

Research Article

Constitutive Model of Rock Uniaxial Damage Based on Rock Strength Statistics

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Based on the assumption that rock strength follows the log normal distribution statistically, this paper establishes a damage constitutive model of rock under uniaxial stress conditions in combination with the Mohr–Coulomb strength criterion and damage mechanical theory. Experiments were carried out to investigate the damage evolution process of rock material, which can be categorized into nondamaging, accelerated growth, constant-speed, similar growth, and speed-reducing growth stages. The evolution process had a good corresponding relationship with the rock stress-strain curves. Based on the statistical damage constitutive model proposed in this paper, a numerical fitting analysis was conducted on the uniaxial compression testing data of laboratory sand rock and on experimental data from other literature, in order to validate the rationality of the constitutive equation and the determination of its parameters and to analyze the effect of internal friction variables on damage variables and compression strength. The research outcomes presented in this paper can provide useful reference for the theory of rock mechanics and for rock engineering.

1. Introduction

Experiments have shown that rock is a type of complex medium. Under the application of external loading, rock presents a very complex nonlinear deformation, and the complex deformation results in complexity and uncertainty of rock engineering accordingly. Many investigators have tried to explain the relationship between the deformation of rock and external loading, i.e., the constitutive relationship of rock. The results of early research into rock mechanics include Hooke's law, Newton's law of viscosity, and Saint Venant's ideal plasticity law. The continuous recognition of progress in understanding the mechanical properties of rock has led investigators to propose a variety of constitutive models by means of a subsequent viscoelastic plastic theory of continuum mechanics, such as that put forward by Liu et al. [1]. Based on rheological theory, Liu et al. [1] proposed a mechanical constitutive model that can describe the rheology of fully saturated rock. Fu et al. [2] proposed an

elastic-plastic constitutive model for soft sedimentary rock by using the conventional elastic-plastic constitutive model and considering the anisotropy and constraint-strain dependence of soft rock. Desai and Salami [3] proposed a constitutive model describing the deformation behavior of rock stress on the basis of the assumption that, in elastic-plastic theory, rock behaviors such as hardening, volumetric variation, stress path, cohesion, tensile strength, and yield behavior vary with average pressure. Li et al. [4] carried out a research in the framework of the theory of plasticity, considered internal friction and cohesion as the variables of the plastic strain function, and proposed a hyperbolic hardening function for kinetic friction and a mixed parabolic exponential equation for kinetic cohesion. In combination with strength theory and the movement of the strength component, Li et al. [4] provided a yield function and proposed a mixed hardening/softening elastic-plastic constitutive model that can predict the stress-strain behavior of soft rock.

Under the long-term application of external load and environmental conditions, many fine cracks and other defects are created inside rock. Owing to the existence of these defects, the mechanical properties of rock differ significantly from metal materials and polymers. The microstructure inside the rock material can cause the deterioration of the mechanical properties of the rock; therefore, the constitutive relationship of the rock material can be investigated from the point of view of damage mechanics. Damage mechanics assumes that the growth and confluence of various defects inside a material can result in the deterioration of its structure and accordingly, affect its mechanical properties.

Lemaitre [5] established a new elastic-plastic damage complex model on the basis of the maximum loss principle, achieving strong coupling between plasticity and damage. Cerrolaza and Garcia [6] established a nonlinear mechanical model to explain the stress-strain relationship of damaged rock during excavation on the basis of damage mechanical theory and in combination with the boundary method. By combining plasticity mechanics and damage mechanics, Unteregger et al. [7] created a constitutive model that could describe the nonlinear mechanical behaviors (including irreversible deformation, strain hardening, and strain softening) and bear complex three-dimensional stress for various types of rocks. Based on the assumption that the crack growth of rock during the postpeak period is the process of cohesive instability and making use of the nonassociated plastic potential function of rock plastic deformation and expansion derived from the dilation angle, Pourhosseini and Shabanimashcool [8] developed a damage constitutive model to describe the nonlinear behaviors of intact rock under static load (including prepeaking elastic behaviors and postpeaking strain-softening behaviors). Zhou and Zhu [9] investigated the plastic friction and the deformation mechanisms of plastic holes on the basis of irreversible thermodynamic theory and damage mechanical theory. They established a plastic friction yield criterion through a parabolic function that included volumetric deformation effects. They then proposed a plastic hole yield criterion by making use of Gurson's criterion and a friction yield model to determine the matrix deformation of porous materials. Combining these two criteria, they developed an elastic-plastic damage constitutive model with two yield surfaces. Based on continuum damage mechanics theory and from the microscopic viewpoint, Shao and Rudnicki [10] created a microcontinuum damage model to describe the brittle rock solid material that is mainly dominated by compression.

Statistical physics theory has been widely used in the evolution and development of microscopic damage mechanics [11–13] and in the establishment of many constitutive models [14–16] that consider rock damage statistics. Guang [17] pointed out that due to the heterogeneity of rock mechanical properties, probability statistical analysis can be applied to analyze rock strength. Based on damage mechanics and probability statistical analysis of rock strength, Guang devised the concept of rock damage and created the constitutive model for it. Based on the theory of damage mechanics and statistical strength theory, Li et al. [18]

established a damage statistical constitutive model to analyze the softening deformation of rock strain. Budiansky [19] pointed out that microscopic damage mechanics used quantum and statistical mechanics to determine the impact of damage on microscopic structures and then inferred their macro-mechanical effects. Meanwhile, microscopic damage mechanics illuminated the quantitative relationship [20] between microscopic damage structure and mechanical parameters.

In the constitutive models that consider the statistics of damage evolution, the investigators defined the types of various microelement strength distributions, including Weibull distribution [21–24], normal distribution [25], power function distribution [26], and so on. To define the microelement damage criterion, they made use of the Mohr–Coulomb (M-C) and Drucker–Prager (D-P) criteria. Based on the fact that rock strength microelements follow a random distribution, and on the M-C rock damage criterion, we have assumed that the strength distribution of microelements complies with a log normal distribution function, studied the statistical damage evolution of microelements in the loading process of rock, and established a damage constitutive equation of rock uniaxial damage. Thus, our research provides references for rock mechanics research and engineering.

2. Establishment of Statistical Damage Constitutive Model

It is assumed that N is the total microelement number of rock and that N_d is the microelement number of damaged rock under a certain load, and then D is defined as the statistical damage variable, i.e.,

$$D = \frac{N_d}{N}. \quad (1)$$

When the internal microelement stress σ_s of rock reaches its strength F , the microelement fails. Assuming that the microelement strength F follows a certain probability distribution, the number of failed microelements at any stress level interval $[\sigma_s, \sigma_s + d\sigma_s]$ can then be determined by

$$dN_d = Np(\sigma_s)d\sigma_s, \quad (2)$$

where p represents the density function of the probability distribution for the microelement strength F .

When loading reaches a certain stress level σ_s , the number of failed microelements is

$$N_d = \int_0^{\sigma_s} Np(x)dx = Np(\sigma_s), \quad (3)$$

where p represents the distribution function of the probability distribution for the microelement strength F .

From Equations (1) and (3), the following equation can be derived:

$$D = p(\sigma_s), \quad (4)$$

which, in turn, can be used to derive the statistical damage evolution equation. For any probability distribution, with

the increase of stress level σ_s , the distribution function value varies from 0 to 1. The value of D reflects the extent of internal damage of the materials: when $D = 0$, it indicates intact material without damage, and when $D = 1$, it indicates completely failed material.

The stress level σ_s can be represented by the common M-C criterion as

$$\sigma_s = \sigma_1^* - \sigma_3^* - (\sigma_1^* + \sigma_3^*) \sin \varphi = c \cos \varphi, \quad (5)$$

where σ_1^* , σ_2^* , and σ_3^* denote the effective principal stresses and $\sigma_1^* \geq \sigma_2^* \geq \sigma_3^*$ and c and φ denote the cohesion and internal friction angle of the material, respectively.

According to Hooke's law, the principal strain can be expressed by

$$\varepsilon_i = \frac{1}{E} [(1 + \mu)\sigma_i^* - \mu(\sigma_1^* + \sigma_2^* + \sigma_3^*)], \quad i = 1, 2, 3, \quad (6)$$

where σ_1^* is the effective principal stress, E is the Young modulus, and μ is the Poisson ratio.

We next introduce the strain equivalent hypothesis [27, 28] in the following equation, where σ_i denotes the apparent stress:

$$\sigma_i^* = \frac{\sigma_i}{1 - D}, \quad i = 1, 2, 3. \quad (7)$$

Substituting Equation (7) into Equation (6), we obtain

$$D = 1 - \frac{1}{E\varepsilon_i} [(1 + \mu)\sigma_i - \mu(\sigma_1 + \sigma_2 + \sigma_3)]. \quad (8)$$

Substituting Equation (8) into Equation (4), the general form of the statistical damage constitutive equation can be expressed as

$$P(\sigma_s) = 1 - \frac{1}{E\varepsilon_i} [(1 + \mu)\sigma_i - \mu(\sigma_1 + \sigma_2 + \sigma_3)]. \quad (9)$$

For uniaxial compression experiments, Equation (8) can be written as

$$\sigma_1 = E\varepsilon_1(1 - D), \quad (10)$$

which can be substituted into Equation (7) to obtain

$$\sigma_1^* = E\varepsilon_1. \quad (11)$$

According to Equations (10) and (11), the apparent stress σ_1 and axial strain can be used to calculate the damage variable D and effective stress σ_1^* .

Equation (9) can be directly utilized to achieve the general form of the constitutive model of uniaxial testing statistical damage as

$$\sigma = E\varepsilon(1 - P(\sigma_s)), \quad (12)$$

where σ and ε are the stress and strain, respectively, for rock specimens under uniaxial compression.

Considering the situation of uniaxial stress and substituting Equation (5) into Equation (12), we obtain

$$\sigma = E\varepsilon\{1 - P[(1 - \sin \varphi)E\varepsilon]\}. \quad (13)$$

Assuming that the rock strength follows a log normal distribution, its probability density function is

$$P(\sigma_s) = \Phi\left[\frac{\ln(\sigma_s/\sigma_s^0)}{\eta}\right] = \Phi[a \ln(\sigma_s) + b], \quad (14)$$

where σ_s^0 and η are distribution parameters, $a = 1/\eta$ and $b = -(\ln \sigma_s^0)/\eta$. Φ is the standard normal distribution function, defined by

$$\begin{cases} \Phi(x) = \int_{-\infty}^x \varnothing(\sigma_s) d\sigma_s, \\ \varnothing(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right). \end{cases} \quad (15)$$

The two statistical parameters in Equation (15) can be determined using the linear fitting [28], peak value [29], and inversion analysis methods [25, 30], among others. We utilized the peak value characteristics of the stress-strain curves and solved these parameters using the extreme value method.

Substituting Equation (14) into Equation (12), we obtain the constitutive equations (16) and (17) of the damage evolution and statistical damage based on the M-C criterion, respectively:

$$D = \Phi(a \ln \varepsilon + b_1), \quad (16)$$

$$\sigma = E\varepsilon[1 - \Phi(a \ln \varepsilon + b_1)], \quad (17)$$

where $b_1 = a \ln[(1 - \sin \varphi)E] + b$.

Considering the full stress-strain curve of uniaxial compression and defining ε_1 and σ_1 as the strain and stress, respectively, at the peak values of the full stress-strain curves, we get $\varepsilon = \varepsilon_1$, $\sigma = \sigma_1$, and $d\sigma_1/d\varepsilon_1 = 0$:

$$\begin{cases} \sigma_1 = E\varepsilon_1[1 - \Phi(a \ln \varepsilon_1 + b_1)], \\ \sigma_1 = aE\varepsilon_1\varnothing(-a \ln \varepsilon_1 - b_1). \end{cases} \quad (18)$$

Equation (18) can be used to solve the parameters a and b_1 in Equation (17).

3. Experimental Validation of Models

To validate the accuracy of the above constitutive model, uniaxial compression tests were conducted in the laboratory for three sand rock specimens as shown in Figure 1. The testing apparatus included the electrohydraulic, servo-controlled MTS Model 815.02 Rock Mechanics Test System (MTS Systems Corp., USA), which was loaded by displacement control with a loading rate of 0.002 mm/s.

Based on the damage equations presented in this paper, the evolution regularity chart for the three rock specimens in the loading process were prepared as shown in Figures 2(a)–2(c). The damage evolution of the rock during the loading process can be divided into four stages: (1) a nondamaged stage at the beginning of loading (OE), which responded to the compression and linear elastic stage in the stress-strain curve of the rock; (2) an accelerated growth stage (EF), which responded to the prepeak yield stage in the stress-strain curve of the rock; (3) a constant-speed-similar growth stage (FG), which responded to the postpeak stage in the stress-strain curve of the rock; and



FIGURE 1: Specimen of sand rock.

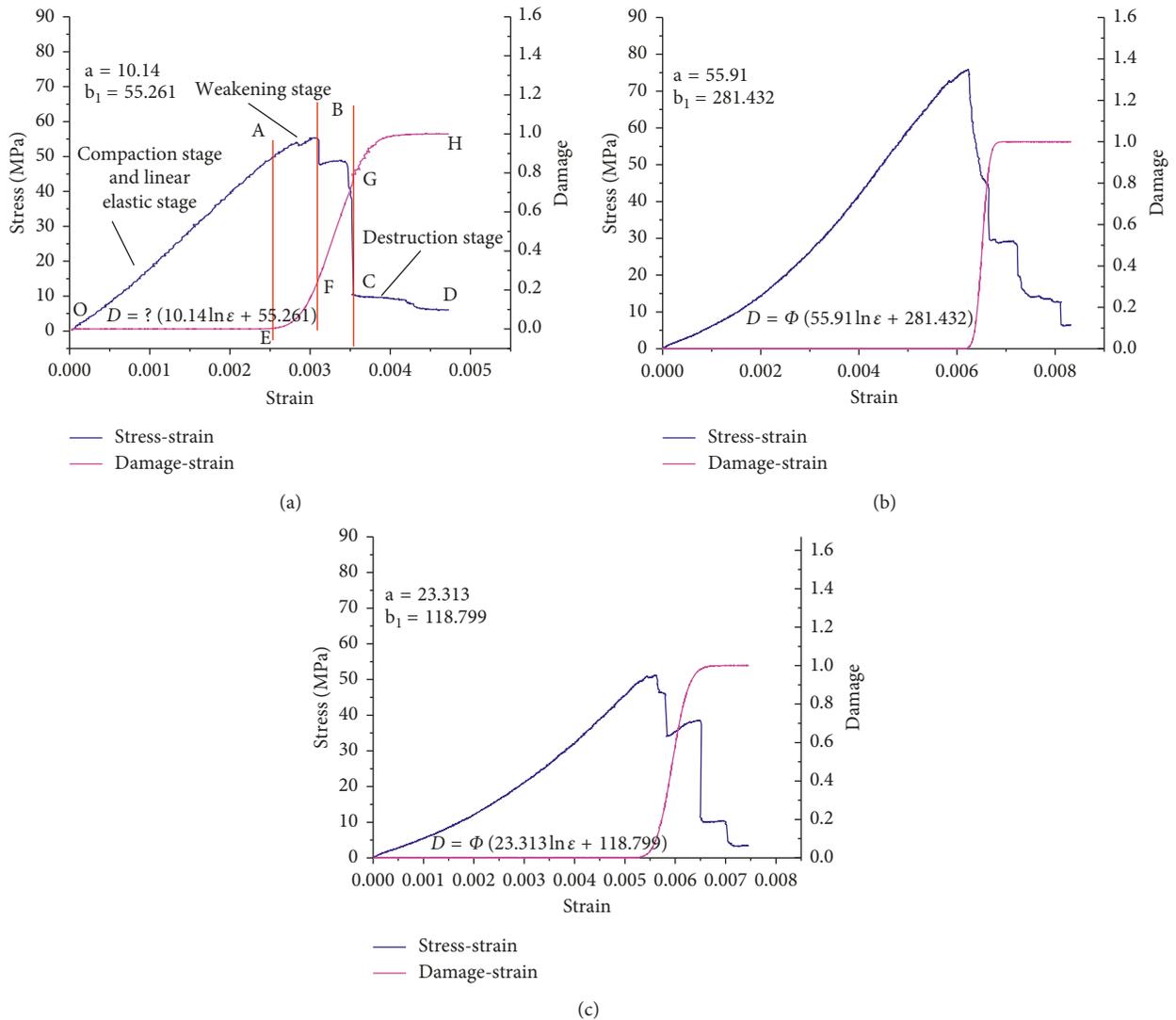


FIGURE 2: Evolution of damage variable. (a) Sample no. 1. (b) Sample no. 2. (c) Sample no. 3.

(4) a speed-reducing growth stage (GH), which responded to the residual stress stage in the stress-strain curve of the rock.

The theoretical and experimental data curves were individually prepared as shown in Figures 3(a)–3(c) based on the damage constitutive model presented in this paper, and the correlation coefficients between modeling data and lab

data are 0.9995, 0.8095, and 0.7867. The statistical constitutive equation matched the real uniaxial compression testing curves of the rock well. This latter observation indicated that the proposed constitutive equation was able to represent the elastic, yield, and softening stages of the uniaxial stress-strain curve of the rock, and that the

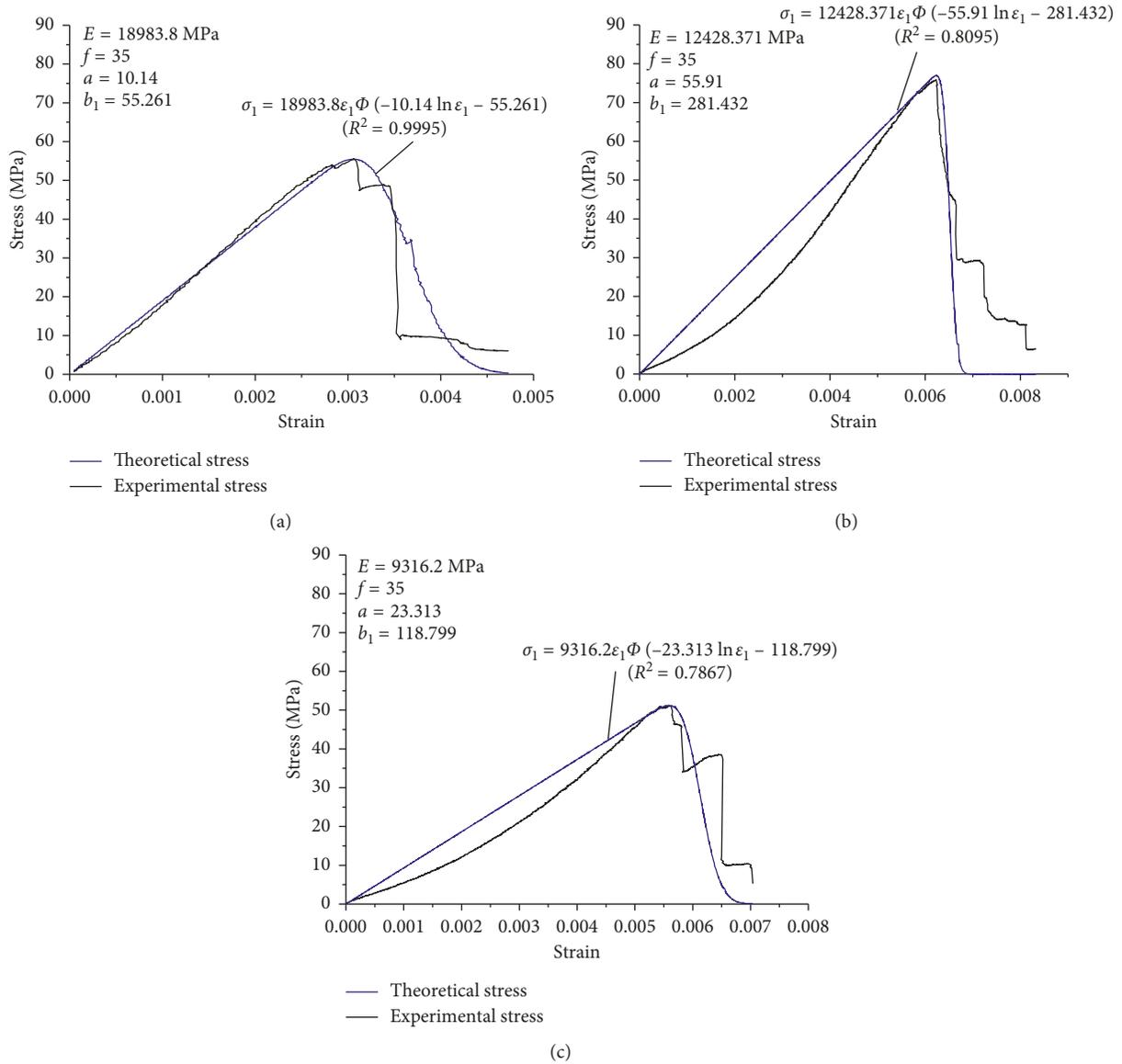


FIGURE 3: Comparison of theoretical results and experimental data. (a) Sample no. 1. (b) Sample no. 2. (c) Sample no. 3.

constitutive model was able to describe the strength and deformation characteristics of the uniaxial stress-strain curve of the rock.

Ji et al. [31] also proposed a constitutive model for two kinds of rocks, but the fitting results were not good. Ji et al. concluded that the constitutive relationship can only partially represent the loading characteristics of rock. Making use of the data presented in that paper and combining it with the constitutive relationship derived in this research, the theoretical curve and experimental data were prepared for validation and results comparison. As shown in Figures 4(a)–4(c), the fitting correlation coefficient between modeling data and lab data with the model of this paper is the highest one, so the rock constitutive equation in this paper provided a better description of the rock constitutive relationship than that in Ji et al.’s previous paper.

4. Impact of Parameters on Constitutive Equations

Using the uniaxial compression and shear testing data of sand rock as an example, the related parameters were $\varphi = 35^\circ$, $a = 10.14$, and $b = -33.34$. When the parameters a and b were constant, the internal friction angle φ value was changed to achieve the stress-strain and damage-strain curves as shown in Figures 5 and 6, respectively.

From the viewpoint of the stress-strain relationship, the internal friction angle φ reflects the uniaxial compression strength of rock, but it does not change the form of the stress-strain curve. The higher the internal friction angle, the higher the peak strength; on the other hand, the smaller the internal friction angle, the lower the peak strength. This observation is consistent with the characteristic represented by the uniaxial compression strength

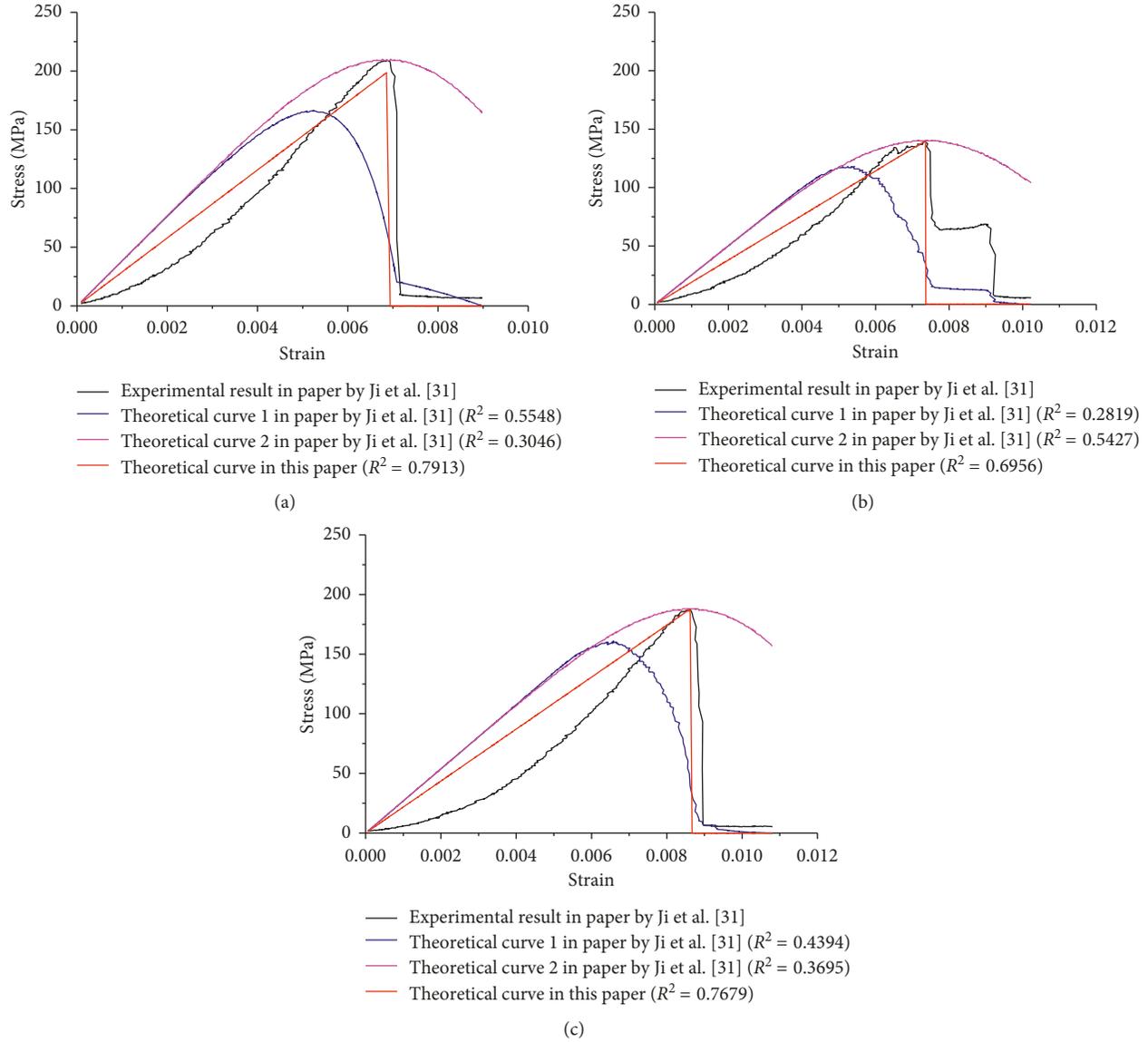


FIGURE 4: Curves of the theoretical fitting from Ji et al. [31]. (a) Sample no. 1. (b) Sample no. 2. (c) Sample no. 3.

Equation (19). Under the conditions of Coulomb's criteria [32]:

$$\sigma_c = 2c \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}, \quad (19)$$

where σ_c represents the peak value of the strength in the uniaxial stress-strain curve and c is the cohesion of the rock material.

From Figure 6, it is observed that with increasing internal friction angle φ , the deformation of rock from nondamaged status to accelerated damage status increased and the evolution process of rock to reach complete damage status was delayed. This means that the compression durability of rock increased with a higher internal friction angle. Under the condition of an identical damage value, the rock strain accordingly increased with increasing internal friction angle φ . Under the identical strain condition, the

damage value of rock decreased with the increasing internal friction angle.

5. Conclusions

In this paper, the general form of the rock statistical damage constitutive model is inferred under the uniaxial stress condition, and the model parameters are determined by using uniaxial compression testing. In addition, the reliability of the proposed constitutive model is checked through analysis of testing data and concluded the following:

- (1) On the basis of strength theory and the random distribution characteristics of rock, in this research, we established a damage evolution equation based on statistics
- (2) Based on the M-C criterion, we established a statistical damage constitutive model, and identified that

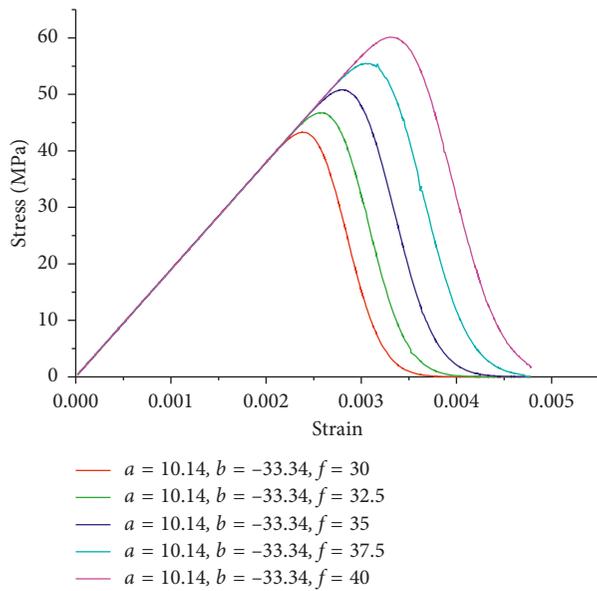


FIGURE 5: Impact of internal friction angle φ on the constitutive model.

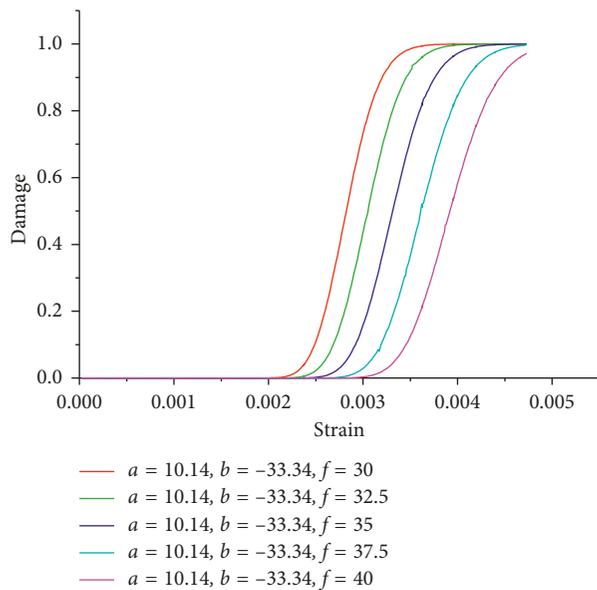


FIGURE 6: Impact of internal friction angle φ on damage evolution.

the model was able to describe the mechanical behaviors of rock under uniaxial compression testing

- (3) The variation of the internal friction angle had a certain impact on the uniaxial compression strength and damage evolution and reflected the compression durability of rock

Data Availability

All of the data supporting the conclusions of the study are available in the article and the authors are willing to share the data underlying the findings of the article.

Disclosure

The abstract of this paper had been accepted by the conference of GeoEdmonton 2018, but the full paper has not been submitted to the conference.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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