Research Article

Preopenness Degree in $RL$-Fuzzy Bitopological Spaces

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Based on the concept of pseudocomplement, we introduce a new representation of preopenness of $L$-fuzzy sets in $RL$-fuzzy bitopological spaces. The concepts of pairwise $RL$-fuzzy precontinuous and pairwise $RL$-fuzzy preirresolute functions are extended and discussed based on the $(i, j)$-$RL$-preopen gradation. Further, we follow up with a study of pairwise $RL$-fuzzy precompactness in $RL$-fuzzy bitopological spaces of an $L$-fuzzy set. We find that our paper offers more general results since $RL$-fuzzy bitopology is a generalization of $L$-bitopology, $RL$-bitopology, and $L$-fuzzy topology.

1. Introduction

The fuzzy set theory was introduced in the year 1965 by Lotfi A. Zadeh [1]. The development of fuzzy set theory, since its introduction, has been dramatic and breathtaking! Thousands of research papers have appeared in various journals devoted entirely to theoretical and application aspects of fuzzy sets. Artificial intelligence, automata theory, computer science, control theory, decision-making, expert systems, medical diagnosis, neural networks, pattern recognition, robotics, and social sciences are a few fields where fuzzy sets find application. Within a short time since its introduction, the fuzzy sets have permeated almost every academic discipline and have made their way into consumer products! Apart from this, it is also used for the construction of machines by way of intelligent robotics (engines, cars, ships, turbines, etc.) and controls (Sendai subway train in Japan, etc.) as well as for military purposes.

In 1991, Bin Shahna [2] introduced the concept of $a$-open and preopen sets in the context of fuzzy sets, and he introduced a preliminary study of fuzzy strong semicontinuity and fuzzy precontinuity as well. Later, the concepts of fuzzy $a$-open sets, fuzzy preopen, fuzzy $a$-continuous mappings, and fuzzy precontinuous mappings have been generalized to the setting of fuzzy bitopological spaces in [3], where some of their fundamental properties have been studied.

Shi [4] presented the concept of an $L$-fuzzy preopen degree of $L$-fuzzy set in $L$-fuzzy topological spaces. In addition, he discussed the fundamental properties of $L$-fuzzy precontinuous and $L$-fuzzy preirresolute mappings. It has been found that Shi’s operator is incredibly useful in introducing other gradations as well as in analyzing many topological characteristics [5].

The concept of $RL$-topology has recently been introduced and studied by H. Li and Q. Li [6] as an extension of $L$-topology. A detailed discussion is also presented concerning $RL$-continuous mapping and $RL$-compactness by means of an inequality. As a generalization of $RL$-topology and $L$-fuzzy topology, H. Li and Q. Li [7] defined $RL$-fuzzy topology. Further investigations are conducted regarding the $RL$-fuzzy compactness in $RL$-fuzzy topological spaces. As a consequence of their work, Zhang et al. [8] presented the Lindelöf property degree as well as the countable $RL$-fuzzy compactness degree of an $L$-subset. It is clear that the gradation of fuzzy compactness and Lindelöf property in the sense of Kubik and Šostak are special cases of the corresponding degrees in $RL$-fuzzy topology.

In this paper, the pseudocomplement of $L$-fuzzy sets is put forward as a basis for the definition of $(i, j)$-$RL$-preopen
degree in RL-fuzzy bitopology. Additionally, we introduce and discuss pairwise RL-fuzzy precontinuous, pairwise RL-fuzzy preirresolute mappings, and pairwise RL-fuzzy precompactness.

2. Preliminaries
Throughout this paper, \((L, \vee, \wedge, ',)\) denotes a complete DeMorgan algebra \([9, 10]\) and \(X\) is a nonempty crisp set. By \(L^X\), we refer to the family of all \(L\)-fuzzy sets presented on \(X\). The greatest and the smallest elements in \(L\) and \(L^X\) are \(1_L, 0_L\), and \(1_L, 0_L\), respectively. For any \(\alpha, \beta \in L\), \(\alpha < \beta\) means that the element \(\alpha\) is wedge below \(\beta\) in \(L\) [11]. \(J(L)\) (resp. \(P(L)\)) denotes the collection of nonzero coprime (resp. nonunit prime) members in \(L\). The greatest minimal collection and the greatest maximal collection of \(a\) are denoted by \(\alpha(a)\) and \(\beta(a)\), respectively. Moreover, \(\alpha^+(a) = \alpha(a) \cap J(L)\) and \(\beta^+(a) = \beta(a) \cap P(L)\). The valued \(L\)-fuzzy set \(A\) is an \(L\)-fuzzy set that achieves the condition \(A \leq A'\). \(\Psi^L_X\) refers to the family of valued \(L\)-fuzzy sets on \(X\), i.e., \(\Psi^L_X = \{A \in L^X: A \leq A'\}\). Moreover, \(\Psi^L_X(A) = \{B \in L^X: B \leq A\}\) for any \(A \in \Psi^L_X\). Let \(\psi : X \rightarrow Y\) be a mapping, \(A \in \Psi^L_X\), and \(B \in \Psi^L_Y\), \(\psi^{-L}(A) \rightarrow L^Y\) with \(D \in \Psi^L_Y(A) \rightarrow \psi^{-L}(D)\), is called an \(L\)-fuzzy function restriction (briefly, \(RL\)-fuzzy function), defined by \(\psi^{-L}_{LA}(A) \rightarrow B\) with \(\psi^{-L}(A) \leq B\). Moreover, \(\psi^{-L}_{LAC}(C) = \vee\{D \in \Psi^L_Y(C): \psi^{-L}(D) \leq C\}\) for each \(C \in \Psi^L_Y(B)\). Clearly, \(\psi^{-L}_{LAC}(C) = A \wedge \psi^{-L}(C)\). Let \(A \in \Psi^L_X\) and \(B \in \Psi^L_Y\), then \(\psi^{-L}(A) \leq B\) if \(B \neq A\) and \(\psi^{-L}(A) = 0\) if \(B = A\). The operation \(\psi^{-L}\) is called the pseudocomplement of \(B\) with respect to \(A\) [6, 7]. The following proposition lists some of its properties:

**Proposition 1** (see [6, 7]). If \(A \in \Psi^L_Y, H, G \in \Psi^L_X(A)\), and \(\{H_i\}_{i \in D} \subseteq \Psi^L_X(A)\), then:

\begin{align*}
(1) \quad &\psi^{-L}_{LA}(A) 
(2) \quad &H \leq G \Rightarrow \psi^{-L}(G) \leq \psi^{-L}(H) 
(3) \quad &\psi^{-L}_{LA}(A) \leq \psi^{-L}_{LA}(A) 
(4) \quad &\psi^{-L}_{LAC}(C) = A \wedge \psi^{-L}(C).
\end{align*}

**Lemma 2** (see [6]). For any \(A \in \Psi^L_X, B \in \Psi^L_Y, \psi^{-L}_{LA} : A \rightarrow B, H \in \Psi^L_X(A),\) and \(\mathcal{C} \subseteq \Psi^L_X(A),\) we have

\[
\forall y \in Y \left(\psi^{-L}_{LA}(H)(y) \wedge G(y)\right) = \forall x \left(\psi^{-L}_{LA}(H)(x) \wedge G(x)\right).
\]

(1)

Equivalently [8],

\[
\forall y \in Y \left(\psi^{-L}_{LA}(H)(y) \vee G(y)\right) = \forall x \left(\psi^{-L}_{LA}(H)(x) \vee G(x)\right).
\]

(2)

A subcollection \(\mathcal{F} \subseteq L^X\) is called an \(L\)-topology \([9, 10, 12]\) (briefly, \(L\)-t) if \(\mathcal{F}\) includes the smallest and the greatest \(L\)-fuzzy sets in \(L^X\). Moreover, the subcollection \(\mathcal{F}\) is closed for every suprema and finite infima. If \(\mathcal{F}\) is an \(L\)-topology on \(X\), then the pair \((X, \mathcal{F})\) is said to be an \(L\)-topological space on \(X\). Furthermore, elements of \(\mathcal{F}\) are said to be open, and their complements are said to be closed. For any mapping \(\psi : (X, \mathcal{F}_1) \rightarrow (Y, \mathcal{F}_2)\), \(\psi\) is said to be \(L\)-continuous iff \(\psi^{-L}(C) \in \mathcal{F}_1\) for every \(C \in \mathcal{F}_2\).

The mapping \(\mathcal{F} : L^X \rightarrow L\) is called an \(L\)-fuzzy topology on the set \(X\) [13–15] if it achieves the following statements:

\begin{align*}
(1) \quad &\mathcal{F}(1_L) = \mathcal{F}(1_L) = 1_L \\
(2) \quad &\mathcal{F}(H_1 \cap H_2) \cap \mathcal{F}(H_1 \cap H_2), \quad \text{for every } H_1, H_2 \in L^X \\
(3) \quad &\forall \mathcal{F}(H_1) \cap \mathcal{F}(H_2), \quad \text{for every } \{H_i\}_{i \in D} \subseteq L^X
\end{align*}

If \(\mathcal{F}\) is an \(L\)-topology on \(X\), then the pair \((X, \mathcal{F})\) is said to be an \(L\)-fuzzy topological space (briefly, \(L\)-t). The degree of openness and the degree of closeness of \(G \in L^X\) are represented by \(\mathcal{F}(G)\) and \(\mathcal{F}^*(G) = \mathcal{F}(G^*)\), respectively. The mapping \(\psi : (X, \mathcal{F}_1) \rightarrow (Y, \mathcal{F}_2)\) is said to be an \(L\)-fuzzy continuous if and only if \(\psi^{-L}(M) \in \mathcal{F}(M)\) for every \(M \in Y\).

**Definition 3** (see [6]). Let \(A \in \Psi^L_X\) and \(\mathcal{R} \subseteq \Psi^L_X(A)\); then \(\mathcal{R}\) is called a relative \(L\)-topology (briefly, \(RL\)-t) \(A\) if it achieves the next conditions:

\begin{align*}
(1) \quad &A \in \mathcal{R} \quad \text{and} \quad H \in \mathcal{R}, \quad \text{for each } H \subseteq A' \\
(2) \quad &H_1 \cap H_2 \in \mathcal{R}, \quad \text{for any } H_1, H_2 \in \mathcal{R} \\
(3) \quad &\forall \mathcal{R}(H_i) \in \mathcal{R}, \quad \text{for any } \{H_i\}_{i \in D} \subseteq \mathcal{R}
\end{align*}

If \(\mathcal{R}\) is an \(RL\)-topology on \(A\), then the pair \((A, \mathcal{R})\) is called an \(RL\)-topological space on \(A\) (briefly, \(RL\)-ts). The relative open \(L\)-fuzzy sets (briefly, \(RL\)-open fuzzy set) are the members of \(\mathcal{R}\) while their pseudocomplements are said to be \(RL\)-closed fuzzy sets, i.e., \(\psi^{-L}(B) \in \mathcal{R}\). The family of all \(R\) \(L\)-closed fuzzy sets with respect to \(\mathcal{R}\) is denoted by \(\langle \mathcal{R} \rangle\). Let \(A \in \Psi^L_X, B \in \Psi^L_Y\), and \(\{A, R_1, R_2\}\) be two \(RL\)-ts. The \(RL\)-fuzzy mapping \(\psi^{-L}_{LA} : A \rightarrow B\) is called an \(RL\)-continuous if and only if \(\psi^{-L}_{LA}(C) \in \langle \mathcal{R} \rangle\) for each \(C \in \mathcal{R}_2\). Equivalently, \(\psi^{-L}_{LA} : A \rightarrow B\) is called an \(RL\)-continuous if and only if \(\psi^{-L}_{LA}(H) \in \mathcal{R}_1\) for each \(H \in \mathcal{R}_2\). If \(\mathcal{R}_1\) and \(\mathcal{R}_2\) are \(RL\)-topologies on \(A \in \Psi^L_X\), then \(\mathcal{R} = \langle \mathcal{R} \rangle\) is called an \(RL\)-bitopological space (briefly, \(RL\)-bts). \(RL\)-open (resp. closed) refers to the open (resp. closed) \(L\)-fuzzy set with respect to \(\mathcal{R}\), where \(i = 1, 2\). In case of \(A = 1_X\), we will get back to \(L\)-topology and \(L\)-bitopology.

The concepts of \(\beta^\alpha\) cover, strong \(\beta^\alpha\) cover, \(\alpha\)-shading, strong-\(\alpha\)-shading, \(\alpha\)-remote collection, and strong \(\alpha\)-remote family [16] are extended to \(RL\)-topological spaces in [17] as follows:
Definition 4 (see [17]). For each \( A \in \mathcal{Y}^L_X \), RL-topology \( \mathcal{R} \) on \( A \), \( H \in \Psi^L_X(A) \), and \( a \in L_0 \), a set \( \mathcal{U} \subseteq \Psi^L_X(A) \) is said to be

1. \( \beta_a \)-cover of \( H \) if \( a \in \beta(H) \cap G(x) \) for all \( x \in X \) and \( \mathcal{U} \) is said to be strong \( \beta_a \)-cover of \( B \) if \( a \in \beta(H) \cap G(x) \)
2. \( Q_a \)-cover of \( B \) if \( \beta_1(H) \cap G(x) \geq \alpha \) for all \( x \in X \)

Definition 5 (see [17]). For each \( A \in \mathcal{Y}^L_X \), RL-topology \( \mathcal{R} \) on \( A \), \( a \in L_1 \), and \( H \in \Psi^L_X(A) \), a family \( \mathcal{F} \subseteq \Psi^L_X(A) \) is said to be

1. \( a \)-shading of \( H \) if \( (\beta_1(H) \cap G(x)) \leq \alpha \) for all \( x \in X \)
2. strong \( \alpha \)-shading of \( H \) if \( \cap_{x \in X} \beta_1(H) \cap G(x) \leq \alpha \)
3. \( \alpha \)-remote family of \( H \) if \( (H(x) \cap \cap_{x \in X} G(x)) \geq \alpha \) for all \( x \in X \)
4. strong \( \alpha \)-remote family of \( H \) if \( \cap_{x \in X} (H(x) \cap \cap_{x \in X} G(x)) \geq \alpha \)

Theorem 6 (see [6]). For each RL-ts \( (A, \mathcal{R}) \), the next statements are valid:

1. \( A \in \mathcal{Y}^L_X \) and \( H \in \mathcal{Y}^L_X \) for every \( H \leq A' \)
2. \( H_1 \cup H_2 \in \mathcal{Y}^L_X \) for every \( H_1, H_2 \in \mathcal{Y}^L_X \)
3. \( \alpha \)-remote family of \( H \) if \( (\beta_1(H) \cap G(x)) \leq \alpha \) for all \( x \in X \)
4. strong \( \alpha \)-remote family of \( H \) if \( \cap_{x \in X} (H(x) \cap \cap_{x \in X} G(x)) \geq \alpha \)

Definition 7 (see [7]). The mapping \( \mathcal{R} : \mathcal{Y}^L_X(A) \rightarrow L \) such that \( A \in \mathcal{Y}^L_X \) is called an RL-fuzzy topology on \( A \) if \( \mathcal{R} \) achieves the next statements:

1. \( \mathcal{R} \) is an RL-fuzzy topology on \( A \), then the pair \( (A, \mathcal{R}) \) is called an RL-fuzzy topology space (briefly, RL-fs). For every \( H \in \Psi^L_X(A) \), \( \mathcal{R}(H) \) is the degree of openness (resp. closeness) of \( H \) relative to \( \mathcal{R} \), respectively. Moreover, if \( \mathcal{R}(H) = 1_1 \) (resp. \( \mathcal{R}(H) = 1_1 \)), then \( \mathcal{R} \) is an RL-topology on \( A \) and \( \mathcal{X}_{\mathcal{R}} : \Psi^L_X(A) \rightarrow L \) is a mapping defined by \( \mathcal{X}_{\mathcal{R}}(H) = 1_1 \) if \( H \in \mathcal{R} \), and \( \mathcal{X}_{\mathcal{R}}(H) = 0_1 \) if \( H \in \mathcal{R} \), then \( \mathcal{X}_{\mathcal{R}} \) introduces a special RL-fs on \( A \).

Theorem 8 (see [7]). For every \( A \in \mathcal{Y}^L_X \) and RL-fs \( \mathcal{R} \) on \( A \). The mapping \( \mathcal{R} : \mathcal{Y}^L_X(A) \rightarrow L \) defined by \( \mathcal{R}(H) = \mathcal{R}(H) \) for every \( H \in \Psi^L_X(A) \) achieves the next statements:

1. \( \mathcal{R}(A) = 1_1 \), for every \( H \leq A' \)
2. \( \mathcal{R}(H) \cap \cup_{H_1} \mathcal{R}(H_1) \geq \mathcal{R}(H_1) \cap \mathcal{R}(H_2) \), for every \( H_1, H_2 \in \Psi^L_X(A) \)
3. \( \mathcal{R}(\cap_{x \in X} H_1) \cap \cup_{H_1} \mathcal{R}(H_1) \geq \mathcal{R}(H_1) \cap \mathcal{R}(H_2) \), for every \( \{H_i\}_{i \in \Omega} \subseteq \Psi^L_X(A) \)

\( \mathcal{R} \) is called an RL-fuzzy cotopology (briefly, RL-fc) on \( A \).

Definition 9 (see [7]). Let \( A \in \mathcal{Y}^L_X \), \( B \in \mathcal{Y}^L_X \) and \( (A, \mathcal{R}_1), (B, \mathcal{R}_2) \) be two RL-fs on \( A \) and \( B \), respectively. The RL-fuzzy mapping \( \Psi_{LA} : A \rightarrow B \) is called an RL-fuzzy continuous if and only if

\[ \mathcal{R}_1(\mathcal{L}_A(C)) \geq \mathcal{R}_1(C) \]

for every \( C \in \mathcal{Y}^L_X(B) \). Further, if \( (A, \mathcal{R}_1) \) and \( (B, \mathcal{R}_2) \) are RL-fs with respect to \( (A, \mathcal{R}_1) \) and \( (B, \mathcal{R}_2) \), respectively, then \( \mathcal{R}_{LA} \) is called an RL-fuzzy continuous if and only if

\[ \mathcal{R}_1(\mathcal{L}_A(C)) \geq \mathcal{R}_1(C) \]

for every \( C \in \mathcal{Y}^L_X(B) \).

Definition 10 (see [17, 18]). Let \( A \in \mathcal{Y}^L_X \), and \( (A, \mathcal{R}) \) be an RL-fs on \( A \). The mapping \( C^\mathcal{R} : \mathcal{Y}^L_X(A) \rightarrow L(\mathcal{Y}^L_X(A)) \) given by

\[ C^\mathcal{R}(H) = \mathcal{R}(\mathcal{L}_A(C) \cap \mathcal{L}_A(G)) \]

for every \( x_1 \in \mathcal{J}^L_X(A) \) and \( H \in \mathcal{Y}^L_X(A) \) is said to be the induced RL-fuzzy closure operator by \( \mathcal{R} \).

Theorem 12 (see [7]). For \( A = 1_L \), the next statements are true:

1. \( H = H' \), \( H \in \mathcal{Y}^L_X(A) \) iff \( H \in L^X \)
2. RL-fc is turned into LF-c
3. \( H \) is RL-fc iff \( H \) is LF-c
3. The $(i, j)$-RL-Preopenness Degree of L-Fuzzy Set

If $\mathcal{R}_1$ and $\mathcal{R}_2$ be two RL-fuzzy topologies on $A \in \Psi_X^L$, the triple $(A, \mathcal{R}_1, \mathcal{R}_2)$ is called an RL-fuzzy bitopological space (briefly, RL-fbts). Moreover, if $S$ any topological property, then we refer to $S$ with respect the RL-fbts $\mathcal{R}_i$ by $\mathcal{R}_iS$. An L-fuzzy set $H \in \Psi_X^L(A)$ of an RL-fbts $(A, \mathcal{R}_1, \mathcal{R}_2)$ is said to be an $(i, j)$-RL-preopen if and only if $H \leq \text{Int}^{\mathcal{R}_i}(\text{Cl}^{\mathcal{R}_j}(H)) \leq \text{Int}^{\mathcal{R}_j}(\text{Cl}^{\mathcal{R}_i}(H)) \leq H$. In the remainder of this paper $i, j = 1, 2$ such that $i \neq j$.

Definition 14. Let $A \in \Psi_X^L$ and $(A, \mathcal{R}_1, \mathcal{R}_2)$ be an RL-fbts on $A$. The $(i, j)$ - RL-preopenness gradation of $H \in \Psi_X^L(A)$ with respect to $\mathcal{R}_i$ and $\mathcal{R}_j$ is the mapping $(i, j) - \mathcal{P} : \Psi_X^L(A) \rightarrow L$ given by

$$(i, j) - \mathcal{P}(H) = \bigvee_{x_i < Hx_j < M} \bigwedge_{y_i < My_j < G} (1 \lambda (\mathcal{R}_i(i, j,G))).$$

The value $(i, j) - \mathcal{P}(H)$ introduces the gradation of $(i, j)$-RL-preopenness of $H$ is and $(i, j) - \mathcal{P}^*(H) = (i, j) - \mathcal{P}((i, j)H)$ introduces the gradation of $(i, j)$-RL-preopenness of $H$.

The next corollary is a direct consequence of the above definition and Definition 10:

Corollary 15. Let $(A, \mathcal{R}_1, \mathcal{R}_2)$ be an RL-fbts on $A \in \Psi_X^L$. Then, for every $H \in \Psi_X^L(A)$, we have

$$(i, j) - \mathcal{P}(H) = \bigwedge_{x_i < Hx_j < M} \bigvee_{y_i < My_j < G} (\mathcal{R}_i(M) \cap \text{Cl}^{\mathcal{R}_j}(H)(y_j)).$$

Theorem 13 (see [7]). For every $A \in \Psi_X^L$ and RL-fbts $(A, \mathcal{R})$, the next conclusions are true:

(1) If $H_1, H_2 \in \Psi_X^L(A)$ and $H_1, H_2$ are RL-fc, then $H_1 \cap H_2$ is RL-fc.

(2) If $H_1, H_2 \in \Psi_X^L(A)$ such that $H_1$ is an RL-fc and $H_2$ is an RL-closed, then $H_1 \cap H_2$ is an RL-fc.

Proof. The next inequality provides the proof:

$$(i, j) - \mathcal{P}(H) = \bigwedge_{x_i < Hx_j < M} \bigvee_{y_i < My_j < G} (\mathcal{R}_i(M) \cap \text{Cl}^{\mathcal{R}_j}(H)(y_j)).$$

Theorem 17. Let $(A, \mathcal{R}_1, \mathcal{R}_2)$ be an RL-fbts on $A \in \Psi_X^L$ and $(i, j) - \mathcal{P}$ be the degree of $(i, j)$ - RL-preopenness with respect to $\mathcal{R}_i$ and $\mathcal{R}_j$ with $i \neq j$. Then, for every $H \in \Psi_X^L(A)$, we have

$$(i, j) - \mathcal{P}(H) \leq (i, j) - \mathcal{P}(H).$$

Proof. The next inequality provides the proof:

$$(i, j) - \mathcal{P}(H) = \bigwedge_{x_i < Hx_j < M} \bigvee_{y_i < My_j < G} (\mathcal{R}_i(M) \cap \text{Cl}^{\mathcal{R}_j}(H)(y_j)).$$

Corollary 18. Let $(A, \mathcal{R}_1, \mathcal{R}_2)$ be an RL-fbts on $A \in \Psi_X^L$ and $(i, j) - \mathcal{P}$ be the degree of $(i, j)$ - RL-preopenness with respect to $\mathcal{R}_i$ and $\mathcal{R}_j$ with $i \neq j$. Then, for every $H \in \Psi_X^L(A)$, we have

$$(i, j) - \mathcal{P}(H) \leq (i, j) - \mathcal{P}(H).$$

Theorem 19. If $(A, \mathcal{R}_1, \mathcal{R}_2)$ be an RL-fbts on $A \in \Psi_X^L$, and $(i, j) - \mathcal{P}$ be the degree of $(i, j)$ - RL-preopenness with respect to $\mathcal{R}_i$ and $\mathcal{R}_j$ with $i \neq j$, then $(i, j) - \mathcal{P}(H \cup H_1) \geq (i, j) - \mathcal{P}(H)$. For every $H_1 \in \Psi_X^L(A)$.

Proof. Let $a \in L$ and $\alpha < (i, j) - \mathcal{P}(H)$. Then for any $i \in \Omega$ and for any $x_i < H_1$, there exists $C_i \in \Psi_X^L(A)$ such that

$$x_i \in C_i, \alpha < \mathcal{P}_{C_i}(C_i), \alpha < \bigwedge_{y_i < C_i, y_j < H_1} (\mathcal{R}_j(i, j,g)).$$

(12)
Hence,

\[
\alpha \leq \bigwedge_{i \in \Omega} R_i(C_i) \leq \bigvee_{i \in \Omega} \left( \bigwedge_{i \in \Omega} \bigwedge_{i < j, i, j \in D_i} \bigwedge_{i < j, i, j \in D_i} \left( \bigwedge_{i \in \Omega} (R_j \cap \left( \bigwedge_{i \in \Omega} \Omega \right) \right) \right).
\]

(13)

By

\[
\left\{ x_i : x_i < \bigvee_{i \in D_i} C_i \right\} = \bigcup_{i \in \Omega} \left\{ x_i : x_i < C_i \right\},
\]

we have

\[
(i, j) - PRL(\bigvee H_i) = \bigwedge_{x_i < H_i, x_i < C_i} \left( R_i(C_i) \wedge \bigwedge_{j < C_j, j < D_i} \bigwedge_{i \in \Omega} \left( \bigwedge_{i \in \Omega} (R_j \cap \left( \bigwedge_{i \in \Omega} \Omega \right) \right) \right)
\]

\[
\geq \bigwedge_{x_i < H_i, x_i < C_i} \left( R_i(C_i) \wedge \bigwedge_{j < C_j, j < D_i} \bigwedge_{i \in \Omega} \left( \bigwedge_{i \in \Omega} (R_j \cap \left( \bigwedge_{i \in \Omega} \Omega \right) \right) \right)
\]

\[
= \bigwedge_{x_i < H_i, x_i < C_i} \left( R_i(C_i) \wedge \bigwedge_{j < C_j, j < D_i} \bigwedge_{i \in \Omega} \left( \bigwedge_{i \in \Omega} (R_j \cap \left( \bigwedge_{i \in \Omega} \Omega \right) \right) \right)
\]

\[
\geq \bigwedge_{i \in \Omega} \left( R_i(C_i) \wedge \bigwedge_{j < C_j, j < D_i} \bigwedge_{i \in \Omega} \left( \bigwedge_{i \in \Omega} (R_j \cap \left( \bigwedge_{i \in \Omega} \Omega \right) \right) \right)
\]

\[
\geq \alpha.
\]

(15)

Then, \((i, j) - PRL(\bigvee H_i) \geq \bigwedge_{i \in \Omega} (i, j) - PRL(H_i).\)

\(\Box\)

Corollary 20. Let \((A, R_i, R_j)\) be an RL-fbts on \(A \in \mathbb{Y}_X\) and \((i, j) - PRL\) be the degree of \((i, j) - RL\)-preopenness with respect to \(R_i\) and \(R_j\) with \(i \neq j\). Then, \((i, j) - PRL^*(\bigvee H_i) \geq \bigwedge_{i \in \Omega} (i, j) - PRL^*(H_i)\) for any \(\{H_i\}_{i \in \Omega} \subseteq \Psi_X^1(A)\).

4. A New Representation of Pairwise Fuzzy Precontinuous (Preirresolute) Functions

Let \((A, R_1, R_2), (B, R_1^*, R_2^*)\) be RL-fbts on \(A \in \mathbb{Y}_X\) and \(B \in \mathbb{Y}_Y\), respectively. The RL-fuzzy mapping \(\Psi_{LA} : A \rightarrow B\) is called pairwise RL-fuzzy continuous (briefly, PRL-fco) if and only if \(\Psi_{LA} : (A, R_1) \rightarrow (B, R_2^*)\) and \(\Psi_{LA} : (A, R_2) \rightarrow (B, R_1^*)\) are RL-fuzzy continuous. In a similar way, we define the concept pairwise RL-fuzzy open mapping (briefly, PRL-fo).

Definition 21. Let \((A, R_1, R_2)\) and \((B, R_1^*, R_2^*)\) be RL-fbts on \(A \in \mathbb{Y}_X\) and \(B \in \mathbb{Y}_Y\), respectively, and \((i, j) - PRL\) be the corresponding \((i, j) - RL\)-preopenness degrees. An RL-fuzzy mapping \(\Psi_{LA} : A \rightarrow B\) is said to be

1. pairwise RL-fuzzy precontinuous (briefly, PRL-fco) if and only if \(R_1^*(M) \leq (i, j) - PRL(\Psi_{LA}(M))\) is true for every \(M \in \Psi_X^1(B)\)

2. pairwise RL-fuzzy preirresolute (briefly, PRL-fprr) if and only if \((i, j) - PRL(\Psi_{LA}(M))\) is true for every \(M \in \Psi_X^1(B)\)

Corollary 22. Let \((A, R_1, R_2)\) and \((B, R_1^*, R_2^*)\) be RL-fbts on \(A \in \mathbb{Y}_X\) and \(B \in \mathbb{Y}_Y\), respectively, and \((i, j) - PRL\) be the corresponding \((i, j) - RL\)-preopenness degrees. Then,

1. \(\Psi_{LA} : BRL-fpco\) if and only if \((i, j) - PRL^*(\Psi_{LA}(M))\) for every \(M \in \Psi_X^1(B)\)

2. \(\Psi_{LA} : BRL-fprr\) if and only if \((i, j) - PRL^*(\Psi_{LA}(M))\) for every \(M \in \Psi_X^1(B)\)

Theorem 23. Let \((A, R_1, R_2)\) and \((B, R_1^*, R_2^*)\) be RL-fbts on \(A \in \mathbb{Y}_X\) and \(B \in \mathbb{Y}_Y\), respectively, and \((i, j) - PRL\) be the corresponding \((i, j) - RL\)-preopenness degrees. Then,

1. \(\Psi_{LA} : (A, R_1, R_2) \rightarrow (B, R_1^*, R_2^*)\) is PRL-fpco if and only if \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_2^*_{[a]} R_2^*_{[a]})\) is PRL-pco for every \(\alpha \in J(L)\)

2. \(\Psi_{LA} : (A, R_1, R_2) \rightarrow (B, R_1^*, R_2^*)\) is PRL-fprr if and only if \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_2^*_{[a]} R_2^*_{[a]})\) is PRL-fprr for every \(\alpha \in J(L)\)

Proof.

1. If \(M \in \mathbb{R}_1^*\), for every \(M \in \Psi_X^1(B)\) and \(\alpha \in J(L)\), then \(R_1^*(M) \geq \alpha\). Since \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_1^*_{[a]} R_2^*_{[a]})\) is PRL-fpco, then \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\), i.e., \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\). Subsequently, \(\Psi_{LA}(M)\) is \((i, j) - RL\)-preopen \(L\)-fuzzy set in \((A, R_1^*_{[a]} R_2^*_{[a]})\). Thus, \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_1^*_{[a]} R_2^*_{[a]})\) is PRL-pco mapping.

Now, if \(R_1^*(M) \geq \alpha\) for every \(M \in \Psi_X^1(B)\) and \(\alpha \in J(L)\), then \(M \in \mathbb{R}_1^*\). Since \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_1^*_{[a]} R_2^*_{[a]})\) is PRL-pco, we have \(\Psi_{LA}(M)\) is \((i, j) - RL\)-preopen in \((A, R_1^*_{[a]} R_2^*_{[a]})\). Then, \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\) for every \(\alpha \in J(L)\) if \(R_1^*(M) \geq \alpha\), where \(J(R_1^*(M)) = -\alpha \in J(L)\) if \(\alpha \leq R_1^*(M)\). Hence \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_1^*_{[a]} R_2^*_{[a]})\) is PRL-fprr.

2. Let \(M\) be an \((i, j) - RL\)-preopen \(L\)-fuzzy set with respect to \((B, R_1^*_{[a]} R_2^*_{[a]})\), then \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\).

Since \(\Psi_{LA} : (A, R_1^*_{[a]} R_2^*_{[a]}) \rightarrow (B, R_1^*_{[a]} R_2^*_{[a]})\) is PRL-fprr, then \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\), so \((i, j) - PRL(\Psi_{LA}(M)) \geq \alpha\), then \(\Psi_{LA}(M)\) is \((i, j) - RL\)-preopen \(L\)-fuzzy set with respect to \((A, R_1^*_{[a]} R_2^*_{[a]})\).
Let \( \psi_{LA} : (A, R_1[a], R_2[a]) \rightarrow (B, R_1^*[a], R_2^*[a]) \) be PRL-pirr.

Now, if \( (i, j) \in R_2(A) \), then \( \psi_{LA} \) is PRL-pirr.

Theorem 24. Let \( (A, R_1, R_2), (B, R_1^*, R_2^*) \) be RL-fbts on \( A \in Y_X \) and \( B \in Y_Y \), respectively. If \( \psi_{LA} : (A, B) \rightarrow (B, C) \) is PRL-fco, then \( \psi_{LA} \) is also PRL-fco.

Theorem 25. Let \( (A, R_1, R_2), (B, R_1^*, R_2^*) \) be two RL-fbts on \( A \in Y_X \) and \( B \in Y_Y \), respectively. If \( \psi_{LA} : (A, B) \rightarrow (B, C) \) is PRL-fprir, then \( \psi_{LA} \) is PRL-fprir.

Theorem 26. Let \( (A, R_1, R_2), (B, R_1^*, R_2^*) \) be RL-fbts on \( A \in Y_X \) and \( B \in Y_Y \), respectively. If \( \psi_{LA} : (A, B) \rightarrow (B, C) \) is PRL-fco, then \( \psi_{LA} \) is PRL-fco.

Proof. Straightforward.

5. A New Representation of Pairwise Fuzzy Precompactness

Definition 27. For any RL-fbt \( (R_1, R_2) \) on \( A \in Y_X \), an L-fuzzy set \( H \in Y_X(A) \) is called a pairwise RL-fuzzy precompact (briefly, PRP-fcom) with respect to \( (R_1, R_2) \) if for every \( R \subseteq Y_X(A) \), the next inequality is true:

\[
\bigwedge_{D \in R} (i, j) - D \leq \bigwedge_{x \in X} \left( \bigvee_{D \in R} H(x) \bigvee_{D \in R} D(x) \right)
\]

where \( 2^R \) denotes the family of all finite subcollections of \( R \).

Theorem 28. Let \( \mathcal{N} \) be an RL-fbt on \( A \in Y_X \). An L-fuzzy set \( H \in Y_X(A) \) is called a PRL-fcom with respect to \( (R_1, R_2) \) if for every \( W \subseteq Y_X(A) \), we have

\[
\bigvee_{D \in R} (i, j) - D \leq \bigvee_{x \in X} \left( \bigwedge_{D \in R} H(x) \bigvee_{D \in R} D(x) \right)
\]

Proof. Straightforward.

Theorem 29. If \( (R_1, R_2) \) is an RL-fbt on \( A \in Y_X \), and \( B \in Y_Y(A) \), then the following conditions are equivalent:

1. \( H \) is a PRL-fcom
2. For every \( (i, j) \in I \), each strong \( \alpha \)-remote family \( \mathcal{R} \) of \( H \) with \( \wedge_{D \in \mathcal{R}} (i, j) - D \leq \alpha' \) has a finite subfamily \( \mathcal{R}' \) which is a \( (\alpha, \beta) \)-remote family of \( H \)
3. For every \( (i, j) \in I \), each strong \( \alpha \)-remote family \( \mathcal{R} \) of \( H \) with \( \wedge_{D \in \mathcal{R}} (i, j) - D \leq \alpha' \), there exists a finite subfamily \( \mathcal{R}' \) of \( \mathcal{R} \) and \( \beta \in \beta'(\alpha) \) with \( \mathcal{R}' \) is a \( (\alpha, \beta) \)-remote family of \( H \)
4. For every \( (i, j) \in I \), each strong \( \alpha \)-shading \( \mathcal{U} \) of \( H \) with \( \wedge_{D \in \mathcal{R}} (i, j) - D \leq \alpha \) has a finite subfamily \( \mathcal{U}' \) which is a \( (\alpha, \beta) \)-shading of \( H \)
5. For every \( (i, j) \in I \), each strong \( \alpha \)-shading \( \mathcal{U} \) of \( H \) with \( \wedge_{D \in \mathcal{R}} (i, j) - D \leq \alpha \), there exists a finite subfamily \( \mathcal{U}' \) of \( \mathcal{U} \) and \( \beta \in \beta'(\alpha) \) with \( \mathcal{U}' \) is a \( (\alpha, \beta) \)-shading of \( H \)
6. For every \( (i, j) \in I \) and \( \beta \in \beta'(\alpha) \), every Q\textsuperscript{0} of \( H \) has a finite subfamily \( \mathcal{U}' \) which is a \( \beta' \)-cover of \( H \)
7. For every \( (i, j) \in I \) and \( \beta \in \beta'(\alpha) \), every Q\textsuperscript{0} of \( H \) has a finite subfamily \( \mathcal{U}' \) which is a \( \beta' \)-cover of \( H \)

Proof. Straightforward.

Theorem 30. Let \( (R_1, R_2) \) be an RL-fbt on \( A \in Y_X \). An L-fuzzy set \( H \in Y_X(A) \) is called a PRL-fcom with respect to \( \mathcal{N} \):

\[
\bigvee_{D \in \mathcal{N}} (i, j) - D \leq \bigwedge_{x \in X} \left( \bigvee_{D \in \mathcal{N}} H(x) \bigwedge_{D \in \mathcal{N}} D(x) \right)
\]

Proof. Straightforward.
Proof. Straightforward.

**Definition 31.** Let \( A \in \mathcal{V}^L_X \), \( B \in \Psi^L_X(A) \), \( \alpha \in J(L) \), and \( (A, \mathcal{R}_1, \mathcal{R}_2) \) be an \( L \)-bts. An \( L \)-fuzzy set \( H \) is said to be an \( \alpha \) -pairwise \( \mathcal{R} \)-fuzzy precompact (briefly, \( aPRL \)-fpcom) if and only if for each \( \beta \in \beta(\alpha) \), \( \beta \) -preopen cover \( \mathcal{U} \) of \( H \) has a finite subfamily \( \mathcal{V} \) which is a \( Q_\beta \) -preopen cover of \( H \).

**Theorem 32.** Let \( A \in \mathcal{V}^L_X \) and \( (A, \mathcal{R}_1, \mathcal{R}_2) \) be an \( L \)-bts. An \( L \)-fuzzy set \( H \in \Psi^L_X(A) \) is \( aPRL \)-fpcom if and only if \( H \) is an \( aPRL \)-fpcom for every \( \alpha \in J(L) \).

**Proof.** If \( H \) is an \( aPRL \)-fpcom, then for every \( \alpha \in L_1 \), \( \beta \in \beta(\alpha) \), and \( \mathcal{U} \) which can be any \( Q_\beta \) -preopen cover of \( H \), we obtain

\[
\bigwedge_{x \in X} \left( \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right) \right).
\]

(19)

and \( \alpha \leq \bigwedge_{x \in X} \left( \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \), so that

\[
\alpha \leq \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right).
\]

(20)

By \( \beta \in \beta(\alpha) \), we have

\[
\beta \leq \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right).
\]

(21)

Then, there is \( \mathcal{V} \in 2^W \) such that \( \beta \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \). It follows that \( \mathcal{V} \) is \( Q_{\beta(i,j)} \)-preopen cover of \( H \).

Now suppose that every \( Q_\beta \) -preopen cover \( \mathcal{U} \) of \( H \) has a finite subfamily \( \mathcal{V} \) which is a \( Q_\beta \) -preopen cover of \( H \) for each \( \beta \in \beta(\alpha) \). Then, \( \alpha \leq \bigwedge_{x \in X} \left( \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \). Hence, \( \alpha \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \). And so, \( \alpha \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{x \in X} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \).

(22)

Therefore,

\[
\bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right).
\]

(25)

**Theorem 33.** Let \( A \in \mathcal{V}^L_X \) and \( (A, \mathcal{R}_1, \mathcal{R}_2) \) be an \( L \)-bts. An \( L \)-fuzzy set \( H \in \Psi^L_X(A) \) is a \( PRL \)-fpcom in \( (A, \mathcal{R}_1, \mathcal{R}_2) \) if \( H \) is an \( aPRL \)-fpcom in \( (A, \mathcal{R}_1[a], \mathcal{R}_2[a]) \) for each \( \alpha \in J(L) \).

**Proof.** If \( H \in \Psi^L_X(A) \) be a \( PRL \)-fpcom in \( (A, \mathcal{R}_1, \mathcal{R}_2) \), then for every family \( \mathcal{U} \subseteq \Psi^L_X(A) \), we have

\[
\bigwedge_{D(x) \in \mathcal{W}} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right)
\]

\[
\bigwedge_{D(x) \in \mathcal{W}} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right)
\]

(26)

Hence, for all \( \alpha \in J(L) \) and \( \mathcal{U} \subseteq (i,j) - \mathcal{P}_{\mathcal{R}i[a]} \), we have that

\[
\alpha \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right).
\]

(27)

Hence, for each \( \beta \in \beta(\alpha) \), there is \( \mathcal{V} \in 2^W \) such that \( \beta \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \). Hence, \( \alpha \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \). Hence, \( \beta \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right) \).

(28)

Then,

\[
\alpha \leq \bigwedge_{x \in X} \left( \left( \mathcal{L}_H(x) \lor D(x) \right) \left( \bigvee_{D(x) \in \mathcal{W}} (\mathcal{L}_H(x) \lor D(x)) \right) \right).
\]

(29)

So that \( H \) is an \( PRL \)-fpcom in \( (A, \mathcal{R}_1, \mathcal{R}_2) \).

**Lemma 34.** Let \( A \in \mathcal{V}^L_X \) and \( (A, \mathcal{R}_1, \mathcal{R}_2) \) be an \( L \)-bitopological space, \( \alpha \in J(L) \), and \( H, G \in \Psi^L_X(A) \). If \( H \) is an \( aPRL \)-fpcom and \( G \) is \( (i,j) \)-\( PRL \)-preopen, then \( H \lor G \) is an \( aPRL \)-fpcom.

As an immediate consequence from the above lemma, we have the following theorem:
Theorem 35. Let $A \in \Psi_{X}^{L}$ and $(\mathcal{A}, \mathcal{R}_1, \mathcal{R}_2)$ be an $L$-fbts, and $H, G \in \Psi_{X}^{L}(A)$. If $H$ is a PRL-fpcom and $(i, j) - \mathcal{F}^{\alpha}(G) = 1_L$, then $H \wedge G$ is a PRL-fpcom.

Lemma 36. Let $A \in \Psi_{X}^{L}$ and $(\mathcal{A}, \mathcal{R}_1, \mathcal{R}_2)$ be an $L$-fbts, $a \in \mathcal{J}(L)$, and $H, G \in \Psi_{X}^{L}(A)$. If $H, G$ are aPRL-fpcom, then $H \vee G$ is aPRL-fpcom.

Theorem 37. Let $A \in \Psi_{X}^{L}$ and $(\mathcal{A}, \mathcal{R}_1, \mathcal{R}_2)$ be an $L$-fbts, and $H, G \in \Psi_{X}^{L}(A)$. If $H, G$ are PRL-fpcom, then $H \vee G$ is PR $L$-fpcom.

Proof. Straightforward.

Lemma 38. Let $(\mathcal{A}, \mathcal{R}_1, \mathcal{R}_2), (B, \mathcal{R}_1, \mathcal{R}_2^*)$ be $L$-fbts’s on $A \in \Psi_{X}^{L}$ and $B \in \Psi_{Y}^{L}$, respectively, $a \in \mathcal{J}(L)$, $H \in \Psi_{X}^{L}(A)$, and $\psi_{LA} : A \rightarrow B$ be a PRL-irresolute function. If $H$ is aPRL-fpcom in $(A, \mathcal{R}_1, \mathcal{R}_2)$, then $\psi_{LA}(H)$ is aPRL-fpcom in $(B, \mathcal{R}_1^*, \mathcal{R}_2^*)$.

Theorem 39. Let $(\mathcal{A}, \mathcal{R}_1, \mathcal{R}_2)$ and $(B, \mathcal{R}_1, \mathcal{R}_2^*)$ be two $L$-fbts’s on $A \in \Psi_{X}^{L}$ and $B \in \Psi_{Y}^{L}$, respectively, $H \in \Psi_{X}^{L}(A)$, and $\psi_{LA} : A \rightarrow B$ be a PRL-fuzzy irresolute function. If $H$ is a PRL-fpcom in $(A, \mathcal{R}_1, \mathcal{R}_2)$, then $\psi_{LA}(H)$ is a PRL-fpcom in $(B, \mathcal{R}_1^*, \mathcal{R}_2^*)$.

Proof. Let $H$ be a PRL-fpcom in $(A, \mathcal{R}_1, \mathcal{R}_2)$. By Theorem 33, we have $H$ which is aPRL-fpcom in $(A, \mathcal{R}_1, \mathcal{R}_2)$ for every $a \in \mathcal{J}(L)$. By Theorem 33, $\psi_{LA} : (A, \mathcal{R}_1, \mathcal{R}_2) \rightarrow (B, \mathcal{R}_1^*, \mathcal{R}_2^*)$ is PRL-irresolute. Therefore, based on Lemma 38, $\psi_{LA}(H)$ is aPRL-fpcom in $(B, \mathcal{R}_1^*, \mathcal{R}_2^*)$. Thus, $\psi_{LA}(H)$ is PRL-fpcom in $(B, \mathcal{R}_1^*, \mathcal{R}_2^*)$.

6. Conclusion

In this paper, we presented the gradation of preopenness of $L$-fuzzy sets in $L$-fuzzy bitopological spaces relied on pseudo-complement. The new gradation is used to extend and characterize pairwise $L$-fuzzy precontinuous and pairwise $L$-fuzzy preirresolute functions. Moreover, we discussed pairwise $L$-fuzzy precompactness of an $L$-fuzzy set in $L$-fuzzy bitopological spaces. We think that our findings present more general results and it will open the way for many other studies.

Data Availability

No data were used to support this study.

Disclosure

O. H. Khalil’s current address is Department of Mathematics, Faculty of Science in Al-Zulfi, Majmaah University, 11952, Saudi Arabia. A. Ghareeb’s current address is Department of Mathematics, College of Science, Al-Baha University, Al-Baha 65799, Saudi Arabia

Conflicts of Interest

The authors declare that they have no conflict of interest.

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