

Research Article

Conformable Sumudu Transform of Space-Time Fractional Telegraph Equation

Amjad E. Hamza ¹, Mohamed Z. Mohamed ^{1,2}, Eltaib M. Abd Elmohmoud ³
and M. Magzoub ³

¹Mathematics Department, Faculty of Sciences, University of Hail, Hail 8151, Saudi Arabia

²Mathematics Department, Faculty of Sciences, Sudan University of Sciences and Technology, Sudan

³Mathematics Department, Faculties of Sciences and Arts, University of Jeddah Branch Al-kamil, Saudi Arabia

Correspondence should be addressed to Amjad E. Hamza; aboaljad2@hotmail.com
and Mohamed Z. Mohamed; lesha-83@hotmail.com

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This paper intends to obtain accurate and convergent numerical solutions of linear space-time matching telegraph fractional equations by means of a double Sumudu matching transformation method. Moreover, the numerical model is equipped to explain the work, the accuracy of the work, and sobriety in its presentation method, and as a result, the proposed method shows an effective and convenient way, to employ proven problems in science and engineering.

1. Introduction

In recent years, a large extent of progress has been made in the differential equation with frequent appearance in physics, chemical and industrial mathematics, processing and control theory, and fluid mechanics. For example, it acquires space-time telegraph fraction equation of the equation classic telegraph by replacing the derivative terms of space-time from the partial derivatives.

The conformable telegraph equations have a wide variety of applications in science and engineering, ideally in optimizing propagation-oriented and propagating electrical communication systems [1, 2]. The solutions obtained from the telegraph equations of fraction of space-time are the terms Mittag-Leffler functions by mixing the Laplace transform and variation iteration method [3]. Recently, many new FDOs have been developed with the exponential and Mittag-Leffler kernels to analyze the systems and models with better memory characteristics. In this sequence, the latest work includes the investigations of the exothermic model

and vibration equation of fractional order with the inclusion of the power law and the Mittag-Leffler law by Kumar et al. [4, 5]. The researchers used the Laplace transform and the Elzaki transform to solve fractional differential equations, in addition to a system of partial differential equations [6, 7]. Caputo-Fabrizio and other fractional operators have been utilized to examine the solution behavior of fractional-order models. In the year 2018, Evirgen and Yavuz [8] investigated an alternative scheme for a nonlinear optimization problem containing the CF operator. With the utilization of the semi-analytical schemes HPSTM and HASTM, the solutions obtained via HPSTM are in good correspondence with the semianalytical solutions obtained by using HASTM [9]. This indicates that one can try to solve partial differential equations using the same techniques for conformable fractional derivatives (CFD) suggested by Khalil [10]. Analytical solutions have been extracted from this conformable Sumudu transform, and conformable fractional derivatives were solved to construct the new exact traveling wave solutions of the three special form of time fractional WKB equations

[11–13], such as the time fractional approximate long-wave equations, the time fractional variant Bossiness equations, and the time fractional Wuzhang system of equations using the generalized $\exp(\phi(\xi))$ expansion [14] method with a conformable derivative sense. They have presented a new cubic B-spline (CBS) approximation technique for the numerical treatment of coupled viscous Burgers' equations arising in the study of fluid dynamics, continuous stochastic processes, acoustic transmissions, and aerofoil flow theory [15]. A finite difference scheme which depends on a new approximation based on an extended cubic B-spline for the second-order derivative is used to calculate the numerical outcomes of time fractional Burgers Equation (22). We have discussed the new definition of double Sumudu transformation and new conformable fractional derivative for converting the nonlinear fractional differential equations into the ODEs [16]. The conformable fractional double Sumudu transform method was applied to solve the time-space of conformable fractional telegraph differential equations. Thus, the aim of this study is to suggest an analytical solution for the one singular conformable fractional telegraph equation by using the conformable double Sumudu decomposition method.

The methodology of this paper is as follows. In Section 2, we present a double definition of Sumudu transformation, its properties, a description of conformable fractional, and fractional space-time telegraph equations. In Section 3, we obtained the exact solutions of the conformable fractional space-time telegraph equations. The conclusion from this paper is in Section 4.

2. Conformable Double Sumudu Transform and Some Properties

Now, we take into account a few definitions and properties of the CDST (conformable double Sumudu transform) which may allow for the finding of even more transformed pairs $f((x^\beta/\beta), (t^\alpha/\alpha)), F(u, v)$ rather than having to consider the following.

Definition 1. Let us assume that $f : (0, \infty) \rightarrow \mathbb{R}$, then the CFD (conformable fractional derivative) of f having order β can be represented by

$$\frac{d^\beta}{dx^\beta} f\left(\frac{x^\beta}{\beta}\right) = \lim_{\epsilon \rightarrow 0} \frac{f(x^\beta/\beta + \epsilon x^{1-\beta}) - f(x^\beta/\beta)}{\epsilon}, \quad \frac{x^\beta}{\beta} > 0, 0 < \beta \leq 1. \quad (1)$$

See [4, 7, 17].

Definition 2 (see [5]). If the given function $f(x^\beta/\beta, t^\alpha/\alpha) : \mathbb{R} \times (0, \infty) \rightarrow \mathbb{R}$, then the CSFPD (Conformable Space Fractional Partial Derivative) with order β , a function $f(x^\beta/\beta, t^\alpha/\alpha)$, is represented by

$$\frac{\partial^\beta}{\partial x^\beta} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) = \lim_{\epsilon \rightarrow 0} \frac{f(x^\beta/\beta + \epsilon x^{1-\beta}, t^\alpha/\alpha) - f(x^\beta/\beta, t^\alpha/\alpha)}{\epsilon}, \quad \frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} > 0, 0 < \alpha, \beta \leq 1. \quad (2)$$

The CSFPD with order α a function $f(x^\beta/\beta, t^\alpha/\alpha)$ is represented by

$$\frac{\partial^\alpha}{\partial t^\alpha} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) = \lim_{\sigma \rightarrow 0} \frac{f(x^\beta/\beta, t^\alpha/\alpha + \sigma t^{1-\alpha}) - f(x^\beta/\beta, t^\alpha/\alpha)}{\sigma}, \quad \frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} > 0, 0 < \alpha, \beta \leq 1. \quad (3)$$

Definition 3. Let $f(x, t)$ be a piecewise continuous function on the given interval $[0, \infty) \times [0, \infty)$ having the exponential order. Consider for some, $\epsilon \in \mathbb{R}, \sup x^\beta/\beta, t^\alpha/\alpha > 0, e^{|f(x^\beta/\beta, t^\alpha/\alpha)|/a} x^\beta/\beta + b t^\alpha/\alpha$. According to these conditions, the CDST is represented by

$$\begin{aligned} S_t^\alpha S_x^\beta \left[f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] &= F(u, v) \\ &= \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-x^\beta/uv - t^\alpha/v} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) dt dx, \end{aligned} \quad (4)$$

where $u, v \in \mathbb{R}, 0 < \alpha, \beta \leq 1$ and the integrals are known to be the CFI (conformable fractional integral) with respect to x^β/β and t^α/α , respectively.

Definition 4 (single conformable Sumudu transform of a function with two variables). CST (conformable Sumudu transform) with respect to x^β/β of $f(x^\beta/\beta, t^\alpha/\alpha)$ can be represented as

$$S_x^\beta \left[f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = F(u, t) = \frac{1}{u} \int_0^\infty e^{-x^\beta/uv} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) dx, \quad (5)$$

where the integral is in the conformable sense with respect to x .

The expression $S_x^\beta[f(x^\beta/\beta, t^\alpha/\alpha)]$ reveals the conformable integral of (5); in the given expression, the subscript x^β/β on S indicates for which variable the CST can be applied.

Homogenously, the CST of the same function $f(x^\beta/\beta, t^\alpha/\alpha)$ with respect to variable t^α/α is defined as

$$S_t^\alpha \left[f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = F(x, v) = \frac{1}{v} \int_0^\infty e^{-t^\alpha/v} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) dt; \quad (6)$$

due to these definitions, the successive transformation is represented by $S_t^\alpha S_x^\beta[f(x^\beta/\beta, t^\alpha/\alpha)]$ presented in (4). If it is

assumed that the function $f(x^\beta/\beta, t^\alpha/\alpha)$ gives adequate constraints (Love 1970), the order of transformation can be altered, as

$$\begin{aligned} & \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-x^\beta/uv - t^\alpha/v\alpha} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) dt dx \\ &= \frac{1}{vu} \int_0^\infty \int_0^\infty e^{-t^\alpha/v\alpha - x^\beta/uv} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) dx dt, \end{aligned} \tag{7}$$

and symbolically, it can be represented as

$$S_x^\beta S_t^\alpha \left[f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = S_t^\alpha S_x^\beta \left[f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = F(u, v). \tag{8}$$

Theorem 5. Let us assume that the two functions $f(x^\beta/\beta, t^\alpha/\alpha)$, $g(x^\beta/\beta, t^\alpha/\alpha)$ have CDST, then

$$\begin{aligned} (1) \quad & S_x^\beta S_t^\alpha [k_1 f((x^\beta/\beta), (t^\alpha/\alpha)) + k_2 g((x^\beta/\beta), (t^\alpha/\alpha))] = k_1 \\ & S_x^\beta S_t^\alpha [f((x^\beta/\beta), (t^\alpha/\alpha))] + k_2 S_x^\beta S_t^\alpha [g((x^\beta/\beta), (t^\alpha/\alpha))], \end{aligned}$$

where k_1 and k_2 are constants

$$\begin{aligned} (2) \quad & S_x^\beta S_t^\alpha [f(\gamma(x^\beta/\beta), \sigma(t^\alpha/\alpha))] = (1/r uv) F((1/\gamma u), (1/\sigma v)), \\ & \text{where } r = \gamma^\beta \sigma^\alpha \\ (3) \quad & S_x^\beta S_t^\alpha [e^{-dx^\beta/\beta - ct^\alpha/\alpha} f((x^\beta/\beta), (t^\alpha/\alpha))] = (1/uv) F((1/u) + d, (1/v) + c) \\ (4) \quad & (-1)^{m+n} S_x^\beta S_t^\alpha [(x^{m\beta}/\beta^m)(t^{n\alpha}/\alpha^n) f((x^\beta/\beta), (t^\alpha/\alpha))] = 1/uv(\partial^{m+n}/\partial u^m \partial v^n) F((1/u), (1/v)) \end{aligned}$$

Lemma 6. The CDST of β th and α th order FPDs (Fractional Partial Derivatives) are defined as follows:

$$S_x^\beta S_t^\alpha \left[{}_x D_\beta f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = \frac{F(u, v)}{u} - \frac{F(0, v)}{u}, \tag{9}$$

$$S_x^\beta S_t^\alpha \left[{}_t D_\alpha f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = \frac{F(u, v)}{v} - \frac{F(u, 0)}{v}, \tag{10}$$

where ${}_x D_\beta f(x^\beta/\beta, t^\alpha/\alpha)$, ${}_t D_\alpha f(x^\beta/\beta, t^\alpha/\alpha)$ means the β th and α th order CFPDs (Conformable Fractional Partial Derivatives), respectively. Similarly, the CDST of mixed FPDs can be expressed.

Theorem 7. Let $0 < \alpha, \beta \leq 1$ and $m, n \in \mathbb{N}$ such that $f(x^\beta/\beta, t^\alpha/\alpha) \in C^l(\mathbb{R}^+, \mathbb{R}^+)$, $l = \max(m, n)$. Also, assume that the CLT (Conformable Laplace Transforms) of the given functions ${}_x D_\beta^{(i)} f(x^\beta/\beta, t^\alpha/\alpha)$ and ${}_t D_\alpha^{(j)} f(x^\beta/\beta, t^\alpha/\alpha)$, $i = 1, \dots, m, j = 1, \dots, n$ exist. Then,

$$S_x^\beta S_t^\alpha \left[{}_x D_\beta^{(m)} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = \frac{F(u, v)}{u^m} - \frac{F(0, v)}{u^m} - \sum_{i=1}^{m-1} \frac{S_t^\alpha [{}_x D_\beta^{(i)} f(0, t)]}{u^{m-i}},$$

$$S_x^\beta S_t^\alpha \left[{}_t D_\alpha^{(n)} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] = \frac{F(u, v)}{v^n} - \frac{F(u, 0)}{v^n} - \sum_{j=1}^{n-1} \frac{S_x^\beta [{}_t D_\alpha^{(j)} f(x, 0)]}{v^{n-j}}. \tag{11}$$

Similarly, the CDLT (Conformable Double Laplace Transform) of mixed partial derivative

$$\begin{aligned} S_x^\beta S_t^\alpha \left[{}_x D_\beta^{(m)} {}_t D_\alpha^{(n)} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \right] &= \frac{1}{u^m v^n} \left[F(u, v) - F(u, 0) - F(0, v) \right. \\ &\quad \left. - \sum_{j=1}^{f(x,t)n-1} v^j S_x^\beta [{}_t D_\alpha^{(j)} f(x, 0)] - \sum_{i=1}^{m-1} u^i S_t^\alpha [{}_x D_\beta^{(i)} f(0, t)] \right] \\ &\quad + \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} v^j u^i {}_t D_\alpha^{(j)} {}_x D_\beta^{(i)} f(0, 0) + u^m v^n F(0, 0), \end{aligned} \tag{12}$$

where ${}_x D_\beta^{(m)}$ and ${}_t D_\alpha^{(n)}$ represent m, n times conformable fractional derivatives of function $f(x^\beta/\beta, t^\alpha/\alpha)$ having order β and α , respectively.

3. Conformable Double Sumudu Transform

Now, we will study the following general time-space conformable fractional telegraph equation:

$$\begin{aligned} \frac{\partial^\beta}{\partial x^\beta} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) &= a \frac{\partial^\alpha}{\partial t^\alpha} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) + b \frac{\partial^\gamma}{\partial t^\gamma} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \\ &\quad + c f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) + h\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right), \end{aligned} \tag{13}$$

$$1 < \alpha, \beta < 2, 0 < \gamma < 1, \frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} > 0, \tag{14}$$

having initial condition:

$$f\left(\frac{x^\beta}{\beta}, 0\right) = k_1 \left(\frac{x^\beta}{\beta}\right), {}_t D_\alpha f\left(\frac{x^\beta}{\beta}, 0\right) = k_2 \left(\frac{x^\beta}{\beta}\right), \tag{15}$$

and boundary:

$$f\left(0, \frac{t^\alpha}{\alpha}\right) = r_1 \left(\frac{t^\alpha}{\alpha}\right), {}_x D_\beta f\left(0, \frac{t^\alpha}{\alpha}\right) = r_2 \left(\frac{t^\alpha}{\alpha}\right), \tag{16}$$

where $h(x^\beta/\beta, t^\alpha/\alpha)$ is the given function and a, b, c are constants.

By taking conformable double Sumudu transform for bout aids of Equation (13),

$$\begin{aligned} & \frac{1}{u^2} F(u, v) - \frac{F(0, v)}{u^2} - \frac{1}{u} S_t^\alpha \left[{}_x D_\beta f \left(0, \frac{t^\alpha}{\alpha} \right) \right] \\ &= a \left[\frac{1}{v^2} F(u, v) - \frac{F(u, 0)}{v^2} - \frac{1}{v} S_x^\beta \left[{}_t D_\alpha f \left(\frac{x^\beta}{\beta}, 0 \right) \right] \right] \\ &+ b \left[\frac{F(u, v)}{v} - \frac{F(u, 0)}{v} \right] + cF(u, v) + \acute{h}(u, v). \end{aligned} \tag{17}$$

Secondly, by using the conformable single Sumudu transform from Equations (15) and (16), one can obtain

$$\left. \begin{aligned} F(0, v) &= K_1(u), {}_t D_\alpha F(0, v) = K_2(u) \\ F(u, 0) &= R_1(v), {}_x D_\beta F(u, 0) = R_2(v) \end{aligned} \right\}. \tag{18}$$

By replacing (9) in (8) and by direct calculation, one can obtain

$$\begin{aligned} F(u, v) - \frac{\acute{B}_1(v)}{u^2} - \frac{1}{u} \left[\acute{B}_2(v) \right] &= a \left[\frac{1}{v^2} F(u, v) - \frac{\acute{f}_1(u)}{v^2} \right. \\ &- \frac{1}{v} \left[\acute{f}_2(u) \right] \left. \right] + b \left[\frac{F(u, v)}{v} - \frac{\acute{f}_1(u)}{v} \right] + cF(u, v) \\ &+ \acute{h}(u, v), \left[\frac{1}{u^2} - \frac{a}{v^2} - \frac{b}{v} - c \right] F(u, v) \\ &= \frac{\acute{B}_1(v)}{u^2} + \frac{\acute{B}_2(v)}{u} - \frac{a\acute{f}_1(u)}{v^2} - \frac{a\acute{f}_2(u)}{v} - \frac{b\acute{f}_1(u)}{v} \\ &+ \acute{h}(u, v), F(u, v) = v^2 \acute{B}_1(v) + uv^2 \acute{B}_2(v) - au^2 \acute{f}_1(u) \\ &- au^2 v \acute{f}_2(u) - bu^2 v \frac{\acute{f}_1(u) + u^2 v^2 \acute{h}(u, v)}{v^2 - au^2 - bu^2 v - cu^2 v^2}. \end{aligned} \tag{19}$$

Applying the inverse, CDST.

Example 8. Consider the homogenous fractional telegraph equation given by [3, 18]:

$${}_x D_\beta^{(2)} f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) - {}_t D_\alpha^{(2)} f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) - {}_t D_\alpha f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) - f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) = 0, \tag{20}$$

with the conditions

$$\left. \begin{aligned} f \left(0, \frac{t^\alpha}{\alpha} \right) &= e^{-t^\alpha/\alpha}, f \left(\frac{x^\beta}{\beta}, 0 \right) = e^{x^\beta/\beta} \\ {}_x D_\beta f \left(0, \frac{t^\alpha}{\alpha} \right) &= e^{-t^\alpha/\alpha}, {}_t D_\alpha f \left(\frac{x^\beta}{\beta}, 0 \right) = -e^{x^\beta/\beta} \end{aligned} \right\}, \tag{21}$$

where ${}_x D_\beta^{(2)}$ and ${}_t D_\alpha^{(2)}$ mean two times conformable fractional derivative of function $f(x^\beta/\beta, t^\alpha/\alpha)$. Applying conformable double Sumudu transform for () yields

$$\begin{aligned} & \frac{1}{u^2} F(u, v) - \frac{F(0, v)}{u^2} - \frac{1}{u} {}_x D_\beta f(0, v) + \frac{1}{v^2} F(u, v) + \frac{F(u, 0)}{v^2} \\ &- \frac{1}{v} {}_t D_\alpha f(u, 0) - \frac{F(u, v)}{v} + \frac{F(u, 0)}{v} - F(u, v) = 0. \end{aligned} \tag{22}$$

Applying conformable single Sumudu transform to conditions expressed in Equation (22) yields

$$\left. \begin{aligned} F(0, v) &= \frac{1}{1+v}, F(u, 0) = \frac{1}{1-u} \\ {}_x D_\beta F(0, v) &= \frac{1}{1+v}, {}_t D_\alpha F(u, 0) = \frac{-1}{1-u} \end{aligned} \right\}. \tag{23}$$

Making some algebraic manipulations in Equation (22), we have

$$F(u, v) = \frac{1}{(1-u)(1+v)}. \tag{24}$$

Thus, we get the solution of Equation (22) as (see Figure 1)

$$f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) = e^{x^\beta/\beta - t^\alpha/\alpha}. \tag{25}$$

Example 9. Regard nonhomogenous space-time fractional telegraph equation stated as [17]

$$\begin{aligned} & {}_x D_\beta^{(2)} f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) - {}_t D_\alpha^{(2)} f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) - {}_t D_\alpha f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) \\ &- f \left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha} \right) = -2 e^{x^\beta/\beta + t^\alpha/\alpha}, \end{aligned} \tag{26}$$

with the constraints

$$\left. \begin{aligned} f \left(0, \frac{t^\alpha}{\alpha} \right) &= e^{t^\alpha/\alpha}, f \left(\frac{x^\beta}{\beta}, 0 \right) = e^{x^\beta/\beta} \\ {}_x D_\beta f \left(0, \frac{t^\alpha}{\alpha} \right) &= e^{t^\alpha/\alpha}, {}_t D_\alpha f \left(\frac{x^\beta}{\beta}, 0 \right) = e^{x^\beta/\beta} \end{aligned} \right\}, \tag{27}$$

where ${}_x D_\beta^{(2)}$ and ${}_t D_\alpha^{(2)}$ mean two times conformable fractional derivative of function $f(x^\beta/\beta, t^\alpha/\alpha)$. Applying conformable double Sumudu transform for () yields

$$\begin{aligned} & \frac{1}{u^2} F(u, v) - \frac{F(0, v)}{u^2} - \frac{1}{u} {}_x D_\beta f(0f, v) + \frac{1}{v^2} F(u, v) \\ &+ \frac{F(u, 0)}{v^2} - \frac{1}{v} {}_t D_\alpha f(u, 0) - \frac{F(u, v)}{v} + \frac{F(u, 0)}{v} \\ &- F(u, v) = \frac{-2}{(1-u)(1-v)}. \end{aligned} \tag{28}$$

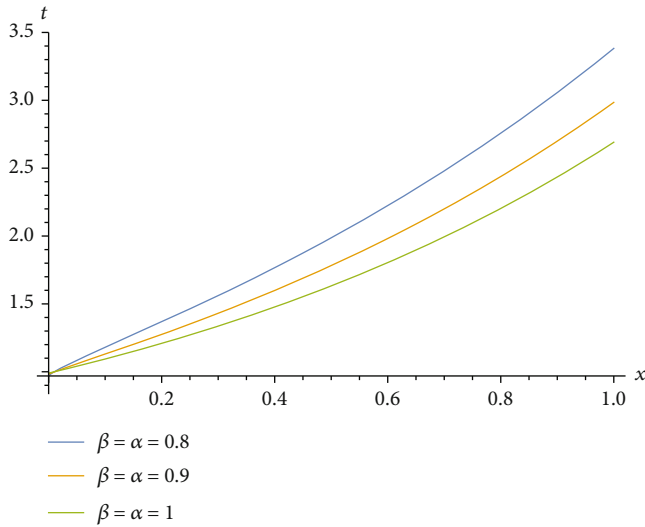


FIGURE 1: The figure shows the solution of (20), when $t = 0.01, 0 \leq x \leq 1$.

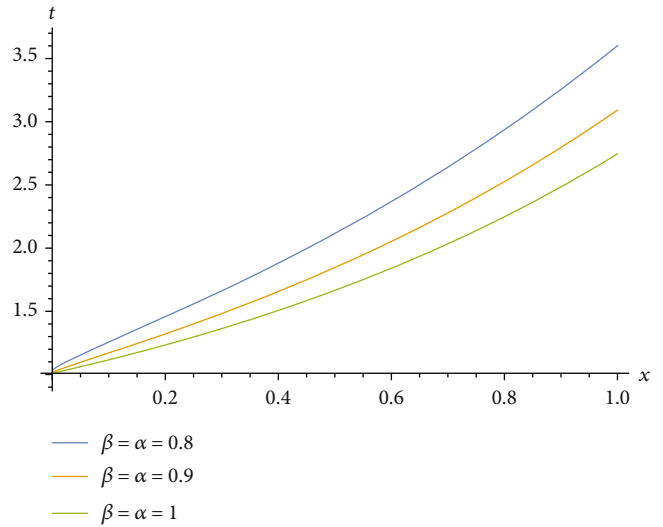


FIGURE 2: The figure shows the solution of (26), when $t = 0.01, 0 \leq x \leq 1$.

Again, employing conformable single Sumudu transform to constraints stated in Equation (28), one can obtain

$$\left. \begin{aligned} F(0, v) = \frac{1}{1-v}, F(u, 0) = \frac{1}{1-u} \\ {}_x D_\beta F(0, v) = \frac{1}{1-v}, {}_t D_\alpha F(u, 0) = \frac{1}{1-u} \end{aligned} \right\}; \quad (29)$$

after some mathematical computation in Equation (28), we get

$$F(u, v) = \frac{1}{(1-u)(1-v)}. \quad (30)$$

So, we are going to get the solution of Equation (28) as (see Figure 2)

$$f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) = e^{x^\beta/\beta + t^\alpha/\alpha}. \quad (31)$$

Example 10. Let us use the telegraph equation [(13)]:

$$\begin{aligned} {}_x D_\beta^{(2)} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) &= {}_t D_\alpha^{(2)} f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) + 4 {}_t D_\alpha f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) \\ &+ 4 f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right), \end{aligned} \quad (32)$$

with the constraints

$$\left. \begin{aligned} f\left(0, \frac{t^\alpha}{\alpha}\right) = 1 + e^{-2t^\alpha/\alpha}, f\left(\frac{x^\beta}{\beta}, 0\right) = 1 + e^{2x^\beta/\beta} \\ {}_x D_\beta f\left(0, \frac{t^\alpha}{\alpha}\right) = 2, {}_t D_\alpha f\left(\frac{x^\beta}{\beta}, 0\right) = -2 \end{aligned} \right\}, \quad (33)$$

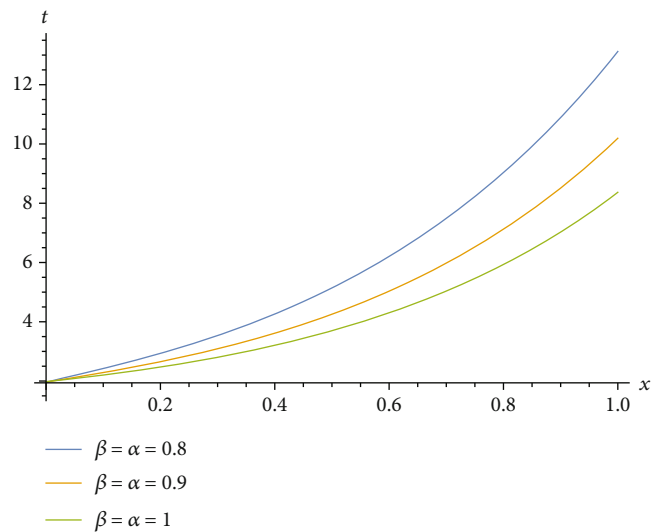


FIGURE 3: The figure shows the solution of (26), when $t = 0.01, 0 \leq x \leq 1$.

where ${}_x D_\beta^{(2)}$ and ${}_t D_\alpha^{(2)}$ mean two times conformable fractional derivative of function $f(x^\beta/\beta, t^\alpha/\alpha)$. Applying conformable double Sumudu transform for () produces

$$\begin{aligned} \frac{1}{u^2} F(u, v) - \frac{F(0, v)}{u^2} - \frac{1}{u} {}_x D_\beta f(0f, v) &= \frac{1}{v^2} F(u, v) - \frac{F(u, 0)}{v^2} \\ - \frac{1}{v} {}_t D_\alpha f(u, 0) + \frac{4F(u, v)}{v} - \frac{4F(u, 0)}{v} &+ 4F(u, v). \end{aligned} \quad (34)$$

Again, employing conformable single Sumudu transform to constraints stated in Equation (34) yields

$$\left. \begin{aligned} F(0, v) = 1 + \frac{1}{1+2v}, F(u, 0) = 1 + \frac{1}{1-2u} \\ {}_x D_\beta F(0, v) = 2, {}_t D_\alpha F(u, 0) = -2 \end{aligned} \right\}; \quad (35)$$

after some mathematical computation in Equation (34), one can obtain

$$F(u, v) = \frac{1}{(1-2u)} + \frac{1}{1+2v}. \quad (36)$$

So, we are going to get the solution of Equation (34) (see Figure 3):

$$f\left(\frac{x^\beta}{\beta}, \frac{t^\alpha}{\alpha}\right) = e^{2x^\beta/\beta} + e^{-2t^\alpha/\alpha}. \quad (37)$$

4. Conclusion

The primary goal is the successful utilization of double Sumudu transforms to the conformable fractional-order and interesting and linear space-time conformable fractional telegraph equations. We have discussed the definition of double Sumudu transformation and new conformable fractional derivative for converting the linear fractional differential equations into the ODEs over time; we presented the general analytical solution of a linear space-time conformable fractional telegraph equations conformable fractional double Sumudu transform. Also, our suggested method is applied successfully for obtaining the general solutions of several linear and nonhomogeneous conformable fractional telegraph equations. Finally, the effects show that our resolved method is efficient and can be applied for finding the general solutions of all cases related to the conformable fractional differential equations.

Data Availability

The results of this study are available from authors upon request of supporting data. Contact author for data requests.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] A. Sevimlican, "An Approximation to Solution of Space and Time Fractional Telegraph Equations by He's Variational Iteration Method," *Mathematical Problems in Engineering*, vol. 2010, Article ID 290631, 10 pages, 2010.
- [2] M. Garg, P. Manohar, and S. L. Kalla, "Generalized Differential Transform Method to Space-Time Fractional Telegraph Equation," *International Journal of Differential Equations*, vol. 2011, Article ID 548982, 9 pages, 2011.
- [3] F. A. Alawad, E. A. Yousif, and A. I. Arbab, "A New Technique of Laplace Variational Iteration Method for Solving Space-Time Fractional Telegraph Equations," *International Journal of Differential Equations*, vol. 2013, Article ID 256593, 10 pages, 2013.
- [4] D. Kumar, J. Singh, K. Tanwar, and D. Baleanu, "A new fractional exothermic reactions model having constant heat source in porous media with power, exponential and Mittag-Leffler laws," *International Journal of Heat and Mass Transfer*, vol. 138, pp. 1222–1227, 2019.
- [5] D. Kumar, J. Singh, and D. Baleanu, "On the analysis of vibration equation involving a fractional derivative with Mittag-Leffler law," *Mathematical Methods in the Applied Sciences*, vol. 43, no. 1, pp. 443–457, 2019.
- [6] M. S. Hashemi, "Invariant subspaces admitted by fractional differential equations with conformable derivatives," *Chaos, Solitons and Fractals*, vol. 107, pp. 161–169, 2018.
- [7] M. Z. Mohamed and T. M. Elzaki, "Applications of new integral transform for linear and nonlinear fractional partial differential equations," *Journal of King Saud University - Science*, vol. 32, no. 1, pp. 544–549, 2018.
- [8] F. Evirgen and M. Yavuz, "An alternative approach for nonlinear optimization problem with Caputo - Fabrizio derivative," *ITM Web of Conferences*, vol. 22, article 01009, 2018.
- [9] H. M. Srivastava, V. P. Dubey, R. Kumar, J. Singh, D. Kumar, and D. Baleanu, "An efficient computational approach for a fractional-order biological population model with carrying capacity," *Chaos, Solitons & Fractals*, vol. 138, article 109880, 2020.
- [10] D. Zhao and M. Luo, "General conformable fractional derivative and its physical interpretation," *Calcolo*, vol. 54, no. 3, pp. 903–917, 2017.
- [11] F. S. Silva, D. M. Moreira, and M. A. Moret, "Conformable Laplace transform of fractional differential equations," *Axioms*, vol. 7, no. 3, p. 55, 2018.
- [12] T. M. Elzaki and E. M. A. Hilal, "Analytical solution for telegraph equation by modified Sumudu transform "Elzaki Transform"," *Mathematics Theory and Modeling*, vol. 2, no. 4, pp. 104–112, 2012.
- [13] T. Akram, M. Abbas, M. B. Riaz, A. I. Ismail, and N. M. Ali, "An efficient numerical technique for solving time fractional Burgers equation," *Alexandria Engineering Journal*, vol. 59, no. 4, pp. 2201–2220, 2020.
- [14] D. Kumar, M. Kaplan, M. R. Haque, M. S. Osman, and D. Baleanu, "A variety of novel exact solutions for different models with the conformable derivative in shallow water," *Frontiers in Physics*, vol. 8, 2020.
- [15] T. Nazir, M. Abbas, and M. K. Iqbal, "New cubic B-spline approximation technique for numerical solutions of coupled viscous Burgers equations," *Engineering Computations*, vol. 38, no. 1, pp. 83–106, 2020.
- [16] A. Akbulut and M. Kaplan, "Auxiliary equation method for time-fractional differential equations with conformable derivative," *Computers & Mathematics with Applications*, vol. 75, no. 3, pp. 876–882, 2017.
- [17] H. Eltayeb and A. Kiliçman, "A note on double Laplace transform and telegraphic equations," *Abstract and Applied Analysis*, vol. 2013, Article ID 932578, 6 pages, 2013.
- [18] A. M. Wazwaz, *Partial Differential Equations and Solitary Waves Theory, Nonlinear Physical Science, Higher Education Press, Beijing, China, Springer, Heidelberg, Germany*, 2009.