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Brief Communication
Simple mathematical and computational wear model for ultra-high-molecular-weight polyethylene total hip replacements

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Abstract: Ultra-high-molecular-weight polyethylene is an important constituent of hip implants. Surgical revisions are required because of implant loosening and osteolysis (destruction or resorption of bone tissue). We develop a mathematical and a computational model to determine implant life (defined as the time when 20% of the implants operating at a given wear rate is revised) based on wear rates and apply them to the data of Sochart (*Clin Orthop Relat Res*, 363:135–50, 1999). No significant difference was found between mathematical and computational model predictions (14.8 and 14.7 years, respectively) from the actual value of 15.0 ± 3.9 years ($\bar{X} \pm 2SE$; $P > 0.05$). We find that an increase in cross-linking does not decrease implant life greatly. However, a large decrease in implant life occurs with an increase in surface roughness, cycles per year and body weight or a decrease in contact stress and femoral head radius.

Key words: UHMWPE hip replacement, implant life, wear model

INTRODUCTION

Ultra-high-molecular-weight polyethylene (UHMWPE) is used for total joint arthroplasty (Wang *et al.* 1995, 1998; Scholes *et al.* 2000; Wang 2001; Endo *et al.* 2002; Lewis *et al.* 2003). Evidence suggests that it is linked to osteolysis (destruction or resorption of bone tissue; Willert *et al.* 1977, 1990; Schmalzried *et al.* 1992; McKellop *et al.* 1995; Wirth *et al.* 1999; Mabrey *et al.* 2002). Research has focused on wear rates of UHMWPE by different material parameters (e.g. molecular weight between cross-links [Wang 2001], femoral head roughness [Wang *et al.* 1998], coefficient of friction, contact stress [Wang *et al.* 2001], tensile toughness, crystallinity and protein concentration [Lewis *et al.* 2003]). Ninety percent of implants survive for more than 10 years (Cornell and Ranawat 1986; Skeie *et al.* 1991; Kavanagh *et al.* 1994; Marston *et al.* 1996) and their subsequent failure is commonly due to implant loosening associated with high torque around the stem axis from forces and moments acting on the hip prostheses (Bergmann *et al.* 1995). The onset of patient pain is a means of detecting prosthetic loosening, requiring implant revision surgery. It is important to detect acetabular loosening early to minimize the degree of bone loss (Sochart 1999).

As far as we are aware, no mathematical or computational relationships between implant revisions and wear rates have been developed. We develop a simple mathematical and computational model for predicting the life of an implant given a set of material and patient parameters. Model results are applied to implant survival of acetabular cups with time from Sochart (1999), allowing for the calculation of implant life (I; defined as the time when 20% of the implants operating at a given wear rate are revised) based on a given wear rate.

General wear is given by

$$k = \frac{Vp}{lx} \tag{1}$$

where k, V, x, l and p are wear factor, volume of wear, sliding distance, load and hardness of the bearing surface, respectively (Rabinowicz 1995). In biomedical engineering,

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volumetric wear rate is corrected for loading and sliding distance and k is

$$k = \frac{V}{FS} \tag{2}$$

where $F = \alpha mg$ and m, body mass (assuming a patient of 82 kg), g, acceleration due to gravity (9.8 m/s^2) and $\alpha = 3.5$, reflecting the 'true' load on the femoral head that may reach levels of 6–8 if the patient is running or ascending stairs (Bergmann *et al.* 1993, 1995). The sliding distance per cycle (S) is

$$S = 2\left(\frac{2\pi r\theta}{360^\circ}\right) \times 10^{-3} \tag{3}$$

where θ is the gait angle in degrees (40°; Nordin and Frankel 2001) and r is the femoral head radius (13 mm).

The wear factor depends on the molecular weight between cross-links (M_c):

$$k_1 = 0.0003(M_c - 4258) \times 10^{-6} \tag{4}$$

where $M_c = 6200$ g/mol (Wang 2001) and the femoral head roughness (R_a):

$$k_2 = 0.00000721 R_a^{0.411}$$

where $R_a = 0.1\ \mu$m (Wang *et al.* 1998). The coefficient of friction (μ) is

$$k_3 = 0.000245\mu^2 \tag{5}$$

where $\mu = 0.07$ (Wang *et al.* 2001). Contact stress (σ) is given by

$$k_4 = 0.00000799\sigma^{-0.653} \tag{6}$$

where $\sigma = 5.3$ MPa (Wang *et al.* 2001). The tensile toughness (U), crystallinity (C) and protein concentration (P) are

$$k_5 = 460U^{0.72}C^{-5.29}P^{-0.66} \tag{7}$$

where U, C and P are 140 MJ/m^3, 49% and 45% (Lewis *et al.* 2003). The average wear rate k_a is

$$k_a = \frac{k_1 + k_2 + k_3 + k_4 + k_5}{5} \tag{8}$$

and the volume of wear is

$$V = k_a FS \tag{9}$$

Multiplying by the number of cycles per year (N, 1.4×10^{-6}; Goldsmith *et al.* 2001) gives the volumetric wear rate (W):

$$W = VN = k_a FSN \tag{10}$$

Dividing the volumetric wear rate by the area of a circle of area corresponding to acetabular cup gives the linear wear rate (L):

$$L = \frac{k_a FSN}{\pi r^2} \tag{11}$$

The relationship between implant time and wear rate (Sochart 1999, Figure 5) is

$$I = -40(L) + 22 \tag{12}$$

($r^2 = 0.95$). Substitution gives

$$I = -40\left(\frac{k_a FSN}{\pi r^2}\right) + 22 \tag{13}$$

We created a replica of the femoral head of an acetabular cup. A cross-sectional photograph of the porcelain mixture was used to map the motion of the femoral head. The centre of the initial femoral head position was derived from the intersection of perpendicular lines drawn from the left side of the curvature (Fig. 1a). We repeated this on the bulge side (right side) to determine the centre of the final femoral head position. The radius of curvature increases initially followed by a decrease suggesting two regions of wear: pre-tunnelling wear (y) and tunnelling wear (x) (Fig. 1). The volume due to pre-tunnelling wear V_y is

$$V_y = \frac{1}{4}V_{x+y} \tag{14}$$

and that of tunnelling wear (V_x) is

$$V_x = V_{x+y} - \frac{1}{4}V_{x+y} = \frac{3}{4}V_{x+y} \tag{15}$$

Consequently, the linear wear rate (proportional to volume) is multiplied by a factor $\beta = 0.75$:

$$I = -40\left(\frac{\beta k_a FSN}{\pi r^2}\right) + 22 \tag{16}$$

A computational model was developed using artificial neural networks and applied to the data of Sochart (1999). The neural networks consist of McCulloch–Pitts neuron layers of non-linear differentiable activation functions, input, output and hidden layers. Each neuron at a given layer is connected to every neuron in the next layer (see Li *et al.* in press for details). Many configurations (1–3 layers; 1–10 neurons) of the network (trained with error back propagation algorithm and momentum learning using 90% of the data) were tested with 10% of the data to obtain the optimal network configuration with the least mean sum of error (Li *et al.* in press). This was characterized by 1 hidden layer of 4 neurons, momentum factor of 0.7, learning rate of 1.0,

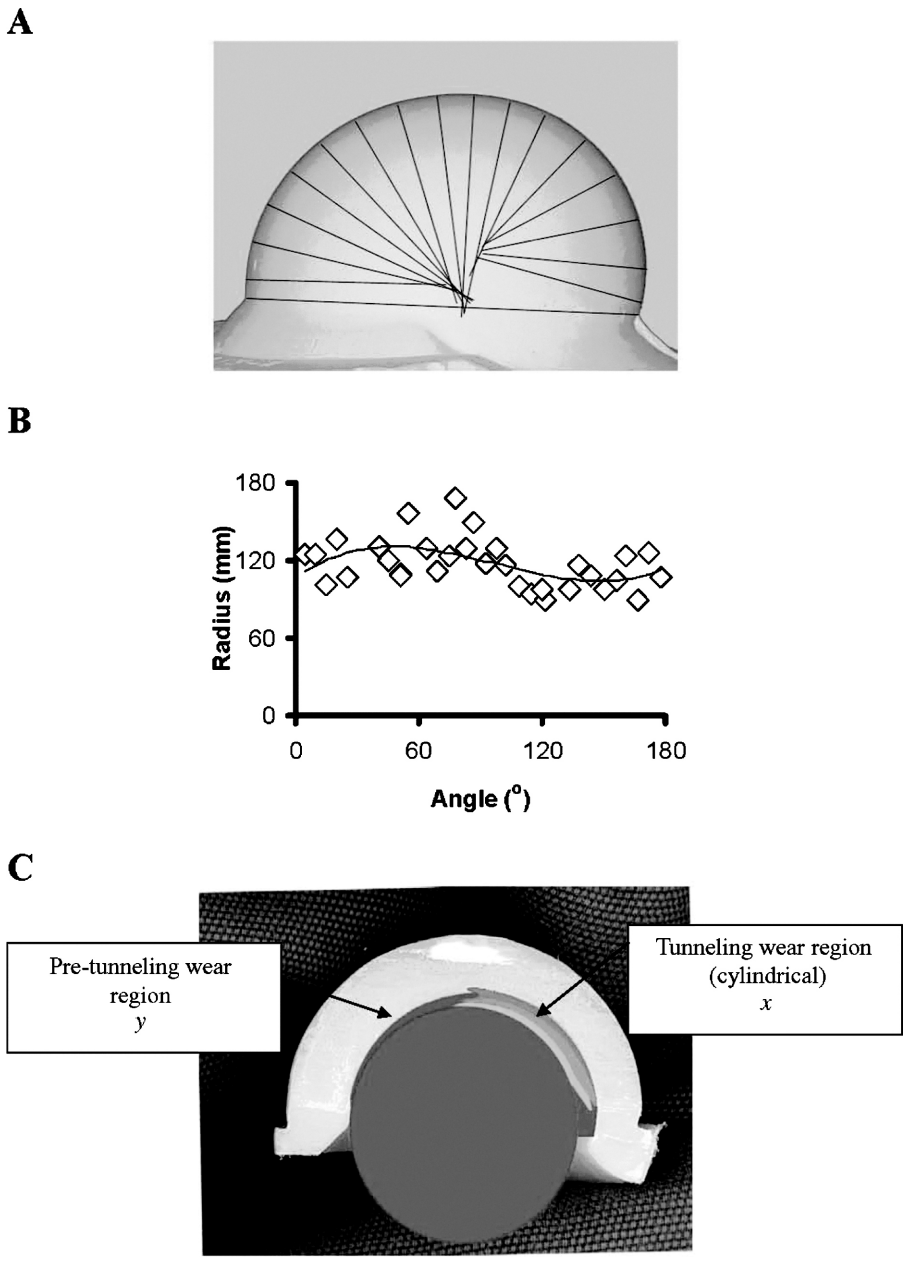

Figure 1 Acetabular cup (a) with perpendicular lines drawn from left to right (the bulge side). (b) The corresponding radius curvature suggests two regions of wear (c): pre-tunnelling (y) and tunnelling wear (x).

sum of error threshold of 0.01 and maximum number of epochs of 10,000.

The implant lives of 14.8 and 14.7 years predicted from our mathematical and computation models, respectively, are not significantly different ($P > 0.05$) from the measured value of 15.0 ± 3.9 years ($\bar{X} \pm$ 2SE; Sochart 1999). We find that an increase in molecular weight between cross-links does not decrease implant life greatly (Fig. 2). This has not been demonstrated in other studies where the molecular weight between cross-links is plotted against k and not implant life (e.g. Muratoglu *et al.* 1999; Wang 2001). Wear rate increases with molecular weight between cross-links associated with a higher degree of rupture. However, cross-linked polyethylenes have a 30% lower wear rate than non-cross-linked polyethylenes because cross-linking reduces the toughness of UHMWPE, which is inversely related to wear rate (Lancaster 1969; Endo *et al.* 2002). A large decrease in implant life occurs with an increase in surface roughness, cycles per year and body weight or a decrease in contact stress and femoral head radius (Fig. 2). The positive relationship between contact stress and implant life is consistent with decreasing wear rate and coefficient of friction (Wang *et al.* 2001). Muratoglu *et al.* (1999) argued that wear rate is independent

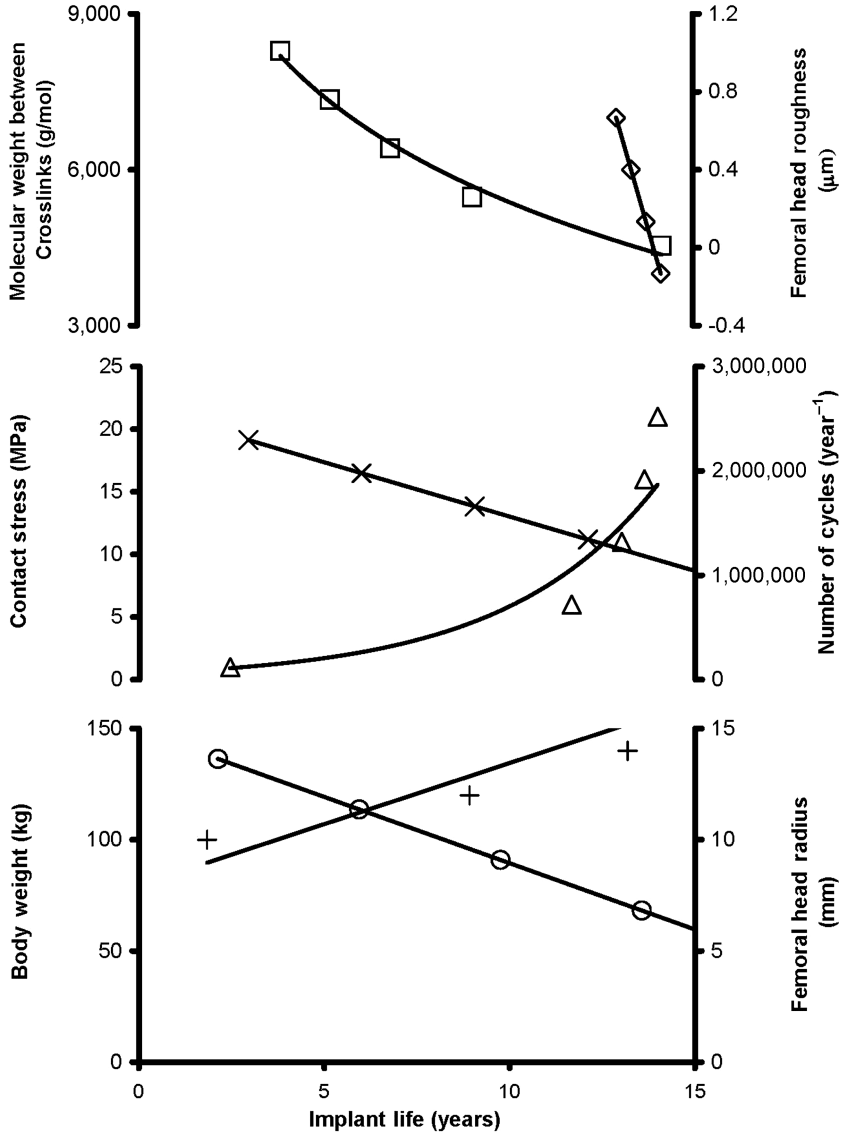

Figure 2 Molecular weight of cross-links (empty diamond), femoral head roughness (empty square), contact stress (empty triangle), number of cycles per year (*X*), body weight (empty circle) and femoral head radius (+) versus implant life.

of surface roughness. This is true only when k_2 is considered. However, a smooth femoral head is associated with a longer implant life when all parameters are considered. Patients should be discouraged from endurance exercise as cycles per year decrease implant life.

Wear path has been assumed to be cylindrical (femoral head travels from its initial to final position linearly; e.g. Devane *et al.* 1995, 1997; Martell *et al.* 2003). However, when the final femoral head position was determined from the bulge side (right side) of the curvature, the radius of curvature in region *y* produces a sphere greater than the femoral head and does not support the linear movement hypothesis of the femoral head because its motion would have to be circular to produce such a curvature in the polyethylene cup from pre-tunnelling and tunnelling wear (Fig. 1).

The initial direction of load deviates from vertical due to the orientation of the implant and mediolateral and anterior–posterior forces in the hip joint during walking (Van Den Bogert *et al.* 1999). The femoral head moves in the direction of the load (Fig. 3A). On the left side, the contact area between the polyethylene cup and the femoral head is reduced over time and the effective direction of load deviates further from the vertical (Fig. 3B). As the contact area continues to decrease (Fig. 3C), the direction of the load may reach a point where pre-tunnelling wear becomes tunnelling wear (Fig. 3D). This occurs near the end of the implant life when the femoral head is close to its final position. Our proposed mechanism of wear path (Fig. 3E) possibly explains why two-dimensional analysis of wear fails to detect 25%–50% of prosthetic hip wear since there are two regions of wear rather than one.

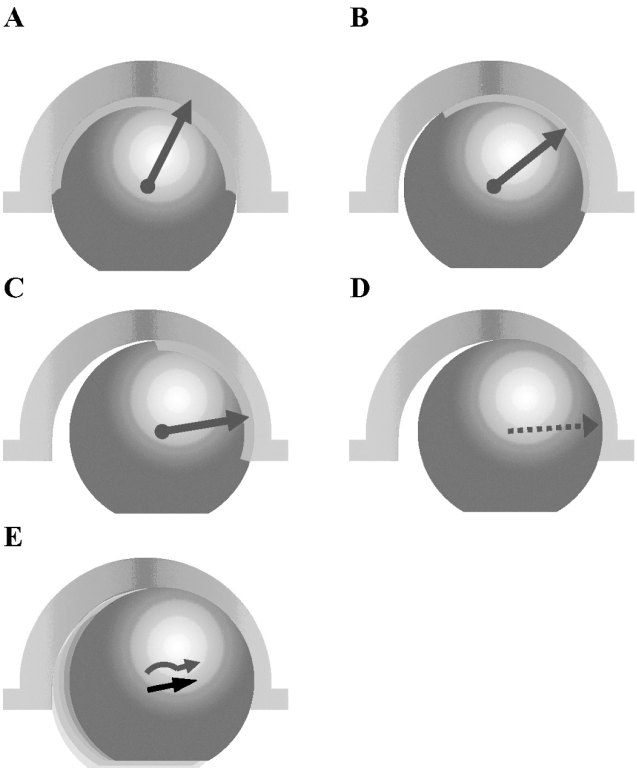

Figure 3 (a) The femoral head moves in the direction of the load. (b) On the left side, the contact area between the polyethylene cup and the femoral head is reduced over time and the effective direction of load deviates further from the vertical. (c) The contact area continues to decrease. (d) The direction of the load may reach a point where pre-tunnelling wear becomes tunnelling wear. (e) Total cylindrical wear (black arrow) and our proposed circular mechanism of wear with time (red arrow).

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