

Research Article

Analytical Assessment of the Effect of Vertical Ground Motion on RC Frames Designed for Gravity Loads with Various Geometric Configurations

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The paper presents an analytical investigation of the effect of vertical ground motion on the selected 13 reinforced concrete (RC) frames with different geometric configurations. For this purpose, earthquake ground motions with various vertical-to-horizontal peak acceleration ratios are selected to which a suitable scale factor is applied to match with seismic hazards of Korea. The methodology involves the evaluation of the structural responses of RC frames subjected to the selected records by means of nonlinear time history analyses. The results from the analysis are compared with results from studies of the case of horizontal-only excitation. The effect of vertical earthquake component on damage of RC frames is considered at both the global and the local levels. The effect of vertical ground motion on axial force, shear demand, and shear capacity of RC columns is investigated to assess failure on a local level. In particular, the shear capacity is evaluated by using both the conservative method of a design code and more realistic predictive approaches. The results of the extensive analyses indicate that vertical ground motion can significantly affect the response of RC members in terms of axial force variation and shear capacity. These results point to the conclusion that vertical ground motion needs to be included in analysis for assessment and design.

1. Introduction

Field observations from recent earthquakes (Northridge earthquake (1994) in USA, Hyogo-ken earthquake (1995) in Japan, Yojakarta earthquake (2006) in Indonesia, and Christchurch earthquake (2011) in New Zealand) report that the vertical component of strong ground motion causes significant damage to RC structures. Furthermore, many recent studies have confirmed the possible destructive effect of vertical ground motion on RC structures, and thus its significance has gradually become of concern in the structural earthquake engineering community.

Mwafy and Elnashai [1] evaluated the effect of vertical ground motion on 12 RC buildings and indicated that an interstorey drift of the collapse limit state was frequently reached when vertical ground motion was included. The study also revealed that the axial compressive force and the curvature ductility demand in columns increased by up to 45% and 58%, respectively. Kunnath et al. [2] examined the seismic

performance of two-span highway bridges with six different structural configurations and found that there was a significant increase in the axial force demand in the columns and moment demands in the girder. Hosseinzadeh [3] investigated the seismic response of a simple RC bridge pier before and after retrofitting, considering both horizontal and vertical ground motions. This analytical study indicated that the maximum axial force, bending moment, and shear force demand of the pier increased by about 30%, 10%, and 15%, respectively, due to vertical ground motion. Kim et al. [4] evaluated the effects of vertical ground motion on bridge piers considering vertical-tohorizontal peak acceleration ratios. One of the notable findings in this study was that the shear capacity of the pier was reduced by 25% when the vertical component of strong ground motion was included. Also, Kim et al. [5] experimentally confirmed the effect of vertical ground motion on RC columns by conducting hybrid simulations with a bridge structure. The first specimen was tested with only horizontal excitation while the second specimen was subjected to combined horizontal and vertical excitation. It was observed that the vertical ground motion significantly affected the axial force variation and spiral strain of the second specimen, which were increased by 98% and 200%, respectively, compared with those of the first specimen. Lee et al. [6] performed a combined experimental and analytical study on the effects of vertical ground motion on the shear capacity in bridge columns. From the experimental study, a considerable tensile force was induced in columns due to vertical ground motion, resulting in degradation of shear capacity. It was also concluded that shear strength models by the current design codes were insufficient to predict the observed shear damage due to the lack of consideration in the axial force fluctuation induced by highfrequency vertical motion.

Many research studies described above lead several design codes including Eurocode (EC8) [7] and ASCE/SEI 7–16 [8] to suggest vertical spectra. However, most of the studies have focused on a structure located in a high seismic area, and thus most design codes in the countries of moderate seismicity still do not account for the effect of vertical ground motion on the structure. Studies considering various geometric configurations in the structure are also sparse. Hence, in this study, three-storey RC frames with different geometric configurations are designed, and the effects of vertical ground motion on those frames are analytically investigated taking into account various vertical-to-horizontal peak ground acceleration (V/H) ratios.

2. Selection of Reinforced Concrete Frames and Response Measures

2.1. Description of Selected Structures. The selected structures consist of simple three-storey RC frames with different geometric configurations, as shown in Figure 1. The prototype structure was originally designed for gravity loads with nonseismic details for the purpose of an experimental study [9]. Since the various geometric configurations are considered in this study, each frame is designed for gravity loads to Korean Building Code [10]. As detailed in Table 1, the considered structural configurations are (i) 5 equal spans with each length varying from 4 m to 8 m, (ii) 5 different ratios of the interior span length (L2) to the exterior span length (L1) varying from 0.57 to 1.60, and (iii) 5 different column heights in the first storey varying from 3.6 m to 4.8 m. It should be noted that the RC frames of SL6, SR100, and SH100 in Table 1 are identical and selected as a reference structure. Thus, a total of 13 structures are considered. A concrete compressive strength of 24 MPa and a rebar yielding strength of 400 MPa are used for all materials. As shown in Figure 1, the cross section of the column is $304.8 \text{ mm} \times 304.8 \text{ mm}$ with four longitudinal reinforcements with the diameter of 19.1 mm. The stirrup with the diameter of 9.53 mm is used with a spacing of 150 mm throughout the length.

The Mid-America Earthquake Center program, Zeus-NL [11], was utilized to perform the analyses for the selected structures. Zeus-NL is an inelastic fiber analysis package which was specifically developed for earthquake engineering applications. Elements capable of modeling material and geometric nonlinearity are available in the program. The sectional stress-strain state is obtained through the integration of the inelastic

material response of the individual fiber. In this study, a bilinear elastoplastic model with kinematic strain-hardening is employed for reinforcements. A nonlinear concrete model with constant confinement modeling based on the work by Mander et al. [12] is also employed. Columns and beams are divided into five and seven elements, respectively, in all analytical models. Mass is deposited at the beam and column connection. The fundamental period of each structure from eigenvalue analysis is shown in Table 1 and has a tendency to increase as the span length and storey height increase due to the increase in mass and decrease in lateral stiffness.

2.2. Limit States and Response Measure. Structural damage or failure may occur due to the attainment of a member or system level limit states. Thus, in this study, the structural response through a nonlinear time history analysis is investigated at both the global and local levels. An interstorey drift ratio is considered as a global failure criterion, while the axial force variation and shear capacity of structural members are monitored to assess failure on a local level.

The interstorey drift limit of each structure from a pushover analysis with a loading profile of first mode shape is estimated, and three limit states termed "serviceability," "damage control," and "collapse prevention" are used [13]. The limit states are defined as the following: (i) serviceability is defined when longitudinal rebar reaches yielding, (ii) damage control is defined when concrete strain reaches the maximum confined stress, and (iii) collapse prevention is defined when concrete strain reaches the ultimate confined strain (ε_{cu}) that is defined in EC8. The maximum strain (ε_{cu}) can be calculated as follows:

$$\varepsilon_{\rm cu} = 0.0035 + 0.1\alpha\omega_{\rm wd},$$

$$\alpha = \left(1 - \frac{\sum b_i^2}{6b_0h_0}\right) \left(1 - \frac{s}{2b_0}\right) \left(1 - \frac{s}{2h_0}\right),$$
(1)

where ω_{wd} is the mechanical volumetric ratio of confining hoops within the critical regions, α is the confinement effectiveness coefficients, b_i is the distance between consecutive engaged bars, and b_0 and h_0 are core dimensions to centerlines of the perimeter hoop. As shown in Figure 2, the first storey drifts corresponding to serviceability, damage control, and collapse prevention for SL6 frame are 0.85%, 1.18%, and 1.78%, respectively. It is assumed that these limit states at the first storey can also be applicable to the remaining stories. Table 1 summarizes each limit state per structure.

To investigate the effect of vertical ground motion on a local level, an axial force variation on columns in the first storey is assessed. The effect of vertical ground motion on the axial force variation is evaluated by considering the ratio of the axial force variation induced by the only vertical excitation to gravity load as shown in the following equation:

$$\frac{\text{AFV}_{\text{H+VGM}} - \text{AFV}_{\text{HGM}}}{\text{gravity load}} \times 100,$$
 (2)

where AFV is the axial force variation which is defined as the difference between the maximum and minimum axial forces on column during the simulation.



FIGURE 1: Elevation of RC frames and section of typical members (units: mm): (a) elevation, (b) column section, and (c) beam section.

Reference name	Span length (m)		Span ratio	Storey height (m)		Storey height ratio	Fundamental	Limit state (interstorey drift ratio (%))		
	L1	L2	L2/L1	H1	H2	H2/H1	period (sec)	Service ability	Damage control	Collapse prevention
Span length										
SL4	4.00	4.00	1.00	3.60	3.60	1.00	0.77	0.83	1.71	2.45
SL5	5.00	5.00	1.00	3.60	3.60	1.00	0.89	0.89 0.84		1.96
SL6	6.00	6.00	1.00	3.60	3.60	1.00	0.99	0.85	1.18	1.78
SL7	7.00	7.00	1.00	3.60	3.60	1.00	1.09	0.83	1.12	1.72
SL8	8.00	8.00	1.00	3.60	3.60	1.00	1.19	0.79	1.13	1.94
Span ratio										
SR057	7.00	4.00	0.57	3.60	3.60	1.00	1.02	0.83	1.37	1.99
SR077	6.50	5.00	0.77	3.60	3.60	1.00	1.00	0.84	1.22	1.88
SR100	6.00	6.00	1.00	3.60	3.60	1.00	0.99	0.85	1.18	1.78
SR127	5.50	7.00	1.27	3.60	3.60	1.00	0.98	0.98 0.85		1.76
SR160	5.00	8.00	1.60	3.60	3.60	1.00	0.96	0.84	1.07	1.74
Storey height										
SH075	6.00	6.00	1.00	4.80	3.60	0.75	1.26	1.04	1.48	1.99
SH080	6.00	6.00	1.00	4.50	3.60	0.80	1.18	0.99	1.41	1.93
SH086	6.00	6.00	1.00	4.20	3.60	0.86	1.11	0.95	1.33	1.88
SH092	6.00	6.00	1.00	3.90	3.60	0.92	1.05	0.90	1.26	1.82
SH100	6.00	6.00	1.00	3.60	3.60	1.00	0.99	0.85	1.18	1.78

TABLE 1: Details and limit states of the selected RC frames.

The shear demand and capacity of columns at the first storey are also investigated as a failure criterion. To consider the shear capacity, shear strength models by ACI318-14 [14], Priestley et al. [15], Sezen and Moehle [16], and Pan and Li [17] are employed. The model proposed by Priestley et al. [15] is composed of three independent components: concrete contribution considering displacement or curvature ductility, shear reinforcement contribution based on the truss mechanism using a 30° angle of inclined shear cracking, and shear resistance of the arch mechanism provided by axial force. Sezen and Moehle [16] also proposed the shear strength model including contributions from the concrete and transverse reinforcement by considering the column cross-sectional dimensions, concrete compressive strength, column aspect ratio, axial load, and displacement ductility demand. The transverse reinforcement contribution is calculated as ACI318-14 [14]. Based on the truss-arch model, Pan and Li [17] proposed the shear strength model that considers both the contributions of concrete and transverse reinforcement to shear strength in the truss model, as well as the contribution of arch action through compatibility of deformation.

3. Selection of Strong Ground Motion

Earthquake ground motion records from PEER NGA database were selected to evaluate the effect on vertical ground motion for RC buildings. The selection criteria are shown below:

- (i) Earthquake magnitude (M_w) of more than 6.0
- (ii) Closest distance to the fault of less than 50 km
- (iii) Peak ground acceleration (PGA) of horizontal ground motion of more than 0.2 g
- (iv) Vertical-to-horizontal peak ground acceleration (V/H) ratio of more than 0.6
- (v) Scale factor of earthquake ground motion record between 0.75 and 1.25

As shown in Table 2, a total of nine records are selected for the analysis. As an example, Figure 3 illustrates horizontal and vertical components of ground motion from the Morongo Valley Fire Station, N. Palm Springs earthquake. As shown in figure, the higher frequency content is observed in vertical ground motion component, compared with horizontal motion. A scale factor to each horizontal ground



FIGURE 2: Limit states of SL6 frame: (a) serviceability and (b) damage control and collapse prevention.

Earthquake	$M_{\rm w}$	Station	Fault dist. (km)	PGA (g) H V		V/H	Time lag (sec)*	Scale factor to HGM	Ref. name
Imperial Valley (1979)	6.5	Chihuahua	7.3	0.270	0.218	0.807	4.950	1.22	IV-CHI
N. Palm Springs (1986)	6.0	Morongo Valley Fire	12.0	0.205	0.395	1.929	1.155	1.02	PS-MVH
1 0		Arleta Fire	8.7	0.308	0.552	1.790	2.780	0.97	NO-ARL
$\mathbf{N}_{\mathbf{r}}$ with \mathbf{r} : $\mathbf{J}_{\mathbf{r}}$ (100.4)	67	Canoga Park	14.7	0.356	0.489	1.374	0.030	1.09	NO-CNP
Northridge (1994)	6.7	N Faring Rd	20.8	0.242	0.191	0.186	2.500	0.96	NO-FAR
		Roscoe Blvd	10.1	0.303	0.306	1.010	-1.590	0.97	NO-ROB
Ch: Ch: Trimer (1000)		TCU055-NS	6.3	0.201	0.167	0.831	-4.285	0.98	CC-TCN
Chi-Chi, Iaiwan (1999)	7.6	TCU089	9.0	0.248	0.191	0.774	5.580	1.08	CC-TCU
Kobe (1995)	6.9	Kobe Univ.	0.9	0.310	0.380	1.220	0.530	0.81	KB-KBU

TABLE 2: Selected ground motions.

*Time interval between vertical and horizontal acceleration peaks.



FIGURE 3: Horizontal and vertical ground motions, Morongo Valley Fire Station, N. Palm Springs earthquake (1986).

motion is applied to match the spectral acceleration value of the maximum considered earthquake (MCE) for the soil class (S_c) of KBC 2016 at the fundamental period (0.99 sec) of a reference structure. Figure 4(a) shows the response acceleration spectra with MCE spectrum retrieved from KBC 2016. In addition, as depicted in Figure 4(b), vertical spectrum proposed by EC8 is well matched with those of the selected records.

4. Analysis Results

4.1. Effect on Global Response. A nonlinear time history analysis with the selected 13 RC frames was performed to investigate the effect of vertical ground motion. Kim et al. [4] showed the distribution of V/H ratio for 452 earthquake ground motion records and indicated that the V/H ratios for 97% of the ground motions is less than 2.0. Thus, for each structure and record, 16 V/H ratios for a fixed horizontal PGA are considered, which range from 0.5 to 2.0 with an increment of 0.1 in this study. Analytical results are compared with the case of horizontal-only excitation. On the global level, the interstorey drift is monitored up to collapse prevention limit as shown in Figure 5. Since P- Δ effects can be significant and could lead to instability of structures at values exceeding the collapse limit, member responses that are exceeding the collapse prevention limit are not included for local levels.

As depicted in Figure 6, the effect on the interstorey drift ratio on RC frames is observed to fluctuate as V/H ratio increases. For most of earthquake records, the effect on the interstorey drift ratio is within the range of $\pm 3\%$. Thus, it could be inferred that the lateral displacement is mainly affected by horizontal ground motion rather than vertical



FIGURE 4: Response spectra of the selected records: (a) horizontal spectra, and (b) normalized vertical spectra.



FIGURE 5: Interstorey drift ratio, SL6.

ground motion. Although including vertical ground motion seems to have a minimal effect on the interstorey drift ratio for most of the records, interstorey drift ratios in some records are significantly affected by the vertical ground motion. For example, the interstorey drift ratio of the structure shown in Figure 6(a) increases up to 36% when the vertical component of earthquake records is considered.

4.2. Effect on Member Response. The effect of vertical ground motion on axial force variation is shown in Figure 7. As illustrated, the axial force variation on the column is significantly affected by vertical ground motion as

V/H ratio increases. The axial force on the column is directly influenced by the magnitude of vertical ground motion, and thus the effect on axial force variation increases considerably as V/H ratio increases. Figure 7(a) indicates that the effect on axial force increases as span length increases. The axial force variations on columns in RC frames with different geometric configurations including span length, span ratio, and storey height increase up to 205.9% (NO-ROB), 223.0% (IV-CHI), and 242.6% (NO-CNP), respectively. Note that the effect shown in Figure 7 is the ratio of contribution of vertical ground motion to the axial force variation, normalized by the dead load as previously given by (2).



FIGURE 6: Effect on interstorey drift ratio: (a) span ratio and (b) storey height ratio.

The significant variations of axial load discussed above also lead to fluctuations in the column shear demand and capacity. Figure 8 presents the effect of vertical ground motion on the shear demand of columns in RC frames with different span length, span ratio, and storey height. As shown in Figure 8, no clear correlation exists between the shear demand and V/Hratio. This is due to the fluctuation of the lateral force caused by highly varying axial force because the vertical component of ground motion has much higher frequency content than the horizontal component. However, the effect on shear demand tends to slightly decrease up to 15% as the span length increases.

Concerning the shear capacity of the column, the shear strength model by ACI318-14 [14] as well as the predictive approaches by Priestley et al. [15], Sezen and Moehle [16], and Pan and Li [17] are utilized. Compared with ACI318-14 model,

the contribution of shear reinforcement in Sezen and Moehle [16] is identical, while that of Priestley et al. [15] is about 1.7 times higher because a 30° crack angle is considered. The contribution of shear reinforcement in Pan and Li [17] which employs the crack angle model proposed by Kim and Mander [18, 19] is about 10% lower than that of ACI318-14 [14]. Moreover, the contribution of concrete in the ACI318-14 equation is more conservative than other models. As shown in Figure 9, the shear capacity of the RC column estimated by using all shear strength models decreases as V/H ratio increases. In particular, the shear strength models by ACI318-14 and Sezen and Moehle approaches show a similar reduction ratio of shear capacity as V/H ratio increases.

Figure 10 shows a clear trend in reduction of shear capacity. The shear capacity is reduced up to 25.8% as the





FIGURE 7: Effect on axial force variation: (a) span length, (b) span length, V/H ratio, (c) span ratio, and (d) storey height ratio.



FIGURE 8: Effect on shear demand.



FIGURE 9: Effect on shear capacity by shear strength models.



FIGURE 10: Effect on shear capacity, ACI318-14: (a) span length, (b) span ratio, and (c) storey height ratio.

vertical motion amplitude increases. The shear capacity also decreases as the span length and ratio increase. As previously mentioned, the effect of vertical ground motion on interstorey drift ratio is minimal, while the effect on axial force variation is significant. Therefore, it is concluded that the axial force variation results in noteworthy reductions in shear capacity.

5. Conclusions

The paper analytically investigates the effect of vertical ground motion on RC frames with different span lengths, variable span ratios, and various column heights by considering various vertical-to-horizontal peak ground acceleration ratios. The most significant findings are summarized below.

It is observed that the effect of vertical ground motion on the interstorey drift ratio of RC frames is minimal within the range of $\pm 3\%$ for most of the selected records, and thus the lateral displacement is mainly affected by horizontal ground motion rather than vertical ground motion. However, in some cases, the effect on interstorey drift ratio increases up to 36.1% or decreases up to 34.1% as V/H ratio increases. Moreover, it is found that vertical ground motion considerably influences the axial force level and variation on RC columns. The axial force variation on the RC columns in the first storey significantly increases up to about 240% when the vertical component of the earthquake ground motion is included. This high variation of axial force leads to a reduction in shear capacity and increases the potential for shear failure as V/H ratio, span length, and span ratio increase. Compared to the response with horizontal-only excitation, shear capacity of vertical members is reduced by up to 25.8%. Thus, neglecting vertical ground motion in the analysis could lead to serious underestimation of demand and overestimation of capacity.

Taking into account the observations from the study described above, it is concluded that RC structures subjected to combined horizontal and vertical components of earthquakes can suffer more damage than those subjected to horizontal-only ground motion. Therefore, it is recommended that vertical ground motion be included in the analysis for reliable seismic assessment of RC buildings in near-fault areas, where V/H ratio is likely to be high.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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