Research Article

A Study on Common Fixed Point of Joint $(A; B)$ Generalized Cyclic $\phi - abc$ Weak Nonexpansive Mappings and Generalized Cyclic $abc; r$ Contractions in Quasi Metric Spaces

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This manuscript proves the existence of a single common fixed point of two joint (A, B) generalized cyclic $\phi - abc$ weak nonexpansive mappings, where A and B are compact sets. The result in particular demonstrated a single fixed point of generalized cyclic $\phi - abc$ weak nonexpansive mappings, without the assumption of a single contracting point. Additionally, it introduces new types of generalized cyclic $abc; r$ contraction mappings and describes the existence of a single fixed point of them in b -metric spaces. Finally, the presented results establish a simpler convergence theorem for a sequence of generalized cyclic $abc; r$ contraction mappings, extend, and generalize some of the previous reported results.

1. Introduction and Preliminaries

In 1922, Banach [1] introduced the concept of contraction mapping and formulated his famous Banach contraction principle, namely, the contraction mapping defined on complete metric space has a unique fixed point.

Definition 1. A self mapping $S : X \longrightarrow X$ on a metric space (X, d) is said to be contraction iff there is a real number $0 < r < 1$ such that

$$d(S(x), S(y)) \leq r d(x, y) \forall x, y \in X. \quad (1)$$

Since 1922 generalizations of the Banach contraction principle have historical achievements.

In 1962, Edelstein [2] introduced the concept of contractive mapping and proved the existence of single (fp) of contractive mapping defined on compact metric space.

Definition 2. A self mapping $S : X \longrightarrow X$ on a metric space (X, d) is said to be contractive iff S fulfills the condition:

$$d(S(x), S(y)) < d(x, y) \forall x \neq y, x, y \in X. \quad (2)$$

In 1972, Chatterjea [3] introduced the Chatterjea type mapping and showed that every Chatterjea mapping on a complete metric space has a single (fp).

Definition 3. Let (X, d) be a metric space. If T is a self mapping on X , then T is said to be Chatterjea contraction type iff there is real number $0 \leq b < 1/2$ such that

$$d(T(x), T(y)) \leq b[d(x, T(y)) + d(y, T(x))] \forall x, y \in X. \quad (3)$$

In 1979, Browder [4] introduced the concept of a nonexpansive mapping and proved the existence of (fp) for nonexpansive mapping defined on bounded closed convex subset of uniformly convex Banach space.

Definition 4. A self mapping $S : X \longrightarrow X$ on a metric space (X, d) is said to be nonexpansive iff S fulfills the condition:

$$d(S(x), S(y)) \leq d(x, y) \forall x, y \in X. \quad (4)$$

In 1997, Alber and Guerre-Delabriere [5] highlighted the weak ϕ contraction type of mapping and proved the existence of (fp) for this mapping in a Hilbert space setting.

In 2001, Rhoades [6] showed analogous results in the case of ϕ -weak contraction mapping on metric spaces.

Definition 5. A self mapping $S : X \longrightarrow X$ on a metric space (X, d) is said to be ϕ -weak contraction iff it fulfills the condition:

$$d(S(x), S(y)) \leq d(x, y) - \phi(d(x, y)) \forall x, y \in X, \quad (5)$$

where ϕ is a continuous nondecreasing function $\phi : [0, \infty[\longrightarrow [0, \infty[$ with $\phi(t) > 0$, for all $t \in]0, \infty[$ and $\phi(0) = 0$.

In 2003, Kirk et al. [7] generalized the above (fp) theorems for mappings satisfying cyclical contractive conditions in various settings.

Definition 6. Let (X, d) be a metric space, j be a positive integer, $\{A_i\}_{i=1}^j$ be a nonempty closed subsets of X , and $X = \bigcup_{i=1}^j A_i$. Then, a self mapping T on X is said to satisfy cyclical contractive condition iff there is a constant $r \in [0, 1)$ such that:

$$d(T(x), T(y)) \leq r d(x, y)$$

for every $x \in A_i, y \in A_{i+1}, i = 1, 2, \dots, j$, where $A_{j+1} = A_1$. (6)

In 2007, Elshobaky [8] generalized the Banach contraction principle for abc type contraction mapping.

Definition 7. Let (X, d) be a metric space, T be a self mapping on X , $a, b, c \in]0, 1[$, and T satisfy the condition:

$$d(T(x), T(y)) \leq a d(x, y) + b d(x, T(x)) + c d(y, T(y)) \forall x, y \in X. \quad (7)$$

- (1) If $a + c + b < 1$, then T is said to be generalized abc contraction
- (2) If $a + c + b = 1$, then T is said to be generalized abc nonexpansive

In 2008, Dutta and Choudhury [8] proved the existence of (fp) for ψ - ϕ -weak contraction mappings.

Definition 8. Let (X, d) be a metric space. If T is a self mapping on X , then T is said to be ψ - ϕ weak contraction on X iff there are two nondecreasing mappings $\psi, \phi : [0, +\infty) \longrightarrow [0, +\infty)$, ψ is continuous, $\psi(t) = 0$ iff $t = 0$ and $\phi(t) = 0$ iff $t = 0$ such that the following is true:

$$\psi(d(T(x), T(y))) \leq \psi(d(x, y)) - \phi(d(x, y)) \forall x, y \in X. \quad (8)$$

In 2009, Choudhury [9] introduced the weakly Chatterjea contractive type mappings and proved that every weakly Chatterjea contractive mapping on a complete metric space has single (fp).

Definition 9. If $T : X \longrightarrow X$ is a self mapping on a metric space X , then T is said to be weakly C -contractive (or weak Chatterjea type contraction) iff

$$d(T(x), T(y)) \leq \frac{1}{2} [d(x, T(y)) + d(y, T(x))] - \phi(d(x, T(y)), d(x, T(y))) \quad \forall x, y \in X, \quad (9)$$

where $\phi : [0, \infty[\times [0, \infty[\longrightarrow [0, \infty[$ is a continuous function satisfying $\phi(x, y) = 0$ iff $x = y = 0$.

In 2009, Bakr [10] proved the existence of (fp) for abc generalized nonexpansive mappings with one contracting point.

Definition 10. Let X be a normed space, C be bounded closed subset of X , $0 \leq t, a, b, c < 1$, $t, b, c \neq 1$, and $a + b + c = 1$. Then, the point $x_0 \in C$ is said to be $abct$ contracting point with respect to C iff there is $\lambda < (1 - \{ta + b + c\}/1 - t)$ satisfying:

$$b\|x_0 - x\| + c\|x_0 - y\| \leq \lambda\|x - y\| \forall x \neq y \in C. \quad (10)$$

This author showed that if X is a normed space in which every bounded sequence has a weakly convergent subsequence and the parallelogram law hold in X , C is a subset of X which containing $ab0r$ -contracting point, and T is a $ab0r$ -generalized-nonexpansive mappings from C into itself, then for every $ab0r$ -contracting point of C , $x_0 \in C$, the following sequence converges strongly to the best approximation element of x_0 in $F(T)$.

$$x_n = (1 - t_n)x_0 + t_n T(x_n). \quad (11)$$

In 2011, Karapinar [11] gave another generalization for cyclic weak ϕ -contraction.

Definition 11. Let (X, d) be a metric space, j be a positive integer, $\{A_i\}_{i=1}^j$ be nonempty closed subsets of X , and $X = \bigcup_{i=1}^j A_i$. Then a self mapping T on X is said to be cyclic weak contraction iff there is a continuous nondecreasing mapping $\phi : [0, +\infty) \longrightarrow [0, +\infty)$ with $\phi(t) = 0$ iff $t = 0$ such that:

- (1) $X = \bigcup_{i=1}^j A_i$ is cyclic representation of X with respect to T

$$d(T(x), T(y)) \leq d(x, y) - \phi(d(x, y)) \quad \text{for all } x \in A_i, y \in A_{i+1}, i = 1, 2, \dots, j, \quad (12)$$

where $A_{j+1} = A_1$.

At the same year, Karapinar and Sadarangani [12] gave (fp) for generalized cyclic ψ - ϕ -weak contraction mappings in complete metric spaces.

Definition 12. Let (X, d) be a metric space, j be a positive integer, $\{A_i\}_{i=1}^j$ be nonempty closed subsets of X , $X = \bigcup_{i=1}^j A_i$, $\psi, \phi : [0, +\infty) \rightarrow [0, +\infty)$ are two nondecreasing mappings, ψ is continuous, $\psi(t) = 0$ iff $t = 0$, and $\phi(t) = 0$ iff $t = 0$. If T is a self mapping on X , then, T is said to be generalized cyclic $\psi - \phi$ -weak contraction mapping iff the following are true:

- (1) $X = \bigcup_{i=1}^j A_i$ is cyclic representation of X with respect to T
- (2) $\psi(d(T(x), T(y))) \leq \psi(d(x, y)) - \phi(d(x, y))$ for all $x \in A_i, y \in A_{i+1}, i = 1, 2, \dots, j$, where $A_{j+1} = A_1$

In 2012, Karapinar and Sadarangani [13] enhanced the corrigendum (fp) theory for cyclic ϕ -weak contraction mapping. Additionally, Karapinar et al. [14] gave some (fp) for generalized $\psi - \phi$ -weak contraction mapping.

At the same year, (2012), Karapinar and Kumar [15] introduced the notion of cyclic weakly Chatterjea type contraction in metric spaces and proved corresponding (fp) theorems.

Definition 13. Let (X, d) be a metric space, j be a positive integer, $\{A_i\}_{i=1}^j$ be nonempty subsets of X , $X = \bigcup_{i=1}^j A_i$, and $T : X \rightarrow X$ be a self mapping on X . Then, T is said to be cyclic weakly Chatterjea type contraction iff:

- (1) $X = \bigcup_{i=1}^j A_i$ is cyclic representation of X with respect to T
- (2) there are a real number $r \in]0, 1/2]$ and a continuous function $\phi : [0, \infty[\times [0, \infty[\rightarrow [0, \infty[$, $\phi(x, y) = 0$ iff $x = y = 0$ such that

$$\begin{aligned} d(T(x), T(y)) &\leq r[d(x, T(y)) + d(y, T(x))] \\ &\quad - \phi(d(x, T(y)), d(y, T(x))) \\ &\quad \forall x \in A_i, y \in A_{i+1}, \\ &\quad i = 1, 2, \dots, j, \text{ where } A_{j+1} = A_1. \end{aligned} \quad (13)$$

In 2013, Bakr [16] discussed the concepts of $\{a, b, c\}$ -ntype and $\{a, b, c\}$ -ctype of mappings and proved the existence of single (fp) for both $\{a, b, c\}$ -ntype and $\{a, b, c\}$ -ctype on closed convex weakly Cauchy subset of normed space.

Definition 14. Let C be a subset of a normed space $(X, \|\cdot\|)$ and T be a self mapping on C satisfying:

$$\begin{aligned} \|T(x) - T(y)\| &\leq a\|x - y\| + b\|y - T(y)\| \\ &\quad + c \max \{\|x - T(x)\|, \|x - T(y)\|\} \quad (14) \\ &\quad \forall x, y \in C \text{ and } a, b, c \in [0, 1]. \end{aligned}$$

- (1) If $0 \leq c < 1/2$ and $a + b + c < 1$, then T is said to be $\{a, b, c\}$ -contraction type mapping

- (2) If $0 < a < 1, 0 < b, 0 \leq c < 1/2$ and $a + b + c = 1$, then T is said to be $\{a, b, c\}$ -nonexpansive type mapping

In this paper, we are introducing the following:

Let $S : X \rightarrow X$ be a self mapping on a metric space (X, d) and a, b, c be three real numbers. Then, we define the real valued function $\Gamma_{S(abc)} : X \times X \rightarrow \mathbb{R}^+$ by.

$$\Gamma_{S(abc)}(x, y) = a d(x, y) + b d(x, S(x)) + c d(y, S(y)) \quad \forall x, y \in X. \quad (15)$$

Definition 15. Let $S, T : X \rightarrow X$ be two self mappings on a metric space (X, d) and a, b, c be three real numbers. Then, we define the real valued function $\Gamma_{S,T(abc)} : X \times X \rightarrow \mathbb{R}^+$ by

$$\Gamma_{S,T(abc)}(x, y) = a d(x, y) + b d(x, S(x)) + c d(y, T(y)) \quad \forall x, y \in X. \quad (16)$$

Definition 16. Let (X, d) be a metric space; $X = A \cup B$ and S be a self mapping on X with

- (1) $S(A) \subset B$ and $S(B) \subset A$ and
- (2) there are real constants $a, b, c \in]0, 1[$ such that

$$d(S(x), S(y)) \leq \Gamma_{S(abc)}(x, y) \quad \forall x \in A, y \in B. \quad (17)$$

- (i) If $a + c + b < 1$, then S is said to be cyclic generalized abc contraction w.r.t (A, B) .
- (ii) If $a + c + b = 1$, then S is said to be cyclic generalized abc nonexpansive w.r.t (A, B) .

Include the definition of lower semicontinuity of the function.

Definition 17. Let X be a linear space. Then, a function f from X into $]-\infty, +\infty]$ is said to be lower semicontinuous if for any real number α , the set $\{x \in X : f(x) \leq \alpha\}$ is closed convex subset of X .

Definition 18. Let $S : X \rightarrow X$ be a self mapping on a metric space (X, d) such that

$$d(S(x), S(y)) \leq \Gamma_{S(abc)}(x, y) - \phi\left(\Gamma_{S(abc)}(x, y)\right) \quad \forall x \in A, y \in B, \quad (18)$$

where ϕ is lower semicontinuous nondecreasing function $\phi : [0, \infty[\rightarrow [0, \infty[$ with $\phi(t) > 0$, for all $t \in]0, \infty[$ and $\phi(0) = 0$.

- (1) If $a + c + b < 1$, then S is said to be generalized cyclic $\phi - abc$ weak contraction w.r.t (A, B) .

- (2) If $a + c + b = 1$, then S is said to be generalized cyclic $\phi - abc$ weak nonexpansive w.r.t (A, B) .

Additionally, we introduce the following fascinating concept of joint mappings:.

Definition 19. Let (X, d) be a metric space with $X = A \cup B$, $S, T : X \longrightarrow X$ be two self mappings, and $a, b, c \in]0, 1[$ be three real numbers satisfying:

- (1) The cyclic condition1: $S(A) \subset B$ and $T(B) \subset A$
- (2) The contractivity condition2:

$$d(S(x), T(y)) \leq \Gamma_{S, T(abc)}(x, y) - \phi\left(\Gamma_{S, T(abc)}(x, y)\right) \forall x \in A, y \in B, \quad (19)$$

where ϕ is lower semicontinuous nondecreasing function $\phi : [0, \infty[\longrightarrow [0, \infty[$ with $\phi(t) > 0$, for all $t \in]0, \infty[$ and $\phi(0) = 0$.

- (i) If $a + c + b < 1$, then S and T are said to be joint (A, B) generalized cyclic $\phi - abc$ weak contraction type
- (ii) If $a + c + b = 1$, then S and T are said to be joint (A, B) generalized cyclic $\phi - abc$ weak nonexpansive type

In 1993, Czerwik [17] verified the principle in a wider class of metric spaces, namely, quasi metric space, these spaces sometimes are called b -metric spaces, for some other types of quasi metric spaces one can see [18].

Definition 20. Let r be a real number, $r \geq 1$, Q be a mapping $Q : X \times X \longrightarrow \mathbb{R}^+$. Then, (X, Q) is called b -metric space iff

- (1) $Q(x, y) = 0$ iff $x = y$
- (2) $Q(x, y) = Q(y, x)$.
- (3) $Q(x, y) \leq r [Q(x, z) + Q(z, y)]$ for all $x, y, z \in X$

In 2017, Bakr [19] adjusted (fp) theorem by using both b -metric spaces and generalized $abc; r$ contraction type of mappings.

Definition 21. Let r be a real number, $r \geq 1$ and (X, Q) be a b -metric space. Then, the mapping S is said to be $abc; r$ -type mapping iff it satisfies:

$$Q(S(u), S(v)) \leq a Q(u, v) + b Q(v, S(v)) + c \max \{Q(u, S(u)), Q(u, S(v))\} \forall u, v \in X, \quad (20)$$

where $a, b, c \in [0, 1]$ are some real numbers with $a + c < 1$ and $r(a + b) + c < 1$.

Now, the generalized cyclic $abc; r$ contraction type mapping in b -metric spaces is defined as follows:.

Definition 22. Let r be a real number, $r \geq 1$ and (X, Q) be a b -metric space. Then, a self mapping S on (X, Q) is said to be generalized cyclic $abc; r$ contraction w.r.t (A, B) iff S satisfies the following:

- (1) $S(A) \subset B$ and $S(B) \subset A$ and
- (2) there are real constants $a, b, c \in]0, 1[$ with

$$\left[r \max \left\{ \frac{a + cr}{1 - (b + cr)}, \frac{a + b}{1 - c} \right\} < 1 \right] \quad (21)$$

and

$$Q(S(x), S(y)) \leq a Q(x, y) + b Q(x, S(x)) + c \max \{Q(y, S(y)), Q(y, S(x))\} \forall x \in A, y \in B. \quad (22)$$

The purposes of this manuscript are designed as follows:

Theorem 23 together with Proposition 24 proves the existence of a single common (fp) for two joint (A, B) generalized cyclic $\phi - abc$ weak nonexpansive type mappings that have compact sets (A, B) . The result in particular presented a single (fp) for generalized cyclic $\phi - abc$ weak nonexpansive mapping, without the assumption of single contracting point, this result showed in Corollary 25.

Additionally, we introduce a new type of generalized cyclic $abc; r$ contraction mapping. Theorem 29 described the existence of a single (fp) of such mappings in b -metric spaces.

Finally, Theorem 31 established a simpler convergence theorem for a sequence of generalized cyclic $abc; r$ contraction mappings. These results are extensions and generalizations of previous results reported in [10, 11, 17, 19, 20].

Since every contraction mapping is nonexpansive and both are cyclic w.r.t (X, X) . We noticed the following:

- (i) The class of all generalized abc nonexpansive mappings is wider than that of generalized abc contraction type
- (ii) The class of all generalized cyclic abc contraction mappings is wider than that of generalized abc contraction type
- (iii) The class of all generalized cyclic abc nonexpansive mappings is wider than that of generalized abc nonexpansive type
- (iv) The class of all generalized cyclic abc nonexpansive mappings is wider than that of generalized cyclic abc contraction type
- (v) The class of all generalized cyclic $\phi - abc$ weak nonexpansive mappings is wider than that of generalized cyclic $\phi - abc$ weak contraction type

- (vi) The class of all generalized $\phi - abc$ weak contraction is wider than that of ϕ weak contraction type
- (vii) The class of all joint (A, B) generalized cyclic $\phi - abc$ weak nonexpansive types is wider than that of joint (A, B) generalized cyclic $\phi - abc$ weak contraction types
- (viii) If S, T are continuous self mappings on (X, d) , then the restriction of the mapping $\Gamma_{S,T(abc)} : A \times B \longrightarrow \mathbb{R}^+$ is continuous

$$\Gamma_{S,T(abc)}(x, y) = a d(x, y) + b d(x, S(x)) + c d(y, T(y)), x \in A, y \in B. \quad (23)$$

- (ix) If A, B are two compact subsets of the metric space (X, d) and S, T are continuous self mappings on X , then the restriction of the mapping $\Gamma_{S,T(abc)} : A \times B \longrightarrow \mathbb{R}^+$ attains its infimum as well as its supremum at some points in $A \times B$

2. Main Results

2.1. Common Fixed Point of Continuous Joint Generalized Cyclic Weak Nonexpansive Mappings. We have the following generalizations:

Theorem 23. *Let (X, d) be a metric space and A, B be two compact subsets of which $X = A \cup B$. If $S, T : X \longrightarrow X$ are two continuous joint (A, B) generalized cyclic $\phi - abc$ weak nonexpansive type of mappings on X , then there is single point $u \in X$ such that $S(u) = T(u) = u \in A \cap B$.*

Proof. Let u_0 be an arbitrarily chosen element in X . Then u_0 is either in A or in B . If u_0 is in B , then define $u_1 = T(u_0) \in A$, $u_2 = S(u_1) \in B$, $u_3 = T(u_2) \in A$, and then by induction define:

$$u_{2n+2} = S(u_{2n+1}) \in B \text{ and } u_{2n+1} = T(u_{2n}) \in A \forall n \geq 0. \quad (24)$$

First, suppose n is an odd natural number. Then

$$\begin{aligned} d(u_{n+1}, u_n) &= d(S(u_n), T(u_{n-1})) \\ &\leq \Gamma_{S,T(abc)}(u_n, u_{n-1}) - \phi(\Gamma_{S,T(abc)}(u_n, u_{n-1})). \end{aligned} \quad (25)$$

Hence

$$\begin{aligned} d(u_{n+1}, u_n) &\leq \Gamma_{S,T(abc)}(u_n, u_{n-1}) \\ &= a d(u_n, u_{n-1}) + b d(S(u_n), u_n) \\ &\quad + c d(T(u_{n-1}), u_{n-1}) \\ &= a d(u_n, u_{n-1}) + b d(u_{n+1}, u_n) + c d(u_n, u_{n-1}) \\ &= (a + c) d(u_n, u_{n-1}) + b d(u_{n+1}, u_n). \end{aligned} \quad (26)$$

Thus,

$$d(u_{n+1}, u_n) \leq \left[\frac{a+c}{1-b} \right] d(u_n, u_{n-1}) = d(u_n, u_{n-1}). \quad (27)$$

Therefore,

$$\begin{aligned} \Gamma_{S,T(abc)}(u_n, u_{n-1}) &\leq (a+c) d(u_n, u_{n-1}) + b d(u_{n+1}, u_n) \\ &\leq (a+c) d(u_n, u_{n-1}) + b d(u_n, u_{n-1}) \\ &= (a+b+c) d(u_n, u_{n-1}) = d(u_n, u_{n-1}). \end{aligned} \quad (28)$$

Hence

$$d(u_{n+1}, u_n) \leq \Gamma_{S,T(abc)}(u_n, u_{n-1}) \leq d(u_n, u_{n-1}). \quad (29)$$

Continuing this process gives

$$\begin{aligned} d(u_{n+1}, u_n) &\leq \Gamma_{S,T(abc)}(u_n, u_{n-1}) \leq d(u_n, u_{n-1}) \\ &\leq \Gamma_{S,T(abc)}(u_{n-1}, u_{n-2}). \end{aligned} \quad (30)$$

Second, by a similar method when n is an even natural number, we obtain

$$\begin{aligned} d(u_{n+1}, u_n) &\leq \Gamma_{S,T(abc)}(u_n, u_{n-1}) \leq d(u_n, u_{n-1}) \\ &\leq \Gamma_{S,T(abc)}(u_{n-1}, u_{n-2}). \end{aligned} \quad (31)$$

Hence, the two sequences $\{d(u_{n+1}, u_n)\}_{n \in \mathbb{N}}$ and $\{\Gamma_{S,T(abc)}(u_{n+1}, u_n)\}_{n \in \mathbb{N}}$ are monotonic nonincreasing and bounded below by 0 with the same infimum, if this infimum is r , then

$$\lim_{n \rightarrow \infty} d(u_{n+1}, u_n) = \lim_{n \rightarrow \infty} \Gamma_{S,T(abc)}(u_n, u_{n-1}) = r. \quad (32)$$

Using the properties of ϕ ,

$$\phi(r) \leq \liminf_{n \rightarrow \infty} \phi(\Gamma_{S,T(abc)}(u_n, u_{n-1})). \quad (33)$$

Taking least upper limits on two sides of the inequalities (25) as $n \longrightarrow \infty$ gives

$$r \leq r - \liminf_{n \rightarrow \infty} \phi(\Gamma_{S,T(abc)}(u_n, u_{n-1})) \leq r - \phi(r), \quad (34)$$

that is $\phi(r) \leq 0$. Thus $\phi(r) = 0$, this gives $r = 0$. Hence,

$$\lim_{n \rightarrow \infty} d(u_{n+1}, u_n) = \lim_{n \rightarrow \infty} \Gamma_{S,T(abc)}(u_n, u_{n-1}) = 0. \quad (35)$$

This implies that

$$\inf \{ \Gamma_{S,T(abc)}(x, y) : x \in A, y \in B \} = 0. \quad (36)$$

Since $\Gamma_{S,T(abc)}$ attains its infimum on $A \times B$, there are $x_0 \in A$ and $y_0 \in B$ such that

$$\Gamma_{S,T(abc)}(x_0, y_0) = 0. \quad (37)$$

This gives

$$\begin{aligned} ad(x_0, y_0) + b d(x_0, S(x_0)) + c d(y_0, T(y_0)) &= \Gamma_{S,T(abc)}(x_0, y_0) \\ &= 0, \end{aligned} \quad (38)$$

that is

$$ad(x_0, y_0) + b d(x_0, S(x_0)) + c d(T(y_0), y_0) = 0, \quad (39)$$

since all are nonnegative real numbers, clearly

$$d(x_0, y_0) = d(x_0, S(x_0)) = d(y_0, T(y_0)) = 0, \quad (40)$$

and we have

$$x_0 = y_0 = S(x_0) = T(y_0). \quad (41)$$

Notice that the converse is also true, if $x_0 = y_0 = S(x_0) = T(y_0)$, then $\Gamma_{S,T(abc)}(x_0, y_0) = 0$ is clear. On the other side, this showed that $x_0 = y_0 \in A \cap B$ is a common (fp) of S and T . If there exists another point $u \in A \cap B$ such that $S(u) = u = T(u)$, then we get

$$\begin{aligned} d(u, y_0) &= d(S(u), T(y_0)) \\ &\leq \Gamma_{S,T(abc)}(u, y_0) - \phi\left(\Gamma_{S,T(abc)}(u, y_0)\right) \\ &\leq a d(u, y_0) + b d(S(u), u) + c d(T(y_0), y_0) \\ &= a d(u, y_0) < d(u, y_0). \end{aligned} \quad (42)$$

This shows that $d(u, y_0) = \Gamma_{S,T(abc)}(u, y_0) = 0$, that is $u = y_0$.

Proposition 24. *The sequence defined iteratively by the induction (24) and by previous theorem notations is convergent to the unique common (fp) of S and T .*

$$\lim_{n \rightarrow \infty} u_n = u. \quad (43)$$

Proof. Let u be the common (fp) of S and T and $\lim_{n \rightarrow \infty} u_n = v$ with $v \neq u$, then

$$\begin{aligned} d(u_{2n+2}, u) &= d(u_{2n+2}, T(u)) = d(S(u_{2n+1}), T(u)) \\ &\leq \Gamma_{S,T(abc)}(u_{2n+1}, u) \\ &\leq a d(u_{2n+1}, u) + b d(S(u_{2n+1}), u_{2n+1}) + c d(T(u), u) \\ &\leq a d(u_{2n+1}, u) + b d(u_{2n+2}, u_{2n+1}) + c d(T(u), u) \\ &\leq a d(u_{2n+1}, u) + b d(u_{2n+2}, u_{2n+1}). \end{aligned} \quad (44)$$

On limiting approach as $n \rightarrow \infty$, equation (35) proves that:

$$d(v, u) \leq a d(v, u). \quad (45)$$

Hence, $(1 - a)d(v, u) \leq 0$, since $a \neq 1$, we get $d(v, u) = 0$, that is, $v = u$.

Corollary 25. *If (X, d) is metric space, $X = A \cup B$, A , and B are compact, and $S : X \rightarrow X$ is continuous generalized cyclic $\phi - abc$ weak nonexpansive mapping w.r.t (A, B) , then there is single point $u \in X$ such that $S(u) = u$. Moreover, for every $u_0 \in X$, we have $\lim_{n \rightarrow \infty} S^n(u_0) = u$.*

2.1.1. Example. Let $X = [-1, 1]$, $A = [-1, 0]$, and $B = [0, 1]$. Define $S : X \rightarrow X$ as:

$$S(v) = \begin{cases} -\frac{v}{3}, & \text{if } v \in A, \\ -\frac{v}{2}, & \text{if } v \in B. \end{cases} \quad (46)$$

It is clear that S is cyclic with respect to the representation $A \cup B$ of X . Endow X with the metric $d(u, v) = |u - v|$. The mapping S is having the unique (fp) $u = 0 \in A \cap B$ and it is generalized cyclic $\phi - (1/4)(1/12)(2/3)$ weak nonexpansive w.r.t (A, B) , where ϕ is defined by $\phi(t) = (1/13)t$. In fact, if $u \in A$ and $v \in B$, then we have

$$\begin{aligned} \Gamma_{S(\frac{1}{4})(\frac{1}{12})(\frac{2}{3})}(u, v) &= \frac{1}{4}d(u, v) + \frac{1}{12}d(u, S(u)) + \frac{2}{3}d(v, S(v)) \\ &= \frac{1}{4}(v - u) + \frac{1}{12}|u - (-\frac{u}{3})| + \frac{2}{3}|v - (-\frac{v}{2})| \\ &= \frac{1}{4}(v - u) + \frac{1}{12}|u + \frac{u}{3}| + \frac{2}{3}|v + \frac{v}{2}| \\ &= \frac{1}{4}(v - u) - \frac{1}{12}\left(u + \frac{u}{3}\right) + \frac{2}{3}\left(v + \frac{v}{2}\right) \\ &= \frac{1}{4}(v - u) - \frac{1}{12}\left(\frac{4u}{3}\right) + \frac{2}{3}\left(\frac{3v}{2}\right) \\ &= \frac{1}{4}(v - u) - \frac{u}{9} + v = \frac{5}{4}v - \frac{13u}{36} \\ &= \frac{1}{36}[45v - 13u]. \end{aligned} \quad (47)$$

Using this, we have

$$\begin{aligned} d(S(u), S(v)) &= \left| -\frac{u}{3} - \left(-\frac{v}{2}\right) \right| = \left| \frac{v}{2} - \frac{u}{3} \right| = \frac{v}{2} - \frac{u}{3} \\ &= \frac{1}{6}(3v - 2u) = \frac{1}{36}(18v - 12u) \\ &= \frac{1}{36}[45v - 13u] - \frac{1}{36}[27v - u] \\ &\leq \frac{1}{36}[45v - 13u] - \frac{1}{36}\left[\frac{45}{13}v - u\right] \\ &= \frac{1}{36}[45v - 13u] - \left(\frac{1}{13}\right)\left(\frac{1}{36}\right)[45v - 13u] \\ &= \Gamma_{S(\frac{1}{4})(\frac{1}{12})(\frac{2}{3})}(u, v) - \frac{1}{13}\Gamma_{S(\frac{1}{4})(\frac{1}{12})(\frac{2}{3})}(u, v) \\ &= \Gamma_{S(\frac{1}{4})(\frac{1}{12})(\frac{2}{3})}(u, v) - \phi\left(\Gamma_{S(\frac{1}{4})(\frac{1}{12})(\frac{2}{3})}(u, v)\right). \end{aligned} \quad (48)$$

2.2. Generalized Results for Generalized $abc; r$ Contraction. Next, we generalize results for generalized $abc; r$ contraction [19] to those of generalized cyclic $abc; r$ contraction in b -metric spaces. We have the following important lemmas:

Lemma 26. *Let S be generalized cyclic $abc; r$ contraction w.r.t (A, B) of the b -metric space X . Then for every $y \in X$ and any natural number $n \in \mathbb{N}$, we have:*

(1) *The inequalities*

$$Q(S^{n+1}(y), S^n(y)) \leq \begin{cases} \left[\frac{a+b}{1-c} \right] Q(S^n(y), S^{n-1}(y)), & \text{if } n \text{ is odd,} \\ \left[\frac{a+cr}{1-(b+cr)} \right] Q(S^n(y), S^{n-1}(y)), & \text{if } n \text{ is even,} \end{cases}$$

(49)

are true

(2) *There are two natural numbers l and m with $n = l + m$ and*

$$Q(S^{n+1}(y), S^n(y)) \leq \left[\frac{a+b}{1-c} \right]^l \left[\frac{a+cr}{1-(b+cr)} \right]^m Q(S(y), y). \quad (50)$$

Proof. Let $y \in X = A \cup B$, $y \in A$ (similar conclusion if $y \in B$). Then $S(y) \in B$, using the contractivity condition of S gives

$$\begin{aligned} Q(S^2(y), S(y)) &= Q(S(y), S(S(y))) \\ &\leq aQ(y, S(y)) + bQ(y, S(y)) \\ &\quad + c \max \{Q(S(y), S(S(y))) + Q(S(y), S(y))\} \\ &= (a+b)Q(y, S(y)) + cQ(S(y), S(S(y))), \end{aligned} \quad (51)$$

hence,

$$Q(S^2(y), S(y)) \leq \left[\frac{a+b}{1-c} \right] Q(S(y), y). \quad (52)$$

For $S(y) \in B$, we have $S(S(y)) \in A$, thus

$$\begin{aligned} Q(S^3(y), S^2(y)) &= Q(S(S^2(y)), S(S(y))) \\ &\leq aQ(S^2(y), S(y)) + bQ(S^2(y), S(S^2(y))) \\ &\quad + c \max \{Q(S(y), S(S(y))), Q(S(y), S(S^2(y)))\} \\ &= aQ(S^2(y), S(y)) + bQ(S^2(y), S^3(y)) \\ &\quad + c \max \{Q(S(y), S(S(y))), r[Q(S(y), S(S(y))) \\ &\quad + Q(S(S(y)), S(S^2(y)))]\} \\ &= aQ(S^2(y), S(y)) + bQ(S^2(y), S^3(y)) \\ &\quad + cr[Q(S(y), S(S(y))) + Q(S(S(y)), S(S^2(y)))] \\ &= (a+cr)Q(S^2(y), S(y)) \\ &\quad + (b+cr)Q(S^2(y), S^3(y)). \end{aligned} \quad (53)$$

Because $r \geq 1$, hence

$$Q(S^3(y), S^2(y)) \leq \left[\frac{a+cr}{1-(b+cr)} \right] Q(S^2(y), S(y)). \quad (54)$$

Using the inequalities (52) and (54) gives

$$Q(S^3(y), S^2(y)) \leq \left[\frac{a+cr}{1-(b+cr)} \right] \left[\frac{a+b}{1-c} \right] Q(S(y), y). \quad (55)$$

For $S^2(y) \in A$, we have $S^3(y) \in B$, and then

$$\begin{aligned} Q(S^4(y), S^3(y)) &= Q(S(S^2(y)), S(S^3(y))) \\ &\leq aQ(S^2(y), S^3(y)) + bQ(S^2(y), S(S^2(y))) \\ &\quad + c \max \{Q(S^3(y), S(S^3(y))), \\ &\quad Q(S^3(y), S(S^2(y)))\} \\ &\leq (a+b)Q(S^2(y), S^3(y)) \\ &\quad + cQ(S^3(y), S(S^3(y))). \end{aligned} \quad (56)$$

Consequently,

$$Q(S^4(y), S^3(y)) \leq \left[\frac{a+b}{1-c} \right] Q(S^3(y), S^2(y)). \quad (57)$$

Using the inequalities (55) and (57) gives

$$Q(S^4(y), S^3(y)) \leq \left[\frac{a+cr}{1-(b+cr)} \right] \left[\frac{a+b}{1-c} \right]^2 Q(S(y), y). \quad (58)$$

For $S^3(y) \in B$, we have $S^4(y) \in A$, and then

$$\begin{aligned} Q(S^5(y), S^4(y)) &= Q(S(S^4(y)), S(S^3(y))) \\ &\leq \left[\frac{a+cr}{1-(b+cr)} \right] Q(S^4(y), S^3(y)). \end{aligned} \quad (59)$$

Using the inequalities (58) and (59) gives

$$Q(S^5(y), S^4(y)) \leq \left[\frac{a+cr}{1-(b+cr)} \right]^2 \left[\frac{a+b}{1-c} \right]^2 Q(S(y), y). \quad (60)$$

Continuing in this process, we see that

$$\begin{aligned} Q(S^6(y), S^5(y)) &\leq \left[\frac{a+cr}{1-(b+cr)} \right]^2 \left[\frac{a+b}{1-c} \right]^3 Q(S(y), y) \\ Q(S^7(y), S^6(y)) &\leq \left[\frac{a+cr}{1-(b+cr)} \right]^3 \left[\frac{a+b}{1-c} \right]^3 Q(S(y), y). \end{aligned} \quad (61)$$

And so on, combining the above inequalities prove formulas (49) and (50).

Lemma 27. Let S be generalized cyclic abc ; r contraction w.r.t any pair (A, B) of X . Then for every $y \in X$ and any natural number $n \in \mathbb{N}$, there is a real number $r_0 < 1$ such that the inequalities

$$Q(S^{n+1}(y), S^n(y)) \leq r_0 Q(S^n(y), S^{n-1}(y)) \quad (62)$$

$$Q(S^{n+1}(y), S^n(y)) \leq r_0^n Q(S(y), y) \quad (63)$$

are true.

Proof. Using Lemma 26 with $r_0 = \max \{[a + cr/1 - (b + cr)], [a + b/1 - c]\}$ completes the proof.

Lemma 28. Let S be generalized cyclic abc ; r contraction w.r.t any pair (A, B) of X . Then for every $y \in X$, the sequence $\{S^n(y)\}_{n \in \mathbb{N}}$ is Cauchy.

Proof. Let m, n be two natural numbers with $m > n$ and y be an arbitrarily element in X . If $y \in A$ or $y \in B$, then we will have the same conclusion

$$\begin{aligned} Q(S^n(y), S^m(y)) &\leq r[Q(S^n(y), S^{n+1}(y)) + Q(S^{n+1}(y), S^m(y))] \\ &\leq r[Q(S^n(y), S^{n+1}(y)) \\ &\quad + r\{Q(S^{n+1}(y), S^{n+2}(y)) \\ &\quad + Q(S^{n+2}(y), S^m(y))\}] \\ &\leq rQ(S^n(y), S^{n+1}(y)) + r^2Q(S^{n+1}(y), S^{n+2}(y)) \\ &\quad + r^2[r\{Q(S^{n+2}(y), S^{n+3}(y)) \\ &\quad + Q(S^{n+3}(y), S^m(y))\}] \\ &\leq rQ(S^n(y), S^{n+1}(y)) + r^2Q(S^{n+1}(y), S^{n+2}(y)) \\ &\quad + r^3Q(S^{n+2}(y), S^{n+3}(y)) \\ &\quad + \dots + r^{m-n}Q(S^{m-1}(y), S^m(y))]. \end{aligned} \quad (64)$$

Using the inequalities (63) gives

$$\begin{aligned} Q(S^n(y), S^m(y)) &\leq rr_0^n Q(S(y), y) \\ &\quad + r^2 r_0^{n+1} Q(S(y), y) + r^3 r_0^{n+2} Q(S(y), y) \\ &\quad + \dots + r^{m-n} r_0^{n+(m-1-n)} Q(S(y), y) \\ &= rr_0^n [Q(S(y), y) + rr_0 Q(S(y), y) \\ &\quad + (rr_0)^2 Q(S(y), y) + \dots + (rr_0)^{(m-1-n)} Q(S(y), y)] \\ &= rr_0^n Q(S(y), y) \left[\frac{1 - (rr_0)^{m-1-n}}{1 - rr_0} \right] \\ &\leq \left[\frac{r}{1 - rr_0} \right] Q(S(y), y) r_0^n. \end{aligned} \quad (65)$$

Clearly, $r_0 < 1$, taking the limit as $n \rightarrow \infty$ completes the proof.

We have the following (fp) results:

Theorem 29. Let (X, Q) be complete b -metric space, S be generalized cyclic abc ; r contraction w.r.t (A, B) . Then, S has single (fp) $u \in A \cap B$. Moreover, for any $y \in X$, the sequence $\{S^n(y)\}_{n \in \mathbb{N}}$ is converging to u .

Proof. Let y be an arbitrarily chosen element of X , in either of the two cases $y \in A$ or $y \in B$ we have the same result, using inequalities (63) of Lemma 27 ensures that

$$\lim_{n \rightarrow \infty} Q(S^{n+1}(y), S^n(y)) = 0, \quad (66)$$

and $\{S^n(y)\}_{n \in \mathbb{N}}$ is Cauchy by Lemma 28. Since X is complete b -metric space, $\{S^n(y)\}_{n \in \mathbb{N}}$ is convergent to some element $u \in X$;

$$\lim_{n \rightarrow \infty} Q(S^n(y), u) = 0. \quad (67)$$

Each subsequence is convergent to the same limit point, since each of A and B contains infinite number of terms of $\{S^n(y)\}_{n \in \mathbb{N}}$. Hence, in A and in B , we construct a subsequence of $\{S^{n_i}(y)\}_{i \in \mathbb{N}}$ that converges to u . Since A and B are closed, we conclude that u is lying in the intersection of A and B . On the other side, if $S^{n_i}(y) \in B$, $u \in A$, (same conclusion if we consider $S^{n_i}(y) \in A$, $u \in B$), use the cyclic condition

$$\begin{aligned} Q(S(u), S^{n_i+1}(y)) &= Q(S(u), S(S^{n_i}(y))) \\ &\leq aQ(u, S^{n_i}(y)) + bQ(u, S(u)) \\ &\quad + c \max \{Q(S^{n_i}(y), S^{n_i+1}(y)), Q(S^{n_i}(y), S(u))\}. \end{aligned} \quad (68)$$

Taking the limit as $i \rightarrow \infty$, the inequalities (66) and (67) give

$$\begin{aligned} \lim_{i \rightarrow \infty} Q(S(u), S^{n_i+1}(y)) &\leq a(0) + bQ(u, S(u)) \\ &\quad + c \max \left\{ 0, \lim_{i \rightarrow \infty} Q(S^{n_i}(y), S(u)) \right\}. \end{aligned} \quad (69)$$

Hence,

$$(1 - c) \lim_{i \rightarrow \infty} Q(S(u), S^{n_i}(y)) \leq bQ(u, S(u)), \quad (70)$$

$$\lim_{i \rightarrow \infty} Q(S(u), S^{n_i}(y)) \leq \frac{b}{1 - c} Q(u, S(u)).$$

Since

$$Q(S(u), u) \leq r[Q(S(u), S^{n_i}(y)) + Q(S^{n_i}(y), u)], \quad (71)$$

we have

$$Q(S(u), u) \leq r \left[\lim_{i \rightarrow \infty} Q(S(u), S^{n_i}(y)) + \lim_{i \rightarrow \infty} Q(S^{n_i}(y), u) \right], \quad (72)$$

that is

$$Q(S(u), u) \leq r \left[\frac{b}{1-c} Q(u, S(u)) + 0 \right]. \quad (73)$$

Accordingly, we have

$$Q(S(u), u) \leq \frac{rb}{1-c} Q(u, S(u)) < Q(u, S(u)). \quad (74)$$

This shows that $Q(S(u), u) = 0$. Consequently $S(u) = u$, that is, u is a (fp) of S . Finally, to prove that such a (fp) is unique, let u and z be two distinct (fps) of S . Noting that both u and z are lying in the intersection of A and B (if $z \in A$, then $z = S(z) \in B$ and vice versa), the following strict inequality gives an obvious contradiction:

$$\begin{aligned} Q(u, z) &= Q(S(u), S(z)) \leq aQ(u, z) + bQ(u, S(u)) \\ &\quad + c \max \{Q(z, S(z)) + Q(z, S(u))\} \\ &= aQ(u, z) + bQ(u, u) + c \max \{Q(z, z) + Q(z, u)\} \\ &= aQ(u, z) + cQ(z, u) = (a+c)Q(u, z) < Q(u, z). \end{aligned} \quad (75)$$

Remark 30. Using the same steps of proof, Theorem (29) can be easily extended to the case when S has $\{A_i\}_{i=1}^j$ (U) cyclic representations.

Now, we are going to give a generalization of Theorem (31) [19], the prove is more simpler and in case of cyclic type mappings.

Theorem 31. If $\{S_n\}_{n \in \mathbb{N}}$ and S are generalized cyclic $abc; r$ contractions on the complete b -metric space (X, Q) w.r.t (A, B) , w_n^* is the unique fixed of each S_n , $S_n(w_n^*) = w_n^* \forall n \in \mathbb{N}$, and $\{S_n(w^*)\}_{n \in \mathbb{N}}$ converges to w^* ,

$$\lim_{n \rightarrow \infty} Q(S_n(w^*), S(w^*)) = 0, \quad (76)$$

where w^* is the unique fixed of S ; $S(w^*) = w^*$, then $\{w_n^*\}_{n \in \mathbb{N}}$ converges to w^* ;

$$\lim_{n \rightarrow \infty} Q(w_n^*, w^*) = 0. \quad (77)$$

Proof. We have

$$\begin{aligned} Q(w_n^*, w^*) &= Q(S_n(w_n^*), S(w^*)) \\ &\leq r[Q(w_n^*, S_n(w^*)) + Q(S_n(w^*), S(w^*))]. \end{aligned} \quad (78)$$

Taking in account that $\{w_n^*\}_{n \in \mathbb{N}} \cup \{w^*\} \subseteq A \cap B$, we have

$$\begin{aligned} Q(w_n^*, S_n(w^*)) &= Q(S_n(w_n^*), S_n(w^*)) \\ &\leq aQ(w_n^*, w^*) + bQ(w^*, S_n(w^*)) \\ &\quad + c \max \{Q(w_n^*, S_n(w_n^*)), Q(w_n^*, S_n(w^*))\} \\ &\leq aQ(w_n^*, w^*) + bQ(w^*, S_n(w^*)) \\ &\quad + cQ(w_n^*, S_n(w^*))Q(w_n^*, S_n(w^*)) \\ &\leq \left[\frac{1}{1-c} \right] [aQ(w_n^*, w^*) + bQ(w^*, S_n(w^*))]. \end{aligned} \quad (79)$$

Substituting from (79) in (78) gives

$$\begin{aligned} Q(w_n^*, w^*) &= Q(S_n(w_n^*), S(w^*)) \\ &\leq r \left[\left[\frac{1}{1-c} \right] [aQ(w_n^*, w^*) + bQ(w^*, S_n(w^*))] \right. \\ &\quad \left. + Q(S_n(w^*), S(w^*)) \right] \\ &\leq r \left[\left[\frac{a}{1-c} \right] Q(w_n^*, w^*) + \left[\frac{b}{1-c} \right] Q(w^*, S_n(w^*)) \right. \\ &\quad \left. + Q(S_n(w^*), S(w^*)) \right] \\ &\leq \left[\frac{ra}{1-c} \right] Q(w_n^*, w^*) + \left[\frac{b}{1-c} + 1 \right] rQ(w^*, S_n(w^*)). \end{aligned} \quad (80)$$

Hence,

$$\left[\frac{1-c-ra}{1-c} \right] Q(w_n^*, w^*) \leq \left[\frac{b-c+1}{1-c} \right] rQ(w^*, S_n(w^*)). \quad (81)$$

Consequently, we have

$$Q(w_n^*, w^*) \leq \left[\frac{b-c+1}{1-c-ra} \right] rQ(w^*, S_n(w^*)). \quad (82)$$

Taking the limit as $n \rightarrow \infty$ and using assumption (76) prove (77).

3. Conclusion

This manuscript indicates the existence of a single common (fp) for two joint (A, B) generalized cyclic $\phi - abc$ weak non-expansive type mappings, where A and B are compact sets. The result presented a single (fp) for generalized cyclic $\phi - abc$ weak nonexpansive mapping, without the assumption of existence a single contracting point. Additionally, it introduces a new type of generalized cyclic $abc; r$ contraction mapping and describes the existence of a single (fp) of such mappings in b -metric spaces. Finally, the presented results establish a simpler convergence theorem for a sequence of generalized cyclic $abc; r$ contraction mappings. These results are extensions and generalizations of previous results reported in [10, 11, 17, 19, 20].

Abbreviations

fp:	Fixed point
fps:	Fixed points
w.r.t:	With respect to
iff:	If and only if
$\{a, b, c\}$ -ntype mappings:	$\{a, b, c\}$ -nonexpansive type mappings
$\{a, b, c\}$ -ctype mappings:	$\{a, b, c\}$ -contraction type mappings.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author has no conflict of interests.

Authors' Contributions

The sole author contributed 100% to the article. The author read and approved the final manuscript.

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