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Research Article

The Marichev-Saigo-Maeda Fractional Calculus Operators Pertaining to the V-Function

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In the present paper, we establish some composition formulas for Marichev-Saigo-Maeda (MSM) fractional calculus operators with V-function as the kernel. In addition, on account of V-function, a variety of known results associated with special functions such as the Mittag-Leffler function, exponential function, Struve's function, Lommel's function, the Bessel function, Wright's generalized Bessel function, and the generalized hypergeometric function have been discovered by defining suitable values for the parameters.

1. Introduction and Preliminaries

Fractional calculus is very old and similar to conventional calculus that has gradually been applied to a variety of fields including technology, science, economics, arithmetic geometry, and computer science. The V-functions are especially valuable functions that provide solutions to a variety of problems developed in terms of fractional order differential, integral, and difference equations. As a result of that, this field has become an area of interest for researchers in recent times. In addition, a majority of scholars (see [1–3]) have investigated the properties, uses, and various extensions of a variety of fractional calculus operators in depth. Also, on other similar themes, there is a lot of activity and coverage all over the world. The research monographs [4, 5] can be consulted.

Kumar [6] has recently characterized the V-function as follows:

$$V(z) = V_n^{a_u,h,b_v}(l,\mu,\zeta,\delta,m,k_u,A_v,B_w,\eta,\nu,\alpha\,;z) = \xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}(z/2)^{n\zeta+h\delta+m}}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\alpha+B_w}\right]}, \quad (1)$$

where

(1) $l,\zeta,\delta,m,\nu,\alpha,k_u(u=1,\cdots,p), A_v(v=1,\cdots,q), B_w(w=1,\cdots,r)$ are real numbers

(2) p, q, and r are natural numbers,

$$a_u, b_v \geq 1 (u=1,\cdots,p\,; v=1,\cdots,q) \quad (2)$$

(3) $\eta>0$, $\Re(\mu)>0$, $\Re(h)>0$, z is a complex variable and ξ is an arbitrary constant

(4) The series on the RHS of (1) converges absolutely if $p<q$ or $p=q$ with $|l(z/2)^{\zeta}|\leq 1$

For descriptions of the series convergence constraints on (1) RHS, simply review [7–10]. The V-function defined by (1) is of a general character since it assimilates and applies a variety of valuable functions such as Macrobert's E-function and the exponential function [11], the generalized Mittag-Leffler function [12–15], Lommel's function [16], Struve's function [17–21], the Wright generalized Bessel function and the Bessel function [22–25], the generalized hypergeometric function [11, 26], and the unified Riemann-Zeta function [27].

Special cases of (1) are as follows:

(i) If we put $w = 1, h = 1, p = P, q = Q, l = -2, \mu = 1, \zeta = 1$, $\delta = 0, m = 0, k_u = 0, A_v = 0, B_1 = -1, \eta = 1, \nu = -1, \alpha = 1$, and $\xi = 1$ in equation (1) then the V-function turns into the generalized hypergeometric function [11]:

$$V_n^{a_u,1,b_v}(-2, 1, 1, 0, 0, 0, 0, -1, 1, -1, 1 ; z) = {}_PF_Q\left(a_P ; b_Q ; z\right) \quad (3)$$

(ii) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = 1, b_2 = 1, l = 1, \mu = 1, \zeta = 2, \delta = 1, m = 0, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = 0, \eta = 1, \nu = 0, \alpha = 1$, and $\xi = 1/(\Gamma(h))$ in equation (1) then the V-function turns into the Bessel function [23]:

$$V_n^{1,h,1,1}(1, 1, 2, 1, 0, 0, 0, 0, 0, 1, 0, 1 ; z) = J_h(z) \quad (4)$$

(iii) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = 1, b_2 = 1, l = 2, \mu = 1, \zeta = 1, \delta = 1, m = 0, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = 0, \nu = 0, \alpha = 1$, and $\xi = 1/(\Gamma(h))$ in equation (1) then the V-function turns into Wright's generalized Bessel function [23]:

$$V_n^{1,h,1,1}(1, 1, 2, 1, 0, 0, 0, 0, 0, \eta, 0, 1 ; z) = J_h^{\eta}(z) \quad (5)$$

(iv) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = 3/2, b_2 = 1, l = 1, \mu = 1, \zeta = 2, \delta = 1, m = 1, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = 1/2, \eta = 1, \nu = 1/2, \alpha = 1$, and $\xi = 1/\Gamma(h)\Gamma(3/2)$ in equation (1) then the V-function turns into Struve's function [23]:

$$V_n^{1,h,3/2,1}(1, 1, 2, 1, 1, 0, 0, 0, 1/2, 1, 1/2, 1 ; z) = H_h(z) \quad (6)$$

(v) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = (\theta + \varepsilon + 3)/2$, $b_2 = (\theta - \varepsilon + 3)/2, l = 1, \mu = 1, \zeta = 2, h = 1, \delta = \theta, m = 1, k_1 = 0, A_1 = 0, A_2 = 0, B_1 = -1, \eta = 1, \nu = -1, \alpha = 1$, and $\xi = 2^{\theta+1}/\{(\theta + \varepsilon + 1)(\theta - \varepsilon + 1)\}$ in equation (1) then the V-function turns into Lommel's function [23]:

$$V_n^{1,1,(\theta+\varepsilon+3)/2,(\theta-\varepsilon+3)/2}(1, 1, 2, \theta, 1, 0, 0, 0, -1, 1, 1, 1 ; z) = S_{\theta,\varepsilon}(z) \quad (7)$$

(vi) If we put $u = 1, v = 1, w = 1, a_1 = 1, b_1 = 1, l = -2, \mu = 1, \zeta = 1, \delta = 0, m = 0, k_1 = 0, A_1 = 0, B_1 = -1, \nu = -1, \alpha = 1$, and $\xi = 1/(\Gamma(h))$ in equation (1) then the V-function turns into Mittag-Leffler function [12, 13]:

$$V_n^{1,h,1}(-2, 1, 1, 0, 0, 0, 0, -1, \eta, -1, 1 ; z) = E_{\eta,h}(z) \quad (8)$$

(vii) If we put $u = 1, v = 1, w = 1, a_1 = a, b_1 = 1, l = -2, \zeta = 1, \delta = 0, m = 0, k_1 = 0, A_1 = 0, B_1 = 0, \eta = 1, \nu = 0, \alpha = 0$, and $\xi = 1$ in equation (1) then the V-function turns into unified Riemann-Zeta function [27]:

$$V_n^{a,h,1}(-2, \mu, 1, 0, 0, 0, 0, 0, 1, 0, 0 ; z) = \phi_a(z, \mu, h) \quad (9)$$

(viii) If we put $u = 1, v = 1, w = 1, a_1 = 1, b_1 = 1, l = 2, \zeta = 1, \delta = 0, m = 0, k_1 = 0, A_1 = 0, B_1 = -1, \eta = 1, \nu = -1, \alpha = 1, h = 1, \mu = 1$, and $\xi = 1$ in equation (1) then the V-function turns into the e^{-z} function:

$$V_n^{1,1,1}(2, 1, 1, 0, 0, 0, 0, -1, 1, -1, 1 ; z) = e^{-z} \quad (10)$$

(ix) If we put $w = 1, p = P, q = Q, l = 2, \mu = 1, \zeta = 1, \delta = 0$, $m = 0, k_u = 0, A_u = 0, B_1 = -1, \eta = 1, \nu = -1, \alpha = 1, h = 1$, and $\xi = \prod_{u=1}^{P}\Gamma(a_u)/\prod_{v=1}^{Q}\Gamma(b_v)$ in equation (1) then the V-function turns into Macrobert's E-function [11]:

$$V_n^{1,1,1}(2, 1, 1, 0, 0, 1, 0, -1, 1, -1, 1 ; z) = E\left[P ; (a_P) ; Q ; \left(b_Q\right) ; z^{-1}\right] \quad (11)$$

(x) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = 1, k_1 = 0, h = 1/2, l = 1, \mu = 1, \zeta = 2, \delta = 0, m = 0, A_1 = 0, A_2 = -1, B_1 = 0, \eta = 1, \nu = -1/2, \alpha = 1$ and $\xi = 1$ in equation (1) then the V-function turns into the $\cos z$ function:

$$V_n^{1,1/,1,1}(1, 1, 2, 0, 0, 0, 0, -1, 0, 1, -1/2, 1 ; z) = \cos z \quad (12)$$

(xi) If we put $u = 1, v = 2, w = 1, a_1 = 1, b_1 = 1, k_1 = 0, h = 1/2, l = 1, \mu = 1, \zeta = 2, \delta = 2, m = 0, A_1 = 0, A_2 = -1$, $B_1 = 0, \eta = 1, \nu = -1/2, \alpha = 1$, and $\xi = 1$ in equation (1) then the V-function turns into the $\sin z$ function:

$$V_n^{1,1/2,1,1}(1, 1, 2, 2, 0, 0, 0, -1, 0, 1, -1/2, 1 ; z) = sinz \quad (13)$$

1.1. Saigo Fractional Calculus Operators. The fractional integral and differential operators with the Gauss hypergeometric function as the kernel are characterized by Saigo [28], and they are notable generalizations of the Riemann-Liouville (R-L) and Erdélyi-Kober fractional calculus operators (see [1]).

For $\wp, \varepsilon, \gamma \in \mathbb{C}$ and $x \in \mathbb{R}^+$ with $\mathfrak{R}(\wp) > 0$, the left-hand and the right-hand sided generalized fractional integral operators connected with Gauss hypergeometric function are defined as below:

$$\left(I_{0+}^{\wp,\varepsilon,\gamma} f\right)(x) = \frac{x^{-\wp-\varepsilon}}{\Gamma(\wp)} \int_0^x (x-t)^{\wp-1} {}_2F_1\left(\wp+\varepsilon,-\gamma;\wp;1-\frac{t}{x}\right) f(t)dt, \tag{14}$$

$$\left(I_{-}^{\wp,\varepsilon,\gamma} f\right)(x) = \frac{1}{\Gamma(\wp)} \int_x^\infty \frac{(t-x)^{\wp-1}}{t^{\wp+\varepsilon}} {}_2F_1\left(\wp+\varepsilon,-\gamma;\wp;1-\frac{x}{t}\right) f(t)dt, \tag{15}$$

respectively. Here, ${}_2F_1(\wp,\varepsilon;\gamma;z)$ is the Gauss hypergeometric function [1] defined for $z \in \mathbb{C}$, $|z| < 1$ and $\wp, \varepsilon \in \mathbb{C}$, $\gamma \in \mathbb{C} \setminus \mathbb{Z}_0^-$ by

$$ {}_2F_1(\wp,\varepsilon;\gamma;z) = \sum_{n=0}^{\infty} \frac{(\wp)_n (\varepsilon)_n}{(\gamma)_n} \frac{z^n}{n!}, \tag{16}$$

where $(z)_n = (z)_{n,1}$. The corresponding fractional differential operators are

$$\left(D_{0+}^{\wp,\varepsilon,\gamma} f\right)(x) = \left(\frac{d}{dx}\right)^k \left(I_{0+}^{-\wp+k,-\varepsilon-k,\wp+\gamma-k} f\right)(x), \tag{17}$$

$$\left(D_{-}^{\wp,\varepsilon,\gamma} f\right)(x) = \left(-\frac{d}{dx}\right)^k \left(I_{-}^{-\wp+k,-\varepsilon-k,\wp+\gamma} f\right)(x), \tag{18}$$

where $k = [\mathfrak{R}(\wp)] + 1$ and $[\mathfrak{R}(\wp)]$ is the integer part of $\mathfrak{R}(\wp)$. Substituting $\varepsilon = -\wp$ and $\varepsilon = 0$ in equations (14)–(17), we get the corresponding R-L and Erdélyi-Kober fractional operators, respectively.

1.2. MSM Fractional Calculus Operators. Marichev [29] introduced and researched fractional calculus operators, which are an extension of the Saigo operators, which were later expanded by Saigo and Maeda [30]. For $\wp, \wp', \varepsilon, \varepsilon', \gamma \in \mathbb{C}$ and $x \in \mathbb{R}^+$ with $\mathfrak{R}(\gamma) > 0$, the left-hand and right-hand sided MSM fractional integral and derivative operators associated with third Appell function F_3 are defined as

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} f\right)(x) = \frac{x^{-\wp}}{\Gamma(\gamma)} \int_0^x \frac{(x-t)^{\gamma-1}}{t^{\wp'}} F_3\left(\wp,\wp',\varepsilon,\varepsilon',\gamma;1-\frac{t}{x},1-\frac{x}{t}\right) f(t)dt, \tag{19}$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} f\right)(x) = \frac{x^{-\wp'}}{\Gamma(\gamma)} \int_x^\infty \frac{(t-x)^{\gamma-1}}{t^{\wp}} F_3\left(\wp,\wp',\varepsilon,\varepsilon',\gamma;1-\frac{x}{t},1-\frac{t}{x}\right) f(t)dt, \tag{20}$$

$$\left(D_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} f\right)(x) = \left(\frac{d}{dx}\right)^q \left(I_{0+}^{-\wp',-\wp,-\varepsilon'+q,-\varepsilon,-\gamma+q} f\right)(x), \tag{21}$$

$$\left(D_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} f\right)(x) = \left(-\frac{d}{dx}\right)^q \left(I_{-}^{-\wp',-\wp,-\varepsilon',-\varepsilon+q,-\gamma+q} f\right)(x), \tag{22}$$

respectively, where $q = [\mathfrak{R}(\gamma)] + 1$, and the third Appell function [29] is defined by

$$F_3\left(\wp,\wp',\varepsilon,\varepsilon',\gamma;x,y\right) = \sum_{m,n=0}^{\infty} \frac{(\wp)_m \left(\wp'\right)_n (\varepsilon)_m \left(\varepsilon'\right)_n}{(\gamma)_{m+n}} \frac{x^m y^n}{m!n!}, \max \cdot \{|x|,|y|\} < 1. \tag{23}$$

The following MSM integral operators are required here [[30], p. 394] to obtain the MSM fractional integration of the generalized V-function.

Lemma 1. *Let $\wp, \wp', \varepsilon, \varepsilon', \gamma, \rho \in \mathbb{C}$ such that $\mathfrak{R}(\wp) > 0$*

(i) $\mathfrak{R}(\rho) > \max\{0, \mathfrak{R}(\wp' - \varepsilon'), \mathfrak{R}(\wp+\wp' + \varepsilon - \gamma)\}$, *then*

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} t^{\rho-1}\right)(x) = \frac{\Gamma(\rho)\Gamma\left(-\wp' + \varepsilon' + \rho\right)\Gamma\left(-\wp-\wp' - \varepsilon + \gamma + \rho\right)}{\Gamma\left(\varepsilon' + \rho\right)\Gamma\left(-\wp-\wp' + \gamma + \rho\right)\Gamma\left(-\wp' - \varepsilon + \gamma + \rho\right)} x^{-\wp-\wp'+\gamma+\rho-1}. \tag{24}$$

(i) If $\mathfrak{R}(\rho) > \max\{\mathfrak{R}(\varepsilon), \mathfrak{R}(-\wp-\wp' + \gamma), \mathrm{Re}\,(-\wp-\varepsilon' + \gamma)\}$, *then*

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma} t^{-\rho}\right)(x) = \frac{\Gamma(-\varepsilon+\rho)\Gamma\left(\wp+\wp' - \gamma + \rho\right)\Gamma\left(\wp+\varepsilon' - \gamma + \rho\right)}{\Gamma(\rho)\Gamma(\wp-\varepsilon+\rho)\Gamma\left(\wp+\wp' + \varepsilon' - \gamma + \rho\right)} x^{-\wp-\wp'+\gamma-\rho} \tag{25}$$

Fractional integral formulas for the V-function are given by [10]. In order to provide unification and extension of such kinds of results on fractional integrals of special functions. The authors derive two unified fractional integrals involving the product of V-function [6] and Appell function [29]. The integrals are further used in establishing two theorems on Saigo-Maeda operators of fractional integration [30].

In this paper, our aim is to study the compositions of the generalized fractional integration operators (18) and (19) with the V-function (1). At least, we establish them as special cases on the main result in connection with various special functions.

2. Fractional Calculus Approach

In this section, we establish image formulas for the V-function involving the left and right sides of the MSM fractional integral operators. These formulas are given by the following theorems.

Theorem 2. *Let* $\wp, \wp', \varepsilon, \varepsilon', \gamma, \rho \in \mathbb{C}$; $l, \zeta, \wp, m, \nu, \alpha, k_u, A_v, B_w \in \Re$; $p, q, r \in \mathbb{N}$; $a_u, b_v \geq 1$, $\eta > 0, \Re(\mu) > 0, \Re(h) > 0, \Re(\sigma) > 0$, *and* $\xi > 0$ *is an arbitrary constant, such that* $\Re(\gamma) > 0$, $\Re(\rho) > \max\{0, \Re(\wp' - \varepsilon'), \Re(\wp + \wp' + \varepsilon - \gamma)\}$, *then for* $t > 0$ *following integral holds true:*

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}V_n^{a_u,h,b_v}(l,\mu,\zeta,\delta,m,k_u,A_v,B_w,\eta,\nu,\alpha;t^{\sigma})\right)\right)(x) = x^{\rho-\wp-\wp'+\gamma-1}$$
$$\times \xi \sum_{n=0}^{\infty} \frac{(-l)^n \prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}(x^{\sigma}/2)^{n\zeta+h\delta+m}\Gamma(\rho+\sigma(n\zeta+h\delta+m))}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\rho+B_w}\right]\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\varepsilon'\right)}$$
$$\times \frac{\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\varepsilon'-\wp'\right)\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)-\varepsilon-\wp-\wp'\right)}{\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\gamma-\varepsilon-\wp'\right)}. \quad (26)$$

Proof. For convenience, we denote the left-hand side of the result (25) by $\Im_1$ using the definitions (1) and taking the left-hand sided MSM fractional integral operator inside the summation, the left-hand side of (25) becomes

$$\Im_1 = \left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}V_n^{a_u,h,b_v}(l,\mu,\zeta,\delta,m,k_u,A_v,B_w,\eta,\nu,\alpha;t^{\sigma})\right)\right)(x)$$
$$= \left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}\xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}(t^{\sigma}/2)^{n\zeta+h\delta+m}}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\alpha+B_w}\right]}\right)\right)(x)$$
$$= \xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\alpha+B_w}\right]2^{\sigma(n\zeta+h\delta+m)}}$$
$$\cdot\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho+\sigma(n\zeta+h\delta+m)-1}\right)\right)(x), \quad (27)$$

by applying the relation (23) in (26), we get RHS of (25). □

The next theorem gives the right-hand MSM fractional integration of the V-function.

Theorem 3. *Let* $\wp, \wp', \varepsilon, \varepsilon', \gamma, \rho \in \mathbb{C}$; $l, \zeta, \wp, m, \nu, \alpha, k_u, A_v, B_w \in \Re$; $p, q, r \in \mathbb{N}$; $a_u, b_v \geq 1$, $\eta > 0, \Re(\mu) > 0, \Re(h) > 0, \Re(\sigma) > 0$, *and* $\xi > 0$ *is an arbitrary constant, such that* $\Re(\gamma) > 0$, $\Re(\rho) > \max\{\Re(\varepsilon), \Re(-\wp-\wp'+\gamma), \Re(-\wp-\varepsilon'+\gamma)\}$, *then for* $t > 0$ *following integral holds true:*

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}V_n^{a_u,h,b_v}(l,\mu,\zeta,\delta,m,k_u,A_v,B_w,\eta,\nu,\alpha;t^{-\sigma})\right)\right)(x) = x^{-\wp-\wp'+\gamma-\rho}$$
$$\times \xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}(x^{-\sigma}/2)^{n\zeta+h\delta+m}\Gamma(-\varepsilon+\rho+\sigma(n\zeta+h\delta+m))}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\rho+B_w}\right]\Gamma(\rho+\sigma(n\zeta+h\delta+m))}$$
$$\times\frac{\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)+\wp+\varepsilon'-\gamma\right)}{\Gamma\left(\rho+\sigma(n\zeta+h\delta+m)-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\sigma(n\zeta+h\delta+m)+\wp-\varepsilon)}. \quad (28)$$

Proof. For convenience, we denote the left-hand side of the result (27) by $\Im_2$ using the definitions (1) and taking the right-hand sided MSM fractional integral operator inside the summation, the left-hand side of (27) becomes

$$\Im_2 = \left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}V_n^{a_u,h,b_v}(l,\mu,\zeta,\delta,m,k_u,A_v,B_w,\eta,\nu,\alpha;t^{-\sigma})\right)\right)(x)$$
$$= \left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}\xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}(t^{-\sigma}/2)^{n\zeta+h\delta+m}}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\alpha+B_w}\right]}\right)\right)(x)$$
$$= \xi\sum_{n=0}^{\infty}\frac{(-l)^n\prod_{u=1}^{p}\left[(a_u)_{n+k_u}\right](h+\eta n+\nu)^{-\mu}}{\prod_{v=1}^{q}\left[(b_v)_{n+A_v}\right]\prod_{w=1}^{r}\left[(h)_{\eta n\alpha+B_w}\right]2^{\sigma(n\zeta+h\delta+m)}}\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-(\rho+\sigma(n\zeta+h\delta+m))}\right)\right)(x), \quad (29)$$

by applying relation (24) in (28), we get R.H.S. of (27). □

3. Special Cases

Here, we present some special cases by choosing suitable values of the parameters in Theorems 3.1 and 2.2. We get certain interesting results concerning the other special functions like the generalized hypergeometric function, the Bessel function, Wright's generalized Bessel function, Struve's function, Lommel's function, the Mittag-Leffler function, the Riemann-Zeta function, the exponential function, and Macrobert's E-function given in the following special cases.

(i) If we put $w = 1, h = 1, p = P, q = Q, l = -2, \mu = 1, \zeta = 1, \delta = 0, m = 0, k_u = 0, A_v = 0, B_1 = -1, \eta = 1, \nu = -1, \alpha = 1$, and $\xi = 1$ in equations (25) and (27), then the V-function turns into the generalized hypergeometric function [11] and applying the result from ([31], equations (25) and (26), page 34)).

$$(\beta)_{\rho n} = \rho^{\rho n}\prod_{i=1}^{\rho}\left[\frac{\beta+i-1}{\rho}\right]_n. \quad (30)$$

As per condition of Theorems 3.1 and 2.2 respectively as below:

$$
\begin{aligned}
&\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}{}_{P}F_{Q}\left(a_P;b_Q;t^{\sigma}\right)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{P+3\sigma}F_{Q+3\sigma}\\
&\cdot\left[\begin{matrix}a_1,a_2,\cdots,a_P;\frac{\rho}{\sigma},\frac{\rho+1}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho+\varepsilon'-\wp'}{\sigma},\frac{\rho+\varepsilon'-\wp'+1}{\sigma}\cdots,\frac{\rho+\varepsilon'-\wp'+\sigma-1}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{\sigma}\cdots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma-1}{\sigma}\\ b_1,b_2,\cdots,b_Q;\frac{\rho+\varepsilon'}{\sigma},\frac{1+\rho+\varepsilon'}{\sigma}\cdots,\frac{\rho+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\gamma-\wp-\wp'}{\sigma},\frac{\rho+\gamma-\wp-\wp'+1}{\sigma}\cdots,\frac{\rho+\gamma-\wp-\wp'+\sigma-1}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{\sigma}\cdots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{matrix}\ x^{\sigma}\right],\\
&\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}{}_{P}F_{Q}\left(a_P;b_Q;t^{-\sigma}\right)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\\
&\quad\times\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{P+3\sigma}F_{Q+3\sigma}\\
&\cdot\left[\begin{matrix}a_1,a_2,\cdots,a_P;\frac{\rho-\varepsilon}{\sigma},\frac{\rho-\varepsilon+1}{\sigma}\cdots,\frac{\rho-\varepsilon+\sigma-1}{\sigma},\frac{\rho+\wp+\wp'-\gamma}{\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{\sigma}\cdots,\frac{\rho+\wp+\wp'-\gamma+\sigma-1}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{\sigma}\cdots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma-1}{\sigma}\\ b_1,b_2,\cdots,b_Q;\frac{\rho}{\sigma},\frac{1+\rho}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{\sigma}\cdots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\wp-\varepsilon}{\sigma},\frac{\rho+\wp-\varepsilon+1}{\sigma}\cdots,\frac{\rho+\wp-\varepsilon+\sigma-1}{\sigma}\end{matrix}\ \Big|\,x^{-\sigma}\right]
\end{aligned}
\tag{31}
$$

(ii) If we put $\sigma=1, u=1, v=2, w=1, a_1=1, b_1=1, b_2=1, l=1, \mu=1, \zeta=2, \delta=1, m=0, k_1=0, A_1=0, A_2=0, B_1=0, \eta=1, \nu=0, \alpha=1$, and $\xi=1/(\Gamma(h))$ in equations (25) and (27) then the V-function turns into the Bessel function [23] and applying the result from ([31], equations (25) and (5.3), page 38) as

$$
(\beta)_{2n}=2^{2n}\left(\frac{\beta}{2}\right)_n\left(\frac{\beta+1}{2}\right)_n,
\tag{32}
$$

under the condition stated in Theorems 3.1 and 2.2, respectively, as below:

$$
\begin{aligned}
&\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}J_h(t)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma+h-1}\times\frac{\Gamma(\rho+h)\Gamma\left(\rho+\varepsilon'-\wp'+h\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'+h\right)}{\Gamma(h+1)\Gamma\left(\rho+\varepsilon'+h\right)\Gamma\left(\rho+\gamma-\wp-\wp'+h\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'+h\right)}\\
&\times{}_6F_7\left[\begin{matrix}\frac{\rho+h}{2},\frac{\rho+h+1}{2},\frac{\rho+\varepsilon'-\wp'+h}{2},\frac{\rho+\varepsilon'-\wp'+h+1}{2},\frac{\rho-\varepsilon-\wp-\wp'+h}{2},\frac{\rho-\varepsilon-\wp-\wp'+h+1}{2}\\ h+1,\frac{\rho+\varepsilon'+h}{2},\frac{\rho+\varepsilon'+h+1}{2},\frac{\rho+\gamma-\wp-\wp'+h}{2},\frac{\rho+\gamma-\wp-\wp'+h+1}{2},\frac{\rho+\gamma-\varepsilon-\wp'+h}{2},\frac{\rho+\gamma-\varepsilon-\wp'+h+1}{2}\end{matrix}\ \Big|-\frac{x^2}{4}\right],\\
&\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}J_h\left(\frac{1}{t}\right)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho-h}\times\frac{\Gamma(\rho-\varepsilon+h)\Gamma\left(\rho+\wp+\wp'-\gamma+h\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma+h\right)}{\Gamma(h+1)\Gamma(\rho+h)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'+h\right)\Gamma(\rho+\wp-\varepsilon+h)}\\
&\times{}_6F_7\left[\begin{matrix}\frac{\rho-\varepsilon+h}{2},\frac{\rho-\varepsilon+h+1}{2},\frac{\rho+\wp+\wp'-\gamma+h}{2},\frac{\rho+\wp+\wp'-\gamma+h+1}{2},\frac{\rho+\wp+\varepsilon'-\gamma+h}{2},\frac{\rho+\wp+\varepsilon'-\gamma+h+1}{2}\\ h+1,\frac{\rho+h}{2},\frac{1+\rho+h}{2},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+h}{2},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+h+1}{2},\frac{\rho+\wp-\varepsilon+h}{2},\frac{\rho+\wp-\varepsilon+h+1}{2}\end{matrix}\ \Big|-\frac{1}{4x^2}\right]
\end{aligned}
\tag{33}
$$

(iii) If we put $u=1, v=2, w=1, a_1=1, b_1=1, b_2=1, l=2, \mu=1, \zeta=1, \delta=1, m=0, k_1=0, A_1=0, A_2=0, B_1=0, \nu=0, \alpha=1$ and $\xi=1/(\Gamma(h))$ in equations (25) and (27) then the V-function turns into Wright's generalized Bessel function [23] using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}J_h^{\eta}(t^{\sigma})\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma(h+1)\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{3\sigma}F_{\eta+3\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho}{\sigma},\frac{\rho+1}{\sigma}\ldots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho+\varepsilon'-\wp'}{\sigma},\frac{\rho+\varepsilon'-\wp'+1}{\sigma},\ldots,\frac{\rho+\varepsilon'-\wp'+\sigma-1}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{\sigma}\ldots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma-1}{\sigma}\\ \frac{h+1}{\eta},\frac{h+2}{\eta}\cdots\frac{h+\eta}{\eta}\frac{\rho+\varepsilon'}{\sigma},\frac{1+\rho+\varepsilon'}{\sigma}\ldots,\frac{\rho+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\gamma-\wp-\wp'}{\sigma},\frac{\rho+\gamma-\wp-\wp'+1}{\sigma},\ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma-1}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{\sigma}\ldots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{matrix}\middle|-\frac{x^{\sigma}}{\eta^{\eta}}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}J_h^{\eta}(t^{-\sigma})\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(h+1)\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{3\sigma}F_{\eta+3\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho-\varepsilon}{\sigma},\frac{\rho-\varepsilon+1}{\sigma}\ldots,\frac{\rho-\varepsilon+\sigma-1}{\sigma},\frac{\rho+\wp+\wp'-\gamma}{\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{\sigma}\ldots,\frac{\rho+\wp+\wp'-\gamma+\sigma-1}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{\sigma}\ldots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma-1}{\sigma}\\ \frac{h+1}{\eta},\frac{h+2}{\eta}\cdots\frac{h+\eta}{\eta},\frac{\rho}{\sigma},\frac{1+\rho}{\sigma}\ldots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{\sigma}\ldots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\wp-\varepsilon}{\sigma},\frac{\rho+\wp-\varepsilon+1}{\sigma}\ldots,\frac{\rho+\wp-\varepsilon+\sigma-1}{\sigma}\end{matrix}\middle|-\frac{x^{-\sigma}}{\eta^{\eta}}\right]\tag{34}$$

(iv) If we put $u=1, v=2, w=1, a_1=1, b_1=3/2, b_2=1, l=1, \mu=1, \zeta=2, \delta=1, m=1, k_1=0, A_1=0, A_2=0, B_1=1/2, \eta=1, \nu=1/2, \alpha=1$, and $\xi=1/\Gamma(h)\Gamma(3/2)$ in equation (25) and (27) then V-function turn into Struve's function [23] using the result (29) under the condition in Theorem 3.1 and 2.2 respectively, we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}H_h(t^{\sigma})\right)\right)(x)=\frac{x^{\rho-\wp-\wp'+\gamma+\sigma(h+1)-1}}{\Gamma(h+3/2)\Gamma(3/2)2^{h+1}}$$
$$\times\frac{\Gamma(\rho+\sigma(h+1))\Gamma\left(\rho+\varepsilon'-\wp'+\sigma(h+1)\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'+\sigma(h+1)\right)}{\Gamma\left(\rho+\varepsilon'+\sigma(h+1)\right)\Gamma\left(\rho+\gamma-\wp-\wp'+\sigma(h+1)\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'+\sigma(h+1)\right)}$$
$$\times{}_{1+6\sigma}F_{2+6\sigma}\left[\begin{matrix}1,\frac{\rho+\sigma(h+1)}{2\sigma},\frac{\rho+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma(h+1)}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma(h+1)+1}{2\sigma},\\ \ldots,\frac{\rho+\varepsilon'-\wp'+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma(h+1)}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma(h+1)+2\sigma-1}{2\sigma}\\ \frac{3}{2},h+\frac{3}{2},\frac{\rho+\varepsilon'+\sigma(h+1)}{2\sigma},\frac{1+\rho+\varepsilon'+\sigma(h+1)}{2\sigma}\ldots,\frac{\rho+\varepsilon'+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma(h+1)}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma(h+1)+1}{2\sigma},\\ \ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+\sigma(h+1)}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{matrix}\middle|-\frac{x^{2\sigma}}{4}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}H_h(t^{-\sigma})\right)\right)(x)=\frac{x^{-\wp-\wp'+\gamma-\rho-\sigma(h+1)}}{\Gamma(h+3/2)\Gamma(3/2)2^{h+1}}$$
$$\times\frac{\Gamma(\rho-\varepsilon+\sigma(h+1))\Gamma\left(\rho+\wp+\wp'-\gamma+\sigma(h+1)\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma+\sigma(h+1)\right)}{\Gamma(\rho+\sigma(h+1))\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(h+1)\right)\Gamma(\rho+\wp-\varepsilon+\sigma(h+1))}$$
$$\times{}_{1+6\sigma}F_{2+6\sigma}\left[\begin{matrix}\frac{\rho-\varepsilon+\sigma(h+1)}{2\sigma},\frac{\rho-\varepsilon+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho-\varepsilon+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma(h+1)}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma(h+1)+1}{2\sigma},\\ \ldots,\frac{\rho+\wp+\wp'-\gamma+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(h+1)}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(h+1)+2\sigma-1}{2\sigma}\\ \frac{3}{2},h+\frac{3}{2},\frac{\rho+\sigma(h+1)}{2\sigma},\frac{1+\rho+\sigma(h+1)}{2\sigma}\ldots,\frac{\rho+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(h+1)}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(h+1)+1}{2\sigma},\\ \ldots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(h+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma(h+1)}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma(h+1)+1}{2\sigma}\ldots,\frac{\rho+\wp-\varepsilon+\sigma(h+1)+2\sigma-1}{2\sigma}\end{matrix}\middle|-\frac{x^{-2\sigma}}{4}\right]\tag{35}$$

(v) If we put $u=1, v=2, w=1, a_1=1, b_1=(\theta+\varepsilon+3)/2, b_2=(\theta-\varepsilon+3)/2, l=1, \mu=1, \zeta=2, h=1, \delta=\theta, m=1, k_1=0, A_1=0, A_2=0, B_1=-1, \eta=1, \nu=-1, \alpha=1$, and $\xi=2^{\theta+1}/\{(\theta+\varepsilon+1)(\theta-\varepsilon+1)\}$ in equations (25) and (27) then V-function turn into Lommel's function [23] using result (29) under the condition in Theorems 3.1 and 2.2, respectively. we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}S_{\theta,\varepsilon}(t^{\sigma})\right)\right)(x)=\frac{x^{\rho-\wp-\wp'+\gamma+\sigma(\theta+1)-1}}{(\theta-\varepsilon+1)(\theta+\varepsilon+1)}$$
$$\times\frac{\Gamma(\rho+\sigma(\theta+1))\Gamma\left(\rho+\varepsilon'-\wp'+\sigma(\theta+1)\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'+\sigma(\theta+1)\right)}{\Gamma\left(\rho+\varepsilon'+\sigma(\theta+1)\right)\Gamma\left(\rho+\gamma-\wp-\wp'+\sigma(\theta+1)\right)\Gamma\rho+\gamma-\varepsilon-\wp'+\sigma(\theta+1))}$$
$$\times{}_{1+6\sigma}F_{2+6\sigma}\left[\begin{array}{l}1,\frac{\rho+\sigma(\theta+1)}{2\sigma},\frac{\rho+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma(\theta+1)}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma(\theta+1)+1}{2\sigma},\\ \frac{\theta-\varepsilon+3}{2},\frac{\theta+\varepsilon+3}{2},\frac{\rho+\varepsilon'+\sigma(\theta+1)}{2\sigma},\frac{1+\rho+\varepsilon'+\sigma(\theta+1)}{2\sigma}\ldots,\frac{\rho+\varepsilon'+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma(\theta+1)}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma(\theta+1)+1}{2\sigma},\\ \ldots,\frac{\rho+\varepsilon'-\wp'+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma(\theta+1)}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma(\theta+1)+2\sigma-1}{2\sigma}\\ \ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma(\theta+1)+2\sigma-1}{2\sigma},0.2cm\frac{\rho+\gamma-\varepsilon-\wp'+\sigma(\theta+1)}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma(\theta+1)+2\sigma-1}{2\sigma}\end{array}\middle|-\frac{x^{2\sigma}}{4}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}S_{\theta,\varepsilon}(t^{-\sigma})\right)\right)(x)=\frac{x^{-\wp-\wp'+\gamma-\rho-\sigma(\theta+1)}}{(\theta-\varepsilon+1)(\theta+\varepsilon+1)}$$
$$\times\frac{\Gamma(\rho-\varepsilon+\sigma(\theta+1))\Gamma\left(\rho+\wp+\wp'-\gamma+\sigma(\theta+1)\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma+\sigma(\theta+1)\right)}{\Gamma(\rho+\sigma(\theta+1))\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(\theta+1)\right)\Gamma(\rho+\wp-\varepsilon+\sigma(\theta+1))}$$
$$\times{}_{1+6\sigma}F_{2+6\sigma}\left[\begin{array}{l}\frac{\rho-\varepsilon+\sigma(\theta+1)}{2\sigma},\frac{\rho-\varepsilon+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho-\varepsilon+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma(\theta+1)}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma(\theta+1)+1}{2\sigma},\\ \frac{\theta-\varepsilon+3}{2},\frac{\theta+\varepsilon+3}{2}\frac{\rho+\sigma(\theta+1)}{2\sigma},\frac{1+\rho+\sigma(\theta+1)}{2\sigma}\ldots,\frac{\rho+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(\theta+1)}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(\theta+1)+1}{2\sigma},\\ \ldots,\frac{\rho+\wp+\wp'-\gamma+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(\theta+1)}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma(\theta+1)+2\sigma-1}{2\sigma}\\ \ldots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma(\theta+1)+2\sigma-1}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma(\theta+1)}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma(\theta+1)+1}{2\sigma}\ldots,\frac{\rho+\wp-\varepsilon+\sigma(\theta+1)+2\sigma-1}{2\sigma}\end{array}\middle|-\frac{x^{-2\sigma}}{4}\right]\tag{36}$$

(vi) If we put $u=1, v=1, w=1, a_1=1, b_1=1, l=-2, \mu=1, \zeta=1, \delta=0, m=0, k_1=0, A_1=0, B_1=-1, \nu=-1, \alpha=1$, and $\xi=1/(\Gamma(h))$ in equation (25) and (27) then the V-function turns into the Mittag-Leffler function [12, 13] using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}E_{h,\eta}(t^{\sigma})\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma(h)\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{1+3\sigma}F_{\eta+3\sigma}$$
$$\cdot\left[\begin{array}{l}1,\frac{\rho}{\sigma},\frac{\rho+1}{\sigma}\ldots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho+\varepsilon'-\wp'}{\sigma},\frac{\rho+\varepsilon'-\wp'+1}{\sigma},\ldots,\frac{\rho+\varepsilon'-\wp'+\sigma-1}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{\sigma}\ldots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma-1}{\sigma}\\ \frac{h}{\eta},\frac{h+1}{\eta},\ldots,\frac{h+\eta-1}{\eta},\frac{\rho+\varepsilon'}{\sigma},\frac{1+\rho+\varepsilon'}{\sigma}\ldots,\frac{\rho+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\gamma-\wp-\wp'}{\sigma},\frac{\rho+\gamma-\wp-\wp'+1}{\sigma},\ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma-1}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{\sigma}\ldots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{array}\middle|-\frac{x^{\sigma}}{\eta^{\eta}}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}E_{h,\eta}(t^{-\sigma})\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(h+1)\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{3\sigma}F_{\eta+3\sigma}$$
$$\cdot\left[\begin{array}{l}1,\frac{\rho-\varepsilon}{\sigma},\frac{\rho-\varepsilon+1}{\sigma}\ldots,\frac{\rho-\varepsilon+\sigma-1}{\sigma},\frac{\rho+\wp+\wp'-\gamma}{\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{\sigma},\ldots,\frac{\rho+\wp+\wp'-\gamma+\sigma-1}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{\sigma}\ldots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma-1}{\sigma}\\ \frac{h}{\eta},\frac{h+1}{\eta},\ldots,\frac{h+\eta-1}{\eta},\frac{\rho}{\sigma},\frac{1+\rho}{\sigma}\ldots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{\sigma},\ldots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\wp-\varepsilon}{\sigma},\frac{\rho+\wp-\varepsilon+1}{\sigma}\ldots,\frac{\rho+\wp-\varepsilon+\sigma-1}{\sigma}\end{array}\middle|-\frac{x^{\sigma}}{\eta^{\eta}}\right]\tag{37}$$

(vii) If we put $u=1, v=1, w=1, a_1=a, b_1=1, l=-2, \zeta=1, \delta=0, m=0, k_1=0, A_1=0, B_1=0, \eta=1, \nu=0, \alpha=0$, and $\xi=1$ in equations (25) and (27) then the V-function turns into the unified Riemann-Zeta function [27] using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get,

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}\phi_a(t^{\sigma},\mu,h)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\sum_{n=0}^{\infty}\frac{(a)_n\Gamma(\rho+\sigma n)\Gamma\left(\rho+\varepsilon'-\wp'+\sigma n\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'+\sigma n\right)x^{\sigma n}}{(h+n)^{\mu}\Gamma\left(\rho+\varepsilon'+\sigma n\right)\Gamma\left(\rho+\gamma-\wp-\wp'+\sigma n\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'+\sigma n\right)},$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}\phi_a(t^{-\sigma},\mu,h)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\sum_{n=0}^{\infty}\frac{(a)_n\Gamma(\rho-\varepsilon+\sigma n)\Gamma\left(\rho+\wp+\wp'-\gamma+\sigma n\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma+\sigma n\right)x^{-\sigma n}}{(h+n)^{\mu}\Gamma(\rho+\sigma n)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma n\right)\Gamma(\rho+\wp-\varepsilon+\sigma n)} \quad (38)$$

(viii) If we put $u=1, v=1, w=1, a_1=1, b_1=1, l=2, \zeta=1, \delta=0, m=0, k_1=0, A_1=0, B_1=-1, \eta=1, \nu=-1, \alpha=1, h=1, \mu=1$, and $\xi=1$ in equations (25) and (27) then V-function turn into e^{-z} function, using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get,

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}e^{-(t^{\sigma})}\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{3\sigma}F_{3\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho}{\sigma},\frac{\rho+1}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho+\varepsilon'-\wp'}{\sigma},\frac{\rho+\varepsilon'-\wp'+1}{\sigma}\cdots,\frac{\rho+\varepsilon'-\wp'+\sigma-1}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{\sigma}\cdots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma-1}{\sigma},\\ \frac{\rho+\varepsilon'}{\sigma},\frac{1+\rho+\varepsilon'}{\sigma}\cdots,\frac{\rho+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\gamma-\wp-\wp'}{\sigma},\frac{\rho+\gamma-\wp-\wp'+1}{\sigma},\ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma-1}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{\sigma}\cdots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{matrix}\middle| -x^{\sigma}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}e^{-(t^{-\sigma})}\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{3\sigma}F_{3\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho-\varepsilon}{\sigma},\frac{\rho-\varepsilon+1}{\sigma}\cdots,\frac{\rho-\varepsilon+\sigma-1}{\sigma},\frac{\rho+\wp+\wp'-\gamma}{\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{\sigma},\ldots,\frac{\rho+\wp+\wp'-\gamma+\sigma-1}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{\sigma}\cdots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma-1}{\sigma}\\ \frac{\rho}{\sigma},\frac{1+\rho}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{\sigma},\ldots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\wp-\varepsilon}{\sigma},\frac{\rho+\wp-\varepsilon+1}{\sigma}\cdots,\frac{\rho+\wp-\varepsilon+\sigma-1}{\sigma}\end{matrix}\middle| -x^{-\sigma}\right] \quad (39)$$

(ix) If we put $w=1, p=P, q=Q, l=2, \mu=1, \zeta=1, \delta=0, m=0, k_u=0, A_u=0, B_1=-1, \eta=1, \nu=-1, \alpha=1, h=1$, and $\xi=\prod_{u=1}^{P}\Gamma(a_u)/\prod_{v=1}^{Q}\Gamma(b_v)$ in equations (25) and (27) then the V-function turns into Macrobert's E-function [11] using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}E\left(P,(a_P);Q,(b_Q);\frac{1}{t^{\sigma}}\right)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\prod_{u=1}^{P}\Gamma(a_u)}{\prod_{v=1}^{Q}\Gamma(b_v)}\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{P+3\sigma}F_{Q+3\sigma}$$
$$\cdot\left[\begin{matrix}a_1,a_2,\cdots,a_P;\frac{\rho}{\sigma},\frac{\rho+1}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho+\varepsilon'-\wp'}{\sigma},\frac{\rho+\varepsilon'-\wp'+1}{\sigma},\ldots,\frac{\rho+\varepsilon'-\wp'+\sigma-1}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{\sigma}\cdots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma-1}{\sigma}\\ b_1,b_2,\cdots,b_Q;\frac{\rho+\varepsilon'}{\sigma},\frac{1+\rho+\varepsilon'}{\sigma}\cdots,\frac{\rho+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\gamma-\wp-\wp'}{\sigma},\ldots,\frac{\rho+\gamma-\wp-\wp'+\sigma-1}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{\sigma}\cdots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma-1}{\sigma}\end{matrix}\middle| -x^{\sigma}\right],$$

$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}E\left(P,(a_P);Q,(b_Q);\frac{1}{t^{-\sigma}}\right)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\frac{\prod_{u=1}^{P}\Gamma(a_u)}{\prod_{v=1}^{Q}\Gamma(b_v)}\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{P+3\sigma}F_{Q+3\sigma}$$
$$\cdot\left[\begin{matrix}a_1,a_2,\cdots,a_P;\frac{\rho-\varepsilon}{\sigma},\frac{\rho-\varepsilon+1}{\sigma}\cdots,\frac{\rho-\varepsilon+\sigma-1}{\sigma},\frac{\rho+\wp+\wp'-\gamma}{\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{\sigma}\cdots,\frac{\rho+\wp+\wp'-\gamma+\sigma-1}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{\sigma}\cdots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma-1}{\sigma}\\ b_1,b_2,\cdots,b_Q;\frac{\rho}{\sigma},\frac{1+\rho}{\sigma}\cdots,\frac{\rho+\sigma-1}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{\sigma}\cdots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma-1}{\sigma},\frac{\rho+\wp-\varepsilon}{\sigma},\frac{\rho+\wp-\varepsilon+1}{\sigma}\cdots,\frac{\rho+\wp-\varepsilon+\sigma-1}{\sigma}\end{matrix}\middle| -x^{-\sigma}\right] \quad (40)$$

(x) If we put $u=1, v=2, w=1, a_1=1, b_1=1, k_1=0, h=1/2, l=1, \mu=1, \zeta=2, \delta=0, m=0, A_1=0, A_2=-1, B_1=0, \eta=1, \nu=-1/2, \alpha=1$, and $\xi=1$ in equations (25) and (27) then the V-function turns into $\cos z$ function, using result (29) under the condition in Theorems 3.1 and 2.2, respectively, we get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}\cos\left(t^{\sigma}\right)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma-1}\times\frac{\Gamma(\rho)\Gamma\left(\rho+\varepsilon'-\wp'\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'\right)}{\Gamma\left(\rho+\varepsilon'\right)\Gamma\left(\rho+\gamma-\wp-\wp'\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'\right)}\times{}_{6\sigma}F_{1+6\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho}{2\sigma},\frac{\rho+1}{2\sigma}\cdots,\frac{\rho+2\sigma-1}{2\sigma},\frac{\rho+\varepsilon'-\wp'}{2\sigma},\frac{\rho+\varepsilon'-\wp'+1}{2\sigma},\dots,\frac{\rho+\varepsilon'-\wp'+2\sigma-1}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+1}{2\sigma}\cdots,\frac{\rho-\varepsilon-\wp-\wp'+2\sigma-1}{2\sigma}\\ \frac{1}{2},\frac{\rho+\varepsilon'}{2\sigma},\frac{1+\rho+\varepsilon'}{2\sigma}\cdots,\frac{\rho+\varepsilon'+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\wp-\wp'}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+1}{2\sigma},\dots,\frac{\rho+\gamma-\wp-\wp'+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+1}{2\sigma}\cdots,\frac{\rho+\gamma-\varepsilon-\wp'+2\sigma-1}{2\sigma}\end{matrix}\middle|-\frac{x^{2\sigma}}{4}\right],$$
$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}\cos\left(t^{-\sigma}\right)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho}\times\frac{\Gamma(\rho-\varepsilon)\Gamma\left(\rho+\wp+\wp'-\gamma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma\right)}{\Gamma(\rho)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'\right)\Gamma(\rho+\wp-\varepsilon)}\times{}_{6\sigma}F_{1+6\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho-\varepsilon}{2\sigma},\frac{\rho-\varepsilon+1}{2\sigma}\cdots,\frac{\rho-\varepsilon+2\sigma-1}{2\sigma},\frac{\rho+\wp+\wp'-\gamma}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+1}{2\sigma},\dots,\frac{\rho+\wp+\wp'-\gamma+2\sigma-1}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+1}{2\sigma}\cdots,\frac{\rho+\wp+\varepsilon'-\gamma+2\sigma-1}{2\sigma}\\ \frac{1}{2},\frac{\rho}{2\sigma},\frac{1+\rho}{2\sigma}\cdots,\frac{\rho+2\sigma-1}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+1}{2\sigma},\dots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+2\sigma-1}{2\sigma},\frac{\rho+\wp-\varepsilon}{2\sigma},\frac{\rho+\wp-\varepsilon+1}{2\sigma}\cdots,\frac{\rho+\wp-\varepsilon+2\sigma-1}{2\sigma}\end{matrix}\middle|-\frac{x^{-2\sigma}}{4}\right] \quad (41)$$

(xi) If we put $u=1, v=2, w=1, a_1=1, b_1=1, k_1=0, h=1/2, l=1, \mu=1, \zeta=2, \delta=2, m=0, A_1=0, A_2=-1, B_1=0, \eta=1, \nu=-1/2, \alpha=1$, and $\xi=1$ in equation (25) and (27) then the V-function turns into sinz function, using result (29) under the condition in Theorems 3.1 and 2.2, respectively. We get

$$\left(I_{0+}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{\rho-1}\sin\left(t^{\sigma}\right)\right)\right)(x)=x^{\rho-\wp-\wp'+\gamma+\sigma-1}\times\frac{\Gamma(\rho+\sigma)\Gamma\left(\rho+\varepsilon'-\wp'+\sigma\right)\Gamma\left(\rho-\varepsilon-\wp-\wp'+\sigma\right)}{\Gamma\left(\rho+\varepsilon'+\sigma\right)\Gamma\left(\rho+\gamma-\wp-\wp'+\sigma\right)\Gamma\left(\rho+\gamma-\varepsilon-\wp'+\sigma\right)}\times{}_{6\sigma}F_{1+6\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho+\sigma}{2\sigma},\frac{\rho+\sigma+1}{2\sigma}\cdots,\frac{\rho+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma}{2\sigma},\frac{\rho+\varepsilon'-\wp'+\sigma+1}{2\sigma}\cdots,\frac{\rho+\varepsilon'-\wp'+\sigma+2\sigma-1}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma}{2\sigma},\frac{\rho-\varepsilon-\wp-\wp'+\sigma+1}{2\sigma}\cdots,\frac{\rho-\varepsilon-\wp-\wp'+\sigma+2\sigma-1}{2\sigma}\\ \frac{3}{2},\frac{\rho+\varepsilon'+\sigma}{2\sigma},\frac{1+\rho+\varepsilon'+\sigma}{2\sigma}\cdots,\frac{\rho+\varepsilon'+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma}{2\sigma},\frac{\rho+\gamma-\wp-\wp'+\sigma+1}{2\sigma}\cdots,\frac{\rho+\gamma-\wp-\wp'+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+\sigma}{2\sigma},\frac{\rho+\gamma-\varepsilon-\wp'+\sigma+1}{2\sigma}\cdots,\frac{\rho+\gamma-\varepsilon-\wp'+\sigma+2\sigma-1}{2\sigma}\end{matrix}\middle|-\frac{x^{2\sigma}}{4}\right],$$
$$\left(I_{-}^{\wp,\wp',\varepsilon,\varepsilon',\gamma}\left(t^{-\rho}\cos\left(t^{-\sigma}\right)\right)\right)(x)=x^{-\wp-\wp'+\gamma-\rho-\sigma}\times\frac{\Gamma(\rho-\varepsilon+\sigma)\Gamma\left(\rho+\wp+\wp'-\gamma+\sigma\right)\Gamma\left(\rho+\wp+\varepsilon'-\gamma+\sigma\right)}{\Gamma(\rho+\sigma)\Gamma\left(\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma\right)\Gamma(\rho+\wp-\varepsilon+\sigma)}\times{}_{6\sigma}F_{1+6\sigma}$$
$$\cdot\left[\begin{matrix}\frac{\rho-\varepsilon+\sigma}{2\sigma},\frac{\rho-\varepsilon+\sigma+1}{2\sigma}\cdots,\frac{\rho-\varepsilon+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma}{2\sigma},\frac{\rho+\wp+\wp'-\gamma+\sigma+1}{2\sigma}\cdots,\frac{\rho+\wp+\wp'-\gamma+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma}{2\sigma},\frac{\rho+\wp+\varepsilon'-\gamma+\sigma+1}{2\sigma}\cdots,\frac{\rho+\wp+\varepsilon'-\gamma+\sigma+2\sigma-1}{2\sigma}\\ \frac{3}{2},\frac{\rho+\sigma}{2\sigma},\frac{1+\rho+\sigma}{2\sigma}\cdots,\frac{\rho+\sigma+2\sigma-1}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma}{2\sigma},\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma+1}{2\sigma}\cdots,\frac{\rho-\gamma+\wp+\wp'+\varepsilon'+\sigma+2\sigma-1}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma}{2\sigma},\frac{\rho+\wp-\varepsilon+\sigma+1}{2\sigma}\cdots,\frac{\rho+\wp-\varepsilon+\sigma+2\sigma-1}{2\sigma}\end{matrix}\middle|-\frac{x^{-2\sigma}}{4}\right] \quad (42)$$

4. Concluding Remark

Due to the generalization of Riemann-Liouville, Weyl, Erdélyi-Kober, and Saigo's fractional calculus operators, MSM fractional calculus operators have a significant advantage; as a result, many writers are referred to as general operators. Now, we will wrap up this paper by highlighting that our most important findings (Theorems 2 and 3) can be deduced as special cases involving well-known fractional calculus operators, as previously stated. The V-function defined in (1), on the other hand, has the property that a number of special functions appear to be the special cases. Several special cases involving integrals relating to the V-function have been uncovered in previous research works by various authors using different arguments.

Data Availability

No data were used to support this study

Conflicts of Interest

There is no conflict of interest regarding the publication of this article.

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