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Smooth and energy saving gait planning for humanoid robot using geodesics

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Abstract. A novel gait planning method using geodesics for humanoid robot is given in this paper. Both the linear inverted pendulum model and the exact Single Support Phase (SSP) are studied in our energy optimal gait planning based on geodesics. The kinetic energy of a 2-dimension linear inverted pendulum is obtained at first. We regard the kinetic energy as the Riemannian metric and the geodesic on this metric is studied and this is the shortest line between two points on the Riemannian surface. This geodesic is the optimal kinetic energy gait for the COG because the kinetic energy along geodesic is invariant according to the geometric property of geodesics and the walking is smooth and energy saving. Then the walking in Single Support Phase is studied and the energy optimal gait for the swing leg is obtained using our geodesics method. Finally, experiments using state-of-the-art method and using our geodesics optimization method are carried out respectively and the corresponding currents of the joint motors are recorded. With the currents comparing results, the feasibility of this new gait planning method is verified.

Keywords: Humanoid robot, gait planning, biped walking, geodesics, Riemannian geometry

1. Introduction

Energy consumption is an important factor which is considered in gait planning of bipedal humanoid robot. Researchers make optimal energy gait planning to ensure that the humanoid robot can walk stably and efficiently. Vanderborght et al. [1] designed a compliance controller to reduce the energy consumption by exploiting the natural dynamics. Park J H et al. [2] used gravity-compensated inverted pendulum mode and computed torque control to make the robot walk more efficiently. Silva et al. [3] studied the required actuator torques by adjusting walking parameters. Chevallereau et al. [4] assumed that a motion defined by ballistic motion and impulsive control at dual support instances will lead to an energetically economical trajectory and give a low energy cost reference trajectories for a biped robot. To extend the minimum-energy walking method to level ground and uphill slopes, Channon et al. [5], Rostami et al. [6] and Roussel et al. [7] have introduced methods of gait planning by minimizing the cost function of energy consumption. Jeon and Park [8] used genetic algorithms to give an energy optimal gait. Many researchers, such as [9, 10], also used genetic algorithms to do optimal gait planning for humanoid robots. Passive walking is also a good way to make the humanoid robot walk energy efficiently. Hass et al. [11] tuned mass distribution of a passive dynamic walker to find the optimal manifolds to maximize walking speed and stability. Geng et al. [12] designed a planar bipedal robot and this robot can exploit its own natural dynamics during critical stages of its walking gait cycle. Kurazume et al. [13] generated a straight legged walking pattern for a biped robot using up and down motion of the upper body, the energy efficient human-like walking is obtained.

Two categories are included in gait planning. One category is to use precise dynamic parameters of a robot, such as mass and inertia of each link [14–16].

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The other category is to use inverted pendulum model [17, 18]. In this paper, both categories are studied using geodesics method. Geodesic is the shortest line between two points in Riemannian space. The straight line in Euclidean space is an example of geodesic. Park [19] defined a distance metric and used it in optimal mechanism design of the manipulator. Park and Ravani [20] generalized the Bezier curve in Riemannian space to solve the problem of workspace fitting. By changing the original constant kinetic energy, Belta and Kumar [21] generated optimal smooth trajectories for a set of mobile robots. Totally speaking, all the existing works are on the rigid body motion with simply one degree-of-freedom. We have successfully implemented the geodesics method in the optimal trajectory planning of multiple degree-of-freedom industrial manipulators [22, 23].

In this paper, we use geodesic method to study optimal energy gait planning on inverted pendulum model and precise dynamic parameters model respectively. At first, the 2-dimensional inverted pendulum model is studied and the energy saving gait is obtained. Then dynamic model of swing leg is established and an optimal energy gait planning in single support phase is studied. The experiments are carried out to verify the geodesic gait planning method.

2. The concept of geodesic

Geodesic is the shortest path between two points in a curved space (Riemannian space) and its equation can be obtained using covariant derivation [22–25]. The straight lines between two points in Euclidean space are intuitive examples of geodesics. On a sphere space (curved space), the images of geodesics are the great circles between two points on the sphere surface. If we regard the energy of the robot system as the curved space, then the distance between two points (start position and end position) in this curved space will be measured by the energy. So the geodesic between two points will be the minimal energy path. Another attribute of geodesic is that the distance from $f(s)$ to $f(t)$ along the geodesic is proportional to $|s - t|$, that is, the covariant derivative of the tangent vector along geodesic is zero. For the multi-rigid robotic system, once again, if we regard the kinetic energy as the curved space, the distance between two points will be measured by kinetic energy. The variation of kinetic energy will be constant, there will be no jerk. So the smooth and energy saving gait of humanoid robot can be obtained using geodesic method.

The equation of geodesic is

$$\frac{d^2\theta_i}{ds^2} + \Gamma^i_{kj}\frac{d\theta_k}{ds}\frac{d\theta_j}{ds} = 0 \tag{1}$$

where s is the arc length of the curve, θ_i is the curve coordinate, for $i, j, k = 1, 2, \ldots, n$. The Christoffel symbols are given in terms of Riemannian metric by

$$\Gamma^i_{kj} = \frac{1}{2}g^{mi}\left(\frac{\partial g_{km}}{\partial \theta_j} + \frac{\partial g_{jm}}{\partial \theta_k} - \frac{\partial g_{kj}}{\partial \theta_m}\right) \tag{2}$$

where g^{mi} is the element of the inverse matrix of the Riemannian metric matrix. Geodesic is the shortest curve between two points on the Riemannian surface and the velocity along geodesic curve remains invariant.

3. Optimal energy gait planning of linear inverted pendulum model

3.1. *Riemannian metric of 2-Dimension Linear Inverted Pendulum Model (2DLIPM)*

The humanoid robot can be simplified as a model of 2-dimension linear inverted pendulum model, as shown in Fig. 1. The mass is m and we assume $m = 1$ in this paper. θ is the rotational angle corresponding to z axis, and the clockwise is the positive direction of θ. h is the variable length of the inverted pendulum. The kinetic energy T of the 2-dimension inverted pendulum system is

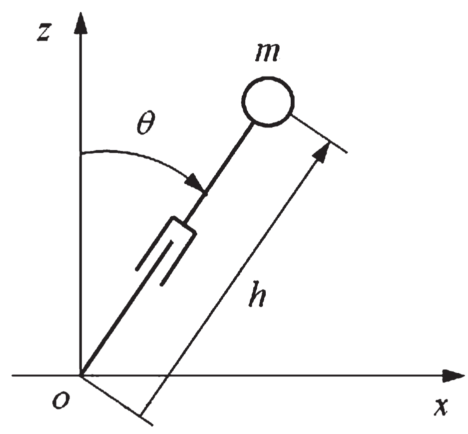

Fig. 1. 2-Dimensional inverted pendulum model.

$$T = \frac{1}{2}m(\dot{h}^2 + h^2\dot{\theta}^2) \tag{3}$$

Now we regard the kinetic energy of the system as Riemannian metric. Ignoring the constant coefficient $\frac{1}{2}$ and assuming $m = 1$, we can rewrite Eq. (3) into the form of Riemannian metric,

$$ds^2 = dh^2 + h^2 d\theta^2 \tag{4}$$

Equation (4) can be presented as a symmetric positive definite quadratic form,

$$ds^2 = \begin{pmatrix} d\theta \; dh \end{pmatrix} \begin{pmatrix} h^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d\theta \\ dh \end{pmatrix} \tag{5}$$

3.2. *Geodesic when kinetic energy as riemannian metric*

The geodesic equation can be obtained according to the kinetic energy metric in Eq (5). We have got the general geodesic equation as Eqs (1) and (2). The matrix $\begin{pmatrix} h^2 & 0 \\ 0 & 1 \end{pmatrix}$ in Eq. (5) is the Riemannian metric matrix, and the elements of this matrix is the Riemannian metric coefficients corresponding to g_{ij}, i, $j = 1, 2$ in Eq. (2). And its inverse matrix can be calculated as $\begin{pmatrix} h^{-2} & 0 \\ 0 & 1 \end{pmatrix}$ and the elements of this inverse matrix correspond to g^{mi}, $m, i = 1, 2$ in Eq. (2). Then we can calculate all the Christoffel symbols of the geodesic, $\Gamma^1_{11} = 0$, $\Gamma^1_{12} = h^{-1}$, $\Gamma^1_{22} = 0$, $\Gamma^2_{11} = -h$, $\Gamma^2_{12} = 0$, $\Gamma^2_{22} = 0$. Substituting all these Christoffel symbols into Eq. (1), then the geodesic equation is obtained,

$$\begin{cases} \ddot{\theta} + \frac{2}{h}\dot{\theta}\dot{h} = 0 \\ \ddot{h} - h\dot{\theta}^2 = 0 \end{cases} \tag{6}$$

We transform Eq. (6) into a standard state space equation in order to solve this differential equation in MatLab program. So the final geodesic equation is

$$\begin{cases} d\theta/ds = \dot{\theta} \\ d\dot{\theta}/ds = -\frac{2}{h}\dot{\theta}\dot{h} \\ dh/ds = \dot{h} \\ d\dot{h}/ds = h\dot{\theta}^2 \end{cases} \tag{7}$$

When the initial condition and final condition are given, we can find a geodesic between this two points. When the inverted pendulum is moving along this geodesic, the kinetic energy is constant. So the walking gait generated from this model will be smooth.

3.3. *Optimal gait planning by Geodesic*

3.3.1. *Geodesics in a whole walking cycle*

The support leg will exchange two times during a whole walking cycle of humanoid robot. A whole walking cycle is shown in Fig. 2.

We give six critical points during the whole walking cycle. In Fig. 2, θ_0 and h_0 is the initial point of the walking, θ_1 and h_1 is the end point of single support phase, θ_2 and h_2 is the initial point of the next single support phase after support leg exchanging. We assume that the leg exchanging is finished in a very short time. θ_3, h_3, θ_4 and h_4 are the points before and after support leg exchanging (double support phase) respectively. θ_5 and h_5 are the final point of the whole walking cycle.

The gaits between these points can be generated with geodesics. Geodesic is the shortest line on Riemannian

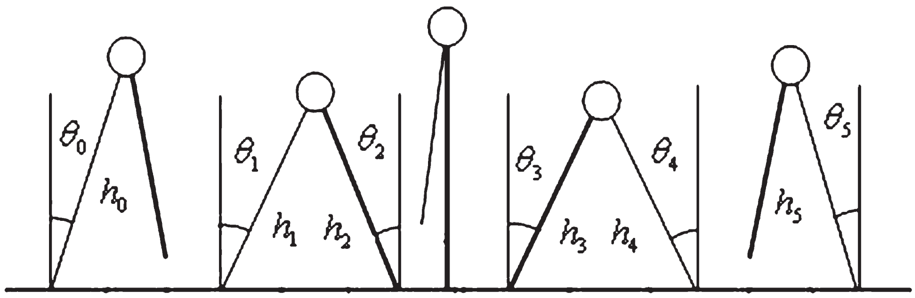

Fig. 2. Gait of a whole walking cycle.

surface between two points. The Riemannian surface is determined by the Riemannian metric. If we define the kinetic energy as the Riemannian metric, then the distance between two points on this surface is measured by the kinetic energy. We can connect these critical points (from θ_0, h_0 to θ_5, h_5) with geodesics on Riemannian surface whose metric is the kinetic energy, then the optimal kinetic energy gait during a whole walking cycle is obtained.

3.4. Gait planning example using geodesics

We consider that the initial and final angles and lengths of inverted pendulum are $\theta = 15^\circ$ and $h = 1$ respectively. The angles and lengths of pendulum during walking cycle are $\theta = 30^\circ$ and $h = 0.8$ respectively. The exact values of all the critical points in Fig. 2 are as below,

$\theta_0 = 0.2618$, $h_0 = 1$; $\theta_1 = 0.5236$, $h_1 = 0.8$; $\theta_2 = -0.5236$, $h_2 = 0.8$; $\theta_3 = 0.5236$, $h_3 = 0.8$; $\theta_4 = -0.5236$, $h_4 = 0.8$; $\theta_5 = -0.2618$, $h_5 = 1$.

The optimal energy geodesic gaits between critical points are expressed in the variables θ and h. The geodesic gaits between states 0 and 1, 2 and 3, 4 and 5 are shown in Figs 3–5 respectively. Gait simulation of the whole walking cycle is shown in Fig. 6. An amazing coincidence is found in Fig. 6 that the optimal kinetic energy gait (from state 2 to state 3) is the gait of 2DLIPM. The gaits between two different height of the center of the mass points are also straight line (from state 0 to 1 and from state 4 to 5). The reason of this coincidence is that the two variables θ and h are decoupled in the 2-dimensional inverted pendulum model.

For the inverted pendulum model, some constraints are applied to limit the motion in a plane in order to get the sole solution [17, 18]. Our geodesic method can get the optimal gait with the given initial and final conditions without other assumptions. For the well-known 2-Dimension Linear Inverted Pendulum Model (2DLIPM), the orbital energy is constant and the height of the center of mass is invariant. Here we obtain this same result naturally by our geodesic method.

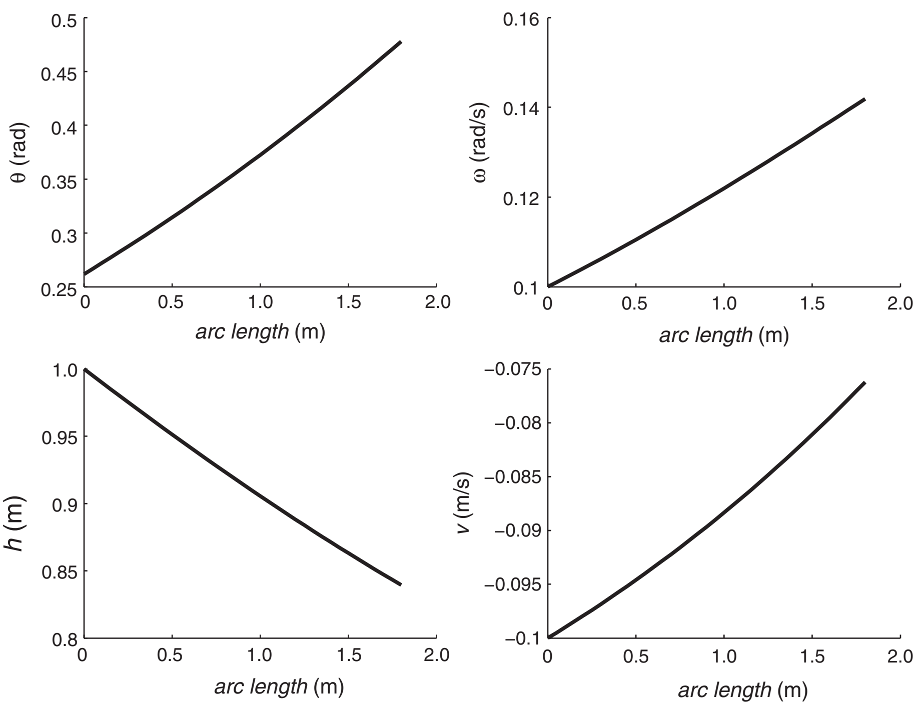

Fig. 3. Geodesic gait from state 0 to state 1.

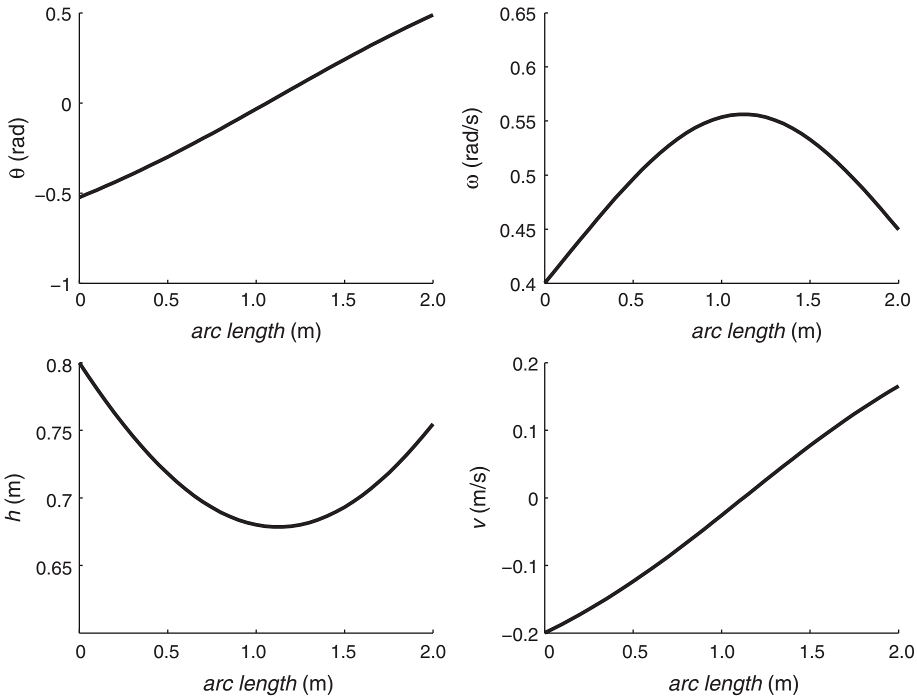

Fig. 4. Geodesic gait from state 2 to state 3.

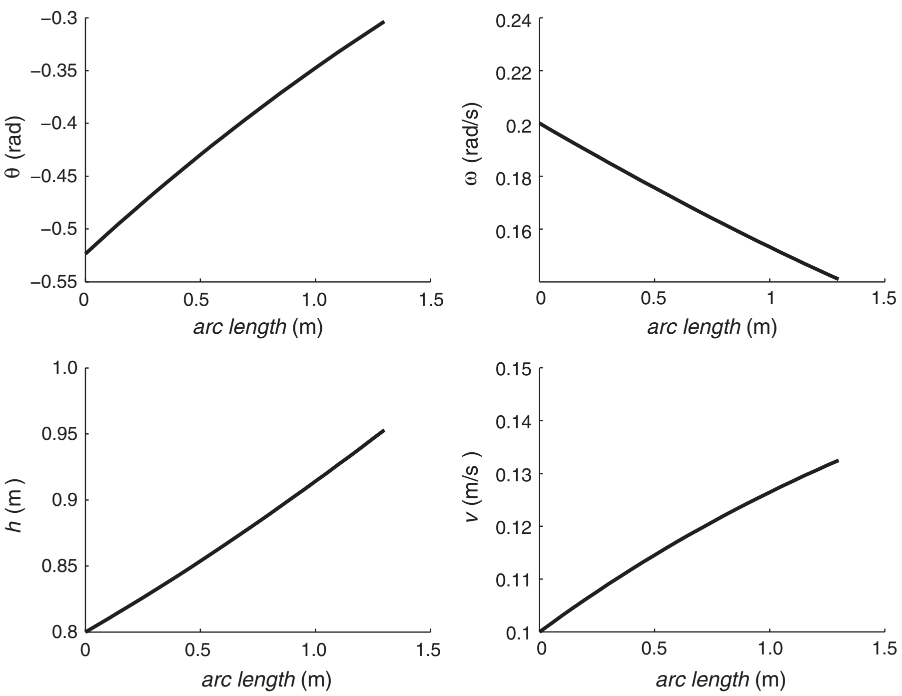

Fig. 5. Geodesic gait from state 4 to state 5.

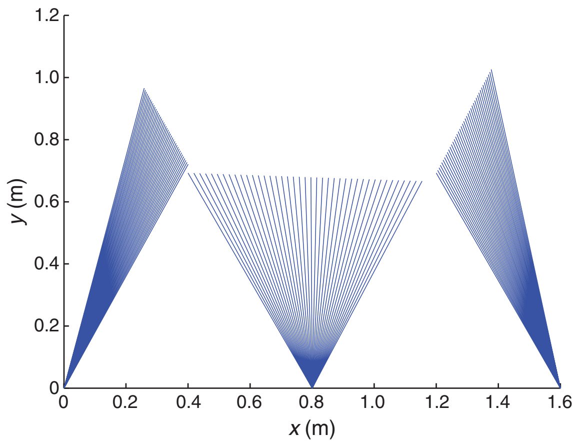

Fig. 6. Geodesic gait simulation of the whole walking cycle.

4. Optimal energy gait planning of swing leg

4.1. The model of swing leg

4.1.1. The kinematic model of swing leg

The RoboErectus AdultSize humanoid robot is made up of PowerCube motors for both legs as shown in Fig. 7(a). For forward walking, the kinematic model of the swing leg is shown in Fig. 7(b). The kinematics of the leg is as follow:

$$\begin{cases} x = -l_1 s_1 - l_2 s_{12} - g_3 s_{123} \\ z = -l_1 c_1 - l_2 c_{12} - g_3 c_{123} \end{cases} \tag{8}$$

4.1.2. The dynamic model of swing leg

The kinetic energy of the swing leg system can be written in Riemannian metric matrix form as follow:

$$ds^2 = \begin{pmatrix} d\theta_1 & d\theta_2 & d\theta_3 \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \end{pmatrix} \tag{9}$$

The elements of the Riemannian metric matrix are:

$$\begin{aligned} m_{11} = {} & m_1 g_1{}^2 + m_2(l_1{}^2 + g_2{}^2 + 2l_1 g_2 c_2) \\ & + m_3(l_1{}^2 + l_2{}^2 + g_3{}^2 + 2l_1 l_2 c_2 \\ & + 2l_1 g_3 c_{23} + 2l_2 g_3 c_3) \end{aligned}$$

$$m_{12} = m_{21}$$

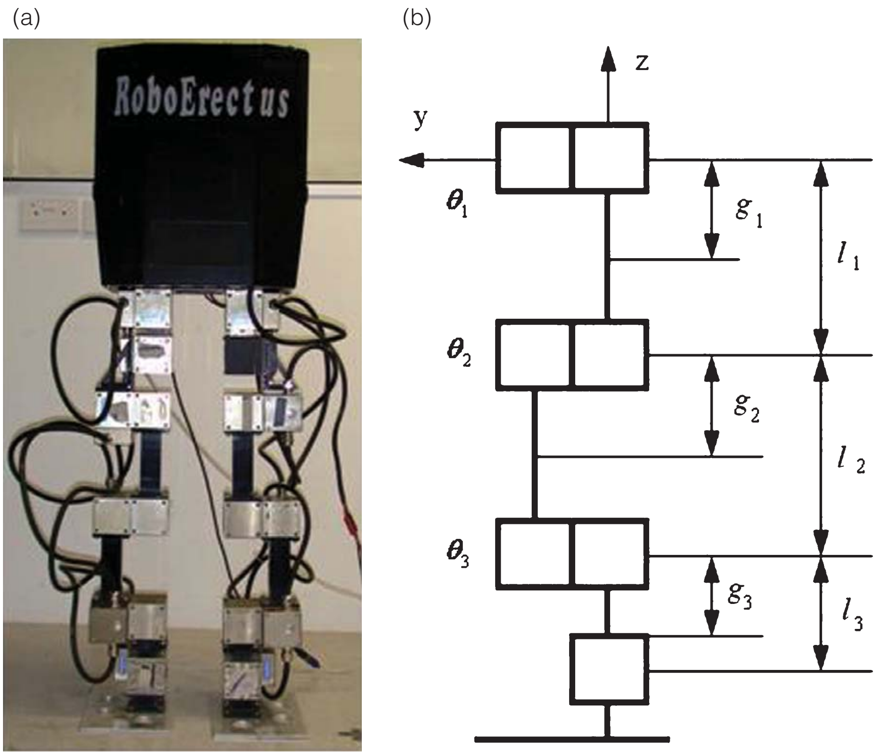

Fig. 7. (a) RoboErectus AdultSize robot (b) model of swing leg.

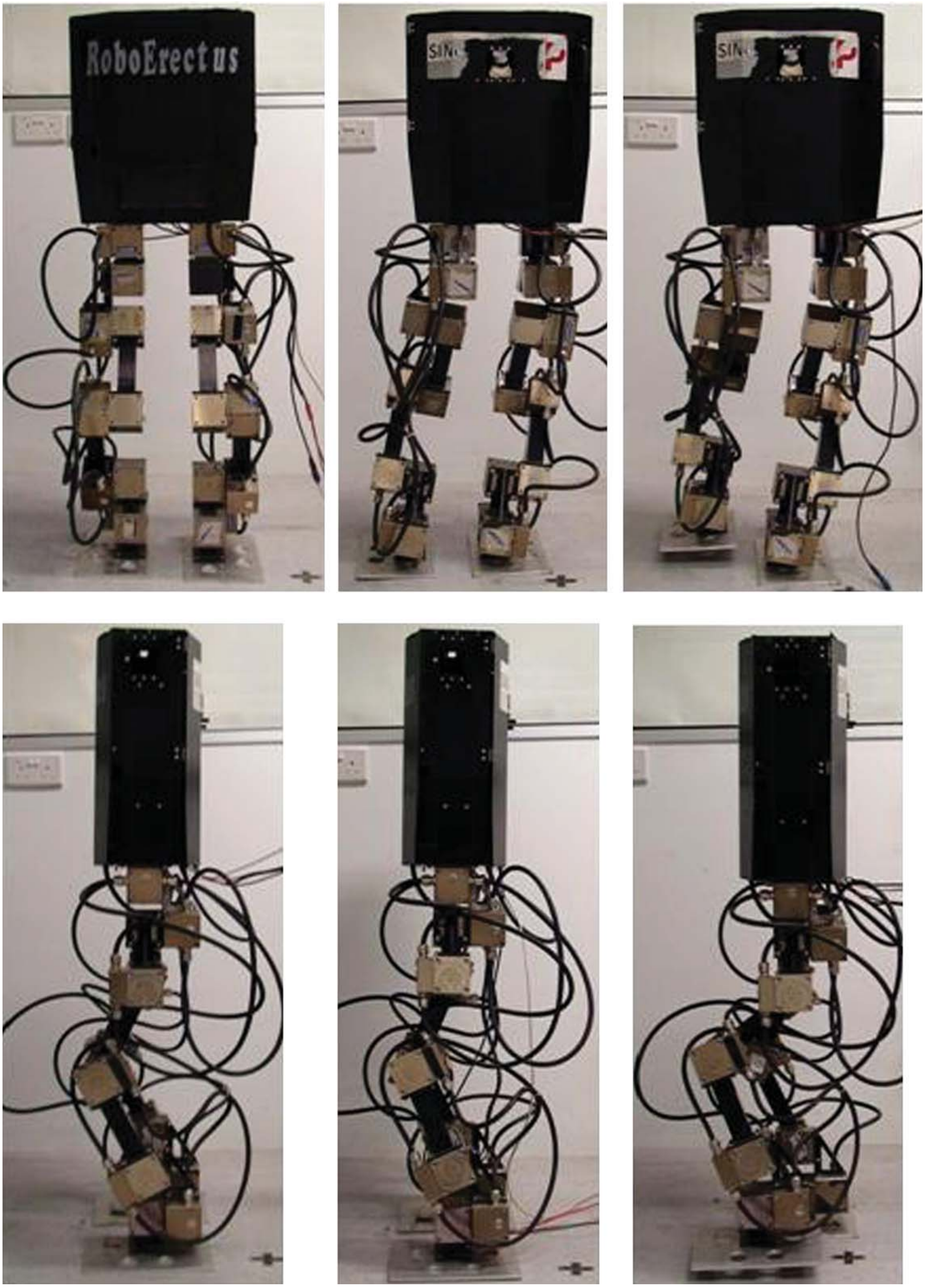

Fig. 8. (a) Idle state (b) State 0 (c) State 1.

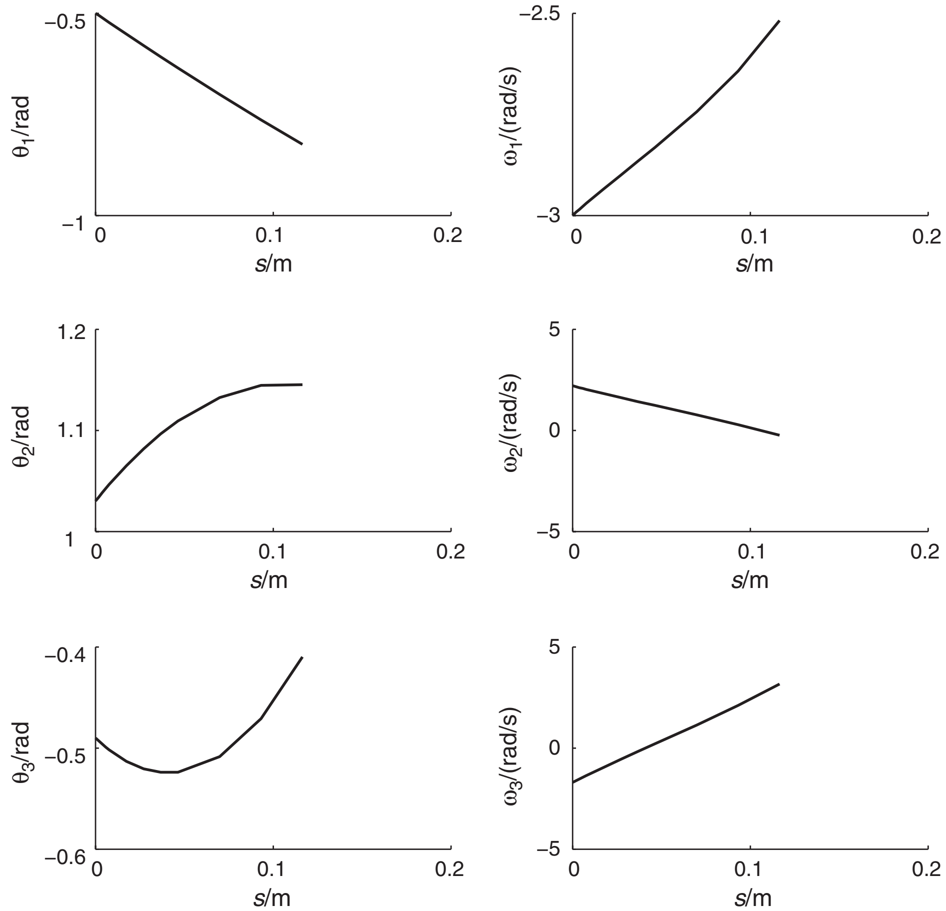

Fig. 9. Geodesic gait of swing leg from state 0 to state 1.

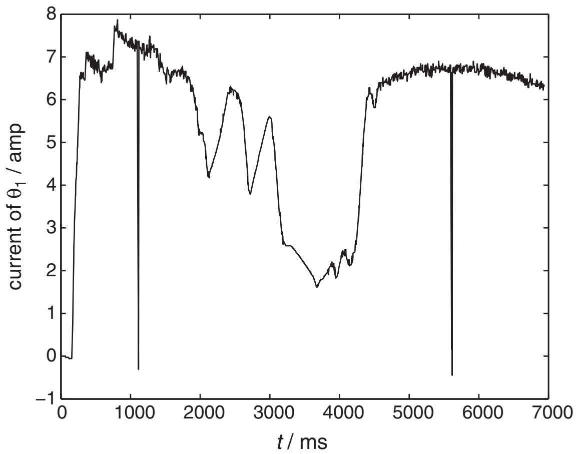

Fig. 10. Current of joint 1 using interpolation.

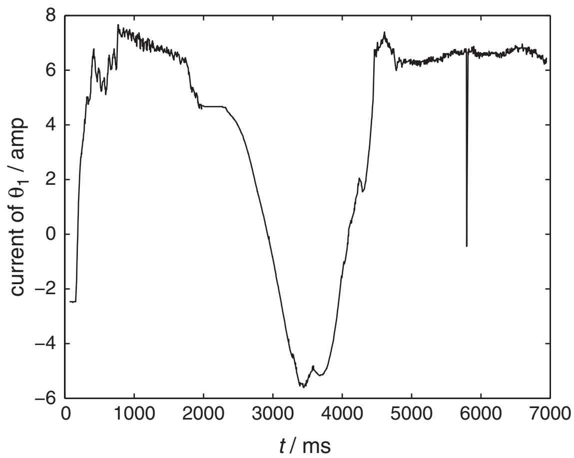

Fig. 11. Current of joint 1 using geodesics.

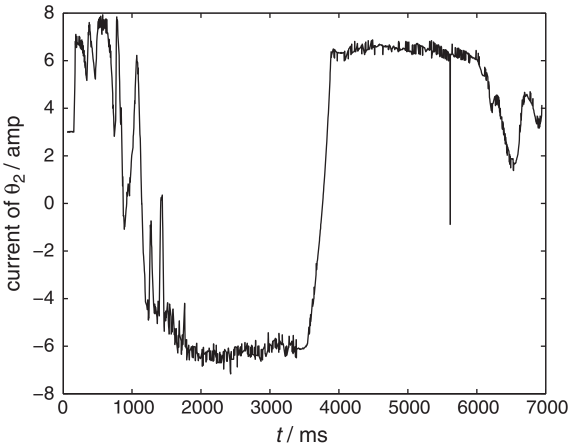

Fig. 12. Current of joint 2 using interpolation.

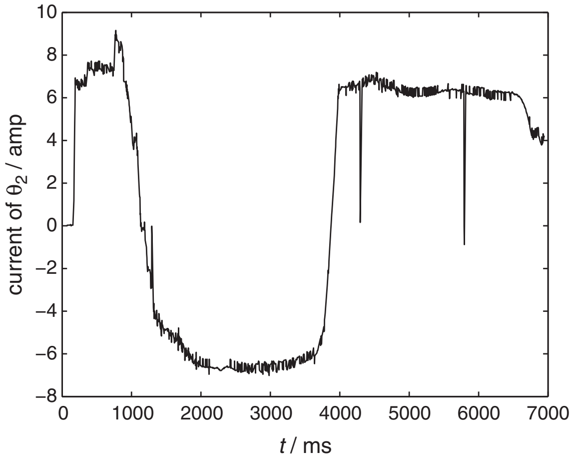

Fig. 13. Current of joint 2 using geodesics.

$$
\begin{aligned}
&= m_2({g_2}^2 + l_1 g_2 c_2) \\
&\quad + m_3({l_2}^2 + {g_3}^2 + l_1 l_2 c_2 + l_1 g_3 c_{23} \\
&\quad + 2 l_2 g_3 c_3) \\
m_{13} &= m_{31} \\
&= m_3({g_3}^2 + l_1 g_3 c_{23} + l_2 g_3 c_3) \\
m_{22} &= m_2 {g_2}^2 + m_3({l_2}^2 + {g_3}^2 + 2 l_2 g_3 c_3) \\
m_{23} &= m_{32} \\
&= m_3({g_3}^2 + l_2 g_3 c_3) \\
m_{33} &= m_3 {g_3}^2
\end{aligned}
$$

where g_i, l_i and m_i, $i = 1, 2, 3$ are the position of center of mass of each link, the length of the linkage and the mass of the linkage respectively. $g_1 = g_2 = 0.1$, $g_3 = 0.07$, $l_1 = l_2 = 0.2$, $l_3 = 0.105$, $m_1 = m_2 = m_3 = 4$kg.

4.1.3. Optimal energy gait planning using geodesics

Now we can use the geodesic method to make the energy optimal gait of walking forward from state 0 to state 1. The idle state, state 0 and state 1 of the RoboErectus AdultSize humanoid robot is shown in Fig. 8. The idle state is the position and orientation of the robot when it starts walking, the height of hip kneels down from 0.685m (stand still) to 0.640m (idle state) in our experiment. The state 0 is: the distance between foot sole and ground in frontal plane is

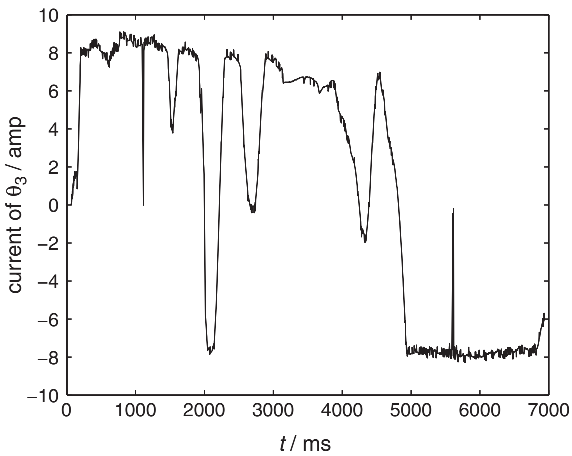

Fig. 14. Current of joint 3 using interpolation.

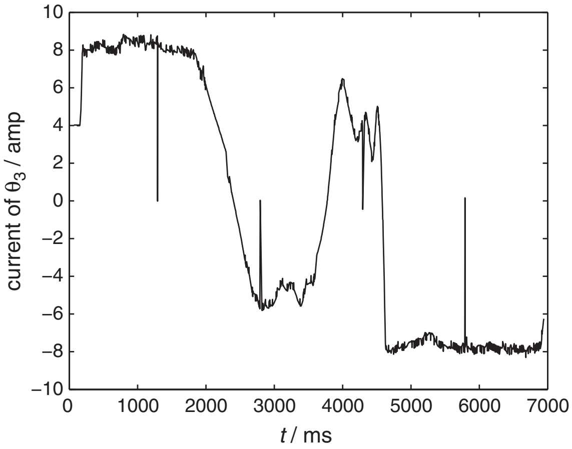

Fig. 15. Current of joint 3 using geodesics.

0.017m, the swing distance in sagittal plane is 0. The state 1 is: the distance between foot sole and ground in frontal plane is 0.04 m, the swing distance in sagittal plane is 0.09 m. According to inverse kinematics, the corresponding joint angles of swing leg for state 0 and state 1 are $\theta_{10} = -0.50$, $\theta_{20} = 1.03$, $\theta_{30} = -0.49$ and $\theta_1 = -0.80$, $\theta_2 = 1.24$, $\theta_3 = -0.41$ respectively. According to the above conditions and the Riemannian metric matrix, the geodesic equation can be obtained and be solved. The results are shown in Fig. 9.

5. Experiments of the optimal energy gait of swing leg

Experiments are carried out on the RoboErectus AdultSize humanoid robot platform in our ARICC Centre. The gait from state 0 to state 1 is generated by geodesic method. We also create the gait using state-of-the-art third-order spline interpolation method [15]. The advantage of our geodesic method is that the giat between two states are energy optimized and obtained directly by solving the geodesic equation, no inverse kinematics are needed. The disadvantage is that the geodesic method is more complicated than the third-order spline interpolation.

The currents of the PowerCube joint motors are recorded during robot swing its left leg. The currents of joint 1 under spline interpolation and under geodesics method are shown in Figs. 10, 11 respectively. The currents of joint 2 and joint 3 before and after geodesic optimization are shown in Figs. 12-15 respectively.

Comparing the two groups of current results, the currents by the geodesics method are a little more smooth than the traditional polynomial interpolating method. This experiment result shows that the smooth and energy saving gait can be obtained by our geodesic method.

6. Conclusion

A new smooth and energy saving gait planning method based on geodesic is introduced in this paper. The optimal kinetic energy gait planning of 2-dimensional inverted pendulum is studied at first and the popular admitted results for this 2D inverted pendulum are obtained. Then we extend geodesics method to the swing leg of the real biped humanoid robot model. The smooth and energy saving gait is attained directly by solving geodesic equations and no inverse kinematics is needed. The novelty of our geodesic method is that we use the geometric attributes of geodesic to naturally obtain the optimal gait while the state-of-the-art method, such as [15], which uses third-order spline interpolation between two states, can not ensure the gait is energy optimal between two states. The disadvantage of our geodesic method is that the calculation is complicated. Finally the gaits planned by geodesics method are carried out on RoboErectus AdultSize humanoid robot [26] successfully and the dynamic efficiency of our method is verified by the experiment results.

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