Hindawi
Advances in Civil Engineering
Volume 2018, Article ID 1217543, 15 pages
https://doi.org/10.1155/2018/1217543

*Research Article*

# Extension Model for Safety Appraisal of Existing Concrete Members Based on an Improved Comprehensive Weighting Method

**Hengyu Zhou, Yongbo Yuan, Chenlong Liu, Chao Wu, and Mingyuan Zhang**

*Faculty of Infrastructure Engineering, Dalian University of Technology, Linggong Road, No. 2, 116024 Dalian, China*

Correspondence should be addressed to Hengyu Zhou; zhy19880607@mail.dlut.edu.cn

Received 17 May 2018; Revised 26 July 2018; Accepted 12 September 2018; Published 10 October 2018

Academic Editor: Roman Wan-Wendner

Copyright © 2018 Hengyu Zhou et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

An effective evaluation model for safety appraisal of existing concrete members plays a significant role in promoting the management of an existing building. This study aims to introduce extension theory into the safety appraisal of existing concrete members based on five indices (bearing capacity, deflection-to-span ratio, cracks, reinforcement corrosion, and concrete carbonation depth) and inspection data. A matter-element model is established for the safety appraisal of existing concrete members based on matter-element theory. The safety appraisal rating is identified by the comprehensive correlation degrees, which can be calculated by the weights and single-index correlation degrees of the five indices. Owing to the one-sidedness in the single-weighting method, a comprehensive weighting method integrating the merits of subjective weight and objective weight is adopted based on game theory. The interval analytic hierarchy process (IAHP) and entropy weight method are, respectively, used to determine the subjective and objective weight of each index. It was found that the subjective weight vector calculated by IAHP consists of interval numbers. Therefore, the traditional comprehensive weighting method based on game theory needs to be improved by the interval number theory. A comparison analysis between the results generated by the proposed model and an analytic hierarchy process-fuzzy comprehensive evaluation model is conducted. The results show that the matter-element extension model based on comprehensive weight is more accurate and rational. The proposed model makes full use of inspection data and gives a clear safety level to decision makers avoiding disorganized data of a single index. Hence, it can serve as guidance for safety appraisal of existing concrete members in the future. Furthermore, the improved comprehensive weighting method has practical merits and high scientific value in terms of safety evaluation and other applications in different research fields.

## 1. Introduction

The safety evaluation of existing buildings facilitates the completion of strengthening, maintenance, and management during the usage phase of an existing building. This study is focused on the safety appraisal of existing concrete members, which is the foundation of the safety evaluation for existing buildings. There has been significant research on the safety degree of structural members. For instance, the concept of the safety degree of structural members was put forward in 1947 [1]. The reliability indicator was first suggested as a unified numerical indicator to measure the safety degree of structural members, and the second-order moment model was established [2]. Moreover, the separation function method was used for the second-order moment model based on the research achievement of Cornell [2]. Partial safety factors were obtained by a separation function method, and the reliability indicator was used to govern the safety class used in the partial factor method [3]. Furthermore, the advanced first-order second-moment method based on a checking point (also known as the JC-method) was proposed. The JC-method was recommended by the Joint Committee on Structural Safety (JCSS) to calculate the safety degrees of structural members [4–6]. As the number of existing buildings continues to increase, the application of the JC-method is recently becoming more focused on existing structural members [7–9]. The JC-method, which was used for the bearing capacity appraisal of structural members, has been widely practiced in many countries and regions, including the US, Canada, the European Union, and China. Since the objects of this study are reinforced concrete members, their safety can be evaluated by bearing capacity appraisal individually according to design specifications. Since existing concrete members have been in service for some time and are typically damaged (e.g., by cracking, reinforcement corrosion and concrete carbonation), their safety appraisal becomes complicated. The effects of these damages have been considered in the specifications of several countries, and the safety appraisal of existing concrete members can be carried out by adjusting the partial safety factors [10–13]. However, the adjustment of partial safety factors significantly depends on the engineers' knowledge and experience.

In China, with the reduction in the number of newly built buildings, the safety of existing buildings has attracted wide attention. The Ministry of Construction enacted the *Standard for appraisal of reliability of civil buildings* [14], which was the first Chinese standard for the detection and safety appraisal of existing buildings. In this standard, there is no partial factor for existing structural members. Thus, the safety appraisal rating of existing concrete members is determined by the lowest appraisal rating of three indices (bearing capacity, deformation, and cracks). It should be noted that damage is not considered in the bearing capacity index. This rating method automatically supposes all these three factors are equally important leading to conservative results of safety appraisal and potentially resulting in unnecessary economic losses. Therefore, an effective safety appraisal method for existing concrete members needs to be explored urgently.

In order to solve this problem, this study introduces the matter-element extension model into the safety appraisal of existing concrete members. Due to their significant impact on the safety of existing concrete members, two additional indices are considered in the proposed rating model: reinforcement corrosion and concrete carbonation depth [15–17]. An extension model, which is composed of the matter-element theory and extension mathematics, is suitable for processing the incompatibility and variability between evaluation indices from both quantitative and qualitative perspectives [18]. Matter-element theory, which was first put forward by the Chinese mathematician Cai Wen, realizes the combination of qualitative and quantitative research methods [19, 20]. Extension mathematics mainly quantifies the process of problem-solving by combining the extension set and correlation function. Extension theory has been widely used for rating evaluations in many research fields [20–23]. The critical step in extension theory is the determination of the weights of the evaluation indices. Two kinds of weighting methods are commonly used: subjective methods and objective methods. In this study, IAHP and the entropy weight method are, respectively, used to determine the subjective and objective weight of each index. However, both of these methods have their limitations when they are used individually. Subjective weight is easily influenced by expert knowledge and prejudices, and its subjectivity could be overemphasized [24]. Meanwhile, the calculated objective weight might not correspond to practical situations [25]. Thus, the comprehensive weight integrating the merits of subjective and objective weights is adopted as the weight of each index in this study [26]. Game theory, a mathematical modelling of strategic interaction between rational and irrational agents, specializes in solving conflicts among two or more participants [27]. The subjective weight and objective weight can be analogous to two participants, and comprehensive weight is regarded as the result of the 'weight' game. In this study, the subjective weights calculated by IAHP are presented by interval numbers. An improved comprehensive weighting method based on game theory and interval number theory which can process interval numbers is used to determine the indices' comprehensive weights.

The objective of this study is to analyse the safety rating of existing concrete members. In the Index Selection and Data Sources section, five indices are selected and the interpretation of each index is presented. The data used in this study are provided by a practical inspection project. In the Methodology section, based on the subjective and objective weight, an improved comprehensive weighting method is proposed. The extension model for safety appraisal of existing concrete member is established by the grading standards of indices and the inspection data in the Index Selection and Data Sources section. In the Results and Discussion section, the weight of each index and the safety rating of two damaged beams are calculated and analysed. To verify the effectiveness of the extension model based on comprehensive weights, a comparison analysis between the results generated by the proposed model and an AHP-Fuzzy comprehensive evaluation model is conducted. In the final section, several conclusions are pointed out. The proposed model makes full use of inspection data and gives a clear safety level to decision makers avoiding disorganized data of a single index. Thus, it can serve as guidance for safety appraisal of existing concrete members in the future. Furthermore, the improved comprehensive weighting method has practical merits and high scientific value in terms of safety evaluation and other applications in different research fields.

## 2. Index Selection and Data Sources

*2.1. Index Selection.* Five indices (i.e., bearing capacity, deflection-to-span ratio, cracks, reinforcement corrosion, and concrete carbonation depth) are selected in the safety appraisal of existing concrete members. The bearing capacity, deflection-to-span ratio, and cracks are mandatory inspection items in the *Standard for appraisal of reliability of civil buildings* [14]. According to practical conditions, the indices should be measurable and have significant impacts on the safety of existing concrete members. Combined with the actual inspection projects in China and extensive literature, another two indices (reinforcement corrosion and concrete carbonation depth) are selected [15–17]. The five indices are described as below:

Bearing capacity (BC): this index reflects the strength, stiffness, and stability of existing concrete members.

According to the standard [14], its value is calculated using Equation (1), which is the global safety factor format [3, 28, 29].

$$\frac{R}{\gamma_0 S}, \tag{1}$$

where $R$ is the resistance of a member, $S$ and $\gamma_0$ are the load effect and coefficient for importance of a structure, respectively. In addition, in the calculation process of the bearing capacity index, the structural members are divided into ten equal segments. The cross section with the minimum resistance/load-effect ratio is defined as the most unfavourable cross section. For each cross section, the software can calculate the ratios of the resistance to load effect under all specified working conditions, and the bearing capacity of each cross section is determined by the minimum value of these ratios. Thus, the bearing capacity of the most unfavourable cross section is the final result of the bearing capacity index [30]. The calculation of the resistance and load effect in this process requires the inspection information of the current status of the existing building, such as reinforcement distribution inspection, material strength inspection, and load distribution inspection, etc.

Deflection-to-span ratio (DE/SP): the value of this index is determined by the ratio of the total deflection-to-span length.

Cracks (CR, mm): the value of this index is determined by the most unfavourable crack width of the member.

Reinforcement corrosion (RC, mv): according to [31], this index represents the degree of the corrosion of reinforcements and can be described by the average electric potential among the measuring points which can be measured by an electric potential survey.

Concrete carbonation depth (CCD): this index is valued by the ratio of the measured carbonation depth ($d$) to the thickness of the concrete cover ($c$).

*2.2. Data Sources.* The survey results of an actual inspection project were used as the data sources in this study. Five indices of twenty-four frame beams were detected. The thicknesses of concrete protective layers of the twenty-four frame beams are all 25 mm. These inspection data are shown in Table 1.

## 3. Methodology

*3.1. Weight Definition.* In order to achieve an effective safety appraisal of existing concrete members, the significant procedure of determining the weights of the five indices is presented in this section. The weighting methods can be divided into two categories: subjective methods and objective methods. Subjective methods, a relatively mature approach, can determine the ranking of each index's weight by the actual situation and expert's experience, but have strong subjective arbitrariness. On the contrary, the objective methods determine the weights according to the initial data and have a strong mathematical theoretical basis. However, it is possible that the objective weights are inconsistent with the actual situation and subjective desires. For the sake of a suitable weight, this study proposes an improved comprehensive weighting method based on game theory and interval number theory, which combines the advantages of both IAHP and the entropy weight method.

*3.1.1. Subjective Weight Based on IAHP.* AHP, a multicriteria decision-making approach, is a simple and useful method for solving complex and ambiguous issues [32, 33]. However, due to the fuzziness of human judgment and uncertainty of many situations, the judgments provided by different decision makers are frequently uncertain and inconsistent. Therefore, the IAHP was proposed based on AHP and interval number theory. The assigned values of the pairwise comparison matrix, which are judged by decision makers, are confined to intervals whose widths reflect the possible attribute values [26, 34].

In IAHP, the establishment of a judgment matrix relies on the experience and knowledge of experts and takes the prior and current information (e.g., inspection data and structural behaviour) into account synthetically. Although, this study is focused on the current safety status of existing concrete members, the prior information is convenient for experts to set up a more realistic judgment matrix.

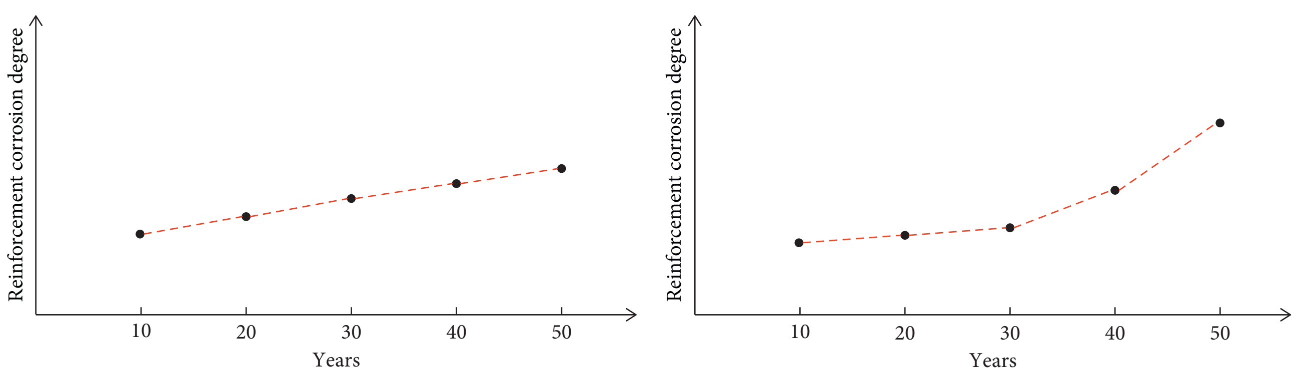
In order to illustrate the role of prior information in the construction of judgment matrices, this analysis takes the reinforcement corrosion index as an example and assumes that a concrete member has been in service for 50 years and the inspection interval is 10 years. Suppose the variation trend of inspection data is as shown in Figure 1(a); it means that the deterioration rate of a material property is slow and the deterioration degree is limited. At this point, the effect of reinforcement corrosion on safety is not obvious. However, supposing the variation trend is as shown in Figure 1(b), the significantly greater slope after 30 years indicates that the deterioration of material and structural behaviour begins to enter a period of rapid development and reinforcement corrosion makes a relatively evident impact on the safety of the existing concrete member [35]. Hence, the experts should improve the importance of the index of reinforcement corrosion in the process of pairwise comparison between each index. The above analysis is only 'the tip of the iceberg'; experienced experts can have a more comprehensive understanding of the current structural behaviour through abundant prior information and knowledge. Therefore, in order to make the subjective weight more close to the actual situation of the project, the acquisition and analysis of the prior information is necessary.

The calculation process of IAHP is as presented below:

*Step 1*. Compare the relative importance of each index pairwise based on the 1~9 scaling rules proposed by Saaty [32], and finally the interval judgment matrix is shown as

Table 1: Inspection data.

| Index | Bearing capacity | Deflection/span ($\times 10^{-3}$) | Cracks | Reinforcement corrosion | Concrete carbonation depth |
|---|---|---|---|---|---|
| Beam 1 | 1.08 | 2.29 | 0.20 | −195 | 0.40 |
| Beam 2 | 0.94 | 2.13 | 0.13 | −178 | 0.32 |
| Beam 3 | 1.07 | 1.89 | 0.17 | −114 | 0.16 |
| Beam 4 | 1.18 | 2.09 | 0.27 | −125 | 0.52 |
| Beam 5 | 1.20 | 2.06 | 0.12 | −184 | 0.55 |
| Beam 6 | 1.21 | 1.89 | 0.09 | −131 | 0.36 |
| Beam 7 | 1.44 | 2.99 | 0.12 | −181 | 0.45 |
| Beam 8 | 1.21 | 2.21 | 0.15 | −124 | 0.49 |
| Beam 9 | 1.31 | 2.45 | 0.17 | −192 | 0.43 |
| Beam 10 | 1.53 | 2.42 | 0.12 | −135 | 0.40 |
| Beam 11 | 1.08 | 2.73 | 0.20 | −119 | 0.53 |
| Beam 12 | 0.86 | 2.06 | 0.47 | −321 | 0.70 |
| Beam 13 | 2.25 | 2.57 | 0.17 | −203 | 0.49 |
| Beam 14 | 1.62 | 1.76 | 0.35 | −245 | 0.57 |
| Beam 15 | 1.60 | 1.97 | 0.25 | −231 | 0.51 |
| Beam 16 | 1.10 | 2.39 | 0.11 | −186 | 0.49 |
| Beam 17 | 1.64 | 2.67 | 0.10 | −137 | 0.58 |
| Beam 18 | 1.63 | 2.22 | 0.15 | −194 | 0.60 |
| Beam 19 | 1.11 | 2.05 | 0.05 | −168 | 0.52 |
| Beam 20 | 1.66 | 2.15 | 0.04 | −171 | 0.37 |
| Beam 21 | 1.63 | 1.77 | 0.06 | −199 | 0.42 |
| Beam 22 | 1.11 | 2.05 | 0.16 | −215 | 0.58 |
| Beam 23 | 1.76 | 1.59 | 0.08 | −234 | 0.50 |
| Beam 24 | 1.73 | 1.71 | 0.12 | −127 | 0.38 |




(a) (b)

Figure 1: The variation trend of inspection data.

$$A = \begin{bmatrix} [1,1] & [\underline{a}_{12}, \overline{a}_{12}] & \cdots & [\underline{a}_{1n}, \overline{a}_{1n}] \\ [\underline{a}_{21}, \overline{a}_{21}] & [1,1] & \cdots & [\underline{a}_{2n}, \overline{a}_{2n}] \\ \vdots & \vdots & & \vdots \\ [\underline{a}_{n1}, \overline{a}_{n1}] & [\underline{a}_{n2}, \overline{a}_{n2}] & \cdots & [1,1] \end{bmatrix}, \quad (2)$$

where $\underline{a}_{ij}$ and $\overline{a}_{ij}$ represent the upper limit and lower limit of the ratio of the relative importance between $i$th element and $j$th element, respectively.

According to the algorithm of interval numbers [34], $A$ can also be expressed as $A = [\underline{A}, \overline{A}]$, where $\underline{A}$ and $\overline{A}$ are shown as

$$\underline{A} = \begin{bmatrix} 1 & \underline{a}_{12} & \cdots & \underline{a}_{1n} \\ \underline{a}_{21} & 1 & \cdots & \underline{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \underline{a}_{n1} & \underline{a}_{n2} & \cdots & 1 \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} 1 & \overline{a}_{12} & \cdots & \overline{a}_{1n} \\ \overline{a}_{21} & 1 & \cdots & \overline{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \overline{a}_{n1} & \overline{a}_{n2} & \cdots & 1 \end{bmatrix}. \quad (3)$$

*Step 2.* Calculate the maximum eigenvalue $\overline{\lambda}_{\max}$ and $\underline{\lambda}_{\max}$ corresponding to $\overline{A}$ and $\underline{A}$, respectively. Then, the normalized eigenvectors $\overline{w}$ and $\underline{w}$ corresponding to $\overline{\lambda}_{\max}$ and $\underline{\lambda}_{\max}$, respectively, are obtained.

*Step 3*. The negative component coefficient $k$ of $\underline{w}$ and the positive component coefficient $m$ of $\overline{w}$ can be calculated using

$$k = \sqrt{\sum_{j=1}^{n}\left(\frac{1}{\sum_{i=1}^{n}\overline{a_{ij}}}\right)},$$
$$m = \sqrt{\sum_{j=1}^{n}\left(\frac{1}{\sum_{i=1}^{n}\underline{a_{ij}}}\right)}. \tag{4}$$

Assuming that the interval judgment matrix $A$ passes the consistency check, $W = [k\underline{w}, m\overline{w}]$ can be regarded as the interval weights of the indices.

*Step 4*. The consistency check can be performed using

$$z^{*} = \sum_{i=1}^{n}\left(\overline{w_i} - \underline{w_i}\right) < \zeta, \tag{5}$$

where $z^*$ represents the consistency of the interval judgment matrix, the smaller the $z^*$, the better the consistency of the judgment matrix; $\zeta$ is the correlation coefficient between $z^*$ and the conformance rate of AHP, and its value can be queried by Table 2.

*3.1.2. Objective Weights Based on the Entropy Weight Method.* The concept of entropy, which is regarded as a parameter for measuring the degree of disorder and randomness, derives from thermodynamics and was first applied to information theory in 1948 [36]. As an objective weighting method, entropy weights are calculated based on the comentropy theory and reflect the available information provided by each index [37]. The calculation process is as follows:

*Step 1*. Establish a matrix $R$ with $n$ indices and $m$ appraisal objects as

$$R = \left(r_{ij}\right)_{m\times n}(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m). \tag{6}$$

*Step 2*. To eliminate the effects of different dimensions, units, and orders of magnitude among indices, the matrix $R$ should be normalized by the extremum method. The standard matrix $B$ can be obtained using Equation (7). The left-side formula in Equation (7) is appropriate for a positive index and the right-side one is appropriate for a negative index.

$$b_{ij} = \frac{r_{ij} - r_{\min}}{r_{\max} - r_{\min}} \text{ or } b_{ij} = \frac{r_{\max} - r_{ij}}{r_{\max} - r_{\min}}. \tag{7}$$

In Equation (7), $r_{ij}$ is the attribute value, $r_{\max}$ is the maximum attribute value, and $r_{\min}$ is the minimum attribute.

*Step 3*. The index's entropy value $H_i$ can be calculated by Equation (8) based on the comentropy theory.

Table 2: Correlation coefficient values.

| $n$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| $\zeta$ | 9376 | 0.8266 | 0.7658 | 0.6660 | 0.6285 | 0.6381 | 0.6215 | 0.5876 |

$$H_i = -\frac{1}{\mathrm{Ln}m}\sum_{j=1}^{m} f_{ij} Lnf_{ij}, \tag{8}$$

where $f_{ij} = b_{ij}/\sum_{j=1}^{m} b_{ij}$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.

*Step 4*. The entropy weight of each index can be calculated as

$$w_i = \frac{1 - H_i}{n - \sum_{i=1}^{n} H_i}, \tag{9}$$

where $i = 1, 2, \ldots, n$.

*3.1.3. Improved Comprehensive Weighting Method.* Game theory is a mathematical model of cooperation and conflict between intelligent rational decision makers [38]. Subjective weights and objective weights can be regarded as the two participants in game theory, which are independent and conflicting. As a cooperative result, the conflicts can be solved by searching for a compromise between subjective and objective weights. The most suitable comprehensive weight is gained by Nash Equilibrium theory. However, the subjective weight calculated by IAHP is an interval weight vector, which is not involved in the traditional comprehensive weighting method based on game theory. Therefore, an improved comprehensive weighting method is proposed in this study, which integrates the interval weights into comprehensive weights. The calculation steps are as outlined below.

*Step 1*. The weights of indices are calculated by $n$ types of weighting methods, and the basic weight vector set $W| = \{w_1, w_2, \ldots, w_n\}$ can be established. The arbitrary linear combination of $n$ vectors is as follows:

$$w = \sum_{k=1}^{n} \alpha_k w_k^T \left(\alpha_k > 0\right), \tag{10}$$

where $w$ is a possible weight vector of set $W$ and $\alpha_k$ is the weight coefficient.

*Step 2*. So as to confirm to the most suitable weight vector $w^*$ of $w$, a compromise should be achieved among $n$ weights based on game theory. The compromise can be considered as optimization of $\alpha_k$, and the objective of the optimization is to minimize the deviation between $w$ and $w_k$. The formula is the following:

$$\min\left\|\sum_{j=1}^{n}\alpha_j \times w_j^T - w_i^T\right\|_2 \quad (i = 1, 2, \ldots, n). \tag{11}$$

According to the differentiation property of the matrix, the condition of optimal first-order derivative in Equation (11) is as

$$\sum_{j=1}^{n}\alpha_j \times w_i \times w_j^T = w_i \times w_i^T. \tag{12}$$

The real number can be regarded as a special case of the interval number, when the lower limit of the interval number is equal to the upper limit. The system of linear equations corresponding to Equation (12) can be expanded using the interval number as

$$\begin{bmatrix} w_1 \cdot w_1^T & w_1 \cdot w_1^T & \cdots & w_1 \cdot w_1^T \\ w_2 \cdot w_1^T & w_2 \cdot w_2^T & \cdots & w_2 \cdot w_2^T \\ \vdots & \vdots & \vdots & \vdots \\ w_n \cdot w_1^T & w_n \cdot w_2^T & \cdots & w_n \cdot w_n^T \end{bmatrix} \begin{bmatrix} [\underline{\alpha_1}, \overline{\alpha_1}] \\ [\underline{\alpha_2}, \overline{\alpha_2}] \\ \vdots \\ [\underline{\alpha_n}, \overline{\alpha_n}] \end{bmatrix} = \begin{bmatrix} w_1 \cdot w_1^T \\ w_2 \cdot w_2^T \\ \vdots \\ w_n \cdot w_n^T \end{bmatrix}. \tag{13}$$

*Step 3.* According to the algorithm of interval number, Equation (13) can be decomposed into two equation sets of mutual independence and two groups of weight coefficients $(\underline{\alpha_1}, \underline{\alpha_2}, \ldots, \underline{\alpha_n})$ and $(\overline{\alpha_1}, \overline{\alpha_2}, \ldots, \overline{\alpha_n})$ can be calculated. After normalization processing, $(\underline{\alpha_1}^*, \underline{\alpha_2}^*, \ldots, \underline{\alpha_n}^*)$ and $(\overline{\alpha_1}^*, \overline{\alpha_2}^*, \ldots, \overline{\alpha_n}^*)$ are obtained. Similarly, $(\underline{\alpha_1}^*, \underline{\alpha_2}^*, \ldots, \underline{\alpha_n}^*)$ and $(\overline{\alpha_1}^*, \overline{\alpha_2}^*, \ldots, \overline{\alpha_n}^*)$ can also be regarded as the two participants in game theory, which are independent and conflicting. Therefore, the compromise between them can be obtained by Nash Equilibrium as

$$\begin{bmatrix} \underline{\alpha} \cdot (\underline{\alpha})^T & \underline{\alpha} \cdot (\overline{\alpha})^T \\ \overline{\alpha} \cdot (\underline{\alpha})^T & \overline{\alpha} \cdot (\overline{\alpha})^T \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \underline{\alpha} \cdot (\underline{\alpha})^T \\ \overline{\alpha} \cdot (\overline{\alpha})^T \end{bmatrix}, \tag{14}$$

where $\underline{\alpha} = (\underline{\alpha_1}^*, \underline{\alpha_2}^*, \ldots, \underline{\alpha_n}^*)$, $\overline{\alpha} = (\overline{\alpha_1}^*, \overline{\alpha_2}^*, \ldots, \overline{\alpha_n}^*)$, $k_1$ and $k_2$ are the weight coefficients of $\overline{\alpha}$ and $\underline{\alpha}$, respectively.

*Step 4.* After normalization processing, $k_1^*$ and $k_2^*$ are obtained. The final optimal weight coefficient $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ of the $n$ weights and the optimal comprehensive weight $w^*$ can be calculated as

$$\begin{aligned} \beta &= (\beta_1, \beta_2, \ldots, \beta_n) = k_1^* \cdot \underline{\alpha} + k_2^* \cdot \overline{\alpha}, \\ w^* &= \sum_{k=1}^{n} \beta_k \cdot w_k^T. \end{aligned} \tag{15}$$

The process of the improved comprehensive weighting method can be summarized as Figure 2.

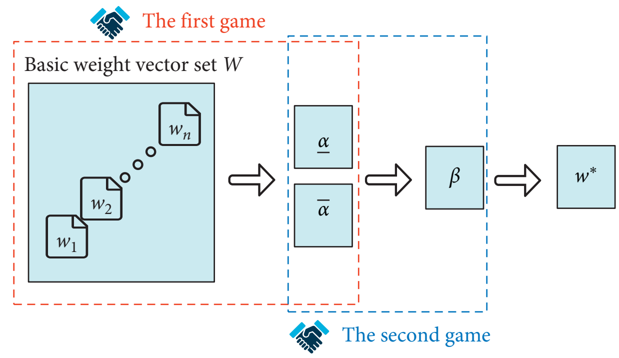



Figure 2: The process of the improved comprehensive weighting method.

### 3.2. Extension Model

*3.2.1. Grading Standard.* According to the grading standard in the *Standard for appraisal of reliability of civil buildings* [14], the safety level of existing concrete members can be divided into $a$, $b$, $c$, and $d$. The levels $a$, $b$, $c$, and $d$ indicate that the member is safe, relatively safe, unsafe, and seriously unsafe, respectively. On the basis of standards and literature [14, 31, 39, 40], the ranges of the five indices, which correspond to the safety levels, are shown in Table 3.

Similarly, the ranges in Table 3 should be normalized by the extremum method (Equation (7)) so as to eliminate the effects of different dimensions, units, and orders of magnitude among indices, as shown in Table 4.

*3.2.2. Extension Theory.* Extension theory, proposed by Cai, is a method of solving problems from both quantitative and qualitative perspectives based on matter-element theory and extension mathematics. The matter-element theory began with the study of the transformation laws and solutions of incompatible or contradictory problems [41, 42]. It is the logical part of the extension theory. The matter-element model is composed of objects, characteristics, and values based on certain characteristics. It has the advantage of convenience for quantifying qualitative indices. On the basis of extension mathematics, problem-solving can be quantified by an extension set and a correlation function. According to the value of the correlation function, we can not only make a judgment on which domain the elements of extension set belong to, but can also distinguish the elements in the same domain to different levels based on their function value [21].

Because of the incompatible characteristic of safety level classification for existing concrete members, the matter-element theory can be introduced into research for solving multisource data decision-making from both qualitative and quantitative perspectives.

*Step 1.* In multisource inspection data, the safety of an existing concrete member, namely, an object $N$, is described by $n$ indices $(c_1, c_2, \ldots c_n)$, which can then be expressed by their corresponding values $(v_1, v_2, \ldots v_n)$. Based on the matter-element theory, the matter-element description of the safety of the existing concrete member can be expressed as

$$R = (N, C, V) = \begin{pmatrix} N, & c_1, & v_1 \\ & c_2, & v_2 \\ & \vdots & \vdots \\ & c_n, & v_n \end{pmatrix}, \tag{16}$$

where $R$ is known as the $n$-dimensional matter-element for describing the object, denoted by $R = (N, C, V)$. Here, the name of the object $N$ (i.e., the safety of the existing concrete member), $n$-dimensional characteristic $C$ (i.e., $n$ indices), and their corresponding value $V$ are called the three elements of the matter-element $R$.

Table 3: Safety grades and corresponding ranges of the five indices.

| Index | Bearing capacity | Deflection/span | Cracks | Reinforcement corrosion | Concrete carbonation depth |
|---|---|---|---|---|---|
| Level $a$ | 1.00~1.10 | 0~1/400 | 0~0.2 | −150~0 | 0~0.25 |
| Level $b$ | 0.95~1.00 | 1/400~1/350 | 0.2~0.35 | −200~−150 | 0.25~0.5 |
| Level $c$ | 0.90~0.95 | 1/350~1/250 | 0.35~0.5 | −350~−200 | 0.5~0.75 |
| Level $d$ | 0.80~0.90 | 1/250~1/150 | 0.5~0.65 | −500~−350 | 0.75~1.0 |

Table 4: Safety grades and corresponding (normalized) ranges of the five indices.

| Index | Bearing capacity | Deflection/span | Cracks | Reinforcement corrosion | Concrete carbonation depth |
|---|---|---|---|---|---|
| Level $a$ | 0.667~1.000 | 0.625~1.000 | 0.692~1.000 | 0.700~1.000 | 0.750~1.000 |
| Level $b$ | 0.500~0.667 | 0.571~0.625 | 0.462~0.692 | 0.600~0.700 | 0.500~0.750 |
| Level $c$ | 0.333~0.500 | 0.400~0.571 | 0.231~0.462 | 0.300~0.600 | 0.250~0.500 |
| Level $d$ | 0~0.333 | 0~0.400 | 0~0.231 | 0~0.300 | 0~0.250 |

For example, the safety of beam 2 in Table 1 can be expressed as

$$R = \begin{pmatrix} \text{the safety of beam 2}, & \text{bearing capacity}, & 0.94 \\ & \text{deflection/Span}, & 2.13 \times 10^{-3} \\ & \text{cracks}, & 0.13\text{mm} \\ & \text{reinforcement corrision}, & -178\text{mv} \\ & \text{concrete carbonation depth}, & 0.32 \end{pmatrix}. \tag{17}$$

*Step 2*. The classical domain matter-element can be defined as

$$R_j = \left(N_j, C, V_j\right) = \begin{pmatrix} N_j, & c_1, & v_{j1,} \\ & c_2, & v_{j2} \\ & \vdots & \vdots \\ & c_n, & v_{jn} \end{pmatrix} = \begin{pmatrix} N_{Cj}, & c_1, & \left[a_{j1}, b_{j1}\right] \\ & c_2, & \left[a_{j2}, b_{j2}\right] \\ & \vdots & \vdots \\ & c_n, & \left[a_{jn}, b_{jn}\right] \end{pmatrix}, \tag{18}$$

where $N_j$ is the safety level, $j$ is the number of levels ranging from 1 to $t$, $c_i$ $(i = 1, 2, \ldots, n)$ is the index of safety appraisal for existing concrete members, and $(a_{ji}, b_{ji})$ is the range of level $N_j$ related to $c_i$ also known as the classical domain.

According to Table 4, the classical domain matter-element for each safety level can be confirmed (Table 5).where $N_P$ is the safety of existing concrete members, $c_i$ $(i = 1, 2, \ldots, n)$ is the index of safety appraisal for existing concrete members, and $(a_{Pi}, b_{Pi})$ $(i = 1, 2, \ldots, n)$ is the whole range of $N_P$ related to $c_i$ (also known as the segmented domain).

*Step 3*. The segmented domain matter-element $R_P$ N be defined as Equation (19), and $R_j$ is the sub-matter-element of

$$R_p = \left(N_P, C, V_p\right) = \begin{pmatrix} N_P, & c_1, & v_{P1} \\ & c_2, & v_{P2} \\ & \vdots & \vdots \\ & c_n, & v_{Pn} \end{pmatrix} = \begin{pmatrix} N_P, & c_1, & \left[a_{P1}, b_{P1}\right] \\ & c_2, & \left[a_{P2}, b_{P2}\right] \\ & \vdots & \vdots \\ & c_n, & \left[a_{Pn}, b_{Pn}\right] \end{pmatrix}. \tag{19}$$

According to Table 4, the segmented domain matter-element of the safety of beam 2 can be expressed as

Table 5: The classical domain matter-element (normalized) for each safety level.

| Level | Classical domain matter-element | Level | Classical domain matter-element |
|---|---|---|---|
| $A$ | $R_a = \begin{pmatrix} a, & \text{BC}, & [0.667, 1.000] \\ & \text{DE/SP}, & [0.625, 1.000] \\ & \text{CR}, & [0.692, 1.000] \\ & \text{RC}, & [0.700, 1.000] \\ & \text{CCD}, & [0.750, 1.000] \end{pmatrix}$ | $c$ | $R_c = \begin{pmatrix} c, & \text{BC}, & [0.333, 0.500] \\ & \text{DE/SP}, & [0.400, 0.571] \\ & \text{CR}, & [0.231, 0.462] \\ & \text{RC}, & [0.300, 0.600] \\ & \text{CCD}, & [0.250, 0.500] \end{pmatrix}$ |
| $B$ | $R_b = \begin{pmatrix} b, & \text{BC}, & [0.500, 0.667] \\ & \text{DE/SP}, & [0.571, 0.625] \\ & \text{CR}, & [0.462, 0.692] \\ & \text{RC}, & [0.600, 0.700] \\ & \text{CCD}, & [0.500, 0.750] \end{pmatrix}$ | $d$ | $R_d = \begin{pmatrix} d, & \text{BC}, & [0, 0.333] \\ & \text{DE/SP}, & [0, 0.400] \\ & \text{CR}, & [0, 0.231] \\ & \text{RC}, & [0, 0.300] \\ & \text{CCD}, & [0, 0.250] \end{pmatrix}$ |

BC, bearing capacity; DE/SP, deflection/span; CR, cracks; RC, reinforcement corrosion; CCD, concrete carbonation depth.

$$R_P = \begin{pmatrix} \text{the safety of beam 2,} & \text{bearing capacity,} & [0, 1] \\ & \text{deflection/span,} & [0, 1] \\ & \text{cracks,} & [0, 1] \\ & \text{reinforcement corrision,} & [0, 1] \\ & \text{concrete carbonation depth,} & [0, 1] \end{pmatrix}. \tag{20}$$

*Step 4*. According to extension theory and the definition of distance [43], $v_{ji} \sqsubset v_{Pi}$, the distance between $v_i$ and $v_{Pi}$ can be calculated as

$$\rho\left[v_i, v_{Pi}\right] = \left| v_i - \frac{a_{Pi} + b_{Pi}}{2} \right| - \frac{b_{Pi} - a_{Pi}}{2}. \tag{21}$$

Then the distance between $v_i$ and $v_{ji}$ can be calculated as

$$\rho\left[v_i, v_{ji}\right] = \left| v_i - \frac{a_{ji} + b_{ji}}{2} \right| - \frac{b_{ji} - a_{ji}}{2}. \tag{22}$$

*Step 5*. Based on Equations (20) and (21), the correlation function is

$$k_j\left(v_i\right) = \begin{cases} \dfrac{\rho\left[v_i, v_{ji}\right]}{\rho\left[v_i, v_{Pi}\right] - \rho\left[v_i, v_{ji}\right]} & \left(\rho\left[v_i, v_{ji}\right] \neq 0\right) \\ -\rho\left[v_i(t), v_{ji}\right] - 1 & \left(\rho\left[v_i, v_{Pi}\right] = \rho\left[v_i, v_{ji}\right]\right) \end{cases} \tag{23}$$

Then, the comprehensive correlative degree of level $j$ can be calculated as

$$k_j(N) = \sum_{i=1}^{n} w_i k_j\left(v_i\right), \tag{24}$$

where $w_i$ is the weight of index $C_i$.

If $K_j = \max K_j(N)$, then the safety level of the existing concrete member belongs to level $j$.

### 3.3. Summary of the Extension Model

*3.3.1. Matter-Element Extension Method.* The matter-element extension method includes several basic steps. Firstly, analysis or evaluation indices are selected and grades are defined (i.e., level $a$, $b$, $c$, $d$). The objects (i.e., the safety of existing concrete members) are described as matter-elements based on inspection data (e.g., Equation (17)). Grade intervals for each index are then also defined, as listed in Table 4. For each grade, the range of values is called the classical domain (Table 5), while the whole range of values for all grades is called the segmented domain (e.g., Equation (20)). Thirdly, the correlation degree for each single index (in other words, how well each index matches the grading standard) is calculated. Finally, the comprehensive correlation degree of matter-elements for each grade is calculated through model integration methods such as the weighted average method. The grade, including the maximum comprehensive correlation degree, defines the safety level within which the matter-element falls. The above process can be summarized as shown in Figure 3.

*3.3.2. Extension Model Based on Comprehensive Weight.* The entire extension model based on comprehensive weight can be summarized as shown in Figure 4.

## 4. Results and Discussion

*4.1. Weight Analysis.* Subjective weight was determined by IAHP and five experts were invited to discuss the establishment of the interval judgment matrices. In many existing buildings, incomplete, obsolete or fragmented prior information on the buildings is predominant [44–46]. In China, most existing buildings for inspection have never been inspected before, and the deficiency of prior information is very common and serious. Hence, the five experts can only make engineering judgment on the basis of current inspection information and structural behaviour.

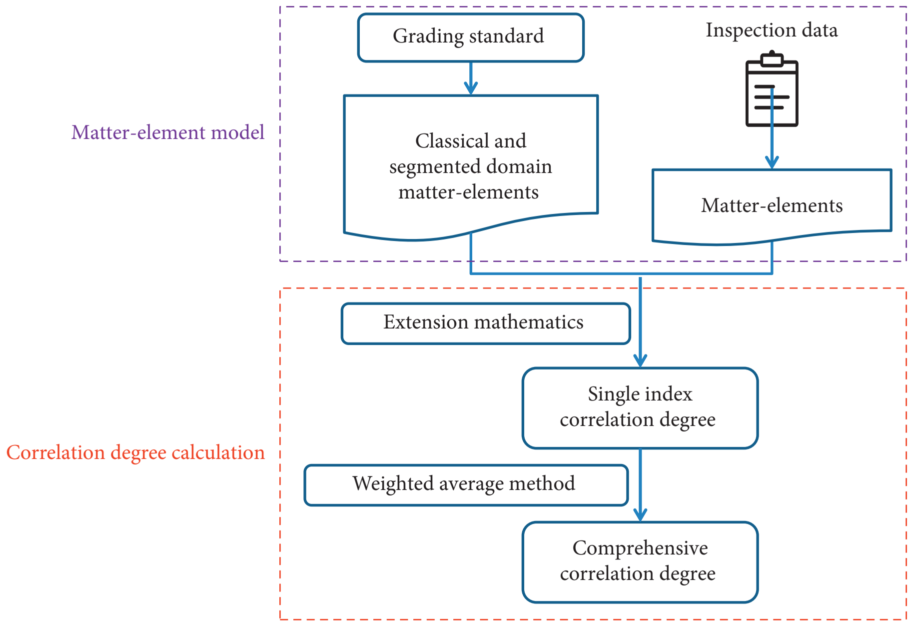



Figure 3: Matter-element extension method.

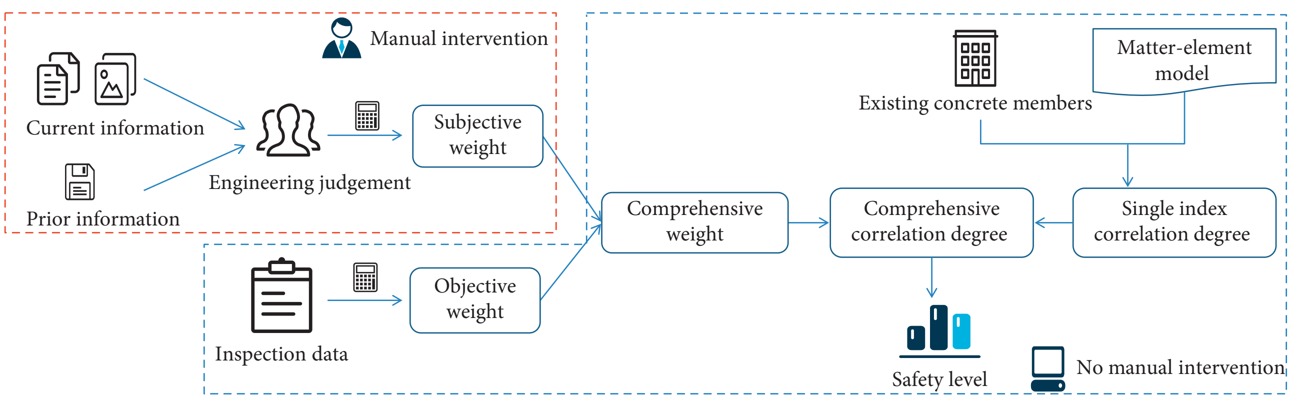



Figure 4: Extension model based on comprehensive weight.

Considering the inspection data and structural behaviour of beam 12, it can be concluded that the cracks, which are in the same direction as that of the main reinforcement, occur primarily owing to reinforcement corrosion and expansion. In addition, the carbonation depth approaches the thickness of the protective layer. The cracks, reinforcement corrosion, and concrete carbonation result in material performance degradation, such as the reduction of yield strength of reinforcement and the degradation of bond performance between reinforcement and concrete. Because the degree of structural deterioration is gradually revealed from the inside out [47, 48], the occurrence of corrosive cracks indicates that the degradation of internal material performance is serious, which leads to a marked decrease of the resistance of existing concrete members. Therefore, the safety of beam 12 is significantly affected by these three indicators, namely, cracks, reinforcement corrosion, and concrete carbonation depth, and in particular by the reinforcement corrosion and concrete carbonation depth [49]. Further, a higher importance must be assigned to these three indicators when establishing the judgment matrix for beam 12. Owing to the lack of prior information, using field recheck inspection and calculation, the experts can only ascertain that the safety of beam 2 is mainly affected by insufficient reinforcement. The other four damage indicators, excluding bearing capacity, are approximately believed to be in the early stage of structural degradation, and the effect of these four indicators on the safety of beam 2 is not obvious.

Based on the above analysis, the judgment matrices of IAHP for beams 2 and 12 are shown in Table 6. The interval judgment matrices satisfy the consistency check. Objective weight was calculated by an entropy weight method using the inspection data in Table 1. The comprehensive weight integrating subjective and objective weights is finally obtained by the optimal weight coefficient $\beta_1$, $\beta_2$, which was calculated by the improved comprehensive weighting method proposed in this study. The weight results are shown in Table 7.

According to Table 7, the weights of IAHP assume bearing capacity and reinforcement corrosion as the most important among all five indices, and deflection-to-span

Table 6: The interval judgment matrix of IAHP.

| Member | Interval judgment matrix | | | | | |
|---|---|---|---|---|---|---|
| | Index | BC | DE/SP | CR | RC | CCD |
| Beam 2 | BC | [1,1] | [6,8] | [6,8] | [5,6] | [5,6] |
| | DE/SP | [1/8,1/6] | [1,1] | [1/2,1] | [1/3,1/2] | [1/3,1/2] |
| | CR | [1/8,1/6] | [1,2] | [1,1] | [1/3,1/2] | [1/3,1/2] |
| | RC | [1/6,1/5] | [2,3] | [2,3] | [1,1] | [1,2] |
| | CCD | [1/6,1/5] | [2,3] | [2,3] | [1/2,1] | [1,1] |
| Beam 12 | BC | [1,1] | [6,8] | [3,4] | [2,3] | [2,3] |
| | DE/SP | [1/8,1/6] | [1,1] | [1/6,1/4] | [1/8,1/6] | [1/8,1/6] |
| | CR | [1/4,1/3] | [4,6] | [1,1] | [1/3,1/2] | [1/3,1/2] |
| | RC | [1/3,1/2] | [6,8] | [2,3] | [1,1] | [1,3] |
| | CCD | [1/3,1/2] | [6,8] | [2,3] | [1/3,1] | [1,1] |

BC, bearing capacity; DE/SP, deflection/span; CR, cracks; RC, reinforcement corrosion; CCD, concrete carbonation depth.

Table 7: Index weights calculated by the three methods.

| Weight | Index | | | | | |
|---|---|---|---|---|---|---|
| | Bearing capacity | Deflection/span | Cracks | Reinforcement corrosion | Concrete carbonation depth | $\beta_1$, $\beta_2$ |
| Subjective weight (beam 2) | (0.583, 0.590) | (0.056, 0.065) | (0.063, 0.076) | (0.134,0.164) | (0.119, 0.141) | 0.656 |
| Objective weight (beam 2) | 0.356 | 0.185 | 0.117 | 0.132 | 0.211 | 0.344 |
| Comprehensive weight (beam 2) | (0.505, 0.510) | (0.100, 0.106) | (0.082, 0.090) | (0.133,0.153) | (0.150, 0.165) | — |
| Subjective weight (beam 12) | (0.381, 0.425) | (0.034, 0.035) | (0.106, 0.114) | (0.220,0.282) | (0.179, 0.213) | 0.718 |
| Objective weight (beam 12) | 0.356 | 0.185 | 0.117 | 0.132 | 0.211 | 0.282 |
| Comprehensive weight (beam 12) | (0.374, 0.405) | (0.076, 0.077) | (0.109, 0.115) | (0.195,0.240) | (0.188, 0.213) | — |

ratio is deemed as the least important index. Obviously, bearing capacity is the most relevant index of the safety of existing concrete members. Although the importance ranking of the five indices conforms to decision makers' intention, the weight value of the deflection-to-span ratio, close to zero, is underestimated due to the strong subjectivity and many biases from various opinions. The entropy weight regards bearing capacity, concrete carbonation depth, and deflection-to-span ratio as the most important and reinforcement corrosion and cracks as the least important. It reveals the valuable information and the inherent law of the inspection data. However, the actual conditions are not considered, causing the results to deviate from the decision maker's intentions. For instance, on the basis of engineering experience, the reinforcement corrosion can cause the expansion of the steel bars, resulting in internal forces. As a consequence, it reduces bond between reinforcement and concrete and greatly accelerates the deterioration of existing concrete members. Therefore, in our opinion, the weight value of reinforcement corrosion should be larger, but 0.132 appears far from practical situations.

There are both advantages and disadvantages in the IAHP and entropy weight methods. The intentions of the decision makers can be flexibly reflected by IAHP, but the inherent law of the data is not considered. Nevertheless, the inherent law and useful information can be revealed using the entropy weight method, but the actual situations are ignored. On the one hand, comprehensive weight integrates the advantages of subjective and objective weights so as to make some abnormal values more reasonable by markedly increasing the objective weight value of reinforcement corrosion and the subjective weight value of the deflection-to-span ratio. On the other hand, since the objective weight narrows the interval width of subjective weight, the interval width of comprehensive weight decreases significantly, meaning that comprehensive weight is more precise than subjective weight. Therefore, the comprehensive weight combines uncertainty and precision and overcomes the one-sidedness of the single weighting method.

The five indices that affect the safety of existing concrete members can be classified as strength index (i.e., bearing capacity), durability indices (i.e., reinforcement corrosion and concrete carbonation depth) and serviceability indices (i.e., deflection-to-span ratio and cracks).

For structural engineering design, the safety of structural members mainly embodies in the safety of bearing capacity. Since existing concrete members have been in service for some time and are potentially damaged, the durability and serviceability indices begin to influence the safety of existing concrete members and make the safety appraisal complicated. Even so, strength is indisputably still the most important index for the safety of existing concrete members, due to the close correlation between bearing capacity and reliability indicator.

The durability indices affect the safety of existing concrete members through deterioration of material properties in a defined work environment. For instance, the corrosive medium in the environment infiltrates into reinforced concrete leading to reinforcement corrosion and material strength reduction. Moreover, $CO_2$ in the air permeates the concrete and reacts with alkaline substances to form carbonate and water, which reduces the alkalinity and strength of concrete and makes concrete lose its protective effect on reinforcement. Thus, reinforcement corrosion and concrete carbonation result in the degradation of the material performance and existing concrete members' resistance, which affects safety. In addition, durability indices have a common feature that the process of damage accumulation, over time, is dynamic and gradual. In other words, only when the damage accumulates to a certain degree, the durability indices begin to have an evident impact on the safety performance of existing concrete members [50]. Therefore, the durability indices have relatively less direct and effective influence on safety than strength index.

Although serviceability indices are focused on the use function and appearance of existing structural members and users' feeling, they still have an impact on safety. However, the serviceability indices can't affect safety independently of durability indices. For example, cracks due to various reasons accelerate the process of reinforcement corrosion, thus affecting safety. As a result, the serviceability indices are considered to have the lowest correlation with safety.

Based on the analysis above, the weight ranking of three types of indicators be as follows in decreasing order: strength index > durability indices > serviceability indices, which is consistent with the comprehensive weight result in Table 7. Certainly, the determination of weights also depends on the requirements of owners or users and the focus of the investigation. The performance of existing concrete members includes safety, use function, and appearance. Assuming that the user requirement and focus of the investigation are appearance and performance, the serviceability indices will apparently be the most important. In consideration of the priority of this research, the comprehensive weight results in Table 7 agree with theory and practical situations.

*4.2. Safety Appraisal of Existing Concrete Members.* Since beams 2 and 12 in Table 1 are obviously damaged, they were selected as numerical examples for analysis. This study assessed the safety of the two damaged beams using the extension model and aforementioned comprehensive weights. Similarly, the inspection data of beams 2 and 12 in Table 1 should be normalized by the extremum method so as to eliminate the effects of different dimensions, units, and orders of magnitude among indices, as shown in Table 8. Figure 5 shows the single index correlation degrees and comprehensive correlation degrees, which are in the form of intervals because of comprehensive weights. It can be distinctly found that the safety level of beams 2 and 12 are $b$ and $d$, respectively.

It should be noted that the weights of the five indices and the safety appraisal results in this study are only suitable for the current status of the existing concrete members. If the twenty-four members in this study need safety appraisal in the future, the inspection data in Table 1 should be updated to the new inspection data. Moreover, the subjective and objective weights need to be adjusted according to the future status and new inspection information. From another point of view, the inspection data in this paper can be regarded as prior information for the future safety appraisal. A comprehensive analysis of prior information and new inspection information is convenient for experts to establish a more realistic judgment matrix, which makes the subjective weight more reasonable. Hence, periodic inspection and new inspection information updating are important for the maintenance of existing concrete members. Note that, in future safety appraisal process, only new inspection data can be used to calculate the objective weights through the entropy weight method, without considering prior inspection data. Similarly, the correlation degree calculation also uses only new inspection data.

Table 8: The normalized inspection data of beams 2 and 12.

| Index | Member | |
|---|---|---|
| | Beam 2 | Beam 12 |
| Bearing capacity | 0.467 | 0.200 |
| Deflection/span | 0.612 | 0.665 |
| Cracks | 0.680 | 0.300 |
| Reinforcement corrosion | 0.800 | 0.277 |
| Concrete carbonation depth | 0.680 | 0.743 |

## 5. Results Verification and Comparison

Basically, the AHP-Fuzzy comprehensive evaluation model is a classical model for evaluation [51]. Tong calculated the weights of the five indices by AHP and established the membership functions [39]. In order to verify the effectiveness of the appraisal method proposed in this study, the safety evaluation of beams 2 and 12 is performed by the AHP-Fuzzy comprehensive evaluation model based on the weights and membership functions in [39]. The results are shown in Figure 6.

According to Figures 5 and 6, the safety levels of beam 2 obtained by the two models are in agreement, meaning that the model proposed in this paper is feasible. However, the safety appraisal results of beam 12 are different, one is $d$ and the other is $c$. On the one hand, according to the inspection data and the grading standard of Table 3, the index value of bearing capacity of beam 12 is 0.86 which suggests that the bearing capacity of beam 12 is seriously insufficient. Through field exploration and original drawings, it was found that the reinforcement of beam 12 is far below design requirements. On the other hand, the index values of reinforcement corrosion, concrete carbonation depth, and cracks of beam 12 are all close to level $d$ suggesting that these three indices signify serious damage. As a result, it is unreasonable that the membership degrees of reinforcement corrosion and concrete carbonation depth of level $d$ are 0, as shown in the evaluation matrix

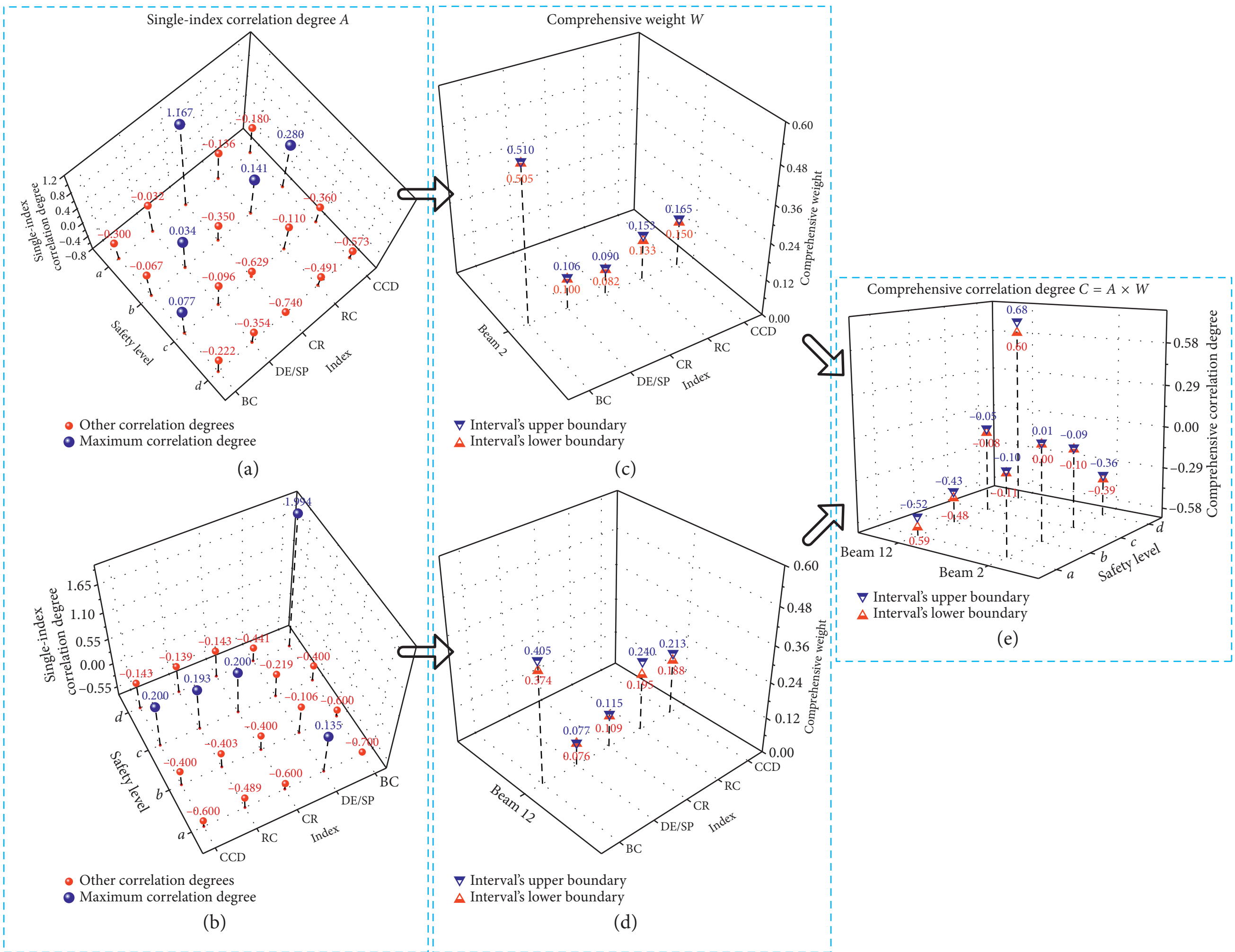



Figure 5: Extension model results. (a, b) The plane axes, respectively, represent safety level and index; the vertical axis represents the single index correlation degree; the blue dot represents the maximum correlation degree of each index and the red dots represent the other correlation degrees. (c, d) The inverse triangle (blue) and regular triangle (red), respectively, represent the upper and lower boundary of the comprehensive weight (which is an interval number). (e) The plane axes represent the safety level and the safety appraisal objects (beams 2 and 12); the vertical axis represents comprehensive correlation degree of each safety level; the inverse triangle (blue) and regular triangle (red), respectively, represent the upper and lower boundary of the comprehensive correlation degree (which is an interval number). Relationship: the comprehensive weight (i.e., (c) and (d)) integrates the single index correlation degree (i.e., (a) and (b)) into a comprehensive correlation degree (i.e., (e)). BCsss, bearing capacity; DE/SP, deflection/Span; CR, cracks; RC, reinforcement corrosion; CCD, concrete carbonation depth.

of Figure 6(b). The primary cause is that an imperfect distribution pattern was selected to determine the membership functions in [39]. Furthermore, reinforcement corrosion and concrete carbonation depth have significant impacts on the safety of existing concrete members based on the weights in Table 7 and Figure 6. The points mentioned above remarkably affect the final ranking of comprehensive membership degrees of level *c* and level *d* and lead to inaccurate results. Therefore, beam 12 needs to be strengthened and repaired immediately, and the level *d* obtained by the extension model is more reasonable. Hence, the result of the AHP-Fuzzy comprehensive evaluation model not on the safe side.

According to the evaluation result of beam 12 calculated by the AHP-Fuzzy comprehensive evaluation model (Figure 6(d)), the comprehensive membership degrees of level *c* and level *d* are 0.48 and 0.47, respectively. These two values are too close leading to potential confusion of decision makers. On the contrary, the results of the extension model (Figure 5(e)) are quite clear.

In conclusion, the reasons for the difference can be attributed to (1) the selection of distribution patterns of membership functions dependent to a large degree on experts' experience and having strong subjectivity as AHP. Nevertheless, the comprehensive correlation degree of the extension model is determined by the distance between inspection data and the range of grading standard (Table 3), which is more consistent with the actual situation avoiding being disturbed by human factors; and (2) the membership functions of different distribution patterns are all

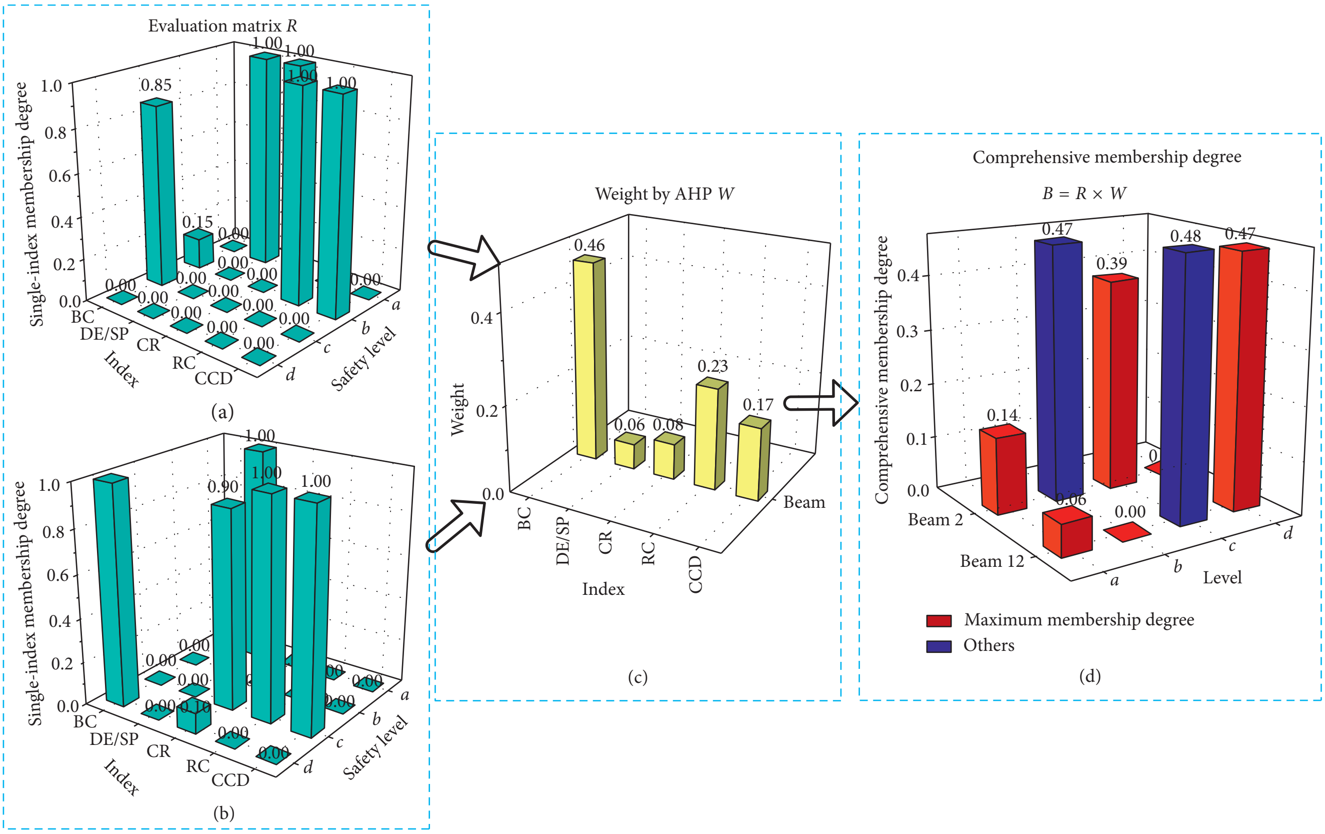



Figure 6: AHP-Fuzzy comprehensive evaluation results. (a, b): The plane axes represent the safety level and index, respectively; the vertical axis represents the single index membership degree; the column height represents the value of the membership degree of each index for each safety level. (c) The yellow column height represents the weight value of each index calculated by AHP. (d): The plane axes represent the safety level and the safety appraisal objects (beams 2 and 12); the vertical axis represents the comprehensive membership degree of each safety level; the blue column represents the maximum membership degree of beams 2 and 12, and the red columns represent other membership degrees. Relationship: the weight (i.e., (c)) integrates the single index membership degree (i.e., (a) and (b)) into the comprehensive membership degree (i.e., (d)). BC: bearing capacity; DE/SP, deflection/span; CR, cracks; RC, reinforcement corrosion; CCD, concrete carbonation depth; AHP, analytic hierarchy process.

approximate descriptions of the research objects; therefore, deviations are inevitable.

## 6. Conclusions

Based on the previous analyses, the following conclusions can be drawn:

(1) The safety appraisal of existing concrete members, which is the foundation of the safety evaluation for existing buildings, plays a significant role in promoting the management of an existing building. Extension theory, proposed by Cai, is a method to solve the problem from both quantitative and qualitative perspectives based on matter-element theory and extension mathematics. In this study, five indices (i.e., bearing capacity, deflection-to-span ratio, cracks, reinforcement corrosion, and concrete carbonation depth) were taken into account, and the application of extension theory was developed to assess the safety of two existing concrete beams. The extension model can better process the incompatibility and variability between indices. Due to the simple calculation and clear results, this model is convenient for the safety appraisal of existing concrete members and can be widely accepted by engineers.

(2) The subjective and objective weights are determined by IAHP and entropy weight method, respectively. IAHP flexibility reflects every expert's opinion obtained by in-depth analysis of structural behaviour and other inspection information. The entropy weight method reflects the inherent information of indices. In order to integrate the advantages of these two weighting methods, the traditional comprehensive weighting method is improved based on game theory and interval number theory. As a cooperative result, the conflicts can be solved by searching for a compromise between subjective weight and objective weight, and the most suitable comprehensive weight is obtained by Nash Equilibrium theory. On the one hand, comprehensive weight can reflect the intentions of decision makers and the inherent law of current inspection data. On the other hand, it

combines the uncertainty of IAHP and the precision of the entropy weight method. Therefore, the improved comprehensive weighting method integrates the advantages of subjective and objective weights so as to overcome the one-sidedness of single weighting method.

(3) The comprehensive weight, calculated by the improved comprehensive weighting method, is adopted in extension model. A comparison of the extension model based on comprehensive weight and the AHP-Fuzzy comprehensive evaluation model was performed. The safety levels of beam 2 obtained by the two models are in agreement, meaning that the extension model is feasible. However, the safety appraisal results of beam 12 are inconsistent, one is $d$ and the other is $c$. According to the inspection data, current situation, and data analysis, the result of the extension model was proved to be correct. The comprehensive weight is more reasonable than single AHP and the selection and inherent defect of distribution patterns of membership functions are the primary causes for the inaccuracy of results of the fuzzy comprehensive evaluation model. However, the comprehensive correlation degree of the extension model is determined by the distance between inspection data and the range of grading standard, which is more consistent with the actual situation. Hence, the extension model based on comprehensive weight is more rational and the results are quite clear avoiding confusing the decision makers.

The proposed model in this study makes full use of inspection data and gives a clear safety level to decision makers avoiding disorganized data of a single index. Thus, it can serve as guidance for safety appraisal of existing concrete members in the future. Furthermore, the improved comprehensive weighting method has practical merits and high scientific value in terms of safety evaluation and other applications in the research fields.

## Abbreviations

AHP: Analytic hierarchy process (a multicriteria decision-making approach)
IAHP: Interval analytic hierarchy process (an improved subjective weighting method based on AHP and interval number theory)
BC: Bearing capacity (an indicator that reflects the strength, stiffness, and stability of structural members)
DE/SP: Deflection-to-span ratio (a damage indicator that affects the safety of existing concrete members)
CR: Cracks (a damage indicator that affects the safety of existing concrete members)
RC: Reinforcement corrosion (a damage indicator that affects the safety of existing concrete members)
CCD: Concrete carbonation depth (a damage indicator that affects the safety of existing concrete members)

## Data Availability

The inspection data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

The authors appreciate those who assisted us in the process of field detection. This work was supported by the "Fundamental Research Funds for the Central Universities" (DUT18JC44).

## References

[1] A. M. Freudenthal, "The safety of structures," *Transactions of the American Society of Civil Engineers*, vol. 112, no. 1, pp. 125–159, 1947.
[2] C. A. Cornell, "Structural safety specifications based on second-moment reliability analysis," *Rapports des Commissions de Travail AIPC*, vol. 4, pp. 235, 1969.
[3] O. Larsson, *Reliability Analysis*, LUND University, Lund, Sweden, 2015.
[4] A. M. Hasofer and N. C. Lind, "Exact and invariant second-moment code format," *Journal of the Engineering Mechanics Division*, vol. 100, no. 1, pp. 111–121, 1974.
[5] N. C. Lind, "Consistent partial safety factors," *Journal of the Structural Division*, vol. 97, no. 6, pp. 1651–1669, 1971.
[6] R. Rackwitz and B. Flessler, "Structural reliability under combined random load sequences," *Computers and Structures*, vol. 9, no. 5, pp. 489–494, 1978.
[7] D. Diamantidis and P. Bazzurro, "Safety acceptance criteria for existing structures," in *Proceedings of Special Workshop on Risk Acceptance and Risk Communication*, Stanford, California, March 2007.
[8] R. D. J. M. Steenbergen and A. Vrouwenvelder, "Safety philosophy for existing structures and partial factors for traffic loads on bridges," *Heron*, vol. 55, no. 2, pp. 123–140, 2010.
[9] M. Pimentel, E. Brühwiler, and J. Figueiras, "Safety examination of existing concrete structures using the global resistance safety factor concept," *Engineering Structures*, vol. 70, pp. 130–143, 2014.
[10] Institution of Structural Engineers, *Appraisal of Existing Structures*, Institution of Structural Engineers, London, UK, 3rd edition, 2010.
[11] Standard CSN 73-0083, *Czechoslovak Code for the Design and Assessment of Building Structures Subjected to Reconstruction*, English Translation by Building Research Institute, Prague, Czechoslovakia, 1986.
[12] ACI 318-08, *Building Code Requirements for Structural Concrete and Commentary*, American Concrete Institute, Detroit, MI, USA, 2008.
[13] D. E. Allen, "Limit states criteria for structural evaluation of existing buildings," *Canadian Journal of Civil Engineering*, vol. 18, no. 6, pp. 995–1004, 1991.
[14] GB 50292-2015, *Standard for Appraisal of Reliability of Civil Buildings*, China Architecture and Building Press, Beijing, China, 2016.
[15] W. J. Zhu and R. François, "Corrosion of the reinforcement and its influence on the residual structural performance of

a 26-year-old corroded RC beam," *Construction and Building Materials*, vol. 51, pp. 461–472, 2014.
[16] N. F. Ortega and S. I. Robles, "Assessment of residual life of concrete structures affected by reinforcement corrosion," *HBRC Journal*, vol. 12, no. 2, pp. 114–122, 2016.
[17] D. W. Li, B. Chen, H. F. Sun et al., "Evaluating the effect of external and internal factors on carbonation of existing concrete building structures," *Construction and Building Materials*, vol. 167, pp. 73–81, 2018.
[18] W. Cai and C. Y. Yang, "Basic theory and methodology on extenics," *Chinese science bulletin*, vol. 58, no. 13, pp. 1190–1199, 2013.
[19] W. Cai, *Matter-Element Model and Application*, Science Technology Literature Press, Beijing, China, 1994.
[20] J. Z. Gong, Y. S. Liu, and W. L. Chen, "Land suitability evaluation for development using a matter-element model: a case study in Zengcheng, Guangzhou, China," *Land Use Policy*, vol. 29, no. 2, pp. 464–472, 2012.
[21] Q. B. Deng and Z. J. Pu, "Extenics-based evaluation of China's Insurance ecological environment," *Energy Procedia*, vol. 5, pp. 2604–2609, 2011.
[22] Y. Guo, Q. Wu, C. Li et al., "Application of the risk-based early warning method in a fracture-karst water source, North China," *Water Environment Research*, vol. 90, no. 3, pp. 206–219, 2018.
[23] Y. Zhang, S. Li, and F. Meng, "Application of extenics theory for evaluating effect degree of damaged mountains based on analytic hierarchy process," *Environmental Earth Sciences*, vol. 71, no. 10, pp. 4463–4471, 2014.
[24] Q. Zou, J. Zhou, C. Zhou, L. Song, and J. Guo, "Comprehensive flood risk assessment based on set pair analysis-variable fuzzy sets model and fuzzy AHP," *Stochastic Environmental Research and Risk Assessment*, vol. 27, no. 2, pp. 525–546, 2013.
[25] F. Jin, L. Pei, H. Chen et al., "Interval-valued intuitionistic fuzzy continuous weighted entropy and its application to multi-criteria fuzzy group decision making," *Knowledge-Based Systems*, vol. 59, pp. 132–141, 2014.
[26] S. X. Wang, L. J. Ge, S. X. Cai, and L. Wu, "Hybrid interval AHP-entropy method for electricity user evaluation in smart electricity utilization," *Journal of Modern Power Systems and Clean Energy*, vol. 6, no. 4, pp. 701–711, 2017.
[27] T. Y. Wu, W. T. Lee, N. Guizani, and T. M. Wang, "Incentive mechanism for P2P file sharing based on social network and game theory," *Journal of Network and Computer Applications*, vol. 41, pp. 47–55, 2014.
[28] A. S. Nowak and K. R. Collins, *Reliability of Structures*, CRC Press, Boca Raton, FL, USA, 2012.
[29] M. M. Szerszen and A. S. Nowak, "Calibration of design code for buildings (ACI 318) part 2: reliability analysis and resistance factors," *ACI Structural Journal*, vol. 100, no. 3, pp. 48109–2125, 2003.
[30] X. L. Gu, D. J. Sun, K. Sun, and G. H. Hu, "Development and application of reliability assessment software for historic masonry structures," *Advanced Materials Research*, vol. 133-134, pp. 1271–1276, 2010.
[31] T GB, *Technical Standard for Inspection of Building Structure*, China Building Industry Press, Beijing, China, 2004.
[32] T. L. Saaty, "How to make a decision: the analytic hierarchy process," *European Journal of Operational Research*, vol. 48, no. 1, pp. 9–26, 1990.
[33] A. Rezaei and S. Tahsili, "Urban vulnerability assessment using AHP," *Advances in Civil Engineering*, vol. 2018, Article ID 2018601, 20 pages, 2018.
[34] T. Entani and K. Sugihara, "Uncertainty index based interval assignment by interval AHP," *European Journal of Operational Research*, vol. 219, no. 2, pp. 379–385, 2012.
[35] Y. S. Yuan, F. P. Jia, and Y. Cai, "The structural behavior deterioration model for corroded reinforced concrete beams," *China civil engineering journal*, vol. 34, no. 3, pp. 47–52, 2001.
[36] C. E. Shannon, "A mathematical theory of communication," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 5, no. 1, pp. 3–55, 2001.
[37] J. Yan, C. Feng, and L. Li, "Sustainability assessment of machining process based on extension theory and entropy weight approach," *International Journal of Advanced Manufacturing Technology*, vol. 71, no. 5–8, pp. 1419–1431, 2014.
[38] B. M. Roger, "Game theory: analysis of conflict," *President and Fellows of Harvard College*, vol. 15, no. 1, pp. 96–112, 1991.
[39] X. Tong, *Reliability Analysis of Concrete Structures Based on Fuzzy Mathematics*, HeFei University of Technology, Hefei, China, 2013.
[40] H. He, *Study on Existing Reinforced Concrete Structure Deterioration Evaluation and Prediction and Maintenance Decision*, Tianjin University, Tianjin, China, 2009.
[41] W. Cai, "Extension theory and its application," *Chinese Science Bulletin*, vol. 44, no. 17, pp. 1538–1548, 1999.
[42] M. H. Wang, Y. K. Chung, and W. T. Sung, "Using thermal image matter-element to design a circuit board fault diagnosis system," *Expert Systems with Applications*, vol. 38, no. 5, pp. 6164–6169, 2011.
[43] Y. Jun, "Application of extension theory in misfire fault diagnosis of gasoline engines," *Expert Systems with Applications*, vol. 36, no. 2, pp. 1217–1221, 2009.
[44] B. Becerik-Gerber, F. Jazizadeh, N. Li, and G. Calis, "Application areas and data requirements for BIM-enabled facilities management," *Journal of construction engineering and management*, vol. 138, no. 3, pp. 431–442, 2011.
[45] C. Nicolle and C. Cruz, "Semantic building information model and multimedia for facility management," in *Proceedings of International Conference on Web Information Systems and Technologies*, Springer, Valencia, Spain, April 2010.
[46] R. Volk, J. Stengel, and F. Schultmann, "Building information modeling (BIM) for existing buildings—literature review and future needs," *Automation in construction*, vol. 38, pp. 109–127, 2014.
[47] C. Alonso, C. Andrade, J. Rodriguez, and J. M. Diez, "Factors controlling cracking of concrete affected by reinforcement corrosion," *Materials and Structures*, vol. 37, no. 7, pp. 435–441, 1998.
[48] T. Vidal, A. Castel, and R. François, "Analyzing crack width to predict corrosion in reinforced concrete," *Cement and Concrete Research*, vol. 34, no. 1, pp. 165–174, 2004.
[49] W. L. Jin and Y. X. Zhao, *Concrete Structural Durability*, Beijing Science Press, Beijing, China, 2002.
[50] W. L. Jin and X. P. Zhong, "Relationship of structural durability with structural safety and serviceability in whole lifecycle," *Journal of Building Structures*, vol. 30, no. 6, pp. 1–7, 2009.
[51] C. G. Lai, X. H. Chen, X. Y. Chen, Z. Wang, X. Wu, and S. Zhao, "A fuzzy comprehensive evaluation model for flood risk based on the combination weight of game theory," *Natural Hazards*, vol. 77, no. 2, pp. 1243–1259, 2015.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
Advances in Multimedia
Advances in Civil Engineering
Journal of Control Science and Engineering
Journal of Robotics
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration