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A model of the smooth pursuit eye movement with prediction and learning

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Smooth pursuit is one of the five main eye movements in humans, consisting of tracking a steadily moving visual target. Smooth pursuit is a good example of a sensory-motor task that is deeply based on prediction: tracking a visual target is not possible by correcting the error between the eye and the target position or velocity with a feedback loop, but it is only possible by predicting the trajectory of the target. This paper presents a model of smooth pursuit based on prediction and learning. It starts from a model of the neuro-physiological system proposed by Shibata and Schaal (Shibata et al., *Neural Networks,* vol. 18, pp. 213–224, 2005). The learning component added here decreases the prediction time in the case of target dynamics already experienced by the system. In the implementation described here, the convergence time is, after the learning phase, 0.8 s.

Keywords: eye movements; smooth pursuit; predictive sensory-motor control; internal models; learning

1. Introduction

One of the most important characteristics of the primate visual system is represented by the space-variant resolution retina with a high-resolution fovea that offers considerable advantages for a detailed analysis of visual objects (Thier and Ilg 2005). The space-variant resolution of the retina requires efficient eye movements for correct vision.

The purpose of *smooth pursuit* eye movements is to minimise the retinal slip, i.e. the target velocity projected onto the retina, stabilizing the image of the moving object on the fovea. Retinal slip disappears once eye velocity catches up to target velocity in smooth pursuit eye movements. In primates, with a constant velocity or a sinusoidal target motion, the smooth pursuit gain, i.e. the ratio of tracking velocity to target velocity, is almost 1.0 (Robinson et al. 1986). This cannot be achieved by a simple visual negative feedback controller because of the long delays (around 100 ms in the human brain), most of which are caused by visual information processing. During maintained smooth pursuit, the lag in eye movement can be reduced or even cancelled if the target trajectory can be predicted (Whittaker and Eaholtz 1982; Wells and Barnes 1998; Fukushima et al. 2002).

Infants gradually learn to predict the motion of moving targets and they pass from a strategy that mainly depends on saccades to one that depends on anticipatory control of smooth pursuit. Before an infant can correctly use smooth pursuit, in fact, they use catch-up saccades to correct the delays of their smooth pursuit. As the smooth pursuit system develops, these saccades become less frequent, but they are still used to catch up if the lag becomes too large. Infants, at 1 month of age, can exhibit smooth pursuit, but only at the speed of $10^\circ\ s^{-1}$ or less and with a low gain (Roucoux et al. 1983). The gain of smooth pursuit improves substantially between 2 and 3 months of age (von Hofsten and Rosander 1997). At 5 months of age, this ability approaches that of adults and the relative proportion of saccades is actually quite adultlike. Other studies investigated horizontal and vertical tracking of moving targets and the vertical tracking was found to be inferior to horizontal tracking at all age levels (Grönqvist et al. 2006). These components are mutually dependent during early development of two-dimensional tracking (Gredeback et al. 2005). These studies demonstrate that the primate smooth pursuit develops with experience.

The objective of this work is to investigate the applicability of smooth pursuit models derived from neuroscience research on humanoid robots (Brooks 1991; Dario et al. 2005) in order to achieve a human-like predictive behaviour able to adapt itself to changes of the environment and to learn from experience.

2. Current models of smooth pursuit

Several models have been developed to simulate smooth pursuit eye movement: the Yasui and Young (1975) model is developed in the velocity space and it is based on the idea that primates can generate an estimation of the target's velocity by summing the retinal slip information and the proprioceptive feedback of the eye velocity. Because of the visual delay in the information processing, the estimated target velocity is always late with respect to the real target

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velocity, so this model is only valid for constant-velocity target movements. An improved smooth pursuit model was suggested by Robinson et al. (1986); their model works as a feed-forward control to cancel the visual delay, but this model cannot achieve zero-lag tracking of a sinusoidal signal. Bahill and McDonald (1983) developed a target-selective adaptive control model that performs zero-latency tracking. This model had the capability to predict future values of the target velocity for sinusoidal or parabolic signal, but it required a priori knowledge of the target motion model. One of the most related works was made by Bradshaw et al. (1997), who employed a Kalman filter for prediction. However, these authors assumed prior knowledge of the target dynamics and, thus, they avoided addressing how unknown target motion can be tracked accurately.

An important biologically plausible smooth pursuit controller has been proposed by Shibata and Schaal (Shibata et al. 2005). This controller learns to predict the visual target velocity in head coordinates, based on fast on-line statistical learning of the target dynamics. The model proposed by Shibata is the best solution to represent predictive behaviour and on-line learning in smooth pursuit eye movement.

According to von Hofsten and Rosander (1997), taking into account previous experience increases the efficiency of smooth pursuit. In this work, a memory-based internal model that stores the model parameters related to the target dynamics is added to Shibata's model. Therefore, after a learning phase, the system can recall already experienced target dynamics parameters and improve its performance, e.g. in terms of convergence speed.

2.1. Neural basis

Many studies have shown that a separate pathway exists, the dorsal pathway, that processes visual motion information. In the monkey brain, the neural pathways that mediates smooth-pursuit eye movements, described in (Thier and Ilg 2005), starts in the primary visual cortex (V1) and extends to the middle temporal (MT) area that serves as the generic visual motion processor. It contributes to smooth pursuit by extracting retinal motion of the target in retinal coordinates (Newsome et al. 1988; Komatsu and Wurtz 1988a, 1988b). By contrast, the middle superior temporal (MST) area seems to contain the explicit representation of object motion in world-centred coordinates (Ilg et al. 2004). Recent works (Kawawaki et al. 2006) demonstrate that this area is responsible for target dynamics prediction. Cortical eye fields are also involved in smooth pursuit (Tian and Lynch 1996); in particular, the frontal eye field can modulate the gain control (Gottlieb et al. 1994; Tanaka and Lisberger 2001, 2002) that determines how strongly pursuit will respond to a given motion stimulus. The gain control works as a link between the visual system and the motor system, therefore motor learning could concern this stage by altering this link (Chou and Lisberger 2004). The dorsal pontine nuclei and the nucleus reticularis tegmenti pontis are the principal recipients of efferent signals originating from the parieto-occipital and frontal areas that contribute to smooth pursuit (Dicke et al. 2004; Ono et al. 2005). They are considered to be intermediary stations that adapt the signal for the extraocular motoneurons. Finally, the cerebellum seems to play a crucial role in supporting the accuracy and adaptation of voluntary eye movements. It uses at least two areas for processing signals relevant to smooth pursuit: the flocculus–paraflocculus complex and the posterior vermis. These areas might be primarily required for the coordination of the vestibular reflex with pursuit behaviour (Rambold et al. 2002) and for pursuit adaptation (Takagi et al. 2000).

2.2. Schaal and Shibata's model

S. Schaal and T. Shibata (Shibata et al. 2005) presented a biologically motivated smooth pursuit controller that predicts visual target velocity in head coordinates, based on fast on-line statistical learning of the target dynamics. They proposed a predictive control model that consists of two sub-systems: (1) a recurrent neural network mapped onto the MST, which receives the retinal slip, i.e. target velocity projected onto the retina with delays, and predicts the current target motion and (2) an inverse dynamics controller (IDC) of the oculomotor system mapped onto the cerebellum and the brainstem.

In the following, x is the target position and $\dot{x}$ is the target velocity. The target state vector is expressed as x in bold, the bar is used to indicate the current estimation of a variable and the hat to indicate the prediction result.

Since the brain cannot observe the target state vector $\boldsymbol{x} = [x\,\dot{x}]^T$ directly, the first part predicts the current target velocity $\hat{\dot{x}}(t)$ from the delayed estimated target state $\bar{x} = (t - \Delta)$. This is calculated from the retinal slip information $\dot{e}(t)$ and the eye velocity $\dot{E}(t)$ as follows:

$$\bar{\dot{x}}(t - \Delta) = \dot{E}(t - \Delta) + \dot{e}(t - \Delta). \tag{1}$$

The estimated target position $\bar{x}(t - \Delta)$ is obtained by integrating $\bar{\dot{x}}(t - \Delta)$. According to neuro-physiological studies (Kawawaki et al. 2006), the MST area predicts only the velocity information about the target dynamics. To predict the target velocity the model uses a second-order linear system to represent the target dynamics:

$$\hat{\dot{x}}(t) = \boldsymbol{w}^T \bar{\boldsymbol{x}}(t - \Delta), \tag{2}$$

where $\boldsymbol{w}$ represents the vector of regression parameters and $\hat{\dot{x}}(t)$ is the predicted target velocity. A recursive least squares algorithm (RLS; Ljung and Soderstrom 1986) is employed for learning, because it is robust and it guarantees convergence. Originally, RLS requires the presence of a target output in the update rules, but the predictor can only utilise the retinal signals as the prediction error. Thus, the algorithm is modified as follows:

$$\mathbf{P}(t) = \frac{1}{\lambda}\left[\mathbf{P}(t-1) - \frac{\mathbf{P}(t-1)x(t)^T\mathbf{P}(t-1)}{\lambda + x(t)^T\mathbf{P}(t-1)x(t)}\right], \quad (3)$$

$$\mathbf{w}(t) = \mathbf{w}(t-1) + \frac{\mathbf{P}(t)x(t)}{\lambda + x(t)^T\mathbf{P}(t)x(t)}\dot{e}(t+1), \quad (4)$$

$$\hat{y}(t) = \mathbf{w}(t)^T\mathbf{x}(t), \quad (5)$$

where $\mathbf{P}$ is the inverted covariance matrix of the input data, $\boldsymbol{x}$ is the input state and λ is the forgetting factor which lies in the [0, 1] interval. For $\lambda = 1$, no forgetting takes place, while for smaller values, the oldest values in the matrix $\mathbf{P}$ are exponentially forgotten. Essentially, the forgetting factor ensures that the prediction of RLS is only based on $1/(1-\lambda)$ data points. This forgetting strategy also enables the predictor to be adaptive to the changes in the target dynamics. Another important element of Equation (4) is that it explicitly shows the requirement for the time alignment of the predictor output and the error since the learning module cannot see at time t. Thus, all variables in Equation (4) are delayed by one time step, which requires the storage of some variables for a short time in memory. The RLS algorithm is implemented in a discrete time domain, so the algorithm upgrades the variables with the new values every discrete step.

The second part of Schaal and Shibata's model is based on theory and experiments showing that the cerebellum and brainstem together act as an IDC of the oculomotor plant (Shidara, Kawano, Gomi and Kawato 1993; Kawato 1999). The model assumes that the IDC has the capability to cancel the dynamics of the eye plant making it valid to write:

$$\dot{E}(t) = \hat{\dot{x}}(t). \quad (6)$$

In accordance with von Hofsten (von Hofsten and Rosander 1997), the prediction in smooth pursuit movements is about 200 ms, so the entire closed-loop delay must be larger of the single visual delay proposed by Schaal and Shibata. In Robinson's model (Robinson et al. 1986), a closed-loop delay of about 150 ms has been proposed and a delay block before the eye plant has been added. In order to simulate a prediction of 200 ms, in this work a delay block before the eye plant has been added, so that the predictor must adapt its dynamics both to visual delay and eye plant dynamics.

2.3. *Implementation of Schaal and Shibata's model*

In this work, the model by Schaal and Shibata has been tested in MATLAB and Simulink by using a sampling frequency of 20 Hz, like in the human visual system. The model was tested on sinusoidal target motions with angular frequency included between 0.5 and 2.5 rad s^{-1} with a 0.1 rad s^{-1} step. The model correctly follows the target dynamics reaching convergence after more than 4 s of simulation (Figure 1). Figure 2 shows the learned values of the vector of regression parameters $\boldsymbol{w}$ in the angular frequency domain. The converging speed is slower than in humans (Shibata et al. 2005), and if a new target dynamics is presented to the model, it is necessary to wait for the system to converge, whether this dynamics has been already presented or not. With the purpose to obtain a developmental model that can take into account previous experiences, in this work it has been supposed that it is possible to store the previously acquired weights, the regression coefficients. These values are placed in a memory block and then used to improve the converging speed of the model.

3. Proposed model of smooth pursuit with prediction and learning

Figure 2 shows that there is a direct relationship between the angular frequency of the target dynamics and the final regression coefficients calculated by the RLS algorithm. Such values depend only on the angular frequency of the target dynamics and on the configuration of the system. Instead, they are independent from the amplitude and the phase of the sinusoidal motion. In this work, a module storing the regression coefficients of already-seen target motions has been added to Shibata and Schaal's model. Figure 3 shows the proposed model block schema. The velocity information ($\dot{v}$) is processed by the V1 and MT areas in order to extract the target slip on the retina ($\dot{e}$). These operations are made by the visual processing module. The estimator state module generates the target velocity estimation according to Equation 1 and it employs the position estimation by integrating the velocity information. The state vector ($\bar{\boldsymbol{x}}$) is sent to the Predictor that provides the next target velocity ($\hat{\dot{x}}$). The IDC generates the necessary torque that allows the eye plant to reach the predicted velocity [Equation (6)]. In general, the system develops an internal representation, i.e. an internal model, of the external environment. The internal model, namely a memory block, has been added to recognise the target dynamics and to provide the correct weight values before the RLS algorithm. For this purpose, the regression coefficients are stored in a neural network, the internal model, for future presentation of learned target dynamics. The Predictor, shown in Figure 3, is the RLS algorithm that minimises the retinal slip, $\dot{e}(t)$, adapting the regression coefficient according the Equation (4). The neural network inputs are a sample series of initial velocity values of the target dynamics and the outputs are the correct regression coefficients of the corresponding target dynamics. Such weights are sent to the predictor module in Equation (4) to guide the RLS algorithm to final values, thus

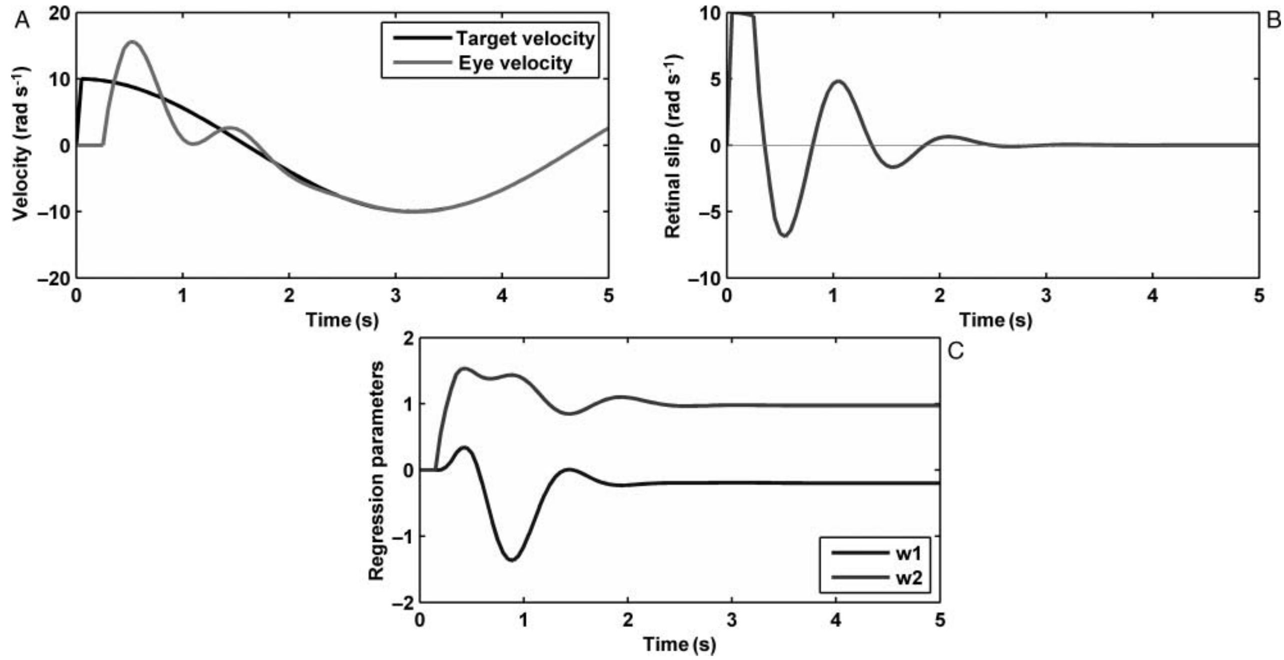

Figure 1. Simulation results in the case of a sinusoidal signal with angular frequency of 1 rad s^{-1} and amplitude of 10 rad using Shibata and Schaal's model. Here A and B show the time course of target and eye velocity and the retinal slip, respectively. After about 4 s, the target velocity and the eye velocity are aligned and the values of the vector of regression parameters w reaches convergence (C) [−0.1987; 0.9751].

improving the converging speed (Figure 4). When the new values are ready from the network, it is necessary to wait for another cycle to verify the correctness of this prediction. If the retinal slip given by RLS is greater than the neural network one, the neural network output is used to predict the target velocity. In the other case, the RLS goes on learning the target dynamics, hence it is necessary to train the neural network

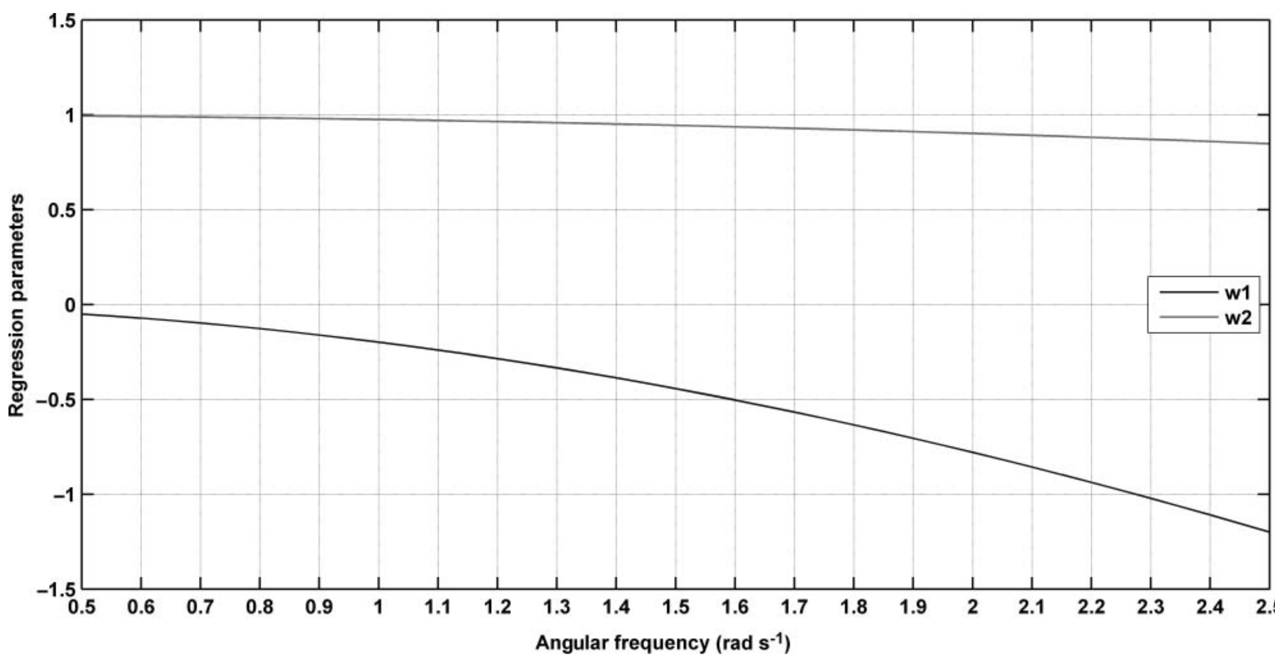

Figure 2. The graph shows the correlation between the values of regression parameters w and the angular frequency of sinusoidal target dynamics.

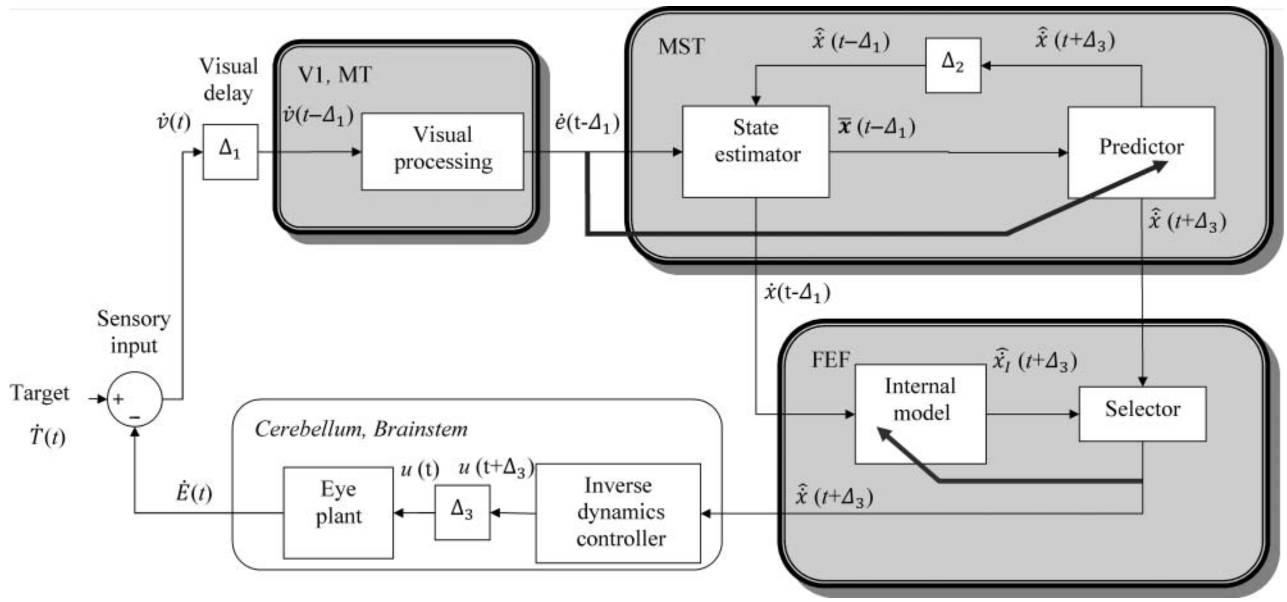

Figure 3. The figure shows a block schema of the proposed model of the smooth pursuit eye movement. An internal model is added to Shibata and Schaal's model to learn the target dynamics and a selector allows to recognise the best prediction between the internal model and the RLS predictor. The internal model is online trained by the values of the regression parameter vector when the RLS reaches convergence.

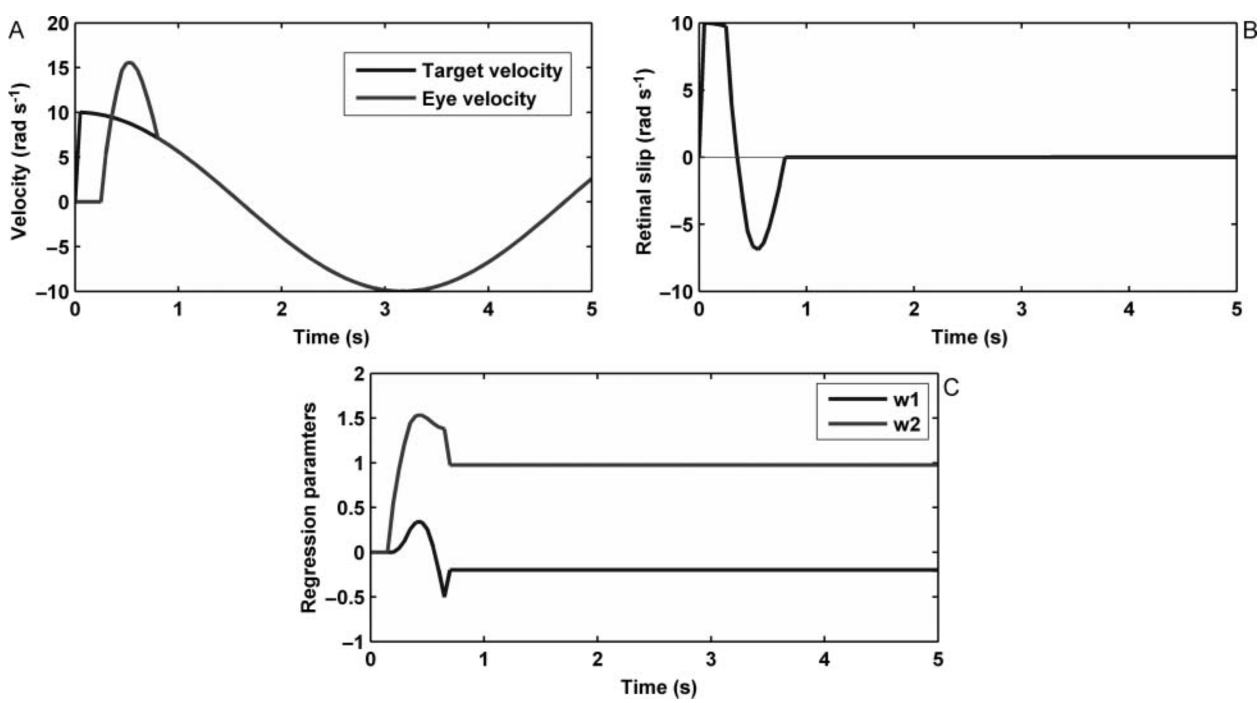

Figure 4. Simulation results in case of a sinusoidal target dynamics with angular frequency of 1 rad s^{-1} and amplitude of 10 rad using the proposed model. Here A and B show the time course of target and eye velocity and the retinal slip respectively. After 0.8 s, the values of the vector of regression parameters w are set to the final values (C) [−0.1987; 0.9751] and the retinal slip reaches zero just after this time.

on the new data. This behaviour is represented by the selector module in block schema. Note that the predictor block in Figure 3 provides a target velocity prediction that overcome the delay in the execution of the movement (Δ_3). The entire close loop delay has been fixed to $\Delta_2 = \Delta_1 + \Delta_3$.

From a neuro-physiological point of view, the visual motion information follows the dorsal pathway and is processed by the primary visual cortex, MT and MST. This area provides the sensory information to guide pursuit movements but may not be able to initiate them. The frontal eye field, in the pre-motor cortex, is more important for initiating pursuit and it is also related with associative memory (Chou and Lisberger 2004). Then, it is possible to suppose that the brain keeps motion information and uses it to obtain a correct smooth pursuit eye movement based on its own previous experience.

4. Implementation of the proposed model of smooth pursuit

In the proposed model, a neural network has been added to associate a specific sequence of velocity values with the correct regression parameters. For this purpose, a simple multi-layer perceptron has been used that maps half seconds of sampled target velocity (with sampling frequency of 20 Hz) onto corresponding weight values.

This network has been developed with the Neural Network Fitting Tool on MATLAB with 10 neurons in the input layer, 25 neurons in the hidden layer and 2 neurons in output layer that correspond with the two regression parameters of the RLS algorithm. It uses the non-linear activation sigmoid function with backpropagation learning rule. In accordance with neuro-physiological studies, the model recognises motion sequences and so it takes only half a second to provide the correct values. Moreover, when the learning is complete, it is possible to obtain correct values also with unknown angular frequency target motions. The model follows a developmental approach, therefore initially the neural network needs to learn by experience. The RLS algorithm learns the target dynamics and reaches convergence. The regression coefficients are used to train the neural network. When the neural network gives as output a new predicted velocity value, the selector module has to compare this value with the real state of the target to verify the correctness of the prediction. So it has to wait one closed-loop delay for the new values of the target state. If the internal model prediction is better than the actual RLS output, the selector module changes the regression parameters in Equation (4) with the neural network output, otherwise it has to wait for the convergence of RLS and to use the regression parameter obtained for the learning of the neural network.

The dimension of the input layer has been chosen from experimental trials. The velocity sample number needs to be a trade-off between motion recognition accuracy (that needs a large number of samples) and system response velocity. Considering the specific system configuration, it has been observed that 10 samples (0.5 s at 20 Hz) are a good solution.

5. Experimental results

The model represents the signal prediction of one axis because it has been proven that the horizontal axis is separate from the vertical axis (Grönqvist et al. 2006). To represent the other axis it is necessary to add another model like this one for the other component of the target dynamics. Moreover, the model predicts only the target velocity, so the position error reaches a constant value after the convergence of the system.

In this work, all the learning experiments start from scratch, i.e., with all initial states including the weights of the learning system set to zero. The model was tested on sinusoidal target motions with the following dynamics:

$$x(t) = A \times \sin(\omega \times t + \varphi), \qquad (7)$$

where $x(t)$ is the target position (expressed in radians) at the time t and A is the amplitude of the dynamics. The angular frequency (ω) has been tested between 0.5 and 2.5 rad s^{-1} with 0.1 rad s^{-1} step. Moreover, the model has been tested with angular phase between 0 and 2π rad with a $\pi/4$-rad step and with amplitude between 4 and 20 rad with a 1-rad step. Figure 1 shows the results of an example of simulation with a sinusoidal motion target with angular frequency of 1 rad s^{-1}: the final values of the vector $\boldsymbol{w}$ are -0.1987 and 0.9751. These values are independent from amplitude or phase of the sinusoidal trajectory. The vector of regression parameters is dependent only on the angular frequency of the sinusoidal motion and on the configuration of the system, like the entire closed loop delay. If the angular frequency changes, it will be necessary to wait so that the model reaches the new steady state. With these configuration properties, Schaal and Shibata's model takes more than 4 s to perfectly cancel the retinal slip. In all experiments, the time necessary for the vector of regression parameters to reach the stable state has been taken into account as converging speed. For this purpose, it has been chosen that the difference between elements of the vector of regression parameters and previous values of itself, $\boldsymbol{e}(k)$, must be less of 10^{-6}.

$$e(k) = w(k) - w(k-1). \qquad (8)$$

For example, for $\omega = 1$ rad s^{-1}, $A = 10$ rad; for $\varphi = 0$, the converging time is 6.75 s. These values are strictly dependent on the initial conditions of the system and on the target dynamics. Figure 5 shows the results of all simulation tests changing the amplitude in a range from 4 to 20 rad with a 1-rad step and the angular frequency in a range from 0.5

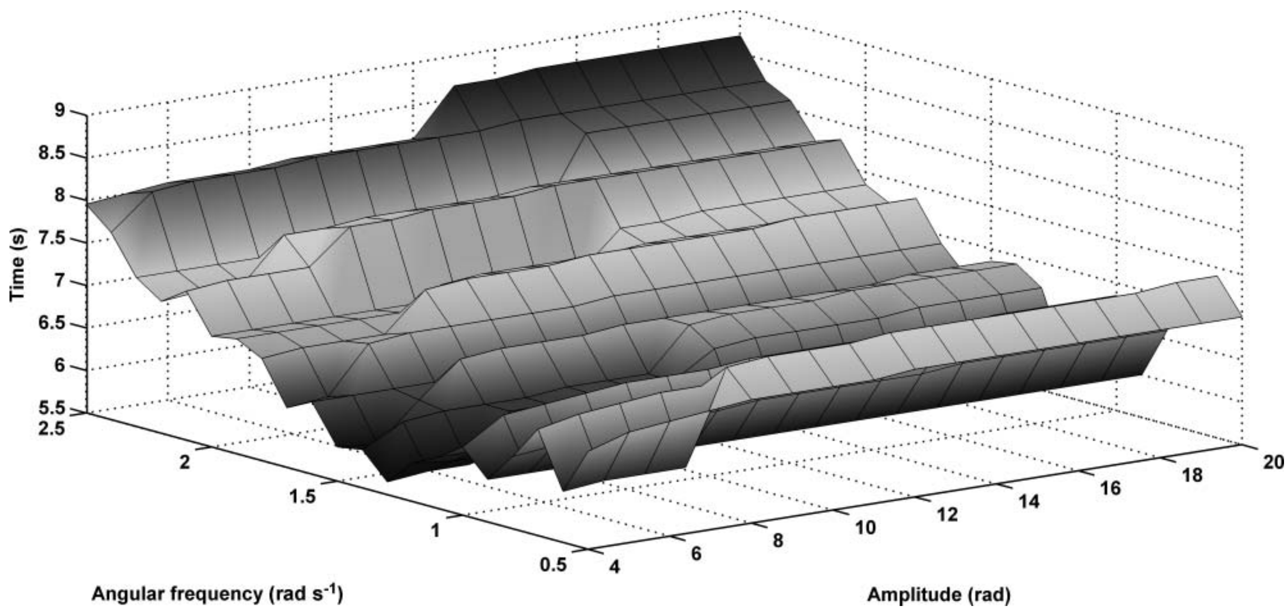

Figure 5. Simulation results in case of a sinusoidal target velocity at several amplitude and angular frequency using Shibata and Schaal's model. The converging time is between 5 and 10 s.

to 2.5 rad s^{-1} with fixed phase. Figure 6 shows the results of all simulation tests changing the phase in a range from 0 rad and 2π rad with a $\pi/4$-rad step and angular frequency in a range from 0.5 and 2.5 rad s^{-1} with fixed amplitude. The converging speed increases as frequency and amplitude increase, moreover there is a periodic trend with the phase change.

In the improved model, it has been assumed that 10 steps are necessary (half a second with sampling frequency of 20 Hz) to recognise precisely the motion. The network outputs the vector of regression parameters and it is placed in Equation (4) for at least 5 s. The model corrects the prediction and the absolute value of retinal slip reaches steady state after 0.8 s (Figure 4). When the new values are

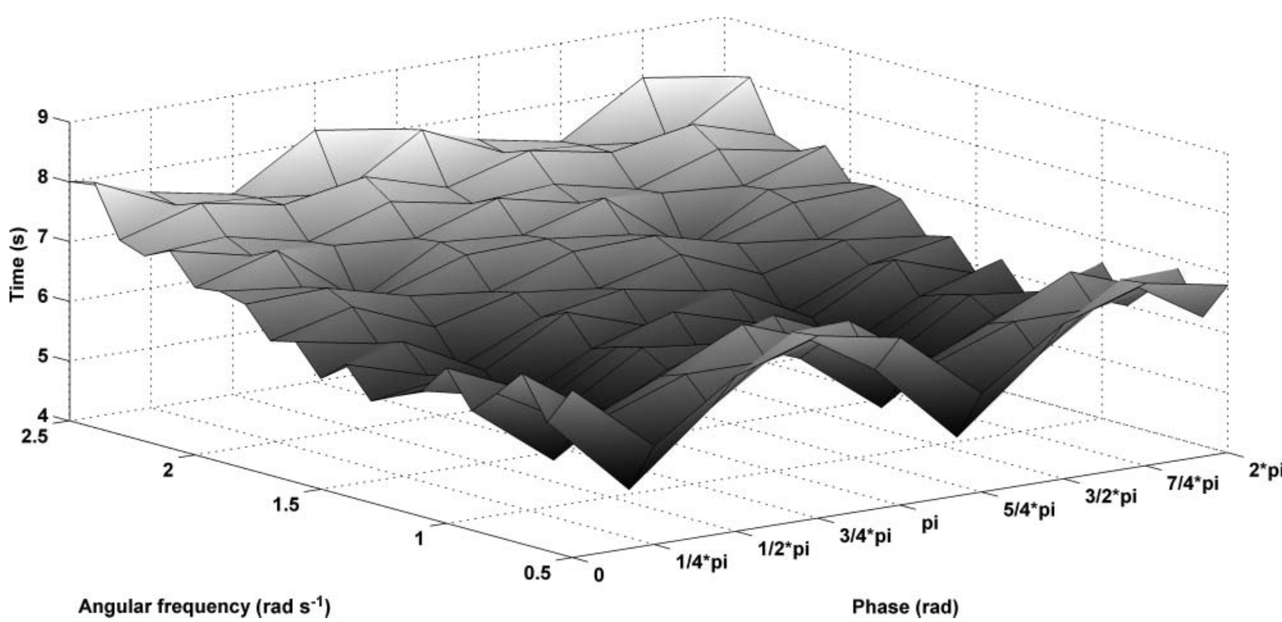

Figure 6. Simulation results in case of a sinusoidal target velocity at several phase and angular frequency using Shibata and Schaal's model. The converging time is between 5 and 9 s. The graph shows a periodic trend of the converging time with the phase change.

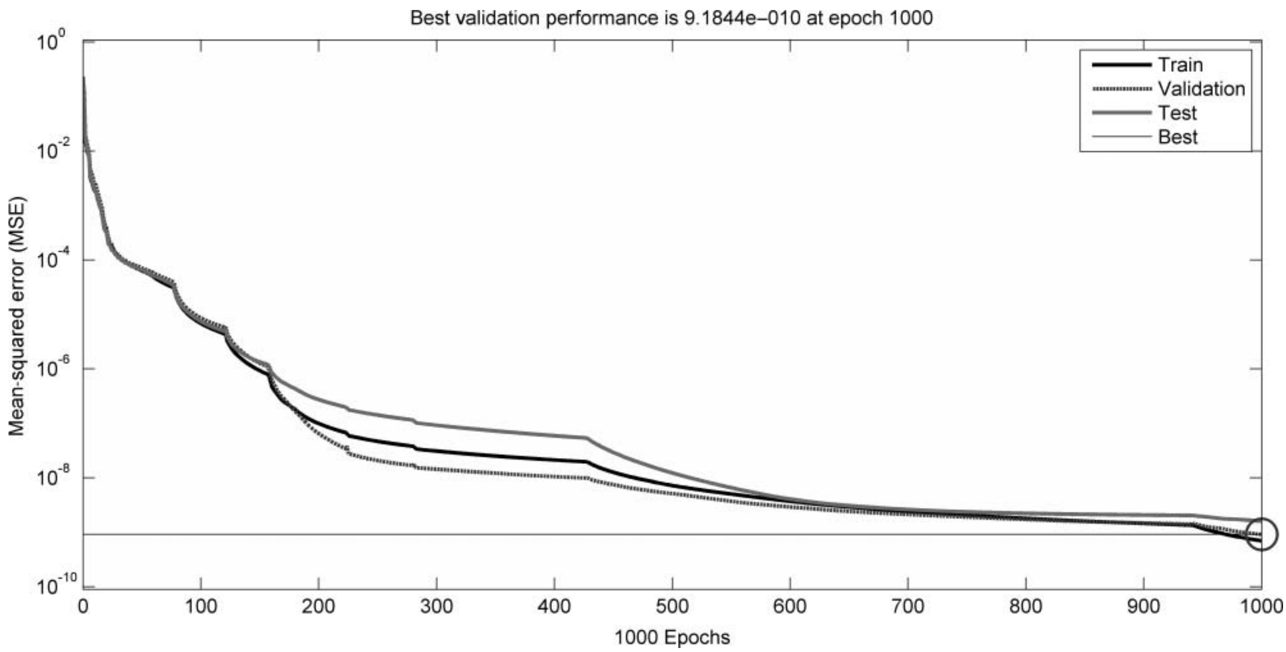

Figure 7. The graph shows the learning phase of the neural network. The mean-squared error is less than 10^{-9} after 1000 epochs and is plotted for training set, validation set and test set of data.

ready from the network, it is necessary to wait for another cycle to verify the correctness of this prediction, so the final converging time is the sum of the time necessary to get 10 samples of target velocity (500 ms at 20 Hz), plus the time to verify that the prediction of the neural network is better than the prediction coming from RLS (one closed loop delay, 200 ms) and the time to move the eye to the correct predicted velocity (100 ms).

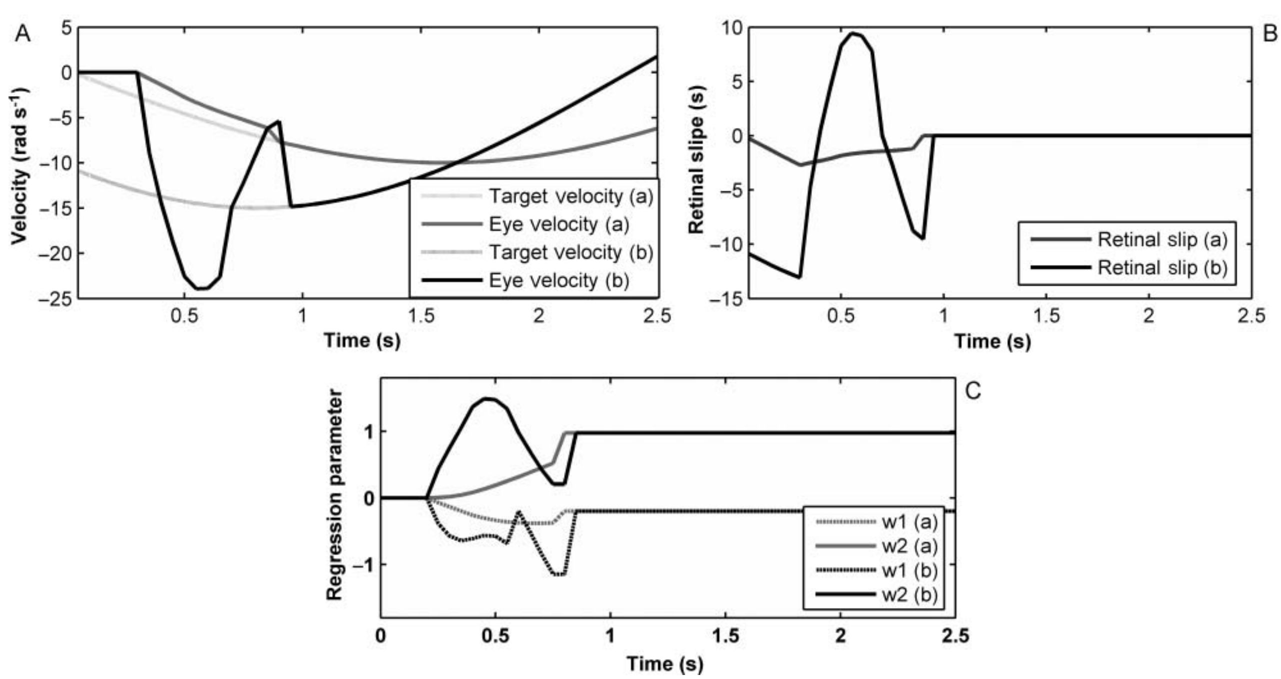

Figure 8. Simulation results in case of two sinusoidal target dynamics with angular frequency of 1 rad s^{-1} and an amplitude of 10 rad and phase of $\pi/2$ (a, the grey line) and amplitude of 15 rad and phase of $3/4\pi$ (b, the black line) using the proposed model. Here A and B show the time course of target and eye velocity and the retinal slip respectively. After 0.8 s, the values of the vector of regression parameters w are set to the final values (C) $[-0.1987; 0.9751]$ and the retinal slip reach zero just after this time independently from the differences of the amplitude and the phase of the target dynamics.

In order to test the hypothesis that the model can recognise the initial value of the target dynamics, the neural network has been trained on a large training set of data. A sinusoidal target dynamics with an angular frequency between 0.5 to 2.5 rad with a 0.1 rad s^{-1} step has been taken into account. The neural network must give the correct value of the regression parameter vector aside from differences in the values of the amplitude and the phase of the sinusoidal target dynamics. So, for each angular frequency, 10 values have been taken (with sampling frequency of 20 Hz) of the target velocity (the derivative of the target position), considering the amplitude of the target position included between 5 and 15 rad with a 2.5-rad step, thus the maximum velocity considered is about 40 rad s^{-1}. Moreover, a different initial phase of the target dynamics in a range between $\pi/2$ and π rad with $\pi/16$-rad step has been taken into account. Nine hundred and forty-five (all combination of angular frequency, phase and amplitude) simulation results have been taken into account and the training set for the neural network is 70% of this matrix. 15% is used for the validation set and another 15% is used for the test set. The input matrices have dimension 10×945 and the output target has 2×945 elements. These values have been shuffled to increase the variability of the training set. The results of the learning are shown in Figure 7. The best mean-squared error is $9.1844 \ \exp(-10)$. Figure 8 shows the results of the model after the learning of the neural network in two different sinusoidal target dynamics. The figure shows that after 0.8 s, the model recognises the dynamics and gets out the correct values of the regression parameter vector aside the different condition in phase or in amplitude.

6. Conclusions

This work demonstrates that the smooth pursuit eye movement in humanoid robots can be modelled as a sensory-motor loop where the visual sensory input can be predicted and where the prediction can be improved by internal models that encode target trajectories already experienced by the system and that are built by learning.

The model of smooth pursuit proposed in this work starts from a model of the neuro-physiological system proposed by Schaal and Shibata, which includes a prediction component to cancel the retinal slip, in the velocity space. The proposed model includes a learning component that decreases the prediction time for a robotic implementation. The internal model is a feed-forward artificial neural network that recognises the initial sequence of target velocity and gives as output the correct values of the regression parameter vector.

When a new dynamics is presented to the proposed model, the predictor module learns to predict the target velocity in order to cancel the retinal slip. Thus the regression parameter vector reaches the steady state. At this point the internal model is upgraded in order to improve the converging time in a next presentation of the same dynamics. In case of sinusoidal target motions, the values of the regression parameter vector depend only on the angular frequency of target dynamics, so the internal model must return the same values aside from the differences of amplitude or phase of target dynamics. This implies the building of a large training set of data that take into account the possibilities of change in amplitude and phase of the target dynamics with the same angular frequency.

The neural network chosen for the internal model provides an improvement for the converging speed of the model but it leads to some considerations. First of all, the dimension of the hidden layer has been chosen a priori, so it might be not optimal for another type of data. Secondly, the neural network requires more computational power than the original model. Thirdly, it needs a lot of memory for storing the training set of data.

In the implementation described in the paper, the convergence time reaches as low as 0.8 s. As a reference, with the same implementation conditions, the convergence time of Schaal and Shibata's model is more than 4 s. This system with this configuration is unable to predict complex dynamics like a sinusoidal sum. Shibata and Schaal (2001) considered in their work these RLS limitations, but an improvement of the system like they suggested would not change the possibility presented here to take into account the converging results.

Such results demonstrate that a memory-based approach can improve the performance of the system. So it is possible to suggest that initially the smooth pursuit system needs to learn the target dynamics. During this phase it requires using a large number of saccades to correct position errors. When the system has built its own internal model of the external environment, it uses its experience to rapidly obtain a zero-phase lag smooth pursuit.

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