

## Research Article

# Research on Fatigue Strain and Fatigue Modulus of Concrete

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Concrete fatigue strain and fatigue modulus evolution play a vital role in the evaluation of the material properties. In this paper, by analyzing the advantages and disadvantages of existing concrete strain analysis methods, the level-S nonlinear fatigue strain model was proposed. The parameters' physical meaning, the ranges, and the impact on the shape of the curve were all discussed. Then, the evolution model of fatigue modulus was established based on the fatigue strain evolution model and the hypothesis of fatigue modulus inversely related fatigue strain amplitude. The results indicate that the level-S model covered all types of fatigue strain evolution. It is very suitable for the description of strain evolution of concrete for its strong adaptability and high accuracy. It was found that the fitting curves coincided with the experimental curves very well, and the correlation coefficients were all above 0.98. The evolution curves of fatigue strain modulus both have three stages, namely, variation phase, linear change stage, and convergence stage. The difference is that the fatigue strain evolution curve is from the lower left corner to the upper right corner, but the fatigue modulus evolution curve is from the upper left corner to the right lower corner.

## 1. Introduction

Fatigue strain of the concrete can truly reflect the variation of the material deformation under the fatigue loads. If we know the relationship of the curve and the cycles, we are able to qualitatively describe the evolution of the material fatigue strain, which will provide the basis for the behavior of the material evaluation. In numerous constant amplitude fatigue tests, it is shown that the total longitude deformation and the residual deformation of the concrete will display the universal and stable three-stage law [1–6], which presents firstly the rapid growth stage, then the stable growth stage, and ultimately the rapid growth stage. This is suitable for not only the ordinary concrete, lightweight aggregate concrete, high strength concrete, or fiber reinforced concrete, but also the compression fatigue, tension fatigue, bending fatigue, uniaxial fatigue, or multiracial fatigue. Chen et al. [7] took advantage of a cubic polynomial fitting curve to get correlation coefficients above 0.937. However, different levels of stress would own different coefficients with nearly an order of

magnitude. Cachim et al. [8] adopted the logarithmic form between the maximum strain rate and the load cycles in the second phase for the concrete. Under the compression fatigue loads, the form was a linear relationship. Xie et al. [9] also got an experienced index formula by fitting the second phase of fatigue strain. Wang et al. [10] fitted the data of compressive fatigue experiment strain and adopted a two-stage nonlinear formula. According to the fatigue strain evolution and methods of current analysis, the following deficiencies were discovered. ① Presently, the linear three-stage fatigue strain equations are simple, but of a low accuracy. While the three-stage nonlinear equation is of high precision, the form is complicated. It is rarely reported about the whole fatigue strain curve with the cycle ratio relationship. ② The evolution and fitting of the fatigue strain curve are aimed at a specific set of experimental data nowadays. As for the conditions that the fatigue stress is smaller than limited stress but greater than the threshold value, there are few researches. ③ A consensus has reached the variation law of the three-stage fatigue strain, and some curve fitting equations have been obtained, but the

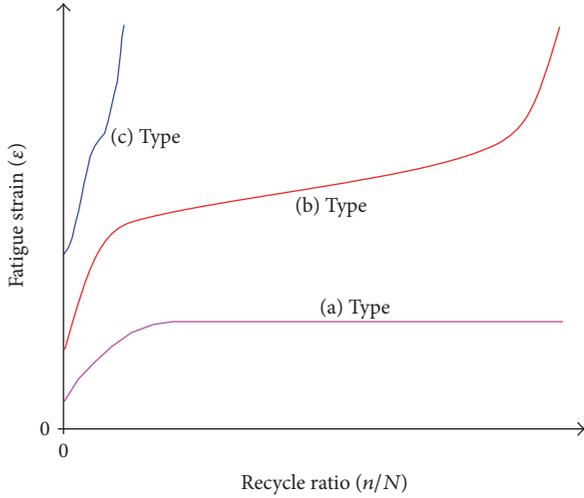


FIGURE 1: Basic types of fatigue strain evolution curve.

initial strain was not taken into consideration basically. In these equations, the significance of each factor is not clear enough, which results in unstable fitting coefficients.

In the literature [11] published by the author, the nonlinear strain evolution model was established. The strain evolution law of concrete under constant amplitude fatigue load and the law of fatigue damage evolution based on strain were studied. Based on the previous research, in this paper, the physical meanings, the ranges, and the impact on the shape of the curve of parameters in the nonlinear strain evolution model were all discussed. The evolution model of fatigue modulus was established under constant amplitude bending fatigue loading based on the fatigue strain evolution model and the hypothesis of fatigue modulus inversely related fatigue strain amplitude. Moreover, the whole process is validated by the experimental data.

## 2. Fatigue Strain Evolution Model

To better describe the three-stage variation law, a fatigue strain evolution model with the simple form was proposed, which could be suitable for different stress types (Figure 1). Moreover, some valuable physical parameters could be obtained via the model, such as the initial strain, instability speed of the third stage (acceleration of fatigue strain), and the proportion of that in the total fatigue life [12]. Via the repeated study on the related mathematical functions, the model could be obtained below.

$$\varepsilon^n = \varepsilon_0 + \alpha \cdot \left( \frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (1)$$

In formula (1),  $\varepsilon_0$  indicate the initial strain and  $\varepsilon^n$  indicate the fatigue strain under  $n$  times of fatigue loading cycle.  $n$  indicate the cycle times of fatigue loads.  $N_f$  indicate the fatigue life.  $\alpha$ ,  $\beta$ , and  $p$  were parameters in the equation.

If the maximum fatigue strain  $\varepsilon_{\max}^n$  or fatigue residual strain  $\varepsilon_{\text{res}}^n$  was regarded as the value of  $\varepsilon^n$ , formula (2) would

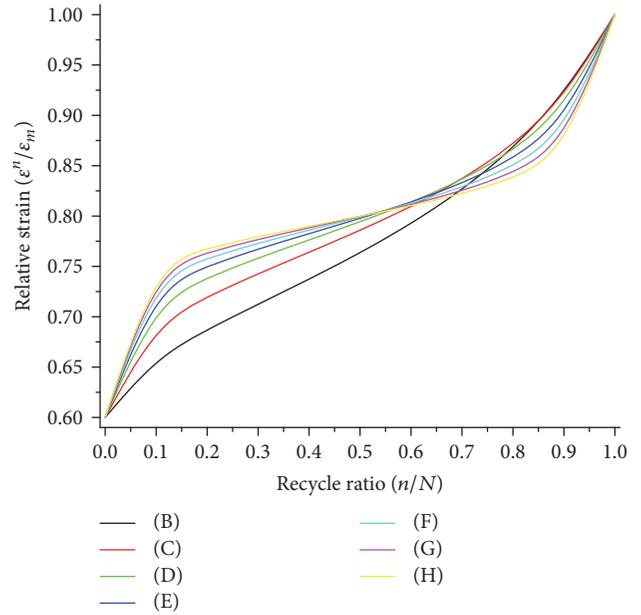


FIGURE 2: Influence of  $p$  on fatigue strain curve. (B)  $p = 2$ ,  $\beta = 1.25$ ; (C)  $p = 3$ ,  $\beta = 1.125$ ; (D)  $p = 4$ ,  $\beta = 1.0625$ ; (E)  $p = 5$ ,  $\beta = 1.03125$ ; (F)  $p = 6$ ,  $\beta = 1.015625$ ; (G)  $p = 7$ ,  $\beta = 1.007813$ ; (H)  $p = 8$ ,  $\beta = 1.003906$ ; (B)~(H)  $\varepsilon_{\max}^0 / \varepsilon_{\max}^f = 0.6$ ,  $\alpha / \varepsilon_{\max}^f = 0.2$ .

be obtained by formula (1). In the same way, if the initial maximum strain  $\varepsilon_{\max}^0$  or initial residual strain  $\varepsilon_{\text{res}}^0$  was regarded as the value of  $\varepsilon_0$ , formula (3) would be obtained.

$$\varepsilon_{\max}^0 = \varepsilon_{\max}^0 + \alpha \left( \frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}, \quad (2)$$

$$\varepsilon_{\text{res}}^n = \varepsilon_{\text{res}}^0 + \alpha \left( \frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (3)$$

Formula (2) was called as the maximum fatigue strain evolution model, while formula (3) was the fatigue residual strain evolution model.

If the upper limit fatigue stress is large, fatigue strain will change fast and increase. The slope of the curve will be large and the curve become the vertical line. That was to say, within a few loading cycles, the strain will reach the unstable state. The three-stage curve will degenerate the c-type in Figure 2 [13]. When the upper limit fatigue stress does not exceed the threshold, the elastic strain will be added to the initial strain and the strain remains unchanged, which is similar to the a-type curve. These two types are both the limit state, so the stresses in most fatigue tests are located between the upper limit and the threshold. In other words, the evolution curves have characteristics of both a-type and c-type. Generally, the b-type curve in Figure 2 can be obtained. According to the actual results, the evolution laws of the b-type curve will be analyzed in this paper.

$\varepsilon_{\max}^0$  and  $\varepsilon_{\text{res}}^0$  indicate, respectively, the initial maximum strain and residual strain. They are mainly caused by some factors, such as the initial defects of the material and preloading of the structure. Due to the difficulty of distinguishing

between the two strains, the actually measured values of the fatigue test can be obtained. Therefore, when the fatigue loading firstly reaches the upper limit, the corresponding maximum strain  $\varepsilon_{\max}^1$  and residual strain  $\varepsilon_{\text{res}}^1$  are adopted in this paper. For example, Lv [14] obtained the formula of  $\varepsilon_{\max}^1$  and  $\varepsilon_{\text{res}}^1$  according to the actual experimental data as  $\varepsilon_{\text{res}}^1 = 0.25(\varepsilon_{\max}^1/\varepsilon_{\text{unstab}})^2$ . Among the formula,  $\varepsilon_{\text{unstab}}$  indicated the total strain of the unstable concrete.

In order to study the range of parameters  $\alpha$ ,  $\beta$ , and  $p$ , including impacts on the evolution law of fatigue strain curves, formulas (2) and (3) are divided by the limited fatigue strain on both sides. So the following are obtained.

$$\frac{\varepsilon_{\max}^n}{\varepsilon_{\max}^f} = \frac{\varepsilon_{\max}^0}{\varepsilon_{\max}^f} + \frac{\alpha}{\varepsilon_{\max}^f} \left( \frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}, \quad (4)$$

$$\frac{\varepsilon_{\text{res}}^n}{\varepsilon_{\text{res}}^f} = \frac{\varepsilon_{\text{res}}^0}{\varepsilon_{\text{res}}^f} + \frac{\alpha}{\varepsilon_{\text{res}}^f} \left( \frac{\beta}{\beta - n/N_f} - 1 \right)^{1/p}. \quad (5)$$

Both formulas are the normalized evolution model of the fatigue strain. In the former one,  $\varepsilon_{\max}^f$  indicates the limited maximum fatigue strain. In the latter one,  $\varepsilon_{\text{res}}^f$  indicates the limited fatigue residual strain.

$\beta$  indicates the destabilizing factor. This parameter depends on  $p$  and  $\alpha$ . If the circulation ratio  $n/N_f$  is equal to 1, the point (1, 1) will locate on the curve of formulas (4) and (5). Via the two formulas, values of  $\beta$  can be deduced as formula (6), which correspond to the maximum fatigue stress and the residual fatigue stress.

$$\beta_1 = \left( \frac{(1 - \varepsilon_{\max}^0/\varepsilon_{\max}^f)}{(\alpha/\varepsilon_{\max}^f)} \right)^{-p} + 1, \quad (6)$$

$$\beta_2 = \left( \frac{(1 - \varepsilon_{\text{res}}^0/\varepsilon_{\text{res}}^f)}{(\alpha/\varepsilon_{\text{res}}^f)} \right)^{-p} + 1.$$

In order to analyze the impacts of  $p$  and  $\alpha$  on the fatigue strain evolution curve, the evolution formula (4) was taken as an example. In the first place, the impact of  $p$  was analyzed via  $\varepsilon_{\max}^0/\varepsilon_{\max}^f$  and  $\alpha/\varepsilon_{\max}^f$ . Then, combined with  $p$  and  $\varepsilon_{\max}^0/\varepsilon_{\max}^f$ , the impact of  $\alpha$  was analyzed further. Ultimately, the impacts curve of  $p$  and  $\alpha$  were shown in Figures 2 and 3.

It could be seen from Figure 2 that parameter  $p$  would influence the convergence speed of the curve using the level-S nonlinear model. With the growing of  $p$ , the convergence speed of the fatigue strain curve in the third stage could become faster, while the stage corresponded to the irreversible third stage of fatigue deformation, which also meant the acceleration stage of unstable crack propagation. The parameter  $p$  was also called the instability speed factor and the suggested values were located in [2, 8].

The effects of different  $\alpha$  values on the curve are shown in Figure 3.

It could be seen from Figure 3 that parameter  $\alpha$  would influence the proportion of the third stage in the total fatigue life. The stage was the so-called acceleration stage of the

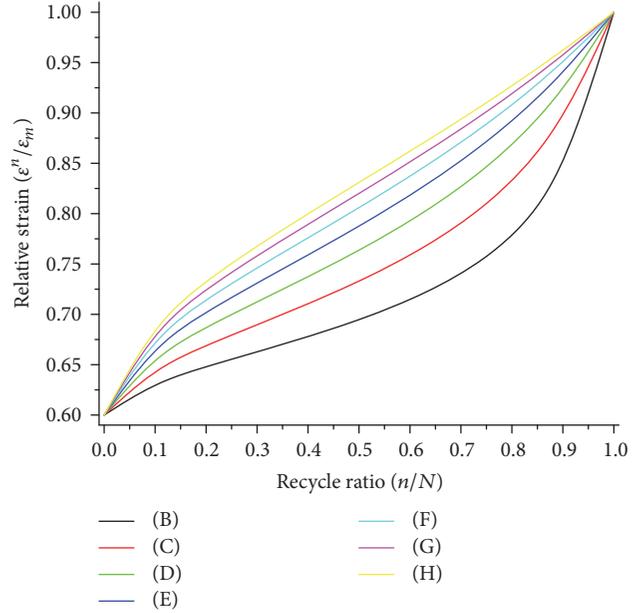


FIGURE 3: Influence of  $\alpha$  on the fatigue strain curve. (B)  $\alpha/\varepsilon_{\max}^f = 0.1$ ,  $\beta = 1.0625$ ; (C)  $\alpha/\varepsilon_{\max}^f = 0.15$ ,  $\beta = 1.140625$ ; (D)  $\alpha/\varepsilon_{\max}^f = 0.2$ ,  $\beta = 1.25$ ; (E)  $\alpha/\varepsilon_{\max}^f = 0.25$ ,  $\beta = 1.390625$ ; (F)  $\alpha/\varepsilon_{\max}^f = 0.3$ ,  $\beta = 1.5625$ ; (G)  $\alpha/\varepsilon_{\max}^f = 0.35$ ,  $\beta = 1.765625$ ; (H)  $\alpha/\varepsilon_{\max}^f = 0.4$ ,  $\beta = 2$ ; (B)~(H)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.6$ ,  $p = 2$ .

fatigue strain. With the growing of  $\alpha$ , the proportion of the acceleration stage could become shorter.  $\alpha/\varepsilon_{\max}^f$  was located in  $(0, 1 - \varepsilon_{\max}^0/\varepsilon_{\max}^f)$ , so  $\alpha$  was located in  $(0, \varepsilon_{\max}^f - \varepsilon_{\max}^0)$ .

The suggested values of the parameters  $\alpha$ ,  $\beta$ ,  $p$  were mainly aimed at the b-type curves which had three stages of evolutions. That was to say, the values for the a-type and c-type were not limited by the paper. In Figure 4, the S-shaped curves by combining various parameters were presented, including nearly different kinds of fatigue strain evolutions for the concrete material.

### 3. Fatigue Modulus Evolution Model

Fatigue modulus  $E$  is the ratio of stress amplitude  $\sigma_{\max} - \sigma_{\min}$  to strain amplitude  $\varepsilon_{\max} - \varepsilon_{\min}$ ; that is,  $E = (\sigma_{\max} - \sigma_{\min})/(\varepsilon_{\max} - \varepsilon_{\min})$ .  $\sigma_{\max}$  and  $\sigma_{\min}$  are the maximum and minimum fatigue stresses. Under constant amplitude fatigue loads, stress amplitude  $\sigma_{\max} - \sigma_{\min}$  remains unchanged. Therefore, the fatigue modulus and fatigue strain amplitude are an inversely proportional relationship; that is, if the fatigue strain amplitude decreases, the fatigue modulus increases. On the contrary, if the fatigue strain amplitude increases, the fatigue modulus will be reduced [14, 15]. The strain evolution curve is the level-S-shaped curve from the lower left corner to the upper right corner, from the inverse relationship. The fatigue strain and fatigue modulus can be shown in Figure 5.

Suppose two curves are symmetrical about the straight line  $y = D$ , and the point in the fatigue strain curve is  $(x, y)$ . Then, the corresponding point on the curve of the fatigue modulus is  $(x, 2D - y)$ . Thus the normalized fatigue modulus

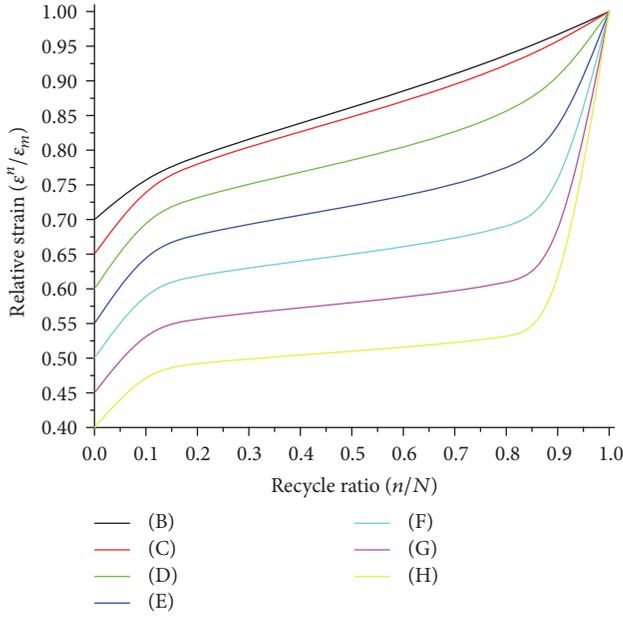


FIGURE 4: Level-S-shaped curve family of fatigue strain. (B)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.7$ ,  $p = 2$ ,  $\alpha/\varepsilon_{\max}^f = 0.25$ ,  $\beta = 1.694444$ ; (C)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.65$ ,  $p = 3$ ,  $\alpha/\varepsilon_{\max}^f = 0.23$ ,  $\beta = 1.283778$ ; (D)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.6$ ,  $p = 4$ ,  $\alpha/\varepsilon_{\max}^f = 0.19$ ,  $\beta = 1.050907$ ; (E)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.55$ ,  $p = 5$ ,  $\alpha/\varepsilon_{\max}^f = 0.17$ ,  $\beta = 1.007695$ ; (F)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.5$ ,  $p = 6$ ,  $\alpha/\varepsilon_{\max}^f = 0.15$ ,  $\beta = 1.000729$ ; (G)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.45$ ,  $p = 7$ ,  $\alpha/\varepsilon_{\max}^f = 0.13$ ,  $\beta = 1.000041$ ; (H)  $\varepsilon_{\max}^0/\varepsilon_{\max}^f = 0.40$ ,  $p = 8$ ,  $\alpha/\varepsilon_{\max}^f = 0.11$ ,  $\beta = 1.000001$ .

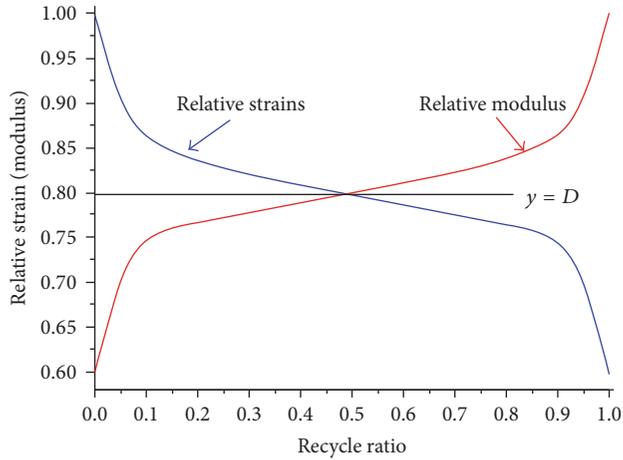


FIGURE 5: Diagram of fatigue strain and fatigue modulus evolution curves.

evolution model can be obtained by the fatigue strain evolution model, as shown in the following equation:

$$\frac{E^n}{E_0} = \frac{\varepsilon_0}{\varepsilon^f} + \frac{\alpha'}{\varepsilon^f} \left( \frac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'} + 2D. \quad (7)$$

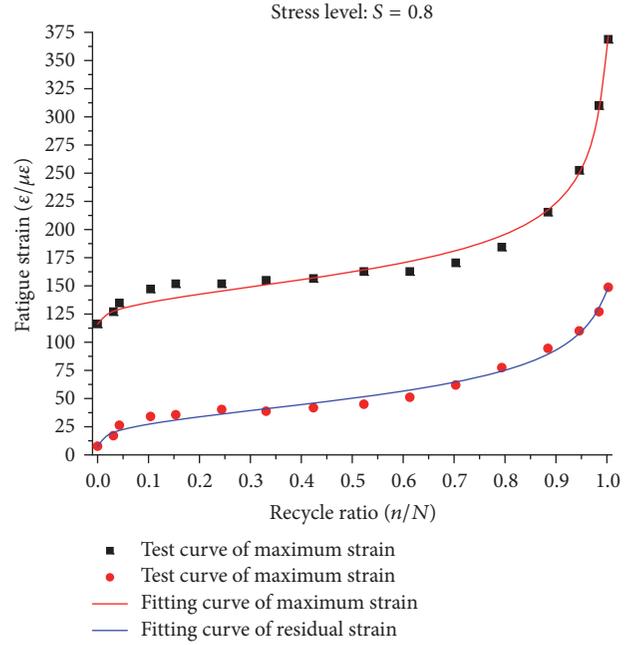


FIGURE 6: Fatigue strain experimental curves and fitting curves of group A.

Therefore, the fatigue modulus evolution model is

$$E^n = E_0 \left( \frac{\varepsilon_0}{\varepsilon^f} + \frac{\alpha'}{\varepsilon^f} \left( \frac{\beta'}{\beta' - n/N_f} - 1 \right)^{1/p'} + 2D \right). \quad (8)$$

In formulas (7) to (8),  $E^n$  is the elastic modulus after fatigue cycle  $n$  times and  $E_0$  is the initial elastic modulus.

## 4. Model Validation

**4.1. Experiment Introduction.** The length, width, and height of the concrete experiment beams are 400 mm, 100 mm, and 100 mm, respectively. Concrete strength is C35. The experimental beam is divided into A, B, and C three groups, with corresponding stress levels  $S_{\max} = \sigma_{\max}/f$  of 0.8, 0.75, and 0.7. The fatigue experiment equipment is the MTS810 material testing machine. The third point (50 + 3 × 100 + 50) bending constant amplitude fatigue load was used. The load frequency is 10 Hz. The fatigue experiment cycle stress ratio  $\sigma_{\min}/\sigma_{\max}$  is 0.1. Fatigue life from the experiments was 37,256 times, 14,905 times, and 631,291 times, and the initial elastic modulus is 50 GPa.

**4.2. Concrete Fatigue Strain Evolution Analysis.** The fitting results of the maximum fatigue strain and fatigue residual strain under different stress levels using the model formulas (2) to (3) are shown in Figures 6–8. Coefficients of the evolutionary model are shown in Table 1. The data in the figure are the average of each group.

From Figures 6–8 and Table 1, fatigue strain evolution equations (2) and (3) can be a good fit to the experimental data. Correlation coefficients were above 0.98. The evolution

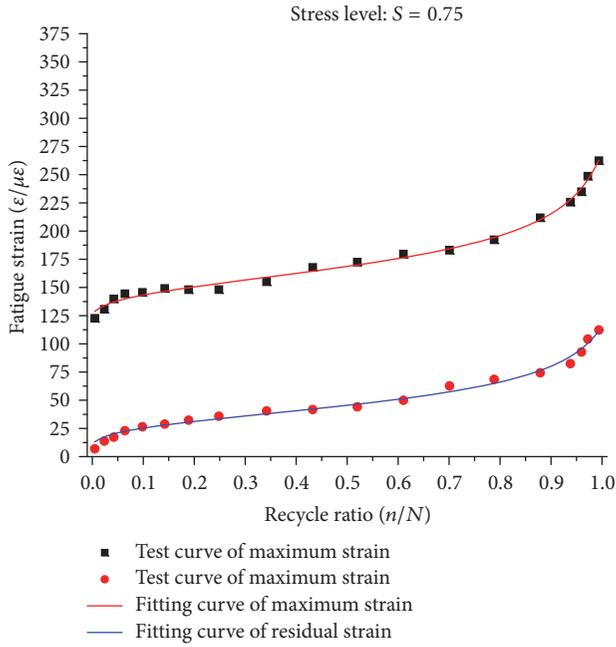


FIGURE 7: Fatigue strain experimental curves and fitting curves of group B.

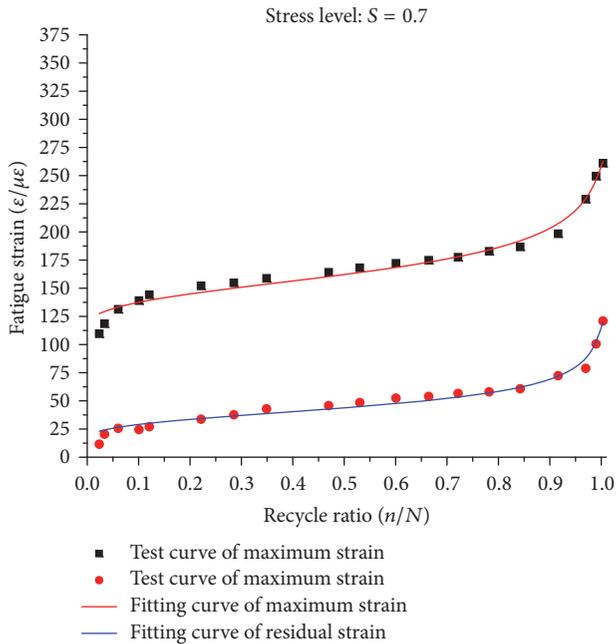


FIGURE 8: Fatigue strain experimental curves and fitting curves of group C.

of maximum fatigue strain and fatigue residual strain has similar three-phase variation and the intermediate stage close to the linear change. The curve converged rapidly when the cycle ratio exceeded approximately 0.9. The strain evolution curve is the level-S-shaped curve from the lower left corner to the upper right corner. Because the experiment measured only the initial maximum strain and has not measured initial

TABLE 1: Coefficient values of fatigue strain.

Groups	S	$\epsilon_{\max}^n$		p	$R^2$
		$\alpha$	$\beta$		
A	0.8	47.0919	1.0182	2.4672	0.9891
B	0.75	48.1478	1.0456	2.7550	0.9935
C	0.7	54.0070	1.2096	3.4928	0.9751

Groups	S	$\epsilon_{\text{res}}^n$		p	$R^2$
		$\alpha$	$\beta$		
A	0.8	43.8164	1.0422	2.7736	0.9889
B	0.75	39.9757	1.0545	2.9135	0.9887
C	0.7	32.5965	1.0171	3.5857	0.9753

TABLE 2: Coefficient values of fatigue modulus.

Stress level	$E^n$				
	$\alpha'$	$\beta'$	$p'$	D	$R^2$
0.8	2.3645	1.0081	30.2698	2.7075	0.9909
0.75	1.5242	1.0762	19.2173	1.8772	0.9935
0.7	1.1918	1.0070	19.0863	1.5847	0.9833

residual strain, the evolution curve of the maximum strain starts from the initial value, but the evolution curve of fatigue residual strain starts from zero. According to  $(0, \epsilon_{\max}^f - \epsilon_{\max}^0)$  and experimental data, the range of  $\alpha$  of groups A, B, and C is  $(0, 250)$ ,  $(0, 130)$ , and  $(0, 165)$ , respectively. Fitting results of  $\alpha$  falls in these ranges, in line with the boundary conditions. When the stress level is from 0.8 to 0.75 and from 0.75 to 0.7, the  $p$  value is increasing, demonstrating that the smaller the stress level is, the faster the fatigue strain evolution curve converges.

4.3. Concrete Fatigue Modulus Evolution Analysis. The fitting results of the fatigue modulus under different stress levels using the model formula (8) are shown in Figures 9–11. Coefficients of evolutionary model (8) are shown in Table 2. The data in figure are the average of each group.

From Figures 9–11 and Table 2, fatigue modulus evolution equation (8) can be a good fit to the experimental data. Correlation coefficients were above 0.97. The evolution of the fatigue modulus also has a three-phase variation, and the intermediate stage is also close to the linear change. The curve converged rapidly when the cycle ratio exceeded approximately 0.9. Fatigue modulus evolution curve is the level-S-shaped curve from the upper left corner to the lower right corner. When the stress level is from 0.8 to 0.75 and from 0.75 to 0.7, the  $p'$  value is increasing, demonstrating that the smaller the stress level is, the faster the fatigue modulus evolution curve converges.

### 5. Conclusions

(1) By analyzing the shortcomings and limitations of existing fatigue strain evolution equations, the level-S nonlinear evolution model of fatigue strain, which contains the initial strain, was constructed. Then, the physical meaning of the

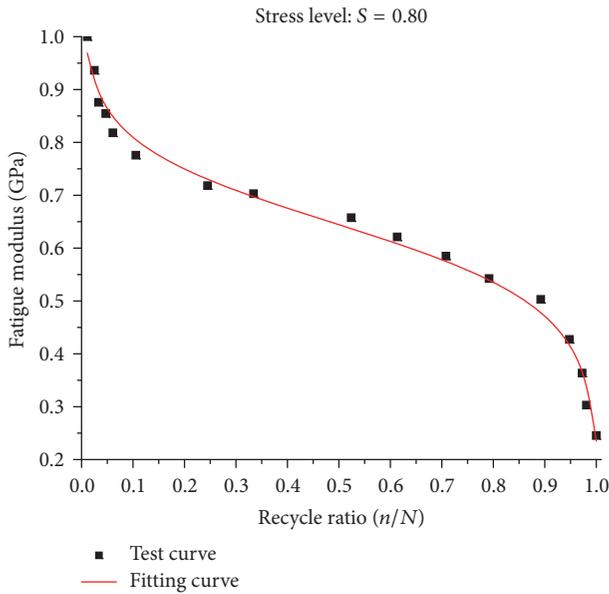


FIGURE 9: Fatigue modulus experimental curves and fitting curves of group A.

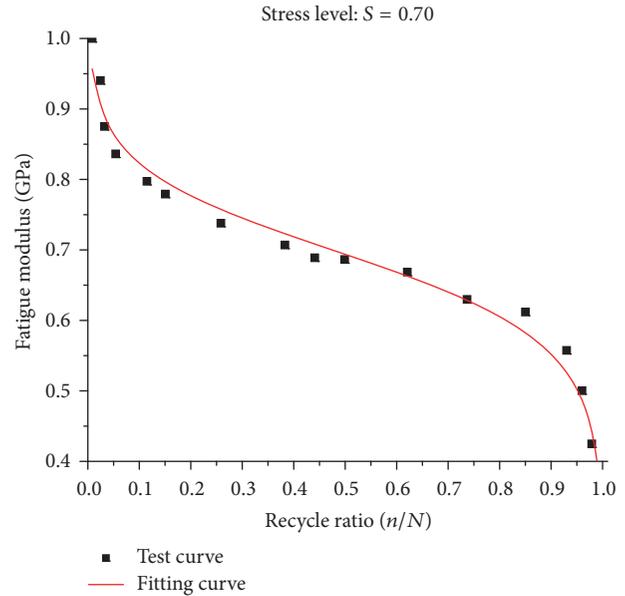


FIGURE 11: Fatigue modulus experimental curves and fitting curves of group C.

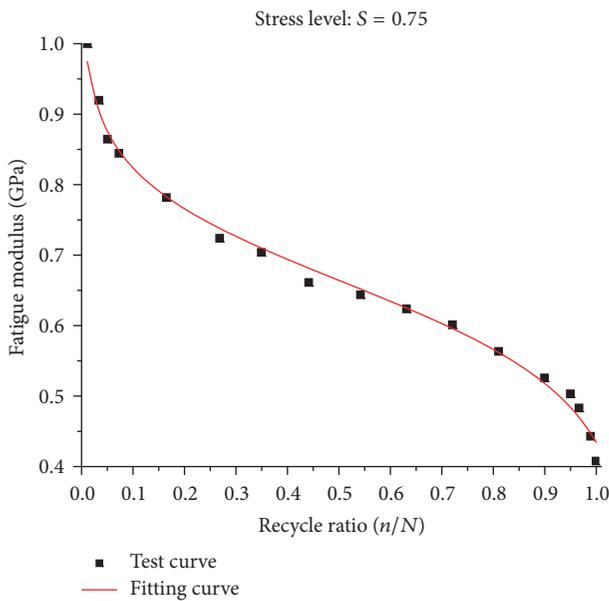


FIGURE 10: Fatigue modulus experimental curves and fitting curves of group B.

parameters and the range and the impact on the shape of the curve were all discussed.

(2) The results show that the level-S model covered all types of fatigue strain evolution and has strong adaptability and high accuracy. The model could be tested for applicability to steel fiber reinforced concrete, glass fiber reinforced concrete, and a variety of characteristics of concrete.

(3) The fatigue modulus evolution model was established under constant amplitude bending fatigue loading based on

fatigue strain evolution model and the hypothesis of fatigue modulus inversely related to fatigue strain amplitude.

(4) The evolution curve of fatigue strain and fatigue modulus both have three stages of variation, and the middle stage is nearly linear. When the recycle ratio is 0.9, two curves converge quickly. The difference is that the fatigue strain evolution curve is from the lower left corner to the upper right corner, but the fatigue modulus evolution curve is from the upper left corner to the right lower corner.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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