

## Research Article

# Prediction of Settling Velocity of Nonspherical Soil Particles Using Digital Image Processing

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Digital image processing (DIP) is used to measure shape properties and settling velocity of soil particles. Particles with diameters of 1 to 10 mm are arbitrarily sampled for the test. The size of each particle is also measured by a Vernier caliper for comparison with the classification results using the shape classification table. The digital images were taken with a digital camera (Canon EOS 100d). Shape properties are calculated by image analysis software. Settling velocity of soil particles is calculated by displacement and time difference of images during settling. The fastest settling particles are spherical shaped. Shape factors well explain the difference of settling velocity by a particle shape. In particular, the aspect ratio has a high negative correlation with residual of settling velocity versus mean diameter. Especially, DIP has a higher applicability than classification using the shape classification table because it can measure a number of particles at once. The settling velocity of soil particles is expressed as a function of mean diameter and aspect ratio.

## 1. Introduction

The settling velocity of sediment particles is one of the key variables in the study of sediment transport and is important in understanding suspension, deposition, mixing, and exchange processes. It is affected by the size, density, shape of sediment particles, and physical properties of fluid [1].

The aim of many researches on the study of the settling velocity is to make precise, general formula for prediction of settling velocity [1–6]. The shape of sediment particles can be divided into ideal spherical and nonspherical. The settling velocity of spherical particles can be predicted to be more than 98% accurate [7]. Recent studies on the settling velocity of spherical particles have been an underway to investigate their acceleration motions analytically [8], especially the effects of initial velocity on spherical particle acceleration in a fluid [9, 10].

A sphere with the nominal diameter of a nonspherical grain has the same weight, so that the difference between

their settling velocities must be due to shape effects alone [11].

To predict settling velocity of nonspherical particles, researchers propose a shape index to explain the effect of the particle shape on settling velocity [11–20]. The shape index of the particle is a coefficient representing various shape properties of the particle such as circularity, sphericity, roundness, and flatness. These shape indexes are calculated by major, intermediate, and minor axes to consider a 3-dimensional shape of particles. Most of the shape indexes well represent the difference in settling velocity according to the particle shape, but they have a poor applicability because the measurement of the 3-dimensional particle shape is inconvenient.

A digital image processing (DIP) can be a proper alternative method for particle shape measuring. The DIP is a method to perform specific operations on digital images to extract information in digital images by computer algorithms. The DIP has been widely used to measure

displacement and shape of particles in geotechnical engineering by many researchers [21–30].

The objective of this paper was to measure particle shape and settling velocity of nonspherical soil by DIP and develop the empirical prediction formula of settling velocity considering the particle shape and diameter. To compare with the classification result using the shape classification table, the particle shape is measured by a Vernier caliper and DIP method. The settling velocity equation uses shape properties measured by the DIP method instead of 3-dimensional shape indexes for high applicability.

## 2. Materials and Methods

**2.1. Materials and Experimental Procedure.** Two different nonspherical soils were used in this study. One is used for formulation of the settling velocity equation using the digital image processing method, and the other is used for verification of the settling velocity equation. The nonspherical soils were classified as SW by the Unified Soil Classification System. Physical properties of nonspherical soils are summarized in Table 1.

In this study, digital image processing is adopted to measure the particle shape properties and the settling velocities of particles. Particles with diameters of 1 to 10 mm are arbitrarily sampled in sample A and B. 343 particles are sampled in sample A and 106 particles in sample B. The size of each particle is also measured by a Vernier caliper for comparison with the classification result using the shape classification table. The experimental procedure is shown in Figure 1.

**2.2. Digital Image Acquisition.** It is important to take a proper digital image of target in digital image processing. The quality of the digital image is influenced by the shooting condition. The apparatus for acquisition of the digital image to get shape properties of soil is shown in Figure 2(a). The digital images were taken with a digital camera (Canon EOS 100d). To separate soil from image background properly, a black background plate is used. The digital camera is installed in the vertical direction of the background plate. Two led lights are used to illuminate uniformly for shooting.

The apparatus for acquisition of the digital image to get settling velocity of soil is shown in Figure 2(b). The dimension of the settling tank is 300 mm (width)  $\times$  80 mm (length)  $\times$  1000 mm (height). A soil particle is settled down under the surface of water by pincette to minimize the effect of surface tension. The digital camera is installed in the horizontal direction of the settling tank. To acquire digital images of settling soil particle, shoot high definition video at 25 frames per second and divide video by frame.

**2.3. Digital Image Processing.** An image analysis software (Media Cybernetics Image Pro Plus, Version 6.0) is used to determine shape properties of soil particles such as diameter, area, perimeter, roundness, and aspect ratio. Description of shape properties of soil particles is summarized in Table 2.

The resolution of the digital image is 0.077 mm/pixel. Procedure of acquisition of soil shape properties is summarized in Figure 3.

To determine settling velocity of soil, the R-programming language is used to analyze the digital image set. An original image is cropped in order to remove unnecessary parts for DIP. The procedure of calculation of settling velocity by DIP is as follows. First, select a digital image set of the settling particle. The digital image set has  $N$  frames of the digital image. Second, perform binarization to segmentation of target (soil particle) and background. After binarization, the soil particle in the digital image is converted to a set of white pixels. To determine the position of the soil particle in each image, calculate centroid of white pixels in each image. The displacement is the difference of the position of the soil particle in two consecutive images, and the time difference is 1/25 second because the original image is taken 25 frames per second. Finally, settling velocity is calculated by dividing the displacement into time difference. The procedure of acquisition of settling velocity is summarized in Figure 4.

## 3. Results and Discussion

**3.1. Classification of the Particle Shape by the Shape Classification Table.** The size of a soil particle could be given by 3-dimensional as the shape of particle is commonly nonspherical. The major (A), intermediate (B), and minor (C) axes of the particle are measured by a Vernier caliper, and each particle is classified by the shape classification table [31]. The shape classification table classifies soil into 9 groups according to elongation ratio ( $=B/A$ ) and flatness ratio ( $=C/B$ ). The shape classification table is plotted in Figure 5(a). The shapes of nonspherical soil are classified into 4 groups such as sphere, short rod, thick plate, and ellipsoid (Figure 5(b)). The other shape groups such as plate, blade, and needle are an extreme case in natural soil particles. Therefore, the soil particles used in this study represent almost all shapes of soil in nature.

**3.2. Shape Properties Measured by Digital Image Processing.** The shape properties of nonspherical soil such as diameter (maximum, mean, and minimum), area, perimeter, roundness, and aspect ratio are measured by digital image processing. The statistic properties of shape properties are shown in Table 3.

The mean diameter of soil particles is 8.41 mm (max), 4.61 mm (average), 1.33 mm (min), and 1.57 mm (SD). The maximum diameter and minimum diameter are 5.92 mm (average) and 2.97 mm (average), respectively, and show similar distribution with mean diameter. The area of soil particles is 19.49 mm<sup>2</sup> (average). The roundness is distributed between 0.6 and 1.0 mostly as shown in Figure 6(a). Sampled soil particles are classified to well-rounded particles. The aspect ratio is 2.24 (average) and is distributed between 1.5 and 3.5 mostly as shown in Figure 6(b). It means that the shape of soil particles is elongated vertically.

TABLE 1: Physical properties of nonspherical soils considered in this study.

Sample	Specific gravity	Liquid limit (%)	Plasticity index (%)	Passing sieve		USCS <sup>a</sup>
				4.75 mm (%)	0.075 mm (%)	
A*	2.60	Nonplastic	Nonplastic	79.58	9.58	SW
B**	2.67	Nonplastic	Nonplastic	95.00	15.66	SW

\*Sample for formulation of the settling velocity equation using digital image processing; \*\* sample for verification of the settling velocity equation using digital image processing; <sup>a</sup>Unified Soil Classification System.

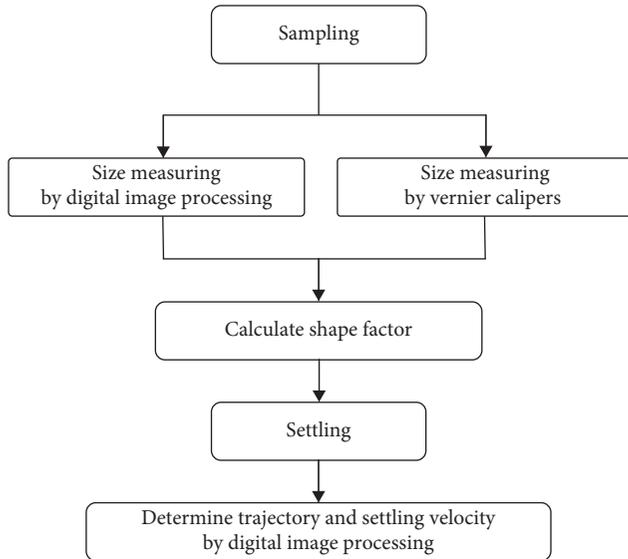


FIGURE 1: Experimental procedure.

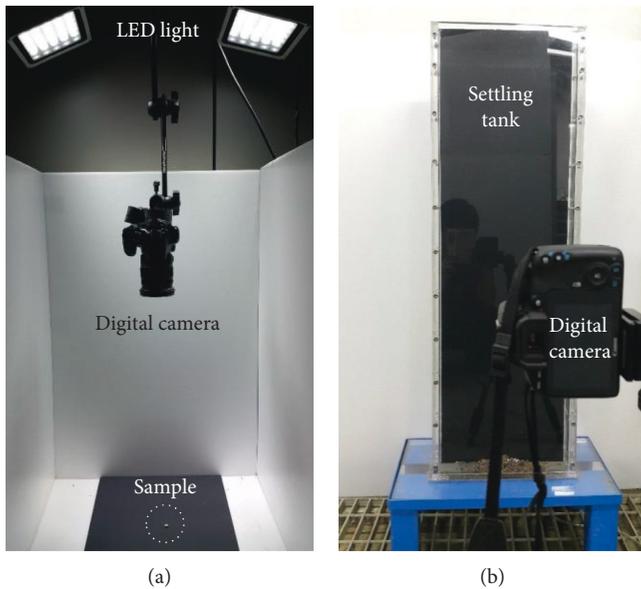


FIGURE 2: Apparatus for the acquisition of the digital image: (a) measuring shape properties; (b) measuring settling velocity.

3.3. Variation of Settling Velocity through Particle Shape Properties by DIP. Figure 7 shows relationship of settling velocity and particle mean diameter measured by the digital image processing method. Settling velocity of soil particle

TABLE 2: Description of shape properties of soil particles.

Shape properties	Description
Area	The area of the particle
Aspect	The ratio between the major axis and the minor axis of the ellipse equivalent to the particle
Major axis	The length of the main axis of the ellipse equivalent to the particle
Minor axis	The length of the minor axis of the ellipse equivalent to the particle
Maximum diameter	The length of the longest line joining two outline points and passing through the centroid
Minimum diameter	The length of the shortest line joining two outline points and passing through the centroid
Mean diameter	The average length of the diameters measured at two degree intervals joining two outline points and passing through the centroid
Maximum radius	The maximum distance between particle's centroid pixel position and its perimeter
Minimum radius	The minimum distance between particle's centroid pixel position and its perimeter
Perimeter	The length of particle's outline
Radius ratio	The ratio between maximum radius and minimum radius for each particle, as determined by maximum radius/minimum radius
Roundness	The roundness of each particle, as determined by $(4 * \pi * \text{area}) / (\text{perimeter}^2)$
Fractal dimension	The fractal dimension of the particle's outline

has a large variation even though the mean diameter of particles is similar to each other. The fastest settling particles are spherical shaped. This means that settling velocity of soil particles is affected by the particle shape, even when particles have the same mean diameter. The particles with mean diameter smaller than 6 mm have a significant variation of settling velocity through a difference in the particle shape. Therefore, particle shape classification by the shape classification table well explains variation of settling velocity through a difference in the particle shape. But particle shape classification by the shape classification table has a poor applicability because it needs 3-dimensional size of every single particle.

As the settling velocity is related with particle diameter and shape, to analysis relationship between settling velocity and particle shape, the relationship between settling velocity and mean diameter of the particle should be excluded. For this reason, the residual of settling velocity versus mean

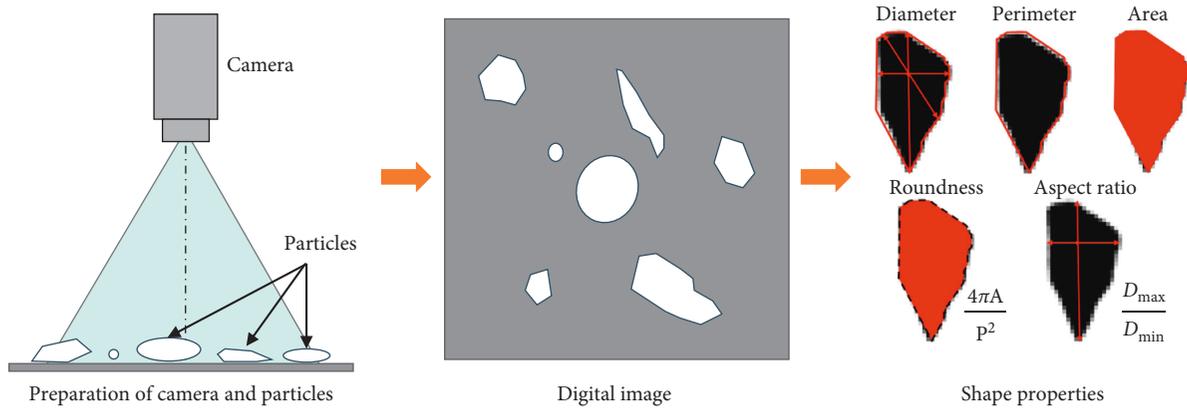


FIGURE 3: Procedure of acquisition of shape properties.

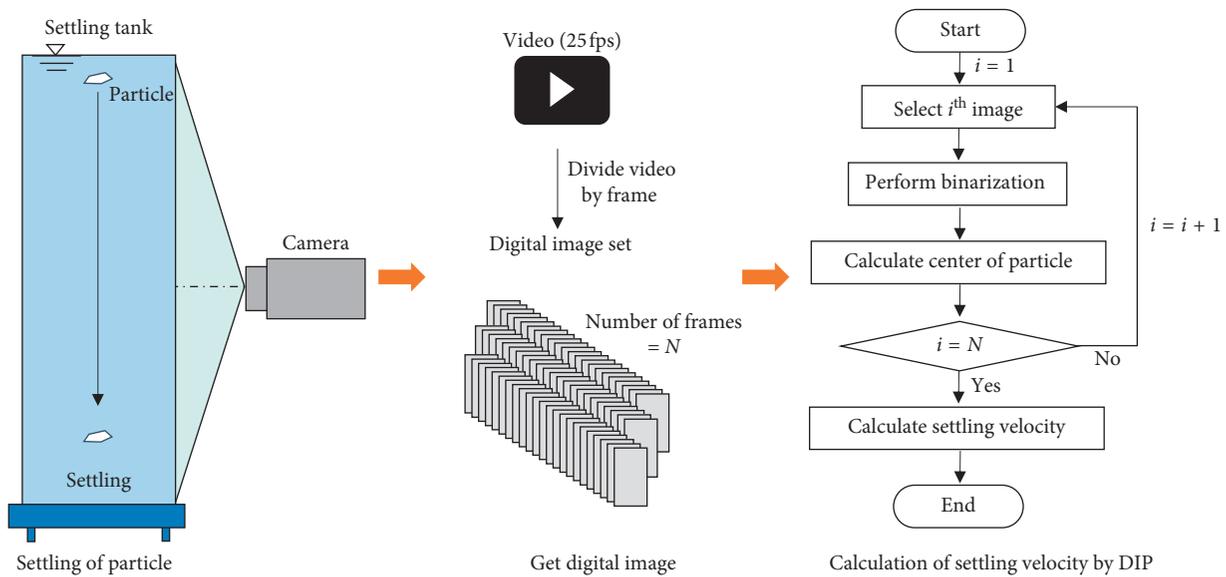


FIGURE 4: Procedure of acquisition of settling velocity.

diameter (Figure 8) is calculated with the regression curve of settling velocity and mean diameter. The residual is distributed independent of mean diameter and varies up to 54%, down to 43% even for the same mean diameter.

The correlation coefficient between shape factors and residual is shown in Figure 9. The aspect ratio has a high negative correlation with residual, and its correlation coefficient is  $-0.72$ . In common with the result using the shape classification table (Figure 7), settling velocity is affected by the particle shape in case of each particle having a same mean diameter. It means that the settling velocity can be explained with shape properties which acquired by a 2-dimensional digital image instead of using the 3-dimensional shape classification method. Especially, DIP has a higher applicability than the shape classification table because it can measure a number of particles at once.

**3.4. Prediction Formula of Settling Velocity by DIP.** The settling velocity of soil particles is a function of diameter and

aspect ratio. The prediction formula for settling velocity (1) is an empirical formula derived with multiple nonlinear regression by Microsoft Excel Solver. The form of the formula was determined in the simplest form by trial and error. The formula was developed for nonspherical soil particles with an average particle size range of 1.33 mm to 8.41 mm:

$$V = 14.5 \sqrt{D_{\text{mean}} \left( \frac{D_{\text{min}}}{D_{\text{max}}} \right)}, \quad (1)$$

where  $V$  is the settling velocity (cm/sec),  $D_{\text{mean}}$  is the mean diameter (mm), and  $D_{\text{min}}/D_{\text{max}}$  is the inverse of aspect ratio.

The coefficient of determination ( $R^2$ ) is 0.82 (Figure 10). This function has a high applicability because it needs only two parameters (mean diameter and aspect ratio) derived from DIP.

To verify the prediction function for settling velocity, 106 soil particles are sampled. Compared with other researches [1, 6, 15, 18], the prediction formula of settling velocity by DIP is much simpler and has a higher accuracy (Figure 11).

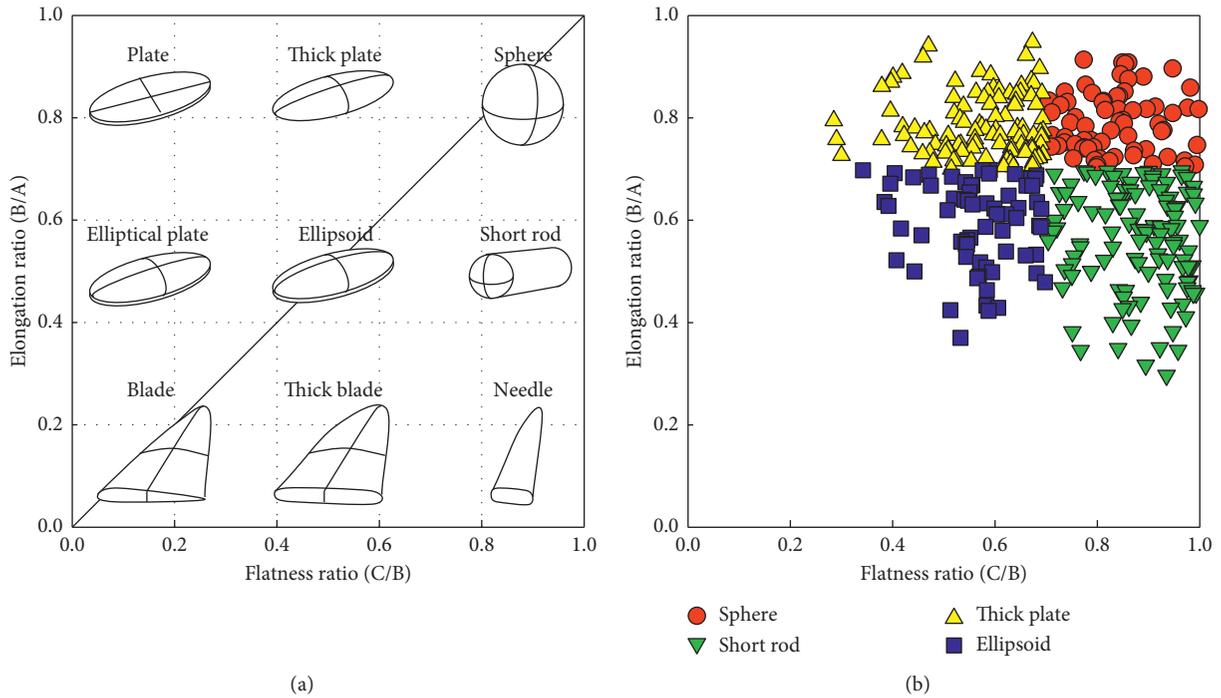


FIGURE 5: Shape classification table and classification result of nonspherical soil: (a) shape classification table [31]; (b) shape of nonspherical soil.

TABLE 3: Statistic properties of shape properties.

	Maximum	Average	Minimum	Standard deviation
Maximum diameter (mm)	11.79	5.92	1.60	2.03
Minimum diameter (mm)	7.49	2.97	0.80	1.36
Mean diameter (mm)	8.41	4.61	1.33	1.57
Area (mm <sup>2</sup> )	58.52	19.49	1.57	12.86
Perimeter (mm)	37.08	17.22	4.43	6.18
Roundness	0.90	0.72	0.20	0.13
Aspect ratio	4.70	2.23	1.18	0.67

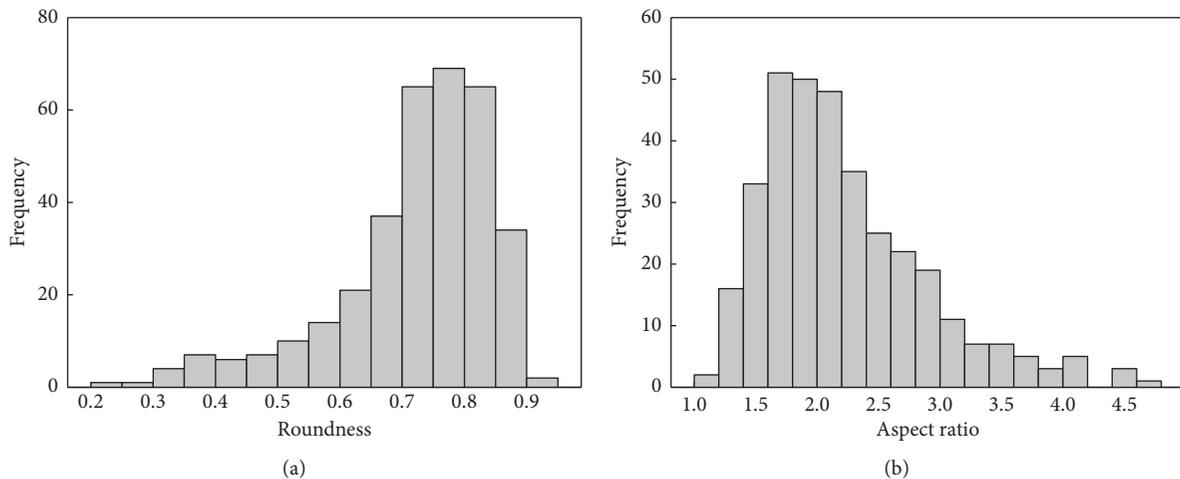


FIGURE 6: Histogram of shape properties of nonspherical soil: (a) roundness; (b) aspect ratio.

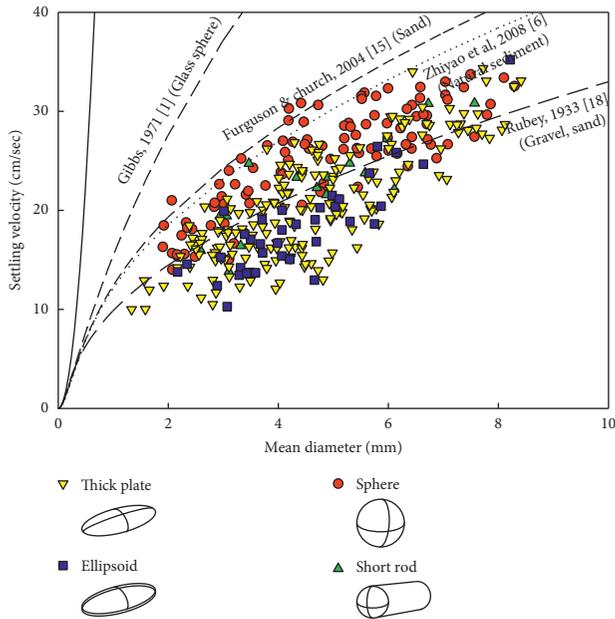


FIGURE 7: Settling velocity of nonspherical soil classified with the shape classification table.

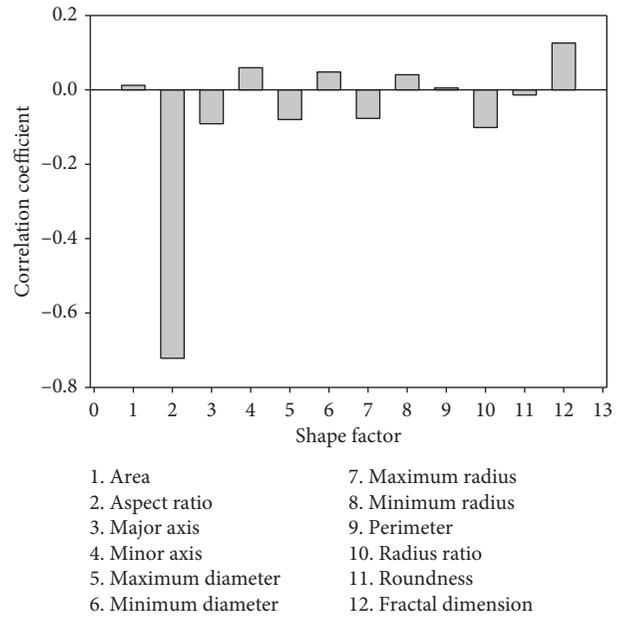


FIGURE 9: Correlation coefficients between shape factors and residuals of settling velocity.

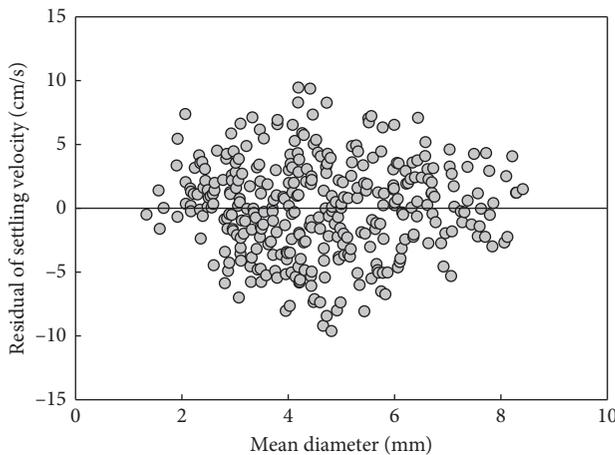


FIGURE 8: Residual of settling velocity versus mean diameter.

### 4. Conclusions

In this study, the digital image processing method has been used as an alternative measure to predict settling velocity of soil particles. The following conclusions were obtained. The shapes of nonspherical soil are classified into 4 groups such as sphere, short rod, thick plate, and ellipsoid. Measurement method of soil shape properties and settling velocity by digital image processing is evaluated as a simple, faster alternative rather than using the shape classification table. The settling velocity of the soil particle has a large variation even though the mean diameter of particles is similar to each other. In case particles have a same mean diameter,

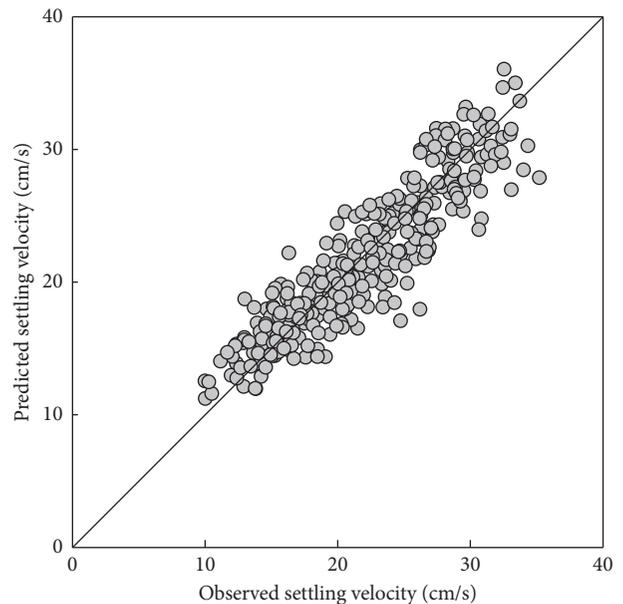


FIGURE 10: Comparison of observed/predicted settling velocity.

sphere shape particles have higher settling velocity than that of other shape particles. The particles smaller than 6 mm have a significant variation of settling velocity through a difference in the particle shape. The settling velocity is affected by mean diameter and aspect ratio measured by digital image processing. The prediction formula for settling velocity is derived with multiple nonlinear regression, and it is simple and accurate compared to results of previous researches.

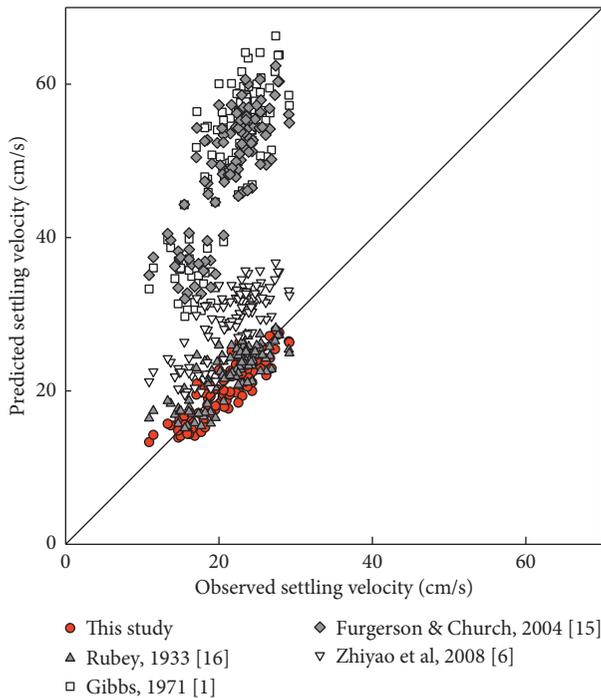


FIGURE 11: Comparison of prediction formula by DIP (this study) and other research studies.

## Data Availability

The data used to support the findings of this study are included within the article.

## Disclosure

This manuscript is based on a part of the first author's master's thesis from Seoul National University [32].

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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