Hindawi
Abstract and Applied Analysis
Volume 2023, Article ID 9505980, 8 pages
https://doi.org/10.1155/2023/9505980

Research Article

Investigation of Fractional Calculus for Extended Wright Hypergeometric Matrix Functions

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Received 20 January 2023; Revised 15 February 2023; Accepted 28 March 2023; Published 28 April 2023

Academic Editor: A. Ghareeb

Throughout this paper, we will present a new extension of the Wright hypergeometric matrix function by employing the extended Pochhammer matrix symbol. First, we present the extended hypergeometric matrix function and express certain integral equations and differential formulae concerning it. We also present the Mellin matrix transform of the extended Wright hypergeometric matrix function. After that, we present some fractional calculus findings for these expanded Wright hypergeometric matrix functions. Lastly, we present several theorems of the extended Wright hypergeometric matrix function in fractional Kinetic equations.

1. Introduction and Preliminaries

Special functions are significant in many disciplines of mathematics nowadays because specific forms of these functions have become vital tools in several sciences such as mathematical physics, probability theory, computer science, and engineering (see [1, 2]).

Special matrix functions demonstrate their relevance in addressing several physics issues, and their applications in statistics, lie groups, and differential equations are developing and becoming an active area in recent projects. Independent research is being conducted on new extensions of special matrix functions such as the beta matrix function, gamma matrix function, and Gaussian hypergeometric matrix function.

In this paper, the null matrix and identity matrix in $\mathbb{C}^{r\times r}$ will be denoted as O and I, respectively. If a matrix $\zeta \in \mathbb{C}^{r\times r}$, then, the spectrum of ζ is the collection of all eigenvalues of ζ and is represented by $\sigma(\zeta)$. A matrix $\zeta \in \mathbb{C}^{r\times r}$ is a positive stable if $\mathrm{Re}\,(\nu) > 0$ for all $\nu \in \sigma\,(\zeta)$.

If $g(z)$ and $h(z)$ are holomorphic functions defined on an open set $D \subseteq \mathbb{C}$ and if ζ is a matrix in $\mathbb{C}^{r\times r}$ such that $\sigma(\zeta) \subset D$ then $g(\zeta)h(\zeta) = h(\zeta)g(\zeta)$ (see [3]). Additionally, if ζ is a matrix in $\mathbb{C}^{r\times r}$ such that $\sigma(\zeta) \subset D$ and $\zeta\eta = \eta\zeta$, then, $g(\zeta)h(\eta) = h(\eta)g(\zeta)$. If ζ is a positive stable matrix in $\mathbb{C}^{r\times r}$, then, the gamma matrix function $\Gamma(\zeta)$ is defined as follows (see [3–6]):

$$\Gamma(\zeta) = \int_0^\infty t^{\zeta-I} e^{-t}\, dt \text{ where } t^{\zeta-I} = e^{(\zeta-I)\ln t}. \tag{1}$$

If ζ and η are positive stable matrices in $\mathbb{C}^{r\times r}$, then, the beta matrix function is defined by (see [3–6])

$$\beta(\zeta,\eta) = \int_0^1 t^{\zeta-I}(1-t)^{\eta-I}\, dt. \tag{2}$$

Also, if ζ, η and $\zeta+\eta$ are positive stable matrices in $\mathbb{C}^{r\times r}$ such that $\zeta\eta = \eta\zeta$ then (see [3, 4])

$$\beta(\zeta,\eta) = \Gamma(\zeta)\Gamma(\eta)\Gamma^{-1}(\zeta+\eta). \tag{3}$$

If ζ is a matrix in $\mathbb{C}^{r\times r}$ such that

$$\zeta + nI \text{ is invertible for all } n \geq 0, \tag{4}$$

then, the version Pochhammer matrix symbol is defined by (see [3])

$$(\zeta)_n = \zeta(\zeta+I)(\zeta+2I)\cdots..(\zeta+(n-1)I) \quad \text{where } n \geq 1 \text{ and } (\zeta)_0 = I. \tag{5}$$

From [7], if ζ, ρ are positive stable matrices in $\mathbb{C}^{r\times r}$ and ζ satisfies condition (4), then the extended Gamma matrix function is defined by

$$\Gamma(\zeta,\rho) = \begin{cases} \int_0^\infty t^{\zeta-I} e^{-It-\rho/t} dt, & \text{if} \quad \rho \neq 0, \\ \Gamma(\zeta), & \text{if} \quad \rho = 0. \end{cases} \tag{6}$$

and the new extended Pochhammer matrix symbol is given by

$$(\zeta,\rho)_n = \begin{cases} \Gamma^{-1}(\zeta)\Gamma(\zeta+nI,\rho), & \text{if} \quad \rho \neq 0, \\ (\zeta)_n, & \text{if} \quad \rho = 0. \end{cases} \tag{7}$$

The new extended Pochhammer matrix symbol is satisfying the following property (see [7]):

$$(\zeta,\rho)_{m+n} = (\zeta)_n(\zeta+nI,\rho)_m. \tag{8}$$

The Gauss hypergeometric matrix function ${}_2F_1(\zeta,\eta, \nu;z)$ is defined as follows (see [3, 8, 9]):

$${}_2F_1(\zeta,\eta;\nu;z) = \sum_{n=0}^{\infty} (\zeta)_n(\eta)_n\left[(\nu)_n\right]^{-1}\frac{z^n}{n!}, \tag{9}$$

where ζ, η and $\nu \in \mathbb{C}^{r\times r}$, ν satisfies the condition (4) and $|z| < 1$.

The Wright hypergeometric matrix function is defined in [10] as follows:

$${}_2R_1^{(\tau)}[\zeta,\eta,\nu;z] = \Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta)_n\Gamma^{-1} \cdot (\nu+n\tau I)\Gamma(\eta+n\tau I)\frac{z^n}{n!}, \tag{10}$$

where $\tau \in R_+$ and ζ, η and ν are positive stable matrix in $\mathbb{C}^{r\times r}$ and ν satisfies the condition (4).

If η and ν are positive stable matrices function in $\mathbb{C}^{r\times r}$ and ν satisfies the condition (4) then the Wright Kummer hypergeometric matrix function is defined in [10] as follows:

$${}_1R_1(\eta,\nu;z) = \Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}. \tag{11}$$

This article is organized into five sections. In Section 2, we will provide a new extension of the Wright hypergeometric matrix function and prove some theorems about integral and derivative formula of the extension of the Wright hypergeometric matrix function ${}_2R_1^\tau[(\zeta,\rho),\eta,\nu;z]$. In Section 3, we state the Mellin matrix transform of the extended Wright hypergeometric matrix function.

In Section 4, we applied certain fractional calculus ideas to the extended Wright hypergeometric matrix function. Lastly, in Section 5, we discuss several applications of ${}_2R_1^\tau[(\zeta,\rho),\eta,\nu;z]$ in fractional kinetic equations.

2. The Extended Wright Hypergeometric Matrix Function

In terms of the generalized Pochhammer matrix symbol $(\zeta,\rho)_n$, we introduce the extended Gauss hypergeometric matrix function ${}_2F_1[(\zeta,\rho),\eta,\nu;z]$ and the extended Wright hypergeometric matrix function ${}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]$ as follows.

Definition 1. Let ζ, η, ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$ and ν satisfies the condition (4) then the extended Gauss hypergeometric matrix function is given by

$${}_2F_1[(\zeta,\rho),\eta;\nu;z] = \sum_{n=0}^{\infty}(\zeta,\rho)_n(\eta)_n\left[(\nu)_n\right]^{-1}\frac{z^n}{n!}. \tag{12}$$

Definition 2. Let ζ, η, ν, and ρ are positive stable matrices in $\mathbb{C}^{r\times r}$ and ν satisfies the condition (4) then the extended Wright hypergeometric matrix function is

$${}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z] = \Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1} \cdot (\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}, \tag{13}$$

where $\tau \in (0,\infty)$.

Remark 3. Several particular remarks of the extended Wright hypergeometric matrix function are mentioned below:

(i) When $\rho = 0$ in (13), we get the Wright hypergeometric matrix function defined in (10)

(ii) If we put $\tau = 1$ and $\rho = 0$ in (13), we get the Gauss hypergeometric matrix function as in (9)

(iii) If $\rho = 0$ and $\zeta = \alpha I, \eta = \beta I, \nu = \gamma I$ (where α, β, and γ are in $\mathbb{C}$) in (13) then we get the Gauss hypergeometric function (see [11])

2.1. Integral and Derivative Formula of ${}_2R_1^\tau[(\zeta,\rho),\eta,\nu;z]$. In this part, we will provide integral representation and derivative formula of the extended Wright hypergeometric matrix function.

Theorem 4. *Let ζ, η, ν, and P be matrices in $\mathbb{C}^{r\times r}$ such that $\nu\eta=\eta\nu$ and ν,η,ν-η, and P are positive stable, then for $|z| < 1$, $\tau\in R_+$, we have*

$$
{}_2R_1^{\tau}[(\zeta,\rho),\eta,\nu\,;z]=\Gamma(\eta)^{-1}\Gamma^{-1}(\nu-\eta)\Gamma(\nu)x\int_0^1 t^{\eta-I}\cdot(1-t)^{\nu-\eta-I}{}_1F_0[(\zeta,\rho),-,-;z]\,dt, \tag{14}
$$

where ${}_1F_0[(\zeta,\rho),-,-;z]=\sum_{n=0}^{\infty}(\zeta,\rho)_n(z^n/n!)$.

Proof. From (2) and (3), we find that

$$
\begin{aligned}
&\Gamma^{-1}(\eta)\Gamma^{-1}(\nu+n\tau I)\Gamma(\nu)\Gamma(\eta+n\tau I)\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\nu-\eta)\Gamma(\nu)\int_0^1 t^{\eta+(n\tau-1)I}(1-t)^{\nu-\eta-I}\,dt.
\end{aligned} \tag{15}
$$

Now, we can write

$$
\begin{aligned}
&{}_2R_1^{(\tau)}[(\zeta,\rho),\eta,\nu\,;z]\\
&\quad=\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\nu-\eta)\Gamma(\nu)\left[\int_0^1 t^{\eta+(n\tau-1)I}\right.\\
&\qquad\left.\cdot(1-t)^{\nu-\eta-I}\sum_{n=0}^{\infty}(\zeta,\rho)_n\frac{z^n}{n!}\,dt\right]\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\nu-\eta)\Gamma(\nu)\left[\int_0^1 t^{\eta-I}\right.\\
&\qquad\left.\cdot(1-t)^{\nu-\eta-I}\sum_{n=0}^{\infty}(\zeta,\rho)_n\frac{(zt^{\tau})^n}{n!}\,dt\right]\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\nu-\eta)\Gamma(\nu)\int_0^1 t^{\eta-I}\\
&\qquad\cdot(1-t)^{\nu-\eta-I}{}_1F_0((\zeta,\rho),-,-;zt^{\tau})\,dt.
\end{aligned} \tag{16}
$$

This complete the proof. □

Theorem 5. *Let ζ, η, ν, κ, and ρ be matrices in $\mathbb{C}^{r\times r}$ such that $\nu\eta=\eta\nu$ and ρ, ν, η, and $\nu+\kappa$ are positive stable. Then, for $|\alpha z|<1$, we have*

$$
\begin{aligned}
&\Gamma^{-1}(\kappa)\Gamma^{-1}(\nu)\Gamma(\nu+\kappa)\int_0^z t^{\nu-I}(z-t)^{\kappa-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\nu\,;\alpha t]dt\\
&\quad=z^{\nu+\kappa-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\nu\,;\alpha z].
\end{aligned} \tag{17}
$$

Proof. We observe that

$$
\begin{aligned}
&\Gamma^{-1}(\kappa)\Gamma^{-1}(\nu)\Gamma(\nu+\kappa)\int_0^z t^{\nu-I}(z-t)^{\kappa-I}{}_2R_1^{\tau}[(\zeta,\rho),\eta,\nu\,;\alpha t]dt\\
&\quad=\Gamma^{-1}(\kappa)\Gamma^{-1}(\nu)\Gamma(\nu+\kappa)\int_0^z t^{\nu-I}(z-t)^{\kappa-I}\\
&\qquad\times\left\{\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}\right.\\
&\qquad\left.\cdot(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{(\alpha t)^n}{n!}\right\}dt,
\end{aligned} \tag{18}
$$

substituting $t=zu$, we find that

$$
\begin{aligned}
&\Gamma^{-1}(\kappa)\Gamma^{-1}(\nu)\Gamma(\nu+\kappa)\int_0^z t^{\nu-I}(z-t)^{\kappa-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\nu\,;\alpha t]dt\\
&\quad=\Gamma^{-1}(\kappa)\Gamma^{-1}(\nu)\Gamma(\nu+\kappa)\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}\\
&\qquad\cdot\left[(\zeta,\rho)_n\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\right.\\
&\qquad\left.\times\int_0^1 z^{\nu-I}u^{\nu-I}z^{\kappa-I}(1-u)^{\kappa-I}u^n\frac{(\alpha z)^n}{n!}z\,du\right]\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\kappa)\Gamma(\nu+\kappa)\beta(\nu,\kappa)z^{\nu+\kappa-I}\\
&\qquad\times\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}(\nu+n\tau I)\Gamma(\eta+n\tau I)\frac{(\alpha z)^n}{n!}\\
&\quad=\Gamma^{-1}(\eta)\Gamma^{-1}(\kappa)\Gamma(\nu+\kappa)\Gamma(\nu)\Gamma(\kappa)\Gamma^{-1}(\nu+\kappa)\\
&\qquad\times z^{\nu+\kappa-I}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}\\
&\qquad\cdot(\nu+n\tau I)\Gamma(\eta+n\tau I)\frac{(\alpha z)^n}{n!}\\
&\quad=z^{\nu+\kappa-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\nu\,;\alpha z],
\end{aligned} \tag{19}
$$

this completes the proof. □

Theorem 6. *Let ζ, η, ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$ then each of the following integrals hold true:*

(i)

$$
\begin{aligned}
&{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\nu\,;z]\\
&\quad=\frac{1}{\tau}\Gamma^{-1}(\tau I)\Gamma^{-1}(\nu-\tau I)\Gamma(\nu)\\
&\qquad\times\int_0^1\left(1-t^{1/\tau}\right)^{\nu-(\tau+1)I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta\,;\tau I\,;tz]\,dt
\end{aligned} \tag{20}
$$

(ii)

$$
\begin{aligned}
&{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\\
&\quad=\Gamma^{-1}(\tau I)\Gamma^{-1}(\nu-\tau I)\Gamma(\nu)\\
&\quad\quad\times\int_0^1 t^{(\tau-1)I}(1-t)^{\nu-(\tau+1)I}\,{}_2R_1^{(\tau)}\\
&\quad\quad\cdot[(\zeta,\rho),\eta;\nu-\tau I;z(1-t)^{\tau}]\,dt
\end{aligned}
\tag{21}
$$

Proof.

(i) let $W=\int_0^1(1-t^{1/\tau})^{\nu-(\tau+1)I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\tau I;tz]\,dt$

From the definition of ${}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]$, we have

$$
\begin{aligned}
W&=\int_0^1\left(1-t^{1/\tau}\right)^{\nu-(\tau+1)I}\Gamma^{-1}(\eta)\Gamma(\tau I)\\
&\quad\times\sum_{n=0}^{\infty}(\zeta,\rho)_n\,\Gamma^{-1}((n+1)\tau I)\Gamma(\eta+n\tau I)\frac{(tz)^n}{n!}\,dt,
\end{aligned}
\tag{22}
$$

put $s=t^{1/\tau}$, and using the definition of beta matrix function, we have

$$
\begin{aligned}
W&=\tau\Gamma^{-1}(\eta)\Gamma(\tau I)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma((n+1)\tau I)\Gamma(\eta+n\tau I)\\
&\quad\times\int_0^1 s^{(n\tau+\tau-1)I}(1-s)^{\nu-(\tau+1)I}\frac{z^n}{n!}\,ds\\
&=\tau\Gamma^{-1}(\eta)\Gamma(\tau I)\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}((1+n)\tau I)\Gamma\\
&\quad\cdot(\eta+n\tau I)\,\beta((n+1)\tau I,\nu-\tau I)\frac{z^n}{n!}\\
&=\tau\Gamma^{-1}(\eta)\Gamma(\tau I)\sum_{n=0}^{\infty}\Big\{(\zeta,\rho)_n\Gamma^{-1}((n+1)\tau I)\Gamma\\
&\quad\cdot(\eta+n\tau I)\Gamma((n+1)\tau I)\Gamma(\nu-\tau I)\Gamma^{-1}(\nu+n\tau I)\frac{z^n}{n!}\Big\}\\
&=\tau\Gamma^{-1}(\nu)\Gamma(\nu-\tau I)\Gamma(\tau I)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z].
\end{aligned}
\tag{23}
$$

This can easily be written as

$$
\begin{aligned}
&{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\\
&\quad=\frac{1}{\tau}\Gamma^{-1}(\tau I)\Gamma^{-1}(\nu-\tau I)\Gamma(\nu)\\
&\quad\quad\times\int_0^1\left(1-t^{1/\tau}\right)^{\nu-(\tau+1)I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\tau I;tz]dt,
\end{aligned}
\tag{24}
$$

and this finishes the proof of (i)

(ii) Let $K=\int_0^1 t^{(\tau-1)I}(1-t)^{\nu-(\tau+1)I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-\tau I;(1-t)^{\tau}z]dt$ by using the definition of ${}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]$, we find that

$$
\begin{aligned}
K&=\int_0^1 t^{(\tau-1)I}(1-t)^{\nu-(\tau+1)I}\Gamma^{-1}(\eta)\Gamma(\nu-\tau I)\\
&\quad\times\sum_{n=0}^{\infty}(\zeta,\rho)_n\Gamma^{-1}(\nu-(1-n)\tau I)\Gamma(\eta+n\tau I)\frac{z^n(1-t)^{\tau n}}{n!}\,dt\\
&=\Gamma^{-1}(\eta)\Gamma(\nu-\tau I)\sum_{n=0}^{\infty}\Big[(\zeta,\rho)_n\Gamma^{-1}(\nu-(1-n)\tau I)\Gamma\\
&\quad\cdot(\eta+n\tau I)\frac{z^n}{n!}\times\int_0^1 t^{(\tau-1)I}(1-t)^{\nu-(\tau-n\tau+1)I}\,dt\Big]\\
&=\Gamma^{-1}(\eta)\Gamma(\nu-\tau I)\sum_{n=0}^{\infty}\Big[\Big[(\zeta,\rho)_n\Gamma^{-1}(\nu+(1-n)\tau I)\Gamma\\
&\quad\cdot(\eta+n\tau I)\frac{z^n}{n!}\times\Gamma(\tau I)\Gamma(\nu-(1-n)\tau I)\Gamma^{-1}(\nu+\tau nI)\Big]\\
&=\Gamma^{-1}(\nu)\Gamma(\tau I)\Gamma(\nu-\tau I)\Big[\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}\\
&\quad\cdot(\zeta,\rho)_n\Gamma^{-1}(\nu+n\tau I)\Gamma(\eta+n\tau I)\frac{z^n}{n!}\Big]\\
&=\Gamma^{-1}(\nu)\Gamma(\tau I)\Gamma(\nu-\tau I)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z],
\end{aligned}
\tag{25}
$$

and this can easily be written as

$$
\begin{aligned}
&{}_2\,R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\\
&\quad=\Gamma^{-1}(\tau I)\Gamma^{-1}(\nu-\tau I)\Gamma(\nu)\times\int_0^1 t^{(\tau-1)I}(1-t)^{\nu-(\tau+1)I}\,{}_2R_1^{(\tau)}\\
&\quad\quad\cdot[(\zeta,\rho),\eta;\nu-\tau I;(1-t)^{\tau}z]dt.
\end{aligned}
\tag{26}
$$

This completes the proof. □

Theorem 7. *Let ζ, η, ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$ then the following derivative formula hold true*

$$
\begin{aligned}
&\frac{d}{dz}\left\{z^{\eta-I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}]\right\}\\
&\quad=z^{\eta-2I}t\Big\{(\nu-I)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-I;\alpha z^{\tau}]\\
&\quad\quad+(\eta-\nu)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}]\Big\}.
\end{aligned}
\tag{27}
$$

Proof. From the definition of extended Wright hypergeometric matrix function, we have

$$
\begin{aligned}
&\frac{d}{dz}\left\{z^{\eta-I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}]\right\}\\
&\quad=\frac{d}{dz}\Big\{\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n\\
&\quad\quad\cdot\frac{\alpha^n z^{\eta+(\tau n-1)I}\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)}{n!}\Big\}
\end{aligned}
$$

$$= \Gamma^{-1}(\eta)\Gamma(\nu)\left\{ \sum_{n=0}^{\infty} (\zeta,\rho)_n (\eta + (\tau n - 1)I) \cdot \frac{\alpha^n z^{\eta+(\tau n-2)I}\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)}{n!} \right\}$$
$$= z^{\eta-2I}\Gamma^{-1}(\eta)\Gamma(\nu)\left\{ \sum_{n=0}^{\infty} (\zeta,\rho)_n \cdot \frac{z^{\tau n}\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\alpha^n(\tau nI+\nu-I+\eta-\nu)}{n!} \right\}$$
$$= z^{\eta-2I}\left\{ (\nu-I)\Gamma^{-1}(\eta)\Gamma(\nu-I)\sum_{n=0}^{\infty} (\zeta,\rho)_n \cdot \frac{(\alpha z^{\tau})^n\Gamma^{-1}(\nu+(\tau n-1)I)\Gamma(\eta+\tau nI)}{n!} + (\eta-\nu)\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_n \cdot \frac{(\alpha z^{\tau})^n\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)}{n!} \right\}$$
$$= z^{\eta-2I}\left\{ (\nu-I)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-I;\alpha z^{\tau}] + (\eta-\nu)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}] \right\}. \tag{28}$$

This completes the proof. □

Theorem 8. *Let ζ, η, ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$, then the following derivative formula hold true:*

$$\left(\frac{d}{dz}\right)^n \left\{ z^{\nu-I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}] \right\} = \Gamma^{-1}(\nu-nI)\Gamma(\nu)z^{\nu-(n+1)I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-nI;\alpha z^{\tau}]. \tag{29}$$

Proof. By using Definition (2) and differentiating term by term under the sign of summation, we have

$$\left(\frac{d}{dz}\right)^n \left\{ z^{\nu-I}\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha z^{\tau}] \right\}$$
$$= \Gamma^{-1}(\eta)\Gamma(\nu)\sum_{m=0}^{\infty}(\zeta,\rho)_m\Gamma^{-1}(\nu+\tau mI)\Gamma \cdot (\eta+\tau mI)\frac{\alpha^m}{m!}\left(\frac{d}{dz}\right)^n z^{\nu+(\tau m-1)I}$$
$$= \Gamma^{-1}(\eta)\Gamma(\nu)\sum_{m=0}^{\infty}(\zeta,\rho)_m\Gamma^{-1}(\nu+(\tau m-n)I)\Gamma \cdot (\eta+\tau mI)\frac{\alpha^m}{m!}z^{\nu+(\tau m-n-1)I}$$
$$= z^{\nu-(n+1)I}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{m=0}^{\infty}(\zeta,\rho)_m\Gamma^{-1} \cdot (\nu+(\tau m-n)I)\Gamma(\eta+\tau mI)\frac{(\alpha z^{\tau})^m}{m!}$$
$$= z^{\nu-(n+1)I}\Gamma^{-1}(\nu-nI)\Gamma(\nu)\,{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-nI;\alpha z^{\tau}]. \tag{30}$$

This finishes the proof, □

3. Mellin Matrix Transform

Definition 9. Let $F(\zeta)$ be a function defined on the set of all positive stable matrices contained in $\mathbb{C}^{r\times r}$, then the Mellin transform is defined as follows:

$$M\{F(\zeta)\colon \zeta \longrightarrow \lambda\} = \int_0^{\infty} \zeta^{\lambda-I}F(\zeta)\,d\zeta. \tag{31}$$

Such that the integral in right hand side exists.

The following lemma will be a useful tool in next theorem.

Lemma 10. *Let ζ, ρ, λ, and $\zeta+\lambda$ are positive stable matrices in $\mathbb{C}^{r\times r}$, then*

$$M\{\Gamma(\zeta,\rho)\colon \rho \longrightarrow \lambda\} = \Gamma(\lambda)\Gamma(\zeta+\lambda). \tag{32}$$

Proof. From (31), the Mellin transform of $\Gamma(\zeta,\rho)$ in ρ is

$$M\{\Gamma(\zeta,\rho)\colon \rho \longrightarrow \lambda\} = \int_0^{\infty}\rho^{\lambda-I}\int_0^{\infty} t^{\zeta-I}e^{-t-(\rho/t)}\,dt\,d\rho. \tag{33}$$

From Fubini theorem with a little calculation (see [12]), we get

$$M\{\Gamma(\zeta,\rho)\colon \rho \longrightarrow \lambda\} = \Gamma(\lambda)\int_0^{\infty} t^{\zeta+\lambda-I}e^{-t}dt = \Gamma(\lambda)\Gamma(\zeta+\lambda). \tag{34}$$

This completes the proof. □

Theorem 11. *Let ζ, η, ν, λ, and $\zeta+\lambda$ be positive stable matrices in $\mathbb{C}^{r\times r}$ and ν satisfies the condition (4), then*

$$M\left\{ {}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\colon \rho \longrightarrow \lambda \right\} = \Gamma(\lambda)\Gamma^{-1}(\zeta)\Gamma(\zeta+\lambda)\,{}_2R_1^{(\tau)}[\zeta+\lambda,\eta;\nu;z]. \tag{35}$$

Proof.

$$\begin{aligned}
&M\left\{{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\colon \rho\longrightarrow\lambda\right\}\\
&=\int_0^\infty \rho^{\lambda-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\,d\rho\\
&=\int_0^\infty \rho^{\lambda-I}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^\infty(\zeta,\rho)_n\Gamma^{-1}\\
&\quad\cdot(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}d\rho\\
&=\Gamma^{-1}(\eta)\Gamma(\nu)\Gamma^{-1}(\zeta)\sum_{n=0}^\infty\Gamma^{-1}(\nu+\tau nI)\Gamma\\
&\quad\cdot(\eta+\tau nI)\frac{z^n}{n!}\int_0^\infty\rho^{\lambda-I}\Gamma(\zeta+nI,\rho)d\rho\\
&=\Gamma^{-1}(\eta)\Gamma(\nu)\Gamma^{-1}(\zeta)\sum_{n=0}^\infty\Gamma^{-1}(\nu+\tau nI)\Gamma\\
&\quad\cdot(\eta+\tau nI)\frac{z^n}{n!}M\{\Gamma(\zeta+nI,\rho)\colon\rho\longrightarrow\lambda\}\\
&=\Gamma^{-1}(\eta)\Gamma(\nu)\Gamma^{-1}(\zeta)\sum_{n=0}^\infty\Gamma^{-1}(\nu+\tau nI)\Gamma\\
&\quad\cdot(\eta+\tau nI)\frac{z^n}{n!}\Gamma(\lambda)\Gamma(\zeta+\lambda+nI)\\
&=\Gamma(\lambda)\Gamma^{-1}(\eta)\Gamma(\nu)\Gamma^{-1}(\zeta)\Gamma(\zeta+\lambda)\times\sum_{n=0}^\infty\Gamma^{-1}\\
&\quad\cdot(\zeta+\lambda)\Gamma(\zeta+\lambda+nI)\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}\\
&=\Gamma(\lambda)\Gamma^{-1}(\eta)\Gamma(\nu)\Gamma^{-1}(\zeta)\Gamma(\zeta+\lambda)\\
&\quad\cdot\sum_{n=0}^\infty(\zeta+\lambda)_n\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\frac{z^n}{n!}\\
&=\Gamma(\lambda)\Gamma^{-1}(\zeta)\Gamma(\zeta+\lambda)_2R_1^{(\tau)}[\zeta+\lambda,\eta;\nu;z].
\end{aligned}\tag{36}$$

This finishes the proof. □

4. Fractional Calculus of the Extended Wright Hypergeometric Matrix Function

In this part, we will prove certain theorems concerning the Riemann-Liouville integral of the Wright hypergeometric matrix function. The fractional integral and derivative of Riemann-Liouville of order μ and $x>0$ are defined as follows (see [13, 14]):

$$I^\mu[f(x)]=\frac{1}{\Gamma(\mu)}\int_0^x(x-t)^{\mu-1}f(t)\,dt,\tag{37}$$

$$D^\mu f(x)=D^n[I^{n-\mu}f(x)],D=\frac{d}{dx}.\tag{38}$$

If ζ is a positive stable matrix in $C^{r\times r}$, such that Re $(\mu)>0$ then the following relation holds true (see [10]):

$$I^\mu\left(x^{\zeta-I}\right)=\Gamma(\zeta)\Gamma^{-1}(\zeta+\mu I)x^{\zeta+(\mu-1)I}.\tag{39}$$

Theorem 12. *Let ζ,η,ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$, $\mu\in\mathbb{C}$ such that* Re $(\mu)>0$*, then for each $|wz^\tau|<1$, we have*

$$\begin{aligned}
&I^\mu\left(x^{\nu-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^\tau]\right)\\
&\quad=\Gamma(\nu)\Gamma^{-1}(\nu+\mu I)x^{\nu+(\mu-1)I}\times{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu+\mu I;wx^\tau].
\end{aligned}\tag{40}$$

Proof. From (37), we find that

$$\begin{aligned}
&I^\mu\left(x^{\nu-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^\tau]\right)\\
&=\frac{1}{\Gamma(\mu)}\int_0^x(x-t)^{\mu-1}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wt^\tau]t^{\nu-I}dt\\
&=\frac{\Gamma^{-1}(\eta)\Gamma(\nu)}{\Gamma(\mu)}\times\sum_{n=0}^\infty(\zeta,\rho)_n\Gamma^{-1}(\nu+\tau nI)\Gamma\\
&\quad\cdot((\eta+\tau nI))\frac{w^n}{n!}\left(\int_0^x(x-t)^{\mu-1}t^{\nu+(\tau n-1)I}\,dt\right)\\
&=\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^\infty\frac{(\zeta,\rho)_n\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI)\,w^n}{n!}I^\mu\left(x^{\nu+(\tau n-1)I}\right)\\
&=\Gamma^{-1}(\nu+\mu I)\Gamma(\nu)\left(\Gamma^{-1}(\eta)\Gamma(\nu+\mu I)\right.\\
&\quad\left.\times\sum_{n=0}^\infty\frac{(\zeta,\rho)_n\Gamma^{-1}(\nu+(\tau n+\mu)I)\Gamma(\eta+\tau nI)(wx^\tau)^n}{n!}\right)x^{\nu+(\nu-1)I}\\
&=\Gamma(\nu)\Gamma^{-1}(\nu+\mu I)x^{\nu+(\mu-1)I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu+\mu I;wx^\tau].
\end{aligned}\tag{41}$$

This completes the proof. □

Theorem 13. *Let ζ, η, ν, and ρ be positive stable matrices in $\mathbb{C}^{r\times r}$ and $\mu\in\mathbb{C}$ such that* Re $(\mu)>0$ *then for each $|wz^\tau|<1$, we have*

$$\begin{aligned}
&D^\mu\left(x^{\nu-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^\tau]\right)\\
&\quad=\Gamma(\nu)\Gamma^{-1}(\nu-\mu I)x^{\nu-(\mu+1)I}\times{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu-\mu I;wx^\tau].
\end{aligned}\tag{42}$$

Proof. From (38), we have

$$\begin{aligned}
&D^\mu\left(x^{\nu-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^\tau]\right)\\
&\quad=\left(\frac{d}{dx}\right)^n\left[I^{n-\mu}\left(x^{\nu-I}{}_2R_1^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^\tau]\right)\right].
\end{aligned}\tag{43}$$

From Theorem (12), we find that

$$D^{\mu}\left(x^{\nu-I}{}_{2}R_{1}^{(\tau)}[(\zeta,\rho),\eta;\nu;wx^{\tau}]\right) = \left(\frac{d}{dx}\right)^{n}\Big\{\Gamma(\nu)\Gamma^{-1}(\nu+(n-\mu)I) \times {}_{2}R_{1}^{(\tau)}[(\zeta,\rho),\eta;\nu+(n-\mu)I;wx^{\tau}]x^{\nu+(n-\mu-1)I}\Big\}. \tag{44}$$

Applying (29), we get the required result in (42). □

5. Applications in Fractional Kinetic Equations

In our time, the fractional kinetic equations have a great importance in deferent branches of applied science such as astrophysics, control system, dynamic system, and mathematical physics.

The standard fractional kinetic equation is defined by

$$N(z)-N_{0}=-C_{0}D_{t}^{-\mu}N(z), \quad \text{where } C>0,\ z>0 \text{ and } \operatorname{Re}(\mu)>0, \tag{45}$$

where $N(z)$ is the rate of reaction, $N_0 = N(z=0)$ and ${}_0D^{-\mu}$ is the Riemann-Liouville fractional integral operator defined in (38). Furthermore, Saxena and Kalla (see [15]) considered the following fractional kinetic equations:

$$N(z)-N_{0}f(z)=-C^{\mu}{}_{0}D_{t}^{-\mu}N(z) \quad \text{where } C \text{ is a constant and } \operatorname{Re}(\mu)>0. \tag{46}$$

The Laplace transform of the Riemann-Liouville fractional integral operator is (see [16])

$$L\left[{}_{0}D_{t}^{-\mu}h(z)\right](p)=p^{-\mu}\bar{h}(p), \tag{47}$$

where $\bar{h}(p)$ is the Laplace transform of $h(z)$.

Theorem 14. *Let ζ, η, ν, and C be positive stable matrices in $\mathbb{C}^{r\times r}$ such that ν is invertible, ν satisfies the condition (4) and $|z|<1$, then the solution of the generalized fractional kinetic matrix equation:*

$$N(z)I-N_{0\,2}R_{1}^{(\tau)}[(\zeta,\rho),\eta;\nu;z]=-C^{\mu}{}_{0}D_{t}^{-\mu}N(z), \tag{48}$$

is given by

$$N(z)I=N_{0}\Gamma^{-1}(\zeta)\Gamma(\nu)\times\sum_{n=0}^{\infty}(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma \cdot(\eta+\tau nI)z^{n}\mathbb{E}_{\mu,n+1}(-C^{\mu}z^{\mu}), \tag{49}$$

where

$$\mathbb{E}_{\mu,n+1}(-C^{\mu}z^{\mu})=\sum_{r=0}^{\infty}(-1)^{r}C^{\mu r}\frac{z^{\mu r}}{\Gamma(\mu r+n+1)}, \tag{50}$$

and called the generalized Mittag-Leffler matrix function (see [17, 18]).

Proof. Applying the Laplace transform on the equation (48) and using (47), we get

$$(I+p^{-\mu}C^{\mu})\bar{N}(p)=N_{0}\,L\left[{}_{2}R_{1}^{(\tau)}[(\zeta,\rho),\eta;\nu;z]\right](p) = N_{0}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma \cdot(\eta+\tau nI)p^{-(n+1)}, \tag{51}$$

now we can write

$$\bar{N}(p)I=N_{0}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}\left[(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma(\eta+\tau nI) \times\sum_{r=0}^{\infty}(-1)^{r}C^{\mu r}p^{-(\mu r+n+1)}\right], \tag{52}$$

where $(I+p^{-\mu}C^{\mu})^{-1}=\sum_{r=0}^{\infty}(-1)^{r}C^{\mu r}p^{-\mu r}$

Taking the inverse Laplace transform, we get

$$\begin{aligned}\bar{N}(z)I&=N_{0}\Gamma^{-1}(\eta)\Gamma(\nu)\sum_{n=0}^{\infty}(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma \cdot(\eta+\tau nI)\times\sum_{r=0}^{\infty}(-1)^{r}C^{\mu r}\frac{z^{\mu r+n}}{\Gamma(\mu r+n+1)}\\&=N_{0}\Gamma^{-1}(\zeta)\Gamma(\nu)\times\sum_{n=0}^{\infty}(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma \cdot(\eta+\tau nI)z^{n}\mathbb{E}_{\mu,n+1}(-C^{\mu}z^{\mu}).\end{aligned} \tag{53}$$

This completes the proof. □

Theorem 15. *Let ζ, η, ν, and C be positive stable matrices in $\mathbb{C}^{r\times r}$ such that ν is invertible, ν satisfies the condition (4), $\alpha \in C$ such that $R(\alpha)$ and $|z|<1$, then the solution of the generalized fractional kinetic matrix equation:*

$$N(z)I-N_{0\,2}R_{1}^{(\tau)}[(\zeta,\rho),\eta;\nu;\alpha^{\mu}z]=-C^{\mu}{}_{0}D_{t}^{-\mu}N(z), \tag{54}$$

is given by

$$N(z)I=N_{0}\Gamma^{-1}(\zeta)\Gamma(\nu)\times\sum_{n=0}^{\infty}(\zeta,\rho)_{n}\Gamma^{-1}(\nu+\tau nI)\Gamma \cdot(\eta+\tau nI)\alpha^{\mu n}z^{n}\mathbb{E}_{\mu,\mu n+1}(-C^{\mu}z^{\mu}). \tag{55}$$

Proof. By using the same steps of proof in the previous theorem, we get the required. □

6. Conclusions

The topic of derivative with fractional parameter has lately attracted the attention of academics. For example, Riemann-Liouville developed the concept of fractional order derivative. Later, Caputo and others adjusted this fractional derivative. Because of their physical features, fractional derivatives have been successfully used to mimic numerous real-world issues. Recently, a derivative based on the classical derivative with a fractional parameter was developed. The derivative has highly fascinating qualities; hence, in this work, we have attempted to present some conclusions concerning fractional calculus of these extended Wright hypergeometric matrix functions as well as certain theorems of the extended Wright hypergeometric matrix function in fractional kinetic equations. As future work, and from a numerical point of view, we aim to employ some of the derived formulas in this paper along with suitable spectral methods to treat numerically the differential equations with polynomial coefficients.

Data Availability

Data supporting this manuscript are available from Scopus, Web of Science, and Google Scholar.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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