Hindawi
Advances in Civil Engineering
Volume 2018, Article ID 3780810, 10 pages
https://doi.org/10.1155/2018/3780810

Research Article

Optimal Cement Mixtures Containing Mineral Admixtures under Multiple and Conflicting Criteria

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Received 6 July 2017; Revised 3 November 2017; Accepted 15 November 2017; Published 17 January 2018

Academic Editor: Tayfun Dede

In modern construction industry, fabrication of sustainable concrete has turned the decision-making process into a challenging endeavor. One alternative is using fly ash and nanostructured silica as cement replacements. In these modern mixtures, proper concrete bulk density, percentage of voids, and compressive strength normally cannot be optimized individually. Hereby, a decision-making strategy on the replacement of those components is presented while taking into account those three performance measurements. The relationships among those components upon concrete fabrication required a design of experiments of mixtures to characterize those mineral admixtures. This approach integrates different objective functions that are in conflict and obtains the best compromise mixtures for the performance measures being considered. This optimization strategy permitted to recommend the combined use of fly ash and nanosilica to improve the concrete properties at its early age.

1. Introduction

In recent years, the environmental damages caused by the production of building materials have compelled the construction industry to seek for sustainable alternatives [1]. The partial replacement of cement by fly ash (FA), a manufacturing waste of the burning coal process, has turned into an increasingly popular alternative. Further, the addition of nanostructured SiO_2 or nanosilica (nS) is highly recommended to counterbalance the loss of concrete compressive strength at early age caused by FA. These nanoparticles improve some valuable concrete properties such as the density, porosity, and compressive strength [2–5]. Of those properties, concrete compressive strength is the most relevant mechanical property and, therefore, the most studied [6]. Moreover, assessment of concrete porosity is necessary as this is related to concrete's durability and permeability [7, 8]. Those characteristics depend on the number, size, and distribution of pores in the cement paste and the aggregates [9].

Hence, a range of values of the mechanical and physical properties of concrete are preferred when mineral admixtures are utilized [10, 11], as the desired characteristics depend on the proposed application. Previous works demonstrated that the specimens with higher compressive strength not necessarily corresponded to the ones with higher density and lower porosity, which are usually the desirable properties in concrete structures [12, 13]. That is why, in some cases, designers have to prioritize, for example, one characteristic over other ones. Therefore, there arises a conflict among the different performance measures of concrete. As a result, one must utilize a multiple criteria optimization method to maximize simultaneously compressive strength and density to minimize the concrete porosity. Finally, the use of this approach helps to design a multifunctional structural material by identifying the mixtures that belong to a Pareto-efficient frontier [14, 15]. The resulting optimized mixtures would become the best compromise among all performance measures between the set of mixtures under evaluation.

Optimization is, therefore, a decision-making tool of great importance issue in the construction industry [16], where simultaneous attention is required for the environmental aspects and design factors. These frequently

contradict each other especially when recycled materials (with their inherent behavioral variability) are involved.

Often, researchers have used regression models to predict performance measures, such as compressive strength, density, and porosity [17–20]. Sometimes, via neural networks, some were able to predict concrete behavior [19]. In addition, although a visual representation of the results facilitates the comparison process, other statistical methodologies can be used to compare the mixtures from a mathematical viewpoint rather than from a more subjective approach. Therefore, researchers have employed a variety of optimization approaches to find the best possible solutions in a single objective [1, 17, 21–26]. Recommendations based on all the performance measures of interest to the user are more appropriate when compared with only the selection of a single solution pertaining to the measured objective.

To address this situation, different attempts to incorporate multiple performance measures can be found [27, 28]. For instance, the ε-constraint method, which is a formal approach to multicriteria optimization, permitted to resolve a multiobjective reliability-based optimum problem of prestressed concrete beams [29]. Most methods provided by the literature require target values—necessarily defined *a priori*—or reduce the multiobjective problem into a single objective optimization problem to find the optimal set. As a consequence, we posit a methodology that does not involve any of the previously mentioned issues that has been developed at the University of Puerto Rico-Mayagüez (UPRM) [13, 14, 30]. This methodology renders the Pareto-optimal solution set by just defining the objectives and their respective (maximization/minimization) directions. Often, when solving a multiple criteria optimization problem, one can find a set of efficient solutions. Such a set is also known as "Pareto-optimal solutions" [14, 30, 31]. These solutions are the best balances among all performance measures under evaluation; they are equally optimal since a gain in one objective results in a sacrifice in at least another objective. The optimal solutions form the Pareto-efficient frontier.

In order to identify those optimal solutions, one utilizes the Pareto-optimality conditions as described in Deb's work [31]. In this work, the author stated that "A solution $x^{(1)}$ is said to dominate the other solution $x^{(2)}$, if both the following conditions are true:

(1) The solution $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives. Thus, the solutions are compared based on their objective function values (or location of the corresponding points ($z^{(1)}$ and $z^{(2)}$) on the objective space).

(2) The solution $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective."

Consequently, based on the said concepts and findings, the present work focuses on the characterization of concrete-containing mineral admixtures and the subsequent multiple criteria optimization. First, a statistical design of experiments for mixtures allowed computing the mixture proportions to evaluate. Subsequently, the optimal tradeoff mixtures among compressive strength, bulk density, and percentage of voids followed. Utilizing the above conditions, we performed a full pairwise comparison between the solutions to eventually find the Pareto-efficient frontier or the nondominated set. Accordingly, we present a decision-making strategy on the replacement of concrete components while taking into account the material's physical and mechanical properties.

2. Experimental Methodology

2.1. Material Selection. Via a sieve analysis (ASTM 136) [32], the experimental proportions of the aggregates were determined as 30% of gravel grade #7 (ASTM C33) [33], 35% of processed aggregate (limestone), and 35% of clean sand. To determine the quantity of polycarboxylate superplasticizer (SP) necessary for each mixture, we took into account the following characteristics of the mixtures: segregation, bleeding, slump, and consistency. This mini-slump test was used for the mixtures at 5, 30, and 60 minutes after the mixing along with a trial-and-error method to find the right proportion of superplasticizer for each mix. The evaluation criteria for slump were 100–152 mm to obtain optimun SP quantity (ASTM C143) [34].

2.1.1. Aggregates. We used gravel as coarse aggregate, with a maximum nominal size of 19.0 mm. Moreover, the processed aggregate (limestone) had a maximum nominal size of 9.5 mm. The fineness modulus of the fine aggregate was 3.0 as given in Table 1.

2.1.2. Portland Cement. We used ordinary Portland cement (OPC) Type I (ASTM C150) [35], which is classified as applicable to general purposes and have fairly high C_3S content for good early strength development with a specific gravity of 3.06.

2.1.3. Fly Ash. The FA class F (ASTM C618) [36] with a specific gravity of 2.38 was selected.

2.1.4. Nanosilica. Nissan Chemical Industries provided the nS used, which was opalescent and odorless amorphous silica dispersed in water, with a particle mean size of 69.40 nm and specific gravity of 2.03. The amounts of nS in the mixes are calculated based on the percentage by weight of solid in the colloidal solution. In the case of these nanoparticles, only 45% by weight is SiO_2. To calculate the amounts of nS for the mixtures, the amount of water and solids is considered, thus making an adjustment to the amounts of both nS and water, in order to then reach the percent established for the design. For instance, the amount of nanosilica was computed as follows:

Mix design k: 9 = (PC: 0.57, FA: 0.40, and nS: 0.03)

Total cementitious quantity: 672 kg/m^3

nS (solids) = 45 ∗ 45% = 20 kg/m^3

FA = 269 kg/m^3

PC = 383 kg/m^3

Sum = 672 kg/m^3

Table 1: Properties of the aggregates.

Aggregate	Apparent specific gravity	Specific gravity (oven-dry)	Specific gravity (SSD)	Absorption (%)	Unit weight (kg/m^3)
Gravel	2.88	2.71	2.77	2.09	1584.70
Limestone	2.77	2.50	2.59	3.86	1740.57
Beach sand	2.65	2.42	2.51	3.48	1460.54

2.1.5. Superplasticizer. The polycarboxylate superplasticizer used followed the ASTM 494 standard [37] and was provided by a company in Puerto Rico.

2.1.6. Water. To prepare the mixtures, we employed tap water at room temperature available at the UPRM Construction Materials Laboratory.

2.2. Fabrication and Testing Procedures. A gear-driven, high-torque transmission 5 L mixer (Globe SP20) manufactured by Globe Food Equipment was used to mix the concrete components. The coarse and fine aggregates were first dry-mixed and then placed into the mixer for 0.25 min at 120 rpm, followed by half of the required water. Then, we added the PC and later the FA (if required by the specific experiment) with the mixture working for 0.25 min at 60 rpm. The nS and SP were diluted in water in order to obtain a uniform particle distribution throughout the mixture and poured into the mixer (when used) for 4.30 min at 120 rpm. The cylindrical molds were filled by the rodding method according to ASTM C192 [38]. We removed the cylinders formwork 24 hours after casting; thereupon, we immersed them into limewater until tested at normal curing conditions (20–23°C and RH = 100%). The temperature (23–25°C) was relatively constant in the laboratory.

Following ASTM C642-13 standard [39], we measured the density and percentage of voids of five specimens at 7 and 28 days of curing. We considered the specimen oven-dry mass, its saturated mass after immersion in water, its saturated mass after boiling, and its immersed apparent mass. These values were used to calculate the bulk density and the volume of permeable pore space or percentage of voids of the specimens. The dimensions of the test cylinders were 76 mm in diameter and 152 mm in length to meet the minimum volume required by the standard.

For the compressive strength test, the dimensions of the test cylinders were 50 mm in diameter and 100 mm in length. We tested the compressive strength of six cylinders at 7 and 28 days of curing, using a 3000 kN Forney universal test machine according to ASTM C39 [40].

2.3. Design of Experiments. In order to generate the different combinations of the cement mixture components, that is, PC, FA, and nS [24, 41], we set up a design of experiments for mixtures. This methodology is explained in detail in our previous publication [3]. In the present work, the upper bound was set at 3% of nS and 40% of FA. In addition, the water-to-binder ratio utilized for all mixtures remained constant at 0.3.

Table 2: Mixture proportion combinations evaluated of cement mixtures.

k	Mixture proportions (PC/FA/nS)	PC kg/m^3	FA kg/m^3	nS kg/m^3	Gravel kg/m^3	Sands kg/m^3
1	0.800/0.20/0.000	538	134	0	1001	431
2	1.000/0.00/0.000	672	0	0	1001	462
3	0.600/0.40/0.000	403	269	0	1001	400
4	0.585/0.40/0.015	393	269	22	1001	396
5	0.770/0.20/0.030	518	134	45	1001	423
6	0.985/0.00/0.015	662	0	22	1001	458
7	0.970/0.00/0.030	652	0	45	1001	454
8	0.785/0.20/0.015	528	134	22	1001	427
9	0.570/0.40/0.030	383	269	45	1001	392

Table 3: Setup of the multiple criteria optimization problem.

Decision variables	PC, FA, and nS
Maximization of	$f_1(x)$: compression strength $f_2(x)$: bulk density
Minimization of	$f_3(x)$: percentage of voids
Subject to these constraints	$0.57 \leq \mathrm{PC} \leq 1.00$ $0.00 \leq \mathrm{FA} \leq 0.40$ $0.00 \leq \mathrm{nS} \leq 0.03$

Table 2 presents the nine-component combinations or mixtures evaluated, as fractions of 1.00 (total mass of the mixture). Each mixture represents a solution k with different characteristics in terms of physical and mechanical properties of the resulting concrete. A multiple criteria optimization method helped us in the decision-making process of recommending some of these mixtures.

In this multiple criteria optimization problem, we were interested in recommending a set of alternatives (k^*) selected from the different mixture proportions of PC, FA, and nS. In view of that, the final decision-making would be based on the following material performance measures: compressive strength, bulk density, and percentage of voids. Naturally, the desired outcomes were higher compressive strength and density and lower percentage of voids. Thus, the strategy for the multiple criteria optimization problem is presented in Table 3.

We intended to restrict the problem described above to a manageable number of sampling experimental solutions generated through a mixture design of experiments (DOE), as mentioned previously. Furthermore, the best tradeoffs among the competing criteria were identified with the application of Pareto-optimality conditions, as advocated in prior research [14, 15, 42]. This method is exact (as opposed to a heuristic approach [18]) and has being utilized

Table 4: Average result of performance measures at day 7.

k	Component fractions (PC/FA/nS)	Compressive strength		Bulk density		Volume of permeable pore space	
		Average MPa	Std. dev. MPa	Average kg/m^3	Std. dev. kg/m^3	Average %	Std. dev. %
1	0.800/0.20/0.000	27.37	1.15	2165.88	37.32	15.59	0.98
2	1.000/0.00/0.000	31.11	7.22	2218.43	34.71	13.85	0.75
3	0.600/0.40/0.000	33.71	3.19	2117.88	13.27	17.42	0.32
4	0.585/0.40/0.015	31.58	5.32	2125.56	29.80	16.58	0.70
5	0.770/0.20/0.030	29.75	5.24	2186.46	16.19	15.93	0.63
6	0.985/0.00/0.015	27.03	5.65	2188.51	37.78	16.00	1.05
7	0.970/0.00/0.030	24.19	8.24	2189.98	14.19	16.04	0.31
8	0.785/0.20/0.015	40.40	2.47	2156.64	6.26	17.00	0.28
9	0.570/0.40/0.030	33.35	4.63	2163.49	23.10	12.49	0.78

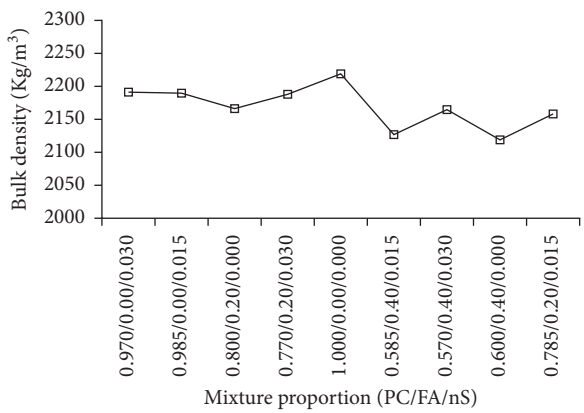

Figure 1: Mean bulk density measured at aging day 7.

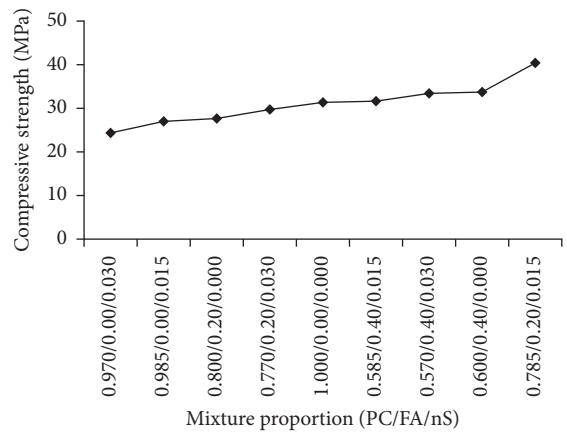

Figure 2: Mean compressive strengths measured at aging day 7.

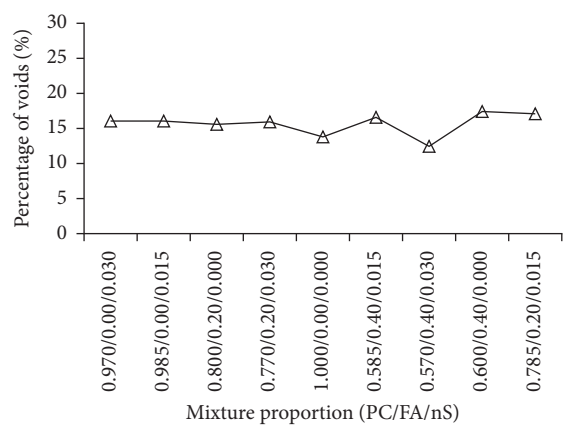

Figure 3: Mean percent of voids measured at aging day 7.

previously to solve engineering and science problems [14, 15]. In this research, we applied the method to find the proportion (or fractional) combinations of a cement mixture that belong to the best possible balances in the presence of the conflicting performance measures, or more formally, the solutions in the Pareto-efficient frontier.

To further demonstrate its straightforwardness, we coded the method in a commercially available spreadsheet program. Utilizing the Pareto-optimality conditions, as aforementioned, we carried out a full pairwise comparison between the solutions to eventually find the Pareto-efficient frontier or the nondominated set. A detailed description of the multiple criteria optimization method utilized in this work can be found in the literature [43].

3. Experimental Results

In this section, we present the experimental results organized for different curing ages. Their graphical representations have been used in the decision-making analysis and the optimization methodology.

3.1. Seven Days of Aging. Table 4 presents the results obtained on aging day 7. One must note that the mean compressive strength was obtained from 6 replicates, whereas the mean bulk density and average percentage of voids were from 5 replicates, due to few experimental flaws. It is apparent that if we consider each one of the performance measures separately, they will aim at different solutions (Figures 1–3). In other words, the performance measures are in conflict. Each one of the mixture combination will represent a solution or alternative k for the multiple criteria optimization problem (Table 4).

Table 5: Matrix A_1 compares all the solutions k from objective f_1.

f_1 versus f_1	37.23	33.49	30.89	33.02	34.85	37.56	40.40	24.19	31.25
37.23	0	1000	1000	1000	1000	−1	−1	1000	1000
33.49	−1	0	1000	1000	−1	−1	−1	1000	1000
30.89	−1	−1	0	−1	−1	−1	−1	1000	−1
33.02	−1	−1	1000	0	−1	−1	−1	1000	1000
34.85	−1	1000	1000	1000	0	−1	−1	1000	1000
37.56	1000	1000	1000	1000	1000	0	−1	1000	1000
40.40	1000	1000	1000	1000	1000	1000	0	1000	1000
24.19	−1	−1	−1	−1	−1	−1	−1	0	−1
31.25	−1	−1	1000	−1	−1	−1	−1	1000	0

Table 6: Matrix A_2 compares all the solutions k from objective f_2.

f_2 versus f_2	2170.43	2117.88	2218.43	2210.75	2149.85	2147.80	2146.33	2179.68	2172.82
2170.43	0	1000	−1	−1	1000	1000	1000	−1	−1
2117.88	−1	0	−1	−1	−1	−1	−1	−1	−1
2218.43	1000	1000	0	1000	1000	1000	1000	1000	1000
2210.75	1000	1000	−1	0	1000	1000	1000	1000	1000
2149.85	−1	1000	−1	−1	0	1000	1000	−1	−1
2147.80	−1	1000	−1	−1	−1	0	1000	−1	−1
2146.33	−1	1000	−1	−1	−1	−1	0	−1	−1
2179.68	1000	1000	−1	−1	1000	1000	1000	0	1000
2172.82	1000	1000	−1	−1	1000	1000	1000	−1	0

Table 7: Matrix A_3 compares all the solutions k from objective f_3.

f_3 versus f_3	15.59	13.85	17.42	16.58	15.93	16.00	16.04	17.00	12.49
15.59	0	1000	−1	−1	−1	−1	−1	−1	1000
13.85	−1	0	−1	−1	−1	−1	−1	−1	1000
17.42	1000	1000	0	1000	1000	1000	1000	1000	1000
16.58	1000	1000	−1	0	1000	1000	1000	−1	1000
15.93	1000	1000	−1	−1	0	−1	−1	−1	1000
16.00	1000	1000	−1	−1	1000	0	−1	−1	1000
16.04	1000	1000	−1	−1	1000	1000	0	−1	1000
17.00	1000	1000	−1	1000	1000	1000	1000	0	1000
12.49	−1	−1	−1	−1	−1	−1	−1	−1	0

Compressive strength, bulk density, and percentage of voids are labeled f_1, f_2, and f_3, respectively. Then, the values of our performance measures, that is, f_1, f_2, and f_3, were utilized to create three matrices A_1, A_2, and A_3 in order to compare all the solutions n in each objective.

In order to assess the first Pareto-optimality condition, the following states were employed [14]:

$$A_j(a,b) = \begin{cases} -1, & f_j(x^a) < f_j(x^b) \\ 0, & f_j(x^a) = f_j(x^b) \\ 1000, & f_j(x^a) > f_j(x^b) \end{cases} \quad (1)$$

For example, if $A_1(37.23, 40.40)$ is evaluated, the result will be −1 for the solution; 37.23 is smaller than solution 40.40 (Table 5). In this context, smaller means better because we are trying to minimize each performance measurement. Then, one can perform the same comparison in each objective for all its solutions (Table 6 and 7).

Now, matrix S is constructed to compare all the objectives and evaluate the second Pareto-optimality condition (Table 8). By this means, one can identify the nondominated solution set using the following conditions:

$$S(a,b) = \begin{cases} 1500, & \sum_{j=1}^{3} A_j(a,b) = 0 \\ 1500, & \sum_{j=1}^{3} A_j(a,b) = 1000 \\ 1500, & \sum_{j=1}^{3} A_j(a,b) = 2000 \\ 3000, & \sum_{j=1}^{3} A_j(a,b) \geq 3000 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Table 8: Matrix S to evaluate the second condition of Pareto.

k	f_1 versus f_2 versus f_3											
1	37.23	2170.43	15.59	1500	3000	0	0	0	0	0	0	0
2	33.49	2117.88	13.85	0	1500	0	0	0	0	0	0	0
3	30.89	2218.43	17.42	0	0	1500	0	0	0	0	3000	0
4	33.02	2210.75	16.58	0	0	0	1500	0	0	0	0	3000
5	34.85	2149.85	15.93	0	3000	0	0	1500	0	0	0	0
6	37.56	2147.80	16.00	0	3000	0	0	0	1500	0	0	0
7	40.40	2146.33	16.04	0	3000	0	0	0	0	1500	0	0
8	24.19	2179.68	17.00	0	0	0	0	0	0	0	1500	0
9	31.25	2172.82	12.49	0	0	0	0	0	0	0	0	1500

Table 9: Mixtures in bold belong to the Pareto-efficient frontier for day 7.

k	Mixture proportions (PC/FA/nS)	f_1 MPa	f_2 kg/m^3	f_3 %
1	0.800/0.20/0.000	27.37	2165.88	15.59
2	**1.000/0.00/0.000**	**31.11**	**2218.43**	**13.85**
3	0.600/0.40/0.000	33.71	2117.88	17.42
4	0.585/0.40/0.015	31.58	2125.56	16.58
5	0.770/0.20/0.030	29.75	2186.46	15.93
6	0.985/0.00/0.015	27.03	2188.51	16.00
7	0.970/0.00/0.030	24.19	2189.98	16.04
8	**0.785/0.20/0.015**	**40.40**	**2156.64**	**17.00**
9	**0.570/0.40/0.030**	**33.35**	**2163.49**	**12.49**

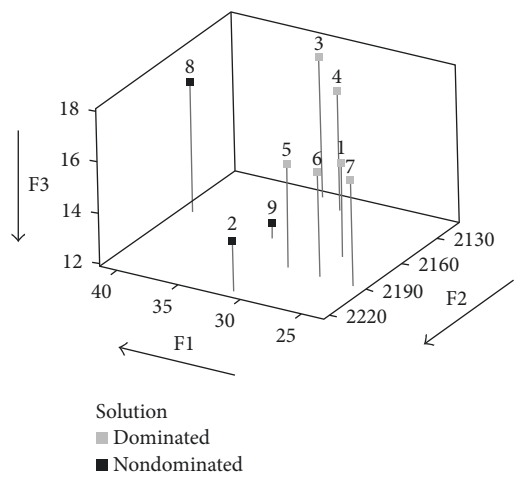

Figure 4: Graphical representation of the results of the solution set evaluated at 7 days (rotated view).

Finally, when we sum each row of matrix S, we can identify the solutions that are part of the Pareto-efficient frontier, that is, the sum associated with that row (solution) is less than 3000 (in this case).

After applying the multiple criteria optimization method, Table 9 shows in bold the efficient solutions for aging day 7. These were the mixtures numbered 2, 8, and 9. Mixture number 2 is the control mixture with only Portland cement (no replacement). We expected that this mixture be in the optimal set since its properties were very competitive during its early age. However, we found particularly interesting that the other two mixtures, that is, 8 and 9, that belong to the Pareto-efficient frontier contained FA and nS. Mixture 8 had 78.5% PC, 20% FA, and 1.5% nS, while mixture 9 possessed 57% PC, 40% FA, and 3% nS. Although mixture 9 had 40% of FA (high level of replacement), the addition of only 3% nS makes it a competitive combination with adequate physical and mechanical properties. In contrast, mixture 3 is made of 60% PC, 40% FA, and no nanoparticles (0% nS), which has a high level of replacement; notwithstanding, this mixture does not belong to the Pareto-efficient frontier. Intriguingly, mixture 1 (20% FA and no nS) had a similar behavior. Hence, the difference between *being and not being* part of the Pareto-efficient frontier appeared to be the presence of the silica nanoparticles. This was a consequential finding that is discussed later.

As we analyzed three performance measures, the results yielded the 3D graph in Figure 4. In addition, one can employ a cone of Pareto dominance to visualize the dominated and nondominated solutions. Figure 4 shows all the solutions k (mixtures) in the criteria space with a rotated view to make easier the visualization of the efficient frontier.

3.2. Twenty-Eight Days of Aging. Table 10 shows the average results of the three performance measures evaluated at 28 days of aging. Clearly, mixture 8 had a higher compressive strength, whereas mixture 6 bears a larger bulk density and lower percentage of voids. This leads, once again, to a conflict between the objectives.

As in the analysis of day 7, at day 28, we sought to maximize the compressive strength and bulk density and to minimize the percentage of voids. Table 11 presents the results obtained from the multiple criteria optimization strategy, which indicates that four solutions belong to the Pareto-efficient frontier: mixtures 2, 6, 7, and 8. The Pareto-optimality conditions can be used to ensure that these sets of mixtures are always better in at least one objective and the same or worse in the other objective.

We did expect mixture number 2 to be part of the Pareto-efficient frontier since it is the control mixture with only PC. Mixtures 6 and 7 contain PC and 1.5% and 3.0% nS, respectively, without any FA, that is, one of the replacements

Table 10: Average result of performance measures at day 28.

k	Mixture proportions (PC/FA/nS)	Compressive strength		Bulk density		Volume of permeable pore space	
		Average MPa	Std. dev. MPa	Average kg/m^3	Std. dev. kg/m^3	Average %	Std. dev. %
1	0.800/0.20/0.000	34.48	5.49	2141.12	52.85	15.77	1.20
2	1.000/0.00/0.000	41.91	9.59	2192.05	42.05	14.75	1.06
3	0.600/0.40/0.000	44.09	2.94	2098.88	19.94	17.26	0.61
4	0.585/0.40/0.015	38.92	9.38	2096.61	45.25	17.17	1.24
5	0.770/0.20/0.030	36.59	5.79	2168.71	27.31	16.06	0.86
6	0.985/0.00/0.015	31.29	3.41	2226.15	28.65	14.25	0.63
7	0.970/0.00/0.030	31.81	4.64	2197.49	22.35	15.35	0.43
8	0.785/0.20/0.015	47.27	6.58	2179.52	12.47	16.12	0.25
9	0.570/0.40/0.030	41.5	3.23	2131.97	18.65	16.02	0.57

Table 11: Optimization results at day 28 where the mixtures in bold belong to the Pareto-efficient frontier.

k	Mixture proportions (PC/FA/nS)	f_1 MPa	f_2 kg/m^3	f_3 %
1	0.800/0.20/0.000	34.48	2141.12	15.77
2	**1.000/0.00/0.000**	**41.91**	**2192.05**	**14.75**
3	0.600/0.40/0.000	44.09	2098.88	17.26
4	0.585/0.40/0.015	38.92	2096.61	17.17
5	0.770/0.20/0.030	36.59	2168.71	16.06
6	**0.985/0.00/0.015**	**31.29**	**2226.15**	**14.25**
7	**0.970/0.00/0.030**	**31.81**	**2197.49**	**15.35**
8	**0.785/0.20/0.015**	**47.27**	**2179.52**	**16.12**
9	0.570/0.40/0.030	41.5	2131.97	16.02

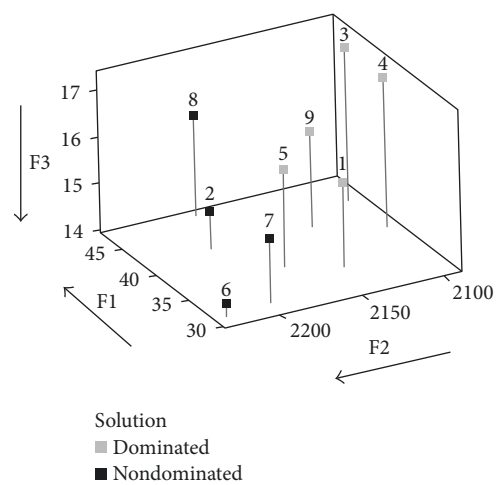

Figure 5: Graphical results of the solution set evaluated at 28 days (rotated view).

of interest. Conversely, mixture 8, which contains 20% FA (with 78.5% PC and 1.5% nS), is also efficient. The solutions can be observed in the criteria space in Figure 5 with a rotated view for visualization convenience.

4. Discussion of Results

The results obtained from the multiple criteria optimization are the best tradeoff mixtures recommended to the decision-makers who can then select a single mixture among the efficient set presented in this work. Naturally, such a decision should be based on the characteristics of the mixtures presented in each performance measures. Also, they should consider the proportion of each component in the mixture. This depends on the user's (or structural designer's) interest about the mineral admixtures and the specific application of each concrete mixture.

The optimization process revealed that mixtures with FA and no nS did not belong to the Pareto-efficient frontier. This behavior was observed throughout the analysis, denoting that the addition of silica nanoparticles is necessary when FA is presented as cement replacement. This beneficial interaction had already been observed in prior works [4]. In such concretes, the nanoparticles do improve the physical and mechanical properties of the resulting concrete. Toutanji et al. [44] discovered that a combination of different supplementary materials, as silica fume, increased the compressive strength. Previous works support these findings [17], where the use of FA and nS was found to improve the concrete microstructure and rate of strength gain. Microstructure analysis of concrete by scanning, scanning transmission, and transmission electron microscopy revealed that nS particles fill the CSH-gel structure voids and act as nuclei, tightly bonded with the CSH particles [45]. This tight packing densifies concrete, protecting it from chemical attacks and leaching, while enhancing its durability and mechanical properties.

As aforementioned, after seven days of aging, there were three efficient mixtures. In the analysis at day 28, the efficient mixtures were four. Examining this pool of efficient mixtures reveals that two were efficient in all the analysis conducted: (a) the control mixture with 100% PC, 0% FA, and 0% nS and (b) the mixture with 78.5% PC, 20% FA, and 1.5% nS. In other words, for the two ages tested, the regular mix (control) can be replaced by mixture 8, which even has higher compression strength at the expense of a slight increase in porosity. This result further proves how

nanosilica can counteract some strength loss induced by the FA presence [2, 5, 17, 46]. This is an important finding as it points to the formulation of a sustainable concrete, that is, one that requires less cement to attain structural behavior, using an industrial waste, that is, fly ash and nanostructured SiO_2 particles. In effect, less consumption of cement would lead to a smaller carbon footprint upon its fabrication, without compromising the structural strength of the mixture.

However, the cost of large amounts of nS, as a construction project would demand, poses an economic challenge. On the other hand, FA is one of the low-priced mineral admixtures, which could balance the final cost of concrete. Therefore, to finally propose the use of nS and FA in structural concrete, we deem critical to an optimization strategy that includes an exhaustive cost analysis.

Finally, the full pairwise comparison between solutions that led us to the Pareto-efficient frontier was implemented in a readily available spreadsheet package. This means that no computational intricacy was required to render a robust analysis of the data to assist in the decision-making process. We recognize that the multiple criteria optimization method, in our case, studied only three material characteristics, that is, compressive strength, density, and void presence. Concrete is a versatile material with numerous potential variables arising from its fabrication process. Nonetheless, the multiple criteria optimization method is a versatile and scalable strategy that could be expanded to include other performance measures more relevant to other specific applications of concrete mixtures.

5. Conclusions

The present work proposes the use of an optimization procedure to determine for nanosilica-containing concrete mixtures the best ones to achieve specific performance measures: concrete compressive strength, bulk density, and percentage of voids (porosity). These performance measurements were measured after 7 and 28 days. The nine proportion combinations evaluated contained different percentages of PC, FA, and nS. The use of the multiple criteria optimization helped find the mixtures that were the best balances among the studied objectives. At day 7, three mixtures were part of the Pareto-efficient frontier. Two of them were mixtures with cement replacement, that is, FA and nS at different levels. On the other hand, four mixtures were part of the Pareto-efficient frontier at day 28. This time, one mixture has cement replacement (FA and nS). In addition, two mixtures were efficient at either day 7 or day 28.

As a consequence, the multiple criteria optimization strategy permitted to recommend the use of FA and nS to improve the concrete properties. However, if the analysis is performed considering only one performance measurement, such as compressive strength, the option of cement replacement by FA is not recommended. Hence, taking into consideration several performance measurements, the use of mineral admixtures is suggested. This is because a mixture with mineral admixtures will be equally optimal than a control mixture with just PC when more properties are considered. Consequently, as the decision-makers know the best tradeoff mixtures for an individual application, the final recommendation is easier to make.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This material is based upon the work supported by the National Science Foundation under Grants nos. HRD 0833112 and 1345156 (CREST program). Additional support was provided by the US Department of Education through Grant no. P120A120097 (MSEIP program). The authors would like to thank the technical personnel of the Nanotechnology Center and the Construction Materials Laboratory of the University of Puerto Rico for their invaluable assistance in the completion of this work.

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