\{(\beta+1)(\beta+2) + 2\beta\right\} \tanh^{\beta+1}(\mu\xi) \\ &\quad \left. - (\beta+1)(\beta+2)\tanh^{\beta+3}(\mu\xi)\right]. \end{aligned} \tag{7}$$

4.3. Csch Function Method (Singular Soliton) [21, 22]

$$\begin{aligned} f(\xi) &= A \operatorname{csch}^{\beta}(\mu\xi), \\ f'(\xi) &= -A\beta\mu \operatorname{csch}^{\beta}(\mu\xi) \cdot \coth(\mu\xi), \\ f''(\xi) &= A\beta\mu^2 \left[(\beta+1) \operatorname{csch}^{\beta+2}(\mu\xi) + \beta \operatorname{csch}^{\beta}(\mu\xi)\right], \\ f'''(\xi) &= -A\beta\mu^3 \left[(\beta+1)(\beta+2) \operatorname{csch}^{\beta+2}(\mu\xi) \right. \\ &\quad \left. + \beta^2 \operatorname{csch}^{\beta}(\mu\xi)\right] \coth(\mu\xi), \end{aligned} \tag{8}$$

where A represent the amplitudes of the solitons and μ represents the solitons width.

We substitute (6), (7), or (8) into the reduced equation (5), balance the terms of the sech, tanh, and csch functions, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $\operatorname{sech}^k(\mu\xi)$, $\tanh^k(\mu\xi)$, or $\operatorname{csch}^k(\mu\xi)$, set to zero their coefficients to get a system of algebraic equations among the unknowns A, μ, and β, and solve the subsequent system.

5. The Application

The starting hypothesis for solving (1) by the aid of traveling waves solution is as follows: introduce the transformations

$$\begin{aligned} E(x,t) &= e^{i\theta(x,t)} u(\xi), \\ N(x,t) &= v(\xi), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \theta &= kx + \omega t + \epsilon_0, \\ \xi &= (x - \rho t + \chi), \end{aligned} \tag{10}$$

where $k, \omega, \rho, \epsilon_0$, and χ are real constants. The parameter ρ represents the soliton velocity.

Substituting (9) and (10) into (1) and decomposing into real and imaginary parts leads to

$$u'' - \left[\omega + k^2 - \alpha_1\right] u - uv = 0, \tag{11}$$

$$2k - \rho = 0, \tag{12}$$

$$\left[12k^2 + \alpha_2\right] v'' - v'''' + 3\left(v^2\right)'' - \left(u^2\right)'' = 0. \tag{13}$$

Substitute (12) in (10), then

$$\xi = (x - 2kt + \chi). \tag{14}$$

Integrating (13) twice with zero constant, (13) can be written as

$$[12k^2 + \alpha_2] v - v'' + 3v^2 - u^2 = 0. \tag{15}$$

5.1. Bright Soliton. Seeking the solution by sech function method as in (6)

$$\begin{aligned} u(\xi) &= A_1 \operatorname{sech}^{\beta_1}(\mu\xi), \\ v(\xi) &= A_2 \operatorname{sech}^{\beta_2}(\mu\xi), \end{aligned} \tag{16}$$

the system of equations in (11) and (15) becomes, respectively,

$$\begin{aligned} &\beta_1\mu^2 \left[(\beta_1 + 1)\operatorname{sech}^{\beta_1+2}(\mu\xi) - \beta_1 \operatorname{sech}^{\beta_1}(\mu\xi)\right] + [\omega \\ &\quad + k^2 - \alpha_1] \operatorname{sech}^{\beta_1}(\mu\xi) - A_2 \operatorname{sech}^{\beta_1+\beta_2}(\mu\xi) = 0, \end{aligned} \tag{17}$$

$$\begin{aligned} &[12k^2 + \alpha_2] A_2 \operatorname{sech}^{\beta_2}(\mu\xi) \\ &\quad + A_2\beta_2\mu^2 \left[(\beta_2 + 1)\operatorname{sech}^{\beta_2+2}(\mu\xi) \right. \\ &\quad \left. - \beta_2 \operatorname{sech}^{\beta_2}(\mu\xi)\right] + 3{A_2}^2 \operatorname{sech}^{2\beta_2}(\mu\xi) \\ &\quad - {A_1}^2 \operatorname{sech}^{2\beta_1}(\mu\xi) = 0. \end{aligned} \tag{18}$$

Equating the exponents and the coefficients of each pair of the sech functions, we find

$$\begin{aligned} 2\beta_1 &= \beta_2 + 2, \\ \beta_1 + \beta_2 &= \beta_1 + 2, \quad \text{then } \beta_1 = \beta_2 = 2. \end{aligned} \tag{19}$$

Thus setting coefficients of (17)-(18) to zero yields set system of equations:

$$\begin{aligned} 4\mu^2 - [\omega + k^2 - \alpha_1] &= 0, \\ [12k^2 + \alpha_2] - 4\mu^2 &= 0, \\ 6\mu^2 - A_2 &= 0, \\ 6A_2\mu^2 + 3{A_2}^2 - {A_1}^2 &= 0. \end{aligned} \tag{20}$$

Solving the system of equations in (20), we get

$$\begin{aligned} A_1 &= \mp 3 [12k^2 + \alpha_2], \\ A_2 &= \frac{3}{2}[12k^2 + \alpha_2], \end{aligned} \tag{21}$$

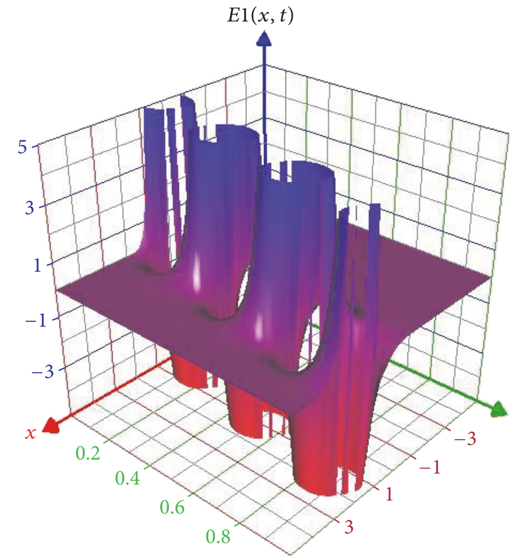

FIGURE 1: The solitary wave of the real part of $E_1(x,t)$ in (23) for $-5 \le x \le 5$, $0 \le t \le 1$.

$$\begin{aligned} \mu &= \mp\sqrt{\frac{[12k^2 + \alpha_2]}{4}}, \\ \omega &= [11k^2 + \alpha_1 + \alpha_2], \end{aligned} \tag{22}$$

$$\begin{aligned} E_1(x,t) &= \mp 3 e^{i(kx + (11k^2+\alpha_1+\alpha_2)t + \epsilon_0)} [12k^2 + \alpha_2] \\ &\quad \cdot \operatorname{sech}^2\left(\sqrt{\frac{[12k^2 + \alpha_2]}{4}}(x - 2kt + \chi)\right), \end{aligned} \tag{23}$$

$$\begin{aligned} N_1(x,t) &= \frac{3}{2}[12k^2 + \alpha_2] \\ &\quad \cdot \operatorname{sech}^2\left(\sqrt{\frac{[12k^2 + \alpha_2]}{4}}(x - 2kt + \chi)\right). \end{aligned} \tag{24}$$

For $k = \alpha_1 = 1$, $\alpha_2 = 4$, $\epsilon_0 = \chi = 0$, the real part of $E_1(x,t) = 48\cos(x+16t)\operatorname{sech}^2\{2(x-2t)\}$, and $N_1(x,t) = 24\operatorname{sech}^2\{2(x-2t)\}$.

Figures 1 and 2 represent the solitary wave of the real part of $E_1(x,t)$ in (23) and $N_1(x,t)$ in (24), respectively, for $-5 \le x \le 5$, $0 \le t \le 1$.

5.2. Dark Soliton. Seeking the solution by tanh function method as in (7)

$$\begin{aligned} u(\xi) &= A_1 \tanh^{\beta_1}(\mu\xi), \\ v(\xi) &= A_2 \tanh^{\beta_2}(\mu\xi), \end{aligned} \tag{25}$$

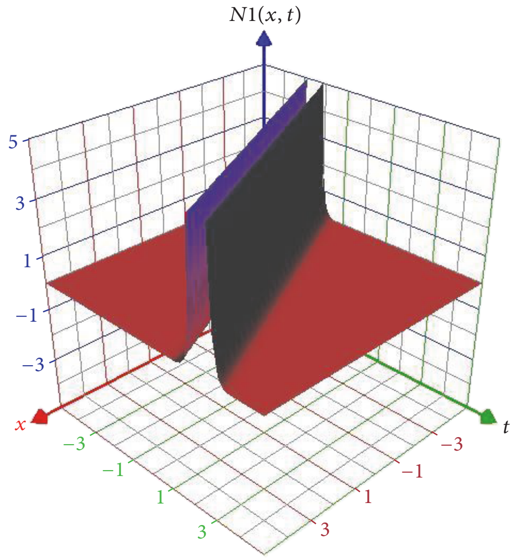

FIGURE 2: The solitary wave $N_1(x,t)$ in (24) for $-5 \le x \le 5$, $0 \le t \le 1$.

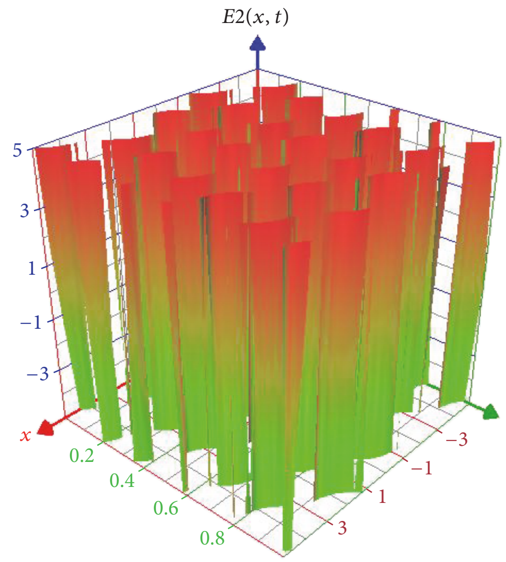

FIGURE 3: The solitary wave of the real part of $E_2(x,t)$ in (32) for $-5 \le x \le 5$, $0 \le t \le 1$.

the system of equations in (11) and (15) becomes, respectively,

$$\begin{aligned}
&\beta_1\mu^2\left[(\beta_1-1)\tanh^{\beta_1-2}(\mu\xi)-2\beta_1\tanh^{\beta_1}(\mu\xi)\right.\\
&\quad\left.+(\beta_1+1)\tanh^{\beta_1+2}(\mu\xi)\right]-\left[\omega+k^2-\alpha_1\right]\\
&\quad\cdot\tanh^{\beta_1}(\mu\xi)-A_2\tanh^{\beta_1+\beta_2}(\mu\xi)=0,
\end{aligned}\tag{26}$$

$$\begin{aligned}
&\left[12k^2+\alpha_2\right]A_2\tanh^{\beta_2}(\mu\xi)\\
&\quad-A_2\beta_2\mu^2\left[(\beta_2-1)\tanh^{\beta_2-2}(\mu\xi)\right.\\
&\quad\left.-2\beta_2\tanh^{\beta_2}(\mu\xi)+(\beta_2+1)\tanh^{\beta_2+2}(\mu\xi)\right]\\
&\quad+3{A_2}^2\tanh^{2\beta_2}(\mu\xi)-{A_1}^2\tanh^{2\beta_1}(\mu\xi)=0.
\end{aligned}\tag{27}$$

Equating the exponents and the coefficients of each pair of the sech functions, we find

$$\begin{aligned}
2\beta_1&=\beta_2+2,\\
\beta_1+\beta_2&=\beta_1+2,\quad\text{then }\beta_1=\beta_2=2.
\end{aligned}\tag{28}$$

Thus setting coefficients of (26)-(27) to zero yields set system of equations:

$$\begin{aligned}
8\mu^2+\left[\omega+k^2-\alpha_1\right]&=0,\\
\left[12k^2+\alpha_2\right]+8\mu^2&=0,\\
6\mu^2-A_2&=0,\\
-6A_2\mu^2+3{A_2}^2-{A_1}^2&=0.
\end{aligned}\tag{29}$$

Solving the system of equations in (29), we get

$$\begin{aligned}
\omega&=\left[11k^2+\alpha_1+\alpha_2\right],\\
\mu&=\mp i\sqrt{\frac{\left[12k^2+\alpha_2\right]}{8}},
\end{aligned}\tag{30}$$

$$\begin{aligned}
A_1&=\pm\frac{3\sqrt{2}}{4}\left[12k^2+\alpha_2\right],\\
A_2&=\frac{-3}{4}\left[12k^2+\alpha_2\right],
\end{aligned}\tag{31}$$

$$\begin{aligned}
E_2(x,t)&=\pm e^{i(kx+(11k^2+\alpha_1+\alpha_2)t+\epsilon_0)}3\sqrt{2}\frac{\left[12k^2+\alpha_2\right]}{4}\\
&\quad\cdot\sec^2\left(\sqrt{\frac{\left[12k^2+\alpha_2\right]}{8}}(x-2kt+\chi)\right),
\end{aligned}\tag{32}$$

$$\begin{aligned}
N_2(x,t)&=-3\frac{\left[12k^2+\alpha_2\right]}{4}\\
&\quad\cdot\sec^2\left(\sqrt{\frac{\left[12k^2+\alpha_2\right]}{8}}(x-2kt+\chi)\right).
\end{aligned}\tag{33}$$

For $k=\alpha_1=1$, $\alpha_2=4$, $\epsilon_0=\chi=0$, the real part of $E_2(x,t)=12\sqrt{2}\cos(x+16t)\sec^2\{\sqrt{2}(x-2t)\}$, and $N_2(x,t)=-12\sec^2\{\sqrt{2}(x-2t)\}$.

Figures 3 and 4 represent the solitary wave of the real part of $E_2(x,t)$ in (32) and $N_2(x,t)$ in (33) for $-5 \le x \le 5$, $0 \le t \le 1$.

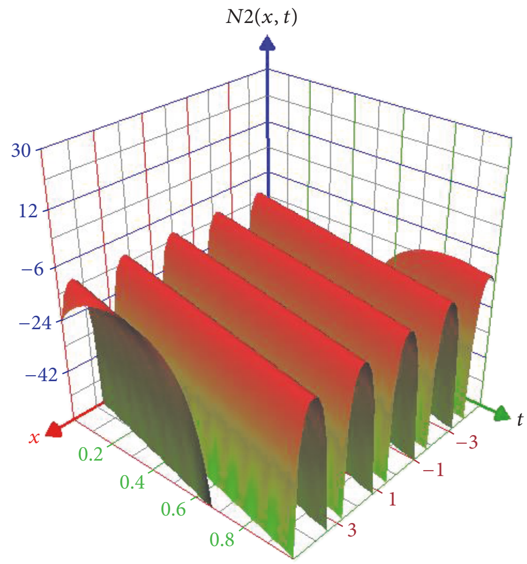

FIGURE 4: The solitary wave $N_2(x,t)$ in (33) for $-5 \le x \le 5$, $0 \le t \le 1$.

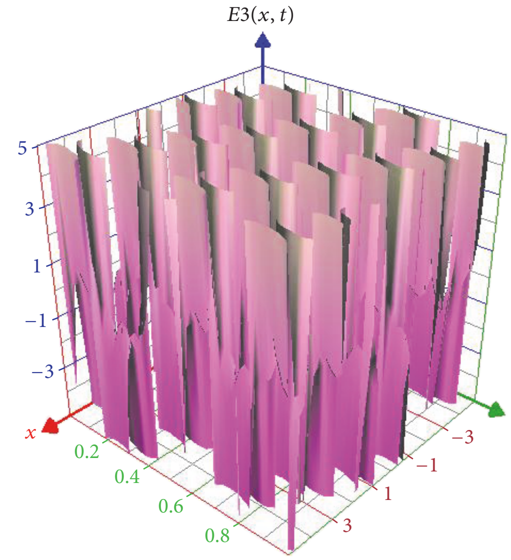

FIGURE 5: The solitary wave of the real part of $E_3(x,t)$ in (40) for $-5 \le x \le 5$, $0 \le t \le 1$.

5.3. Singular Soliton. Seeking the solution by sech function method as in (8)

$$\begin{aligned} u(\xi) &= A_1 \coth^{\beta_1}(\mu\xi), \\ v(\xi) &= A_2 \coth^{\beta_2}(\mu\xi), \end{aligned} \tag{34}$$

the system of equations in (11) and (15) becomes, respectively,

$$\begin{aligned} &\beta_1\mu^2\left[(\beta_1+1)\coth^{\beta_1+2}(\mu\xi)+\beta_1\coth^{\beta_1}(\mu\xi)\right]+\left[\omega\right. \\ &\quad\left.+k^2-\alpha_1\right]\coth^{\beta_1}(\mu\xi)-A_2\coth^{\beta_1+\beta_2}(\mu\xi)=0, \end{aligned} \tag{35}$$

$$\begin{aligned} &\left[12k^2+\alpha_2\right]A_2\coth^{\beta_2}(\mu\xi) \\ &\quad+A_2\beta_2\mu^2\left[(\beta_2+1)\coth^{\beta_2+2}(\mu\xi)\right. \\ &\quad\left.+\beta_2\coth^{\beta_2}(\mu\xi)\right]+3{A_2}^2\coth^{2\beta_2}(\mu\xi) \\ &\quad-{A_1}^2\coth^{2\beta_1}(\mu\xi)=0. \end{aligned} \tag{36}$$

Equating the exponents and the coefficients of each pair of the sech functions, we find

$$\begin{aligned} 2\beta_1 &= \beta_2+2, \\ \beta_1+\beta_2 &= \beta_1+2, \quad \text{then } \beta_1=\beta_2=2. \end{aligned} \tag{37}$$

Thus setting coefficients of (35)-(36) to zero yields set system of equations:

$$\begin{aligned} 4\mu^2+\left[\omega+k^2-\alpha_1\right]&=0, \\ \left[12k^2+\alpha_2\right]+4\mu^2&=0, \\ 6\mu^2-A_2&=0, \\ 6A_2\mu^2+3{A_2}^2-{A_1}^2&=0. \end{aligned} \tag{38}$$

Solving the system of equations in (38), we get

$$\begin{aligned} A_1 &= \pm 3\left[12k^2+\alpha_2\right], \\ A_2 &= -3\frac{\left[12k^2+\alpha_2\right]}{2}, \\ \mu &= \mp i\sqrt{\frac{\left[12k^2+\alpha_2\right]}{4}}, \\ \omega &= 11k^2+\alpha_1+\alpha_2, \end{aligned} \tag{39}$$

$$\begin{aligned} E_3(x,t) &= \mp e^{i(kx+(11k^2+\alpha_1+\alpha_2)t+\epsilon_0)}6\left(6k^2+1\right) \\ &\quad\cdot\cot^2\left(\sqrt{\frac{6k^2+1}{2}}(x-2kt+\chi)\right), \end{aligned} \tag{40}$$

$$\begin{aligned} N_3(x,t) &= 3\left(6k^2+1\right) \\ &\quad\cdot\cot^2\left(\sqrt{\frac{6k^2+1}{2}}(x-2kt+\chi)\right). \end{aligned} \tag{41}$$

For $k=\alpha_1=1$, $\alpha_2=4$, $\epsilon_0=\chi=0$, the real part of $E_3(x,t)=42\cos(x+16t)\cot^2\{\sqrt{7/2}(x-2t)\}$, and $N_3(x,t)=21\cot^2\{\sqrt{7/2}(x-2t)\}$.

Figures 5 and 6 represent the solitary wave of the real part of $E_3(x,t)$ in (40) and $N_3(x,t)$ in (41) for $-5 \le x \le 5$, $0 \le t \le 1$.

5.4. Modified Simple Equation Method. This section will analyze (11) and (15) by the modified simple equation method; assume that solutions are of the form [23]

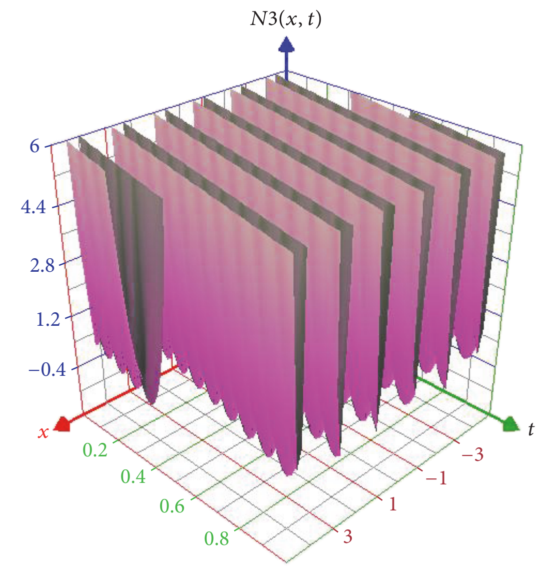

FIGURE 6: The solitary wave of $N_3(x,t)$ in (41) for $-5 \le x \le 5$, $0 \le t \le 1$.

$$u(\xi) = \sum_{i=0}^{m} A_i \left(\frac{\psi_\xi}{\psi}\right)^i, \qquad v(\xi) = \sum_{i=0}^{n} B_i \left(\frac{\psi_\xi}{\psi}\right)^i, \tag{42}$$

where the parameters m, n can be found by balancing the highest-order linear term with the nonlinear terms in (11) and (15), respectively.

In (11), we balance u'' with uv, to obtain $m+2 = m+n$, and then $n = 2$. While in (15), We balance v'' with u^2, to obtain $n+2 = 2m$, and then $m = 2$.

Then

$$\begin{aligned}
u(\xi) &= A_0 + A_1 \frac{\psi_\xi}{\psi} + A_2 \frac{\psi_\xi^{\ 2}}{\psi^2},\\
u_\xi &= A_1 \frac{\psi_{\xi\xi}}{\psi} - A_1 \frac{\psi_\xi^{\ 2}}{\psi^2} + 2A_2 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2} - 2A_2 \frac{\psi_\xi^{\ 3}}{\psi^3},\\
u_{\xi\xi} &= A_1 \frac{\psi_{\xi\xi\xi}}{\psi} + 2A_2 \frac{\psi_{\xi\xi}^{\ 2}}{\psi^2} + 2A_2 \frac{\psi_{\xi\xi\xi}\psi_\xi}{\psi^2}\\
&\quad - 3A_1 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2} - 10A_2 \frac{\psi_{\xi\xi}\psi_\xi^{\ 2}}{\psi^3} - 2A_1 \frac{\psi_\xi^{\ 3}}{\psi^3}\\
&\quad - 2A_2 \frac{\psi_\xi^{\ 4}}{\psi^4},\\
v(\xi) &= B_0 + B_1 \frac{\psi_\xi}{\psi} + B_2 \frac{\psi_\xi^{\ 2}}{\psi^2},\\
v_\xi &= B_1 \frac{\psi_{\xi\xi}}{\psi} - B_1 \frac{\psi_\xi^{\ 2}}{\psi^2} + 2B_2 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2} - 2B_2 \frac{\psi_\xi^{\ 3}}{\psi^3},\\
v_{\xi\xi} &= B_1 \frac{\psi_{\xi\xi\xi}}{\psi} + 2B_2 \frac{\psi_{\xi\xi}^{\ 2}}{\psi^2} + 2B_2 \frac{\psi_{\xi\xi\xi}\psi_\xi}{\psi^2} - 3B_1 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2}\\
&\quad - 10B_2 \frac{\psi_{\xi\xi}\psi_\xi^{\ 2}}{\psi^3} - 2B_1 \frac{\psi_\xi^{\ 3}}{\psi^3} - 2B_2 \frac{\psi_\xi^{\ 4}}{\psi^4},
\end{aligned} \tag{43}$$

where A_0, A_1, A_2, B_0, B_1, and B_2 are constants to be calculated.

Substitute (43) in (11) and (15), respectively, to get

$$\begin{aligned}
&\left[A_1 \frac{\psi_{\xi\xi\xi}}{\psi} + 2A_2 \frac{\psi_{\xi\xi}^{\ 2}}{\psi^2} + 2A_2 \frac{\psi_{\xi\xi\xi}\psi_\xi}{\psi^2} - 3A_1 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2}\right.\\
&\quad \left. - 10A_2 \frac{\psi_{\xi\xi}\psi_\xi^{\ 2}}{\psi^3} - 2A_1 \frac{\psi_\xi^{\ 3}}{\psi^3} - 2A_2 \frac{\psi_\xi^{\ 4}}{\psi^4}\right] - \left[\omega + k^2\right.\\
&\quad \left. - \alpha_1\right]\left[A_0 + A_1 \frac{\psi_\xi}{\psi} + A_2 \frac{\psi_\xi^{\ 2}}{\psi^2}\right] - B_0 \left(A_0 + A_1 \frac{\psi_\xi}{\psi}\right.\\
&\quad \left. + A_2 \frac{\psi_\xi^{\ 2}}{\psi^2}\right) - B_1 \left(A_0 \frac{\psi_\xi}{\psi} + A_1 \frac{\psi_\xi^{\ 2}}{\psi^2} + A_2 \frac{\psi_\xi^{\ 3}}{\psi^3}\right)\\
&\quad - B_2 \left(A_0 \frac{\psi_\xi^{\ 2}}{\psi^2} + A_1 \frac{\psi_\xi^{\ 3}}{\psi^3} + A_2 \frac{\psi_\xi^{\ 4}}{\psi^4}\right) = 0,\\
&\left[12k^2 + \alpha_2\right]\left[B_0 + B_1 \frac{\psi_\xi}{\psi} + B_2 \frac{\psi_\xi^{\ 2}}{\psi^2}\right] - \left[B_1 \frac{\psi_{\xi\xi\xi}}{\psi}\right.\\
&\quad + 2B_2 \frac{\psi_{\xi\xi}^{\ 2}}{\psi^2} + 2B_2 \frac{\psi_{\xi\xi\xi}\psi_\xi}{\psi^2} - 3B_1 \frac{\psi_\xi \psi_{\xi\xi}}{\psi^2}\\
&\quad \left. - 10B_2 \frac{\psi_{\xi\xi}\psi_\xi^{\ 2}}{\psi^3} - 2B_1 \frac{\psi_\xi^{\ 3}}{\psi^3} - 2B_2 \frac{\psi_\xi^{\ 4}}{\psi^4}\right] + 3B_0 \left(B_0\right.\\
&\quad \left. + B_1 \frac{\psi_\xi}{\psi} + B_2 \frac{\psi_\xi^{\ 2}}{\psi^2}\right) + 3B_1 \left(B_0 \frac{\psi_\xi}{\psi} + B_1 \frac{\psi_\xi^{\ 2}}{\psi^2}\right.\\
&\quad \left. + B_2 \frac{\psi_\xi^{\ 3}}{\psi^3}\right) + 3B_2 \left(B_0 \frac{\psi_\xi^{\ 2}}{\psi^2} + B_1 \frac{\psi_\xi^{\ 3}}{\psi^3} + B_2 \frac{\psi_\xi^{\ 4}}{\psi^4}\right)\\
&\quad - A_0 \left(A_0 + A_1 \frac{\psi_\xi}{\psi} + A_2 \frac{\psi_\xi^{\ 2}}{\psi^2}\right) - A_1 \left(A_0 \frac{\psi_\xi}{\psi}\right.\\
&\quad \left. + A_1 \frac{\psi_\xi^{\ 2}}{\psi^2} + A_2 \frac{\psi_\xi^{\ 3}}{\psi^3}\right) - A_2 \left(A_0 \frac{\psi_\xi^{\ 2}}{\psi^2} + A_1 \frac{\psi_\xi^{\ 3}}{\psi^3}\right.\\
&\quad \left. + A_2 \frac{\psi_\xi^{\ 4}}{\psi^4}\right) = 0.
\end{aligned} \tag{44}$$

In (44) equating expressions at (ψ^j $j = 0, -1, -2, -3, -4$) to zero, we get the following system of equations:

$$\begin{aligned}
&\psi_{\xi\xi} + \frac{[2A_1 + A_2B_1 + A_1B_2]}{10A_2}\psi_\xi = 0,\\
&2A_2{\psi_{\xi\xi}}^2 + 2A_2\psi_{\xi\xi\xi}\psi_\xi - 3A_1\psi_\xi\psi_{\xi\xi}\\
&\quad - \left\{\left[\omega + k^2 - \alpha_1\right]A_2 + A_2B_0 + A_1B_1 + A_0B_2\right\}{\psi_\xi}^2\\
&\quad = 0,\\
&\psi_{\xi\xi\xi} - \frac{\left[\omega + k^2 - \alpha_1\right]A_1 - B_0A_1 - B_1A_0}{A_1}\psi_\xi = 0,\\
&\left[\omega + k^2 - \alpha_1\right] + B_0 = 0,\\
&\left[12k^2 + \alpha_2\right]B_0 + 3{B_0}^2 - {A_0}^2 = 0,\\
&2B_2 + 3{B_2}^2 - {A_2}^2 = 0,\\
&B_2 = -2,\\
&\psi_{\xi\xi} + \frac{\{2B_1 + 6B_1B_2 - 2A_1A_2\}}{10B_2}\psi_\xi = 0,\\
&2B_2{\psi_{\xi\xi}}^2 + 2B_2\psi_{\xi\xi\xi}\psi_\xi - 3B_1\psi_\xi\psi_{\xi\xi} - \left\{\left[12k^2 + \alpha_2\right]B_2\right.\\
&\quad \left. + 6B_0B_2 + 3{B_1}^2 - 2A_0A_2 - {A_1}^2\right\}{\psi_\xi}^2 = 0,\\
&B_1\psi_{\xi\xi\xi} - \left\{\left[12k^2 + \alpha_2\right]B_1 + 6B_1B_0 - 2A_0A_1\right\}\psi_\xi = 0.
\end{aligned} \tag{45}$$

Obviously when solving the system of (45), we conclude that equations can be satisfied simultaneously for the following constraints. Hence, the modified simple equation method does not produce the soliton solution in general case:

$$\begin{aligned}
A_0 &= 0,\\
B_0 &= 0,\\
B_1 &= A_1 = 0,\\
A_2 &= 2\sqrt{2},\\
B_2 &= -2,\\
\omega &= \alpha_1 - k^2,\\
\alpha_2 &= -12k^2.
\end{aligned} \tag{46}$$

Then we will solve the following ordinary differential equation:

$$\psi_{\xi\xi} = 0, \tag{47}$$

and therefore

$$\psi = a_0 + a_1\xi, \tag{48}$$

where a_0, a_1 are arbitrary constants.

And

$$\begin{aligned}
u(x,t) &= 2\sqrt{2}\frac{{a_1}^2}{\left[a_0 + a_1(x - 2kt + \chi)\right]^2},\\
v(x,t) &= -2\frac{{a_1}^2}{\left[a_0 + a_1(x - 2kt + \chi)\right]^2}.
\end{aligned} \tag{49}$$

Finally solutions become

$$\begin{aligned}
&E_4(x,t)\\
&\quad = 2\sqrt{2}e^{i(kx + \{\alpha_1 - k^2\}t + \epsilon_0)}\left\{\frac{{a_1}^2}{\left[a_0 + a_1(x - 2kt + \chi)\right]^2}\right\},\\
&N_4(x,t) = -2\left\{\frac{{a_1}^2}{\left[a_0 + a_1(x - 2kt + \chi)\right]^2}\right\}.
\end{aligned} \tag{50}$$

6. Conclusion

In this paper the dispersive bright, dark, and singular soliton solutions to SBE with Kerr law of nonlinearity were studied. The sech, tanh, csch, and the modified simplest equation method have been successfully applied to find solitons solutions for the coupled Schrödinger-Boussinesq equations. Several constraint conditions were assuring the existence of such solitons with Kerr law nonlinearity. The modified simple equation method does not produce the soliton solution in general case. Solutions by three methods are plotted in figures for the real and imaginary parts for $E(x,t)$ and $N(x,t)$. Compatibility in figures shape between the solutions of $E(x,t)$ and $N(x,t)$ by the same method sometimes appeared. Solutions may be important for the conservation laws for dispersive optical solitons. Those research outcomes will be soon disseminated.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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