Research Article

Flexural Modelling and Finite Element Analysis of FRC Beams Reinforced with PVA and Basalt Fibres and Their Validation

Tehmina Ayub ¹, Sadaqat Ullah Khan ², and Nasir Shafiq³

¹Department of Civil Engineering, NED University of Engineering and Technology, Karachi 75270, Pakistan

²Department of Urban and Infrastructure Engineering, NED University of Engineering and Technology, Karachi 75270, Pakistan

³Civil and Environmental Engineering Department, Universiti Teknologi Petronas, 31750 Seri Iskandar, Malaysia

Correspondence should be addressed to Tehmina Ayub; tehminaa@hotmail.com

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A flexural capacity model for fibre-reinforced concrete (FRC) beams reinforced with PVA and basalt fibres is suggested for the rectangular beam sections. The proposed models are based on the concept of equivalent stress block parameters for both compressive and tensile stresses, similar to Eurocode and ACI code. The parameters are defined by allowing the conversion of the stress-strain models into equivalent rectangular stress blocks, similar to Eurocode 2. The flexural model is suggested to determine the loading capacity of 21 FRC beams containing up to 3% volume fraction of PVA and basalt fibres without reinforcing bars. In order to investigate the accuracy of the proposed flexure models, finite element analysis (FEA) of the same beams was carried out using the compressive and tensile stress-strain curves. Furthermore, 21 FRC beams subjected to three-point bending were tested. The results of the flexural models showed good agreement with the load-carrying capacity of the tested FRC beams, and the results of FEA of all beams showed a good correlation with the experimental results in terms of the maximum load, load versus midspan deflection patterns, and the maximum tensile strains.

1. Introduction

Fibre-reinforced concrete (FRC) is concrete containing fibres. FRC is a well-established material, which is commonly used in several civil engineering-related structures. Steel fibre is the most commonly studied, but due to some drawbacks of steel fibres such as cost [1] and durability [2, 3], other fibres are also considered. During the last decade, PVA and basalt fibres are studied and found to be advantageous in terms of improving structural behaviour [3–9]. The compressive and tensile stress-strain models for PVA and basalt fibre-reinforced concrete have already been explored by the authors [10, 11]; however, the application of these models for the prediction of flexure behaviour is missing in the literature.

The modelling of the compressive stress-strain behaviour of FRC containing PVA and basalt fibres is reported in [10], which revealed that the addition of both fibre types increased the compressive strength and corresponding

strain and stiffness, and there is a substantial increase in the post-peak energy absorption capacity (also called “Toughness”). The tensile stress-strain behaviour of FRC reinforced with PVA and basalt fibres is also different from steel fibres [11]. Therefore, the response of PVA and basalt fibres in concrete is different and cannot be predicted through the available model of steel fibre-reinforced concrete (SFRC) [12]. The reason of unsuitability of available models is that the post-peak stress-strain response of FRC solely depends on the fibre properties including fibre modulus of elasticity, elongation, and bond characteristics [11]. Hence, there is a need to develop the flexural model for both PVA and basalt fibre-reinforced concretes.

In this paper, the analytical flexural models are proposed which are based on the compressive and tensile stress-strain laws derived from experimental results, similar to the methodology followed by Barros and Figueiras [13]. The proposed flexural model is based on the traditional equivalent rectangular stress block concept for both compressive

TABLE 1: Material and mixture composition used in the experimental program.

Materials		Series “P”		Series “S”		Series “M”	
Ordinary Portland cement (OPC)		450 kg/m ³		405 kg/m ³		405 kg/m ³	
Silica fume		—		45 kg/m ³		—	
Metakaolin		—		—		45 kg/m ³	
Fine aggregate		670 kg/m ³					
Coarse aggregates		600 kg/m ³ (aggregate size: <10 mm), 500 kg/m ³ (aggregate size: 10 to 20 mm)					
Fibre content	0	13 kg/m ³	26 kg/m ³	13 kg/m ³	26 kg/m ³	13 kg/m ³	26 kg/m ³
		26 kg/m ³	52 kg/m ³	26 kg/m ³	52 kg/m ³	26 kg/m ³	52 kg/m ³
		39 kg/m ³	78 kg/m ³	39 kg/m ³	78 kg/m ³	39 kg/m ³	78 kg/m ³
		(PVA)	(Basalt)	(PVA)	(Basalt)	(PVA)	(Basalt)

Note. In series, “P” represents plain concrete (without silica fume or metakaolin), “S” represents silica fume concrete, and “M” represents metakaolin concrete.

and tensile stress distributions across the beam cross section by converting the predicted stress-strain distributions to equivalent stress block by employing conversion parameters for the flexural models. The modelling procedure described in this paper is applicable to FRC structural elements subjected to flexural loading. It is to be noted that this paper is an allied of the previous work published [14] in which flexural test results of 21 FRC beams containing PVA and basalt fibres (1–3% by volume) were presented. This paper specifically focuses on the flexural modelling of the similar beams based on the suggested compressive [10] and tensile [11] stress-strain behaviours and their validation through experimental and finite element analysis approach. The endeavour is to verify the results obtained from the analytical flexural model with the results obtained from 3D nonlinear finite element program “ATENA 3D” and with the published experimental data by the authors [14].

2. Behaviour and Modelling of FRC Material Properties

It was earlier stated that the proposed analytical flexural model is based on the compressive and tensile stress-strain laws derived from experimental results, similar to the methodology followed by Barros and Figueiras [13]. The compressive stress-strain behaviour of FRC containing PVA and basalt fibres is reported in [10], and the tensile stress-strain behaviour of FRC containing PVA and basalt fibres is reported in [11]. Based on the compressive and fracture responses, the analytical expressions for predicting the compressive and tensile stress-strain were proposed and used in this paper for the development of flexural modelling. For the experimental validation of the flexural model, it is mentioned in the earlier part that the experimental work and results used in this paper have already been published in [14]. However, for the reader’s interest, the summary of the flexural test performed on 21 FRC beams containing PVA and basalt fibres (1–3% by volume) is given first.

3. Concrete Material and Mix Details

FRC mixes were made up of three different mix binders. The first mix binder type was made by utilising 100% cement volume. The remaining two types were blended cement

system containing 10% volume of silica fume or metakaolin with 90% cement volume. For each of three binder mixes, 7 beams of the size 100 × 200 × 1500 mm were cast. The first beam (labelled as control beam) contained no fibres, while the remaining 6 beams were cast using FRC containing a volume fraction of 1%, 2%, and 3% PVA and basalt fibres. All 21 beams were tested to failure under three-point flexural loading. A complete detail of the mix design, investigation of the flexure test results of the beams, has been investigated by the authors and can be found in [14].

Table 1 presents the mix composition used in the experimental program. A total of three series were prepared. The average unit weight of the concrete mixes was about 2450 kg/m³. The fibre volume percentages, fibre type, and binder type were the main variables that were varied to evaluate their influence on the behaviour of the FRC composites. Binder content and water-to-binder (w/b) ratio were fixed as 450 kg/m³ and 0.4. The fracture energy and the postcracking behaviour were evaluated from the flexure tests performed on 21 beam specimens of these three series. The detail of the flexure test can be found in [14]. The compression and tension behaviours were determined using specimens of all three series. For compressive behaviour, three cylinders of size 100 × 200 mm from each 21 mixes were cast as per ASTM C 192/C 192 M standard [15]. For tension test, notched specimens of the size 100 × 70 × 600 mm were cast [11]. The detail is given in the forthcoming section.

Ordinary Portland cement (OPC) was used in all mixes. Kuraray RF 4000 polyvinyl alcohol (PVA) fibres and chopped basalt fibres were used as micro reinforcements for the concrete. The physical properties of both fibres are given in [14], whereas chemical composition of chopped basalt fibres is given in [10]. Fibres were added as 1, 2, and 3% by the volume of concrete mix.

4. Compressive Behaviour

The aim of uniaxial compression testing of FRC cylindrical specimens was to define a stress-strain law to simulate the complete compression behaviour of the FRC analysed. Therefore, 100 × 200 mm size cylindrical specimens were tested under deformation control condition. Testing detail and complete procedure of the compressive stress-strain modelling are given in [10]. The compression tests results of the FRC are given in Table 2, while the description of the suggested expressions is given in the subsequent section.

TABLE 2: Compression test results of plain concrete with and without fibres at 28 days.

Series	Cylinder IDs	Experimental results				Calculated results			
		f'_{cf} (MPa)	ϵ_{of} (mm/mm)	ϵ_{uf} (mm/mm)	E_{it} (MPa)	E_c (MPa)	E_{cf} (MPa)	$TR_f^{(e)}$	Avg. f'_{cf} (MPa)
“P”	Cylinder 1	74.53	0.002372	0.012199	43,135	41,754	31,426	0.159	73.43
	Cylinder 2	72.34	0.002204	0.011287	42,709	42,301	32,829	0.149	—
	Cylinder 1	75.61	0.002687	0.022323	43,329	40,022	28,139	0.186	73.97
	Cylinder 2	72.34	0.002646	0.021088	42,696	39,220	27,339	0.178	0.455
	Cylinder 1	75.23	0.002936	0.024516	43,243	38,562	25,618	0.167	0.455
	Cylinder 2	78.88	0.003078	0.019840	43,933	38,955	25,629	0.184	0.909
	Cylinder 1	74.56	0.002998	0.022076	43,102	38,065	24,871	0.236	0.909
	Cylinder 2	76.92	0.002826	0.022500	43,553	39,631	27,217	0.225	1.364
	Cylinder 1	74.20	0.002335	0.019264	43,059	41,909	31,778	0.186	1.364
	Cylinder 2	74.76	0.002406	0.019029	43,167	41,574	31,073	0.178	13.89
	Cylinder 1	78.02	0.002469	0.019525	43,772	42,208	31,599	0.167	27.78
	Cylinder 2	78.77	0.002544	0.016495	43,913	41,932	30,965	0.184	27.78
“S”	Cylinder 1	77.34	0.002560	0.014745	43,631	41,351	30,210	0.236	41.67
	Cylinder 2	75.35	0.002412	0.016063	43,443	41,525	30,709	0.225	41.67
	Cylinder 1	82.33	0.002319	0.014198	44,590	44,850	35,500	0.174	81.17
	Cylinder 2	80.01	0.002376	0.022226	44,168	43,592	33,675	0.229	—
	Cylinder 1	81.58	0.002600	0.019254	44,442	43,580	33,379	0.170	82.17
	Cylinder 2	82.75	0.002811	0.019839	44,653	41,491	29,438	0.199	0.455
	Cylinder 1	84.51	0.002660	0.017231	44,954	42,967	31,772	0.299	0.909
	Cylinder 2	82.27	0.002351	0.019068	44,552	44,545	34,993	0.237	0.909
	Cylinder 1	82.24	0.002693	0.020304	44,534	42,040	30,538	0.283	82.01
	Cylinder 2	81.78	0.002521	0.017923	44,452	43,060	32,441	0.297	1.364
	Cylinder 1	83.64	0.002464	0.022310	44,812	44,103	33,943	0.170	13.89
	Cylinder 2	81.65	0.002430	0.01131	44,454	43,711	33,601	0.199	13.89
“M”	Cylinder 1	82.39	0.002571	0.022026	44,575	42,908	32,046	0.299	83.51
	Cylinder 2	84.63	0.002633	0.019979	44,975	43,186	32,143	0.237	27.78
	Cylinder 1	81.83	0.002672	0.015939	44,459	42,046	30,624	0.283	81.98
	Cylinder 2	82.14	0.002727	0.018415	44,516	41,797	30,122	0.297	41.67
	Cylinder 1	85.23	0.002381	0.011832	45,109	45,308	35,798	0.149	84.85
	Cylinder 2	84.47	0.002316	0.017960	44,973	45,613	36,478	0.190	—
	Cylinder 1	84.91	0.002619	0.012112	45,039	43,378	32,422	0.166	0.455
	Cylinder 2	83.46	0.002400	0.012304	44,781	44,552	34,777	0.191	0.455
	Cylinder 1	84.42	0.002654	0.018596	44,938	42,977	31,807	0.216	85.48
	Cylinder 2	86.53	0.002678	0.017777	45,309	43,468	32,312	0.217	0.909
	Cylinder 1	81.74	0.002711	0.015828	44,444	41,774	30,152	0.253	1.364
	Cylinder 2	84.07	0.002778	0.017877	44,862	42,071	30,264	0.259	1.364
“MB”	Cylinder 1	85.71	0.002552	0.017760	45,179	44,108	33,586	0.166	84.80
	Cylinder 2	83.89	0.002437	0.014156	44,858	44,398	34,425	0.191	13.89
	Cylinder 1	86.20	0.002530	0.017112	45,251	44,420	34,070	0.216	27.78
	Cylinder 2	87.91	0.002607	0.018787	45,549	44,392	33,723	0.217	27.78
	Cylinder 1	87.30	0.002535	0.015883	45,429	44,725	34,437	0.253	41.67
	Cylinder 2	87.85	0.002760	0.012901	45,525	43,319	31,831	0.259	41.67

(a) Strain corresponding to peak stress, f'_{cf} , (b) ultimate strain, (c) initial tangent modulus of elasticity is the slope at origin of the stress-strain curve, (d) secant modulus elasticity, (e) refer to Ayub et al. [10], and (f) reinforcing index by volume of fibres. Note: Each mix ID is distinguished based on the concrete type, fibre type, and fibre volume fraction. * Mix ID PP-1: first letter “P” represents plain concrete, second letter “P” represents “PVA fibre,” and the number after hyphen represents “1%” fibre volume. * Mix ID SP-2: first letter “S” represents silica fume concrete, second letter “P” represents “PVA fibre,” and the number after hyphen represents “2%” fibre volume.

4.1. Compressive Stress-Strain Model. For plain concrete labelled as “control” and FRC, two equations are proposed for the current experimental results of the control [16] and FRC [10] cylindrical specimens. The first equation was proposed for $0 < \varepsilon \leq \varepsilon_{c,lim}$, where $\varepsilon_{c,lim}$ is the strain corresponding to limiting the stress ($f_{c,lim}$) level of $0.96 f'_c$ in the falling branch. The second equation was proposed for $f_c > f_{c,lim}$ or $f_c > 0.96 f'_c$. A general description of the proposed model is given in Figure 1.

As there is no significant influence of fibre addition on the ascending branch of the compressive stress-strain curve, therefore, the following expression was found suitable for the current experimental data presented in Table 2 (with and without fibres).

For $0 < \varepsilon_c \leq \varepsilon_{c,lim}$,

$$f_{cf} = \frac{n\beta f'_{cf} (\varepsilon_{cf}/\varepsilon_{of})}{n\beta - 1 + (\varepsilon_{cf}/\varepsilon_{of})^{n\beta}} \quad \text{for } 0 \leq \varepsilon_{cf} < \varepsilon_{cf,lim}. \quad (1)$$

In (1), “ f'_{cf} ” and “ ε_{of} ” are the maximum compressive strengths of the FRC and its corresponding strain. “ n ” and “ β ” are the material parameters, where “ n ” contributes to the toughness of the stress-strain curve, and “ β ” depends on the shape of the experimental stress-strain curve. The value of “ n ” used in the current investigation is 3, and the expression of “ β ” was same as recommended by Hsu and Hsu [17] and is given as follows:

$$\beta = \left(\frac{f'_c}{65.23} \right)^3 + 2.59 \quad (\text{for control specimens}), \quad (2)$$

$$\beta = \frac{1}{1 - (E_{cf}/E_{it})} \quad (\text{for FRC specimens}), \quad (3)$$

$$f_c = \frac{f_{c,lim}}{1 + 0.1(n-1) \left[\left\{ (\varepsilon_c/\varepsilon'_c) - 1 \right\} / \left\{ (\varepsilon_{c,lim}/\varepsilon'_c) - 1 \right\} \right]^{[1+0.1(n-1)]}} \quad (\text{no fibre specimen}). \quad (5)$$

For FRC containing PVA fibres, the suggested expression for descending branch ($\varepsilon_{cf} > \varepsilon_{cf,lim}$) is as follows:

$$f_{cf} = \frac{f_{c,lim}}{1 + \left(n - 0.1V_f^2 \right) \times \left\{ (\varepsilon_{cf}/\varepsilon_{of}) - (\varepsilon_{c,lim}/\varepsilon_{of}) \right\}^{(1+0.1V_f)}} \quad (\text{PVA fibre specimens}). \quad (6)$$

For FRC containing basalt fibres, the expression for descending branch ($\varepsilon_{cf} > \varepsilon_{cf,lim}$) is as follows:

$$f_{cf} = f_{c,lim} \times \exp \left[\left(n - 1 - 0.1V_f^2 \right) \times \left\{ \frac{\varepsilon_{cf}}{\varepsilon_{of}} - \frac{\varepsilon_{c,lim}}{\varepsilon_{of}} \right\}^{(1-0.1V_f)} \right] \quad (\text{basalt fibre specimens}). \quad (7)$$

FIGURE 1: Description of the proposed compressive stress-strain model.

where f'_c is the compressive strength of the control specimens (without fibres) in (2). In (3), “ E_{cf} ” is the secant modulus of elasticity f'_{cf}/ε_{of} . “ E_{it} ” is the initial tangent modulus, which slightly decreased by the addition of fibres. The analysis of the results of “ E_{it} ” shows that the expression suggested by Mansur et al. [18] suits well for the current data of FRC. The expression suggested by Mansur et al. [18] for the cylinder size of 100×200 mm is as follows:

$$E_{it} = (10,300 - 400V_f) \sqrt[3]{f'_{cf}} \quad (\text{in MPa}). \quad (4)$$

For $\varepsilon_c > \varepsilon_{c,lim}$, the following expressions are suggested for the control specimens (without fibres):

FIGURE 2: Equivalent rectangular compressive stress block (adopted from ACI 363R-92 [19] and ACI 318-08 [20]).

TABLE 3: Estimated coefficients k_2 , k_1 , α_1 , and β_1 .

Series	Cylinder IDs	Coefficients k_2 , k_1 , α_1 , and β_1 using compressive stress-strain relationship								Coefficients k_2 , k_1 , α_1 , and β_1 using experimental results							
		Cylinder 1				Cylinder 2				Cylinder 1				Cylinder 2			
		k_2	k_1	α_1	β_1	k_2	k_1	α_1	β_1	k_2	k_1	α_1	β_1	k_2	k_1	α_1	β_1
"P"	P-0	0.424	0.640	0.68	0.85	0.469	0.559	0.54	0.94	0.491	0.597	0.55	0.98	0.504	0.551	0.59	0.84
	PP-1	0.373	0.699	0.84	0.75	0.377	0.671	0.80	0.75	0.436	0.646	0.67	0.87	0.440	0.639	0.65	0.88
	PP-2	0.359	0.713	0.89	0.72	0.348	0.698	0.90	0.70	0.399	0.718	0.81	0.80	0.393	0.696	0.80	0.79
	PP-3	0.348	0.668	0.86	0.70	0.368	0.689	0.84	0.74	0.427	0.627	0.66	0.85	0.400	0.716	0.81	0.80
	PB-1	0.433	0.63	0.65	0.87	0.416	0.656	0.67	0.83	0.447	0.648	0.65	0.89	0.470	0.641	0.58	0.94
	PB-2	0.391	0.72	0.83	0.78	0.392	0.702	0.76	0.78	0.470	0.626	0.63	0.94	0.440	0.688	0.66	0.88
	PB-3	0.390	0.68	0.78	0.78	0.417	0.628	0.64	0.83	0.423	0.747	0.75	0.85	0.651	0.673	0.44	1.30
"S"	S-0	0.454	0.580	0.57	0.91	0.431	0.601	0.63	0.86	0.432	0.625	0.65	0.86	0.410	0.655	0.72	0.82
	SP-1	0.385	0.679	0.79	0.77	0.365	0.682	0.84	0.73	0.419	0.687	0.74	0.84	0.444	0.648	0.66	0.89
	SP-2	0.374	0.719	0.86	0.75	0.382	0.697	0.82	0.76	0.369	0.762	0.93	0.74	0.406	0.686	0.76	0.81
	SP-3	0.375	0.683	0.82	0.75	0.388	0.690	0.80	0.78	0.449	0.703	0.70	0.90	0.410	0.699	0.77	0.82
	SB-1	0.412	0.64	0.70	0.82	0.435	0.575	0.56	0.87	0.399	0.679	0.68	0.80	0.398	0.655	0.70	0.80
	SB-2	0.382	0.72	0.84	0.76	0.390	0.653	0.71	0.78	0.329	0.776	0.78	0.66	0.350	0.595	0.72	0.70
	SB-3	0.376	0.69	0.82	0.75	0.370	0.727	0.83	0.74	0.367	0.774	0.77	0.73	0.375	0.669	0.76	0.75
"M"	M-0	0.399	0.805	0.91	0.80	0.403	0.801	0.89	0.81	0.472	0.767	0.73	0.94	0.454	0.664	0.66	0.91
	MP-1	0.378	0.716	0.85	0.76	0.404	0.650	0.72	0.81	0.518	0.771	0.67	1.04	0.439	0.691	0.71	0.88
	MP-2	0.373	0.690	0.83	0.75	0.373	0.705	0.85	0.75	0.373	0.729	0.88	0.75	0.412	0.729	0.80	0.82
	MP-3	0.371	0.718	0.87	0.74	0.363	0.712	0.88	0.73	0.386	0.746	0.87	0.77	0.409	0.717	0.79	0.82
	MB-1	0.384	0.72	0.84	0.77	0.417	0.635	0.65	0.83	0.323	0.704	0.70	0.65	0.372	0.558	0.64	0.74
	MB-2	0.402	0.65	0.73	0.80	0.385	0.656	0.72	0.77	0.448	0.678	0.68	0.90	0.330	0.558	0.72	0.66
	MB-3	0.387	0.71	0.82	0.77	0.37	0.693	0.80	0.74	0.410	0.712	0.71	0.82	0.335	0.641	0.81	0.67

k_1 is the ratio of average to maximum compressive stress in beam = area under the curve $\times \epsilon_{cu} / (k_3 f'_c)$, k_2 is the ratio of depth to compressive resultant to neutral axis depth, and k_3 is the ratio of maximum stress in beam to maximum stress in corresponding axially loaded cylinder = 0.85 [19].

Note that in (5), the term " f_c " represents the compressive strength of plain concrete, whereas the term " f_{cf} " in (6) and (7) represents the compressive strength of FRC. Using the experimental results given in Table 2 and (1)–(7), the complete investigation can be found in [10, 16].

4.2. Compressive Stress Block. For FRC, the designed compression stress distribution is simplified using conversion factors " α_1 " and " β_1 " applied to the design compressive strength and its corresponding depth within the beam so that a rectangular stress block is generated from a parabolic stress distribution to a rectangular stress distribution, as recommended in ACI 363R-92 [19]. A typical equivalent rectangular block, adopted from [19], is shown in Figure 2. In the following figure, the value of ultimate strain is taken as

0.0035, instead of 0.003 recommended in ACI 363R-92 [19] and ACI 318-08 [20]. The reason for selecting the value of 0.0035 is based on the results of the compressive strain corresponding to the maximum compressive strength. Canadian standard CSA [21] and European standard Eurocode 2 [22] also suggest the value of ultimate strain as 0.0035. Since the compressive behaviour of FRC will not vary significantly from that of normal concrete or concrete without fibres, the same concept can be used for FRC in compression. The notations used in this research for the equivalent rectangular stress block parameters at any stress state are shown in Figure 2.

The coefficient k_3 which had been developed already in ACI 363R-92 [19] is equal to 0.85. This coefficient helps in defining the peak stress of in-place stress-strain relationship of concrete, as defined in the preceding section. The proposed

TABLE 4: Tension test results at 28 days.

Series	Specimen ID	Direct tension test result							
		At cracking		Postcracking		At cracking		Postcracking	
		Load (P_{cr}) (kN)	Elongation (mm)	Load (P_{pc}) (kN)	Elongation (mm)	Stress (f'_c) (MPa)	Strain (ϵ_{cr}) (mm/mm)	Stress (f_{pc}) (MPa)	Strain (ϵ_{pc}) (mm/mm)
"P"	P-0	26.30	0.02428	—	—	5.367	0.0001214	—	—
	PP-1	26.44	0.02321	10.617	0.0363	5.396	0.0001160	2.167	0.0001817
	PP-2	30.91	0.02483	17.442	0.1015	6.308	0.0001242	3.559	0.0005077
	PP-3	26.63	0.02539	21.677	0.1009	5.436	0.001269	4.424	0.0005047
	PB-1	27.27	0.02428	—	—	5.564	0.0001214	—	—
	PB-2	31.23	0.02578	—	—	6.374	0.0001289	—	—
	PB-3	27.95	0.03046	—	—	5.703	0.0001523	—	—
"S"	S-0	27.02	0.02482	—	—	5.515	0.0001241	—	—
	SP-1	29.15	0.02201	10.72	0.04157	5.949	0.0001100	2.188	0.0002078
	SP-2	30.73	0.03151	15.672	0.08832	6.271	0.0001576	3.198	0.0004416
	SP-3	31.24	0.03703	18.623	0.08505	6.375	0.0001851	3.801	0.0004253
	SB-1	29.76	0.030212	—	—	6.073	0.0001511	—	—
	SB-2	31.37	0.036938	—	—	6.402	0.0001847	—	—
	SB-3	28.80	0.024795	—	—	5.879	0.0001240	—	—
"M"	M-0	30.24	0.02505	—	—	6.172	0.0001252	—	—
	MP-1	31.35	0.02745	11.928	0.04299	6.399	0.0001373	2.434	0.0002150
	MP-2	32.08	0.03542	17.861	0.04268	6.547	0.0001771	3.645	0.0002134
	MP-3	29.84	0.03377	19.28	0.06781	6.089	0.0001688	3.935	0.0003390
	MB-1	29.16	0.02525	—	—	5.952	0.0001262	—	—
	MB-2	33.53	0.03408	—	—	6.843	0.0001704	—	—
	MB-3	34.68	0.03097	—	—	7.078	0.0001549	—	—

stress-strain relationships were selected to derive the coefficients k_1 and k_2 . Since the proposed compressive stress-strain model for the ascending branch is an indefinite integral, therefore, area under the curve and the centroid of area for all types of FRC up to ultimate compressive strain ϵ_{cu} were estimated with the help of the trapezoidal rule. The centroid of the area relative to extreme fibre strain yielded the coefficient k_2 . The coefficient k_1 was determined such that the area of compressive stress-strain relationship was set equal to an equivalent rectangular stress distribution over the entire compressive zone. The estimated areas under the curve and the centroid of areas for all FRC, using experimental results and proposed compressive stress-strain relationships, are presented in [23]. The values of coefficients k_1 and k_2 after following procedures are shown in Table 3. In the same table, the results of coefficients are compared with coefficients obtained from experimental compressive stress-strain curves to assess the precision of compressive stress-strain relationship. The proposed rectangular compressive stress block is defined by a width equal to $\alpha_1 f'_c$ and depth $\beta_1 c$, as done in ACI 318-08 [20]. The proposed rectangular compressive stress block is shown in Figure 2, while the estimation of coefficients α_1 and β_1 using coefficients k_3 , k_1 , and k_2 is as follows:

$$\begin{aligned}\alpha_1 \beta_1 &= k_1 k_3, \\ \beta_1 &= 2k_2,\end{aligned}\quad (8)$$

where the coefficient k_3 is 0.85. The estimated coefficients k_1 and k_2 are given in Table 3. It is to be noted that the values of coefficient β_1 , according to ACI 318-08 [20], should be greater than or equal to 0.65 for $f'_c > 28$ MPa.

5. Tensile Behaviour

The aim of the uniaxial tension testing of the FRC specimens was to define a stress-strain law to simulate the complete tensile behaviour. Therefore, $70 \times 70 \times 600$ mm size notched specimens were tested under deformation control condition. The testing detail and the complete procedure of the tensile stress-strain modelling are given in [11]. The tension tests results of FRC are given in Table 4, while the description of the suggested expressions is given in the subsequent section.

5.1. Tensile Stress-Strain Model. For the prediction of the tensile stress-strain ($f_t - \epsilon_t$) behaviour of FRC reinforced with PVA and basalt fibres, two models are proposed. The complete investigation of the proposed model can be found in [11]. For the elastic behaviour in tension, a new formulation is suggested with an assumption that when the composite specimen is stressed as shown in Figure 3, the PVA and basalt fibres will be firmly bonded with the matrix so that no slippage can occur at the fibre-matrix interface.

Considering the idealized stress-strain behaviour shown in Figure 3, the following model is suggested for the "elastic behaviour in tension," that is, the pre-peak branch of the tensile stress-strain curve:

$$f_t = \frac{\omega (\epsilon_t / \epsilon_{cr}) f'_c}{\omega - 1 + (\epsilon_t / \epsilon_{cr})^\omega}. \quad (9)$$

In (9), f'_c is the maximum tensile strength (at cracking), ϵ_{cr} is the cracking strain corresponding to f'_c , and ω is the

FIGURE 3: General response of concrete under direct tension [11].

material parameter. The parameter ω strongly depends on the initial modulus of elasticity (E_{it}), tensile strength of FRC (f'_t), and its corresponding cracking strain (ϵ_{cr}). The parameter ω decides the shape of the pre-peak branch of the tensile stress-strain curves. The expression for ω is expressed as

$$\omega = c f'_t \left[\frac{E_{it}}{(f'_t / \epsilon_{cr})} \right] \text{ or } \omega = c E_{it} \epsilon_{cr}. \quad (10)$$

In (10), c is a coefficient that depends on the composite/matrix type. The coefficient c decides the required shape of the initial branch of the tensile stress-strain curve. In this research, a suitable value of c for concrete was found as 1.5 for a concrete-based composite. E_{it} is the initial tangent modulus of elasticity of tensile stress-strain curves of FRC and is the same as given in Table 2 for the compressive stress-strain curves of FRC. The proposed model for the pre-peak branch is unique, as no such model was proposed to capture the curvilinear shape of the curve.

For the postcracking behaviour, the mathematical model can be found in [11, 24].

6. Development of the Flexural Capacity Model for the Rectangular FRC Beam Section

In general, a beam element subjected to flexural loads may experience three basic stages of stress states, as shown in Figure 4. The stages assume that the element will reach the first cracking strain in tension before reaching the compression inelastic strain and are expressed as follows:

- (1) Stage I: linear elastic stress-strain relationship in both compression and tension, that is, stress state before the material cracks in tension
- (2) Stage II: linear elastic stress-strain relationship in compression while tension is in inelastic distribution,

that is, the stress state after the material has reached cracking strain before compression strain (corresponding to inelastic stress) is reached

- (3) Stage III: inelastic stress distribution for both compression and tension stress-strain fields, that is, the stress state after the material has cracked in tension and crushed in compression

6.1. Elastic Stage (Stage I) and Cracking Stage Models (Stage II). The elastic stage and cracking stage models are bit similar. The difference between these two stages is that concrete tensile strain ϵ_t just reached the maximum and the compressive strain ϵ_c was within the elastic range at this Stage I (refer to Figure 4(a)), while at the cracking stage, the concrete tensile strain ϵ_t just reached to yield strain, while the compressive strain was within the elastic range (refer to Figure 4(b)).

At first, the results were estimated for the cracking stage or Stage II condition (refer to Figure 4(b)); that is, at this stage, the concrete tensile stresses will be equivalent to f'_{tf} . It is assumed that the postcracking stress σ_{pc} or f_{pc} was negligible or zero at cracking. Therefore, the flexural load at cracking, that is, F_t , was calculated using the governing stress f'_{tf} assuming that the whole cross section was effective at cracking. F_t (refer to Figure 4(b)) is given as follows:

$$F_t = \frac{1}{2} f'_t (\text{area of tension zone}), \quad (11)$$

$$F_t = \frac{1}{2} f'_t \left(b \times \frac{h}{2} \right). \quad (12)$$

In (12), b and h are the width and height of the rectangular beam section. Flexural capacity (M_{cr}) of FRC using cracking stress σ_{cc} or f'_t (refer to Figure 4(b)) is as follows:

$$M_{cr} = \frac{1}{2} f'_t \left(b \times \frac{h}{2} \right) \times \left(\frac{2}{3} h \right) \text{ or } F_t \times \left(\frac{2}{3} h \right). \quad (13)$$

Using (13), the cracking load (P_{cr}) for the beam subjected to single point load acting at the centre of the beam can be calculated as follows:

$$M_{cr} = P_{cr} \times \frac{L}{4}. \quad (14)$$

In (14), L represents the beam span.

6.2. Ultimate Stage Model (Stage III). Using the approach similar to Eurocode 2 [22] and ACI 318-08 [20], an ultimate stage model for the flexural capacity of FRC beams reinforced with PVA and basalt fibres is suggested for rectangular sections using equivalent stress blocks for both tensile and compressive stresses. The parameters are defined by allowing the conversion of the stress-strain models into equivalent rectangular stress blocks, similar to Eurocode 2 [22]. For the flexural modelling of FRC, the suggested stress-strain model is presented in Figure 4(c). In order to determine the flexural capacity at ultimate, the depth of the

FIGURE 4: Three stages of stress-strain response in flexure. (a) Stage I: elastic behaviour in compression and tension. (b) Stage II: elastic behaviour in compression and plastic behaviour in tension (cracking stage). (c) Stage III: nonlinear behaviour in compression and plastic behaviour in tension (ultimate stage).

FIGURE 5: Typical loading arrangement of beams [14].

neutral axis “ c ” was determined by considering the strain diagram shown in Figure 4(c) as follows:

$$\epsilon_{tu} = \left(\frac{h-c}{c} \right) \times \epsilon_{cu}. \quad (15)$$

In (15), ϵ_{tu} is the ultimate tensile strain corresponding to σ_{pc} . ϵ_{cu} is the ultimate compressive strain assumed as 0.0035, h is the depth of the rectangular beam section, and c is the

depth of the neutral axis from the extreme compression fibre (refer to Figure 4(c)).

After determining the neutral axis depth, the next step was to determine the tension force $F_{t(pc)}$. As failure of FRC beams occurred from the tension side, the dominating tension force was estimated as follows:

$$F_{t(pc)} = \sigma_{pc} \times b(h-c). \quad (16)$$

TABLE 5: Comparison of the predicted and experimental results in flexure (at cracking).

Specimen IDs	Experimental results			Beam cross section			Predicted results at cracking				Predicted/experimental flexural load	Proposed stress distribution at cracking
	Cracking load (kN)	Deflection (mm)	Strain at cracking	Width (b) (mm)	Height (h) (mm)	Beam span (L) (mm)	σ_{cc} (MPa)	Area of tension zone (mm ²)	F_{icc} (kN)	Lever arm (mm)	Flexural load at cracking (kN)	
P-0	10.45	0.227	0.00001216	100	200	1400	5.103	100×100	25.515	133.33	9.72	0.93
PP-1	11.68	0.262	0.00000877	100	200	1400	5.687	100×100	28.435	133.33	10.83	0.93
PP-2	12.10	0.289	0.00000075	100	200	1400	5.689	100×100	28.445	133.33	10.84	0.90
PP-3	10.88	0.282	0.00001581	100	200	1400	5.301	100×100	26.505	133.33	10.10	0.93
PB-1	10.70	0.221	0.00001900	100	200	1400	5.359	100×100	26.795	133.33	10.21	0.95
PB-2	12.44	0.272	0.00000152	100	200	1400	5.390	100×100	26.95	133.33	10.27	0.83
PB-3	10.99	0.230	0.00000023	100	200	1400	5.201	100×100	26.005	133.33	9.91	0.90
S-0	10.99	0.232	0.00001114	100	200	1400	5.488	100×100	27.44	133.33	10.45	0.95
SP-1	11.82	0.255	0.00000102	100	200	1400	6.062	100×100	30.31	133.33	11.55	0.98
SP-2	12.459	0.263	0.000001416	100	200	1400	6.064	100×100	30.32	133.33	11.55	0.93
SP-3	11.462	0.254	0.0000014	100	200	1400	5.658	100×100	28.29	133.33	10.78	0.94
SB-1	11.93	0.243	0.00000710	100	200	1400	5.746	100×100	28.73	133.33	10.94	0.92
SB-2	12.53	0.283	0.00000237	100	200	1400	5.756	100×100	28.78	133.33	10.96	0.87
SB-3	11.43	0.271	0.000002083	100	200	1400	5.559	100×100	27.795	133.33	10.59	0.93
M-0	11.833	0.241	0.000000416	100	200	1400	5.963	100×100	29.815	133.33	11.36	0.96
MP-1	12.08	0.230	0.00001316	100	200	1400	6.265	100×100	31.325	133.33	11.93	0.99
MP-2	12.804	0.250	0.0000129	100	200	1400	6.267	100×100	31.335	133.33	11.94	0.93
MP-3	11.61	0.243	0.00001007	100	200	1400	5.851	100×100	29.255	133.33	11.14	0.96
MB-1	12.31	0.260	0.00000403	100	200	1400	5.947	100×100	29.735	133.33	11.33	0.92
MB-2	14.134	0.292	0.00000533	100	200	1400	5.961	100×100	29.805	133.33	11.35	0.80
MB-3	14.557	0.300	0.000003728	100	200	1400	5.747	100×100	28.735	133.33	10.95	0.75

TABLE 6: Summary of the initial material input parameters, FEA parameters, and results.

Series	Beam IDs	Trial parameters					FE results for the monitoring points			
		Cylinder ^(a) compressive strength (f_{cu}) (MPa)	Elastic ^(b) modulus (E) (MPa)	Tensile ^(c) strength (f_t) (MPa)	Fracture ^(d) energy (G_f) (N/mm)	Prescribed ^(e) displacement (mm)	1 Load (kN)	2 Deflection (mm)	3 Maximum ^(f) strain at bottom (mm/mm)	4 Maximum strain at top (mm/mm)
"P"	P-0	73.43	43.00	3.62	0.11	0.01	10.416	0.2199	0.0001238	0.00009936
	PP-1	73.97	38.00	4.68	0.0068	0.01	11.968	0.2599	0.0001151	0.0001189
	PP-2	77.06	36.00	4.908	0.0163	0.01	12.048	0.2899	0.0001239	0.0001284
	PP-3	75.74	32.50	3.8	0.09	0.01	10.264	0.275	0.0001693	0.0001266
	PB-1	74.48	41.00	4.45	0.061	0.01	10.632	0.2199	0.00009742	0.0001001
	PB-2	78.39	40.00	4.58	0.125	0.01	12.352	0.2699	0.0001582	0.0001227
	PB-3	76.34	41.70	4.26	0.0853	0.01	10.982	0.2299	0.0001326	0.0001043
"S"	S-0	81.17	42.50	4.80	0.06	0.01	11.512	0.2299	0.0001019	0.0001045
	SP-1	82.17	39.2	4.75	0.0193	0.01	11.594	0.2499	0.0001107	0.0001141
	SP-2	83.39	38.3	4.82	0.0693	0.01	11.788	0.2599	0.0001151	0.0001188
	SP-3	82.01	42.00	3.5	0.15	0.01	11.214	0.2499	0.0001119	0.0001666
	SB-1	82.64	41.00	4.71	0.009	0.01	11.85	0.2599	0.0001573	0.0001181
	SB-2	83.51	38.00	4.60	0.115	0.01	12.168	0.2799	0.0001660	0.0001276
	SB-3	81.98	37.00	4.55	0.09	0.01	11.420	0.2699	0.0001615	0.0001232
"M"	M-0	84.85	41.80	4.95	0.04	0.01	11.820	0.2399	0.0001064	0.0001091
	MP-1	84.19	44.00	4.90	0.064	0.01	11.906	0.2299	0.0001020	0.0001043
	MP-2	85.48	43.00	5.25	0.06	0.01	12.684	0.2499	0.0001108	0.0001135
	MP-3	82.91	41.25	3.60	0.1	0.01	10.474	0.2399	0.0001352	0.0001032
	MB-1	84.80	40.00	5.15	0.04	0.01	12.268	0.2599	0.0001152	0.0001185
	MB-2	87.06	43.00	5.72	0.112	0.01	14.11	0.2899	0.0001816	0.0001312
	MB-3	87.56	40.50	5.98	0.05	0.01	14.326	0.2999	0.0001329	0.0001367

Note. Results of average cylinder compressive strength^(a) (f'_{cu}) and modulus of elasticity^(b) were same as given in Table 2. The results of tensile strength^(c) (f_t) are taken from Table 4. The results of fracture energy^(d) were calculated using the formula given in Table 2.12 in [26]. Prescribed displacement^(e) is provided on trial basis. The values of maximum strain^(f) at the bottom and at the top were estimated by the results of the lateral displacements (in mm) by the distance between the monitoring point 3 and 4, which was 60 mm.

Using tension force $F_{t(pc)}$ (refer to Figure 4(c)), the ultimate moment capacity was determined as follows:

$$M_u = F_{t(pc)} \times \left(h - \frac{a}{2} \right), \quad (17)$$

where depth of the proposed rectangular stress block " a " was determined as follows:

$$a = \beta_1 c. \quad (18)$$

In (18), the value of β_1 is estimated from the stress-strain behaviour as explained in the forthcoming section. In order to validate this model, the results of the ultimate load corresponding to the strain values of 0.0035 were compared with the numerical results, as discussed in [10].

The results of the ultimate loads were found to be close to the numerical results. Thus, proposed rectangular stress block mode concept can be used for the practical designing of the structural member made of FRC.

7. Flexure Test Results and Analysis of FRC Beam

The flexure testing of FRC reinforced with PVA and basalt fibres is shown in Figure 5, and the test results are presented

in Table 5. All 21 beams failed in flexure exhibiting I-mode of failure. The reason for mode "I" failure is the application of single point load at the middle of the beam span, as mentioned by Sorankom and Mobasher [25]. The data required from flexural testing were failure load, failure modes, and complete load-deflection response of all the beams along with strain results, which are presented in Table 5. Further details of flexural testing and results can be found in [14].

The flexure strength of FRC beams reinforced with PVA and basalt fibres (without reinforcing bars) was estimated for cracking stage (Stage II) using flexural model shown in Figure 4. The comparison of the predicted results of the flexural strength with the experimental result (at cracking) is given in Table 5. As the beams were not containing conventional longitudinal reinforcement, it could not be possible to gather experimental data at ultimate.

8. Load-Deflection Curves under Flexure

The load-deflection response of all simply supported FRC beams is discussed in detailed by the authors in [14]. In this paper, those load-deflection behaviours are validated

FIGURE 6: Finite element meshes and monitoring point definition.

FIGURE 7: Typical crack pattern at beam failure.

TABLE 7: Comparison of finite element (FE) analysis and experimental results.

Series	Beam IDs	Finite element analysis (FEA) results			Experimental results			Mathematical model results	Experimental/FEA results		
		Load (kN)	Deflection (mm)	Tensile strain (mm/mm)	Load (kN)	Deflection (mm)	Tensile strain (mm/mm)	Load (kN)	Load (kN)	Deflection (mm)	Tensile strain (mm/mm)
"P"	P-0	10.416	0.2199	0.0001238	10.45	0.227	0.0001310	9.72	0.9967	0.9687	0.9450
	PP-1	11.67	0.2599	0.0001151	11.68	0.262	0.0001243	10.83	0.9991	0.9920	0.9260
	PP-2	11.954	0.2799	0.0001239	12.10	0.289	0.0001286	10.84	0.9879	0.9685	0.9635
	PP-3	10.264	0.275	0.0001693	10.88	0.282	0.0001708	10.10	0.9434	0.9752	0.9912
	PB-1	10.632	0.2199	0.0000974	10.70	0.221	0.0001034	10.21	0.9936	0.9950	0.9422
	PB-2	12.352	0.2699	0.0001582	12.44	0.272	0.0001590	10.27	0.9929	0.9923	0.9950
	PB-3	10.982	0.2299	0.0001326	10.99	0.230	0.0001326	9.91	0.9993	0.9996	1.0000
"S"	S-0	10.85	0.2299	0.0001019	10.99	0.232	0.0001020	10.45	0.9873	0.9909	0.9990
	SP-1	11.594	0.2499	0.0001107	11.82	0.255	0.0001297	11.55	0.9809	0.9800	0.8535
	SP-2	11.788	0.2599	0.0001151	12.459	0.263	0.0001468	11.55	0.9461	0.9882	0.7841
	SP-3	11.214	0.2499	0.0001119	11.462	0.254	0.0001736	10.78	0.9784	0.9839	0.6446
	SB-1	11.60	0.2399	0.0001063	11.93	0.243	0.0001610	10.94	0.9723	0.9872	0.6602
	SB-2	12.168	0.2799	0.0001660	12.19	0.283	0.0001699	10.96	0.9982	0.9890	0.9770
	SB-3	11.420	0.2699	0.0001615	11.43	0.271	0.0001666	10.59	0.9991	0.9959	0.9694
"M"	M-0	11.820	0.2399	0.0001064	11.833	0.241	0.0001258	11.36	0.9989	0.9954	0.8458
	MP-1	11.906	0.2299	0.0001020	12.08	0.230	0.0001431	11.93	0.9856	0.9996	0.7128
	MP-2	12.684	0.2499	0.0001108	12.804	0.250	0.0001838	11.94	0.9906	0.9996	0.6028
	MP-3	10.474	0.2399	0.0001108	11.06	0.243	0.0001737	11.14	0.8882	0.9461	0.9505
	MB-1	12.268	0.2599	0.0001152	12.31	0.260	0.0001549	11.33	0.9966	0.9996	0.7437
	MB-2	14.11	0.2899	0.0001816	14.134	0.292	0.0001831	11.35	0.9983	0.9928	0.9918
	MB-3	14.326	0.2999	0.0001329	14.557	0.300	0.0001528	10.95	0.9841	0.9997	0.8698

Note. Experimental results of the beams are borrowed from [14], and mathematical model results are borrowed from Table 5.

through finite element modelling. In order to validate the compressive and tensile stress-strain curve models, finite-element software ATENA 3D [26] was utilized. The results of the compressive and tensile stress-strain curve models were used as preliminary input parameters.

Three material types were defined as follows:

- (1) Concrete material (for plain and FRC)
- (2) Steel (to model loading rod and supports)
- (3) Fibre reinforcement (for FRC only)

For plain and FRC beams, material properties were essentially required. In order to define plain and FRC material, the results of the material properties obtained from the compressive and tensile stress-strain curve models were used as input parameters for the FEA. The summary of input parameters is given in Table 6.

The beams were modelled as a three dimensional solid object characterized by a material model "SBETA" by defining the user-defined material properties and material laws proposed by the researchers earlier [10, 11].

— Beam P-0 (experimental)
— Beam P-0 (simulation)

(a)

— Beam PP-1 (experimental)
— Beam PP-1 (simulation)

(b)

— Beam PB-1 (experimental)
— Beam PB-1 (simulation)

(c)

— Beam PP-2 (experimental)
— Beam PP-2 (simulation)

(d)

— Beam PB-2 (experimental)
— Beam PB-2 (simulation)

(e)

— Beam PP-3 (experimental)
— Beam PP-3 (simulation)

(f)

FIGURE 8: Continued.

FIGURE 8: Experimental versus simulation load-deflection curve for series “P.” (a) Beam “P-0” (without fibres). (b) Beam “PP-1” (1% PVA fibres). (c) Beam “PB-1” (1% basalt fibres). (d) Beam “PP-2” (2% PVA fibres). (e) Beam “PB-2” (2% basalt fibres). (f) Beam “PP-3” (3% PVA fibres). (g) Beam “PB-3” (3% basalt fibres).

It is to be noted that the aim of this investigation was to assess the accuracy of the predictive models as illustrated earlier for the compressive and tensile stress-strain behaviour of FRC containing PVA and basalt fibres in predicting the flexural response of plain and FRC beams using finite element analysis approach, which will be assessed by comparing with the experimental results of the flexural behaviour of 21 FRC beams.

All beams were modelled in full-scale mode. The dimensions of the modelled beam were $100 \times 200 \times 1500$ mm. It was assumed that FRC beams were simply supported at the bottom on semicircular steel plates, and the displacement in the direction of the beam length was set as zero (refer to Figure 6). A finite element mesh was created as a three dimensional cubical structure comprised “brick” element type with a cube size of $25 \times 25 \times 25$ mm, as shown in Figure 7. The selection of this mesh type is based on the literature review presented in [27], which recommends this mesh type for the better quality of analysis results considering the speed and memory requirements. The generated FE mesh is shown in Figure 7.

The support restrained was modelled as typical roller and hinged; that is, the movement of modelled beam was vertically and horizontally restrained. The material properties given in Table 6 were assigned to the beams, which included cylindrical compressive strength, tensile strength, modulus of elasticity, fracture energy, etc. The results of FE analysis, that is, force, displacement, and bottom strain, in the generated model were monitored to have information about the state of structure after the application of prescribed loadings. The information collected by monitoring the predefined 4 monitoring points (refer to Figure 6) is as follows:

- (1) The first monitoring point was located at the middle of the beam at the top of the loading rod to monitor

the maximum load attained by the beam before failure.

- (2) The second monitoring point was located at the middle of the beam near its bottom surface, where largest vertical displacements can be expected. The deflections will be monitored at this location.
- (3) The third and fourth monitoring points were located at 30 mm on the right and left side of the middle of the beam near its bottom surface to monitor the lateral displacement (similar to the tested beam given in Nasir et al. 2016). The reason for selecting 30 mm was the length of strain gauges used to measure the strain at the middle of the bottom face. The actual length of the strain gauges was 60 mm. Therefore, the results of strain in the FE analysis were calculated by placing 2 monitoring points for displacements 60 mm apart, by subtracting their values and dividing by 0.06 m.

For the fracture mechanism, smeared crack approach was adopted. A typical critical stress location failure pattern in the analysed beam, representative of all FRC beams, is shown in Figure 7. Three types of results were obtained from 3D nonlinear FE analysis using ATENA 3D:

- (1) Maximum load (monitoring point 1) versus deflection (monitoring point 2)
- (2) Failure mode (or crack pattern at failure) and the post-peak response of the concrete in the presence and absence of the fibres
- (3) Lateral displacement between monitoring points 3 and 4, which were 60 mm apart. By dividing lateral displacement as given by ATENA 3D with 60 mm, strain was estimated corresponding to the maximum load versus deflection

— Beam S-0 (experimental)
— Beam S-0 (simulation)

(a)

— Beam SP-1 (experimental)
— Beam SP-1 (simulation)

(b)

— Beam SB-1 (experimental)
— Beam SB-1 (simulation)

(c)

— Beam SP-2 (experimental)
— Beam SP-2 (simulation)

(d)

— Beam SB-2 (experimental)
— Beam SB-2 (simulation)

(e)

— Beam SP-3 (experimental)
— Beam SP-3 (simulation)

(f)

FIGURE 9: Continued.

FIGURE 9: Experimental versus simulation load-deflection curve for series “S.” (a) Beam “S-0” (without fibres). (b) Beam “SP-1” (1% PVA fibres). (c) Beam “SB-1” (1% basalt fibres). (d) Beam “SP-2” (2% PVA fibres). (e) Beam “SB-2” (2% basalt fibres). (f) Beam “SP-3” (3% PVA fibres). (g) Beam “SB-3” (3% basalt fibres).

The comparison of the 3D nonlinear finite element analysis results with the experimental results is presented in the forthcoming section; however, the FE analysis results and final parameters to obtain these results are summarised in Table 6.

9. Comparison of Finite Element Analysis Results with Experimental and Mathematical Model Results

The comparison of the finite element analysis (FEA) results with the experimental and mathematical results is presented in Table 7, which shows that the similarity of FEA results in terms of load, deflection, and tensile strain with the experimental results is more than 90%. However, the results predicted with the flexural model do not match well with the finite element analysis and experimental results. This confirms that the 3D finite element modelling is significantly accurate compared to the mathematical model. However, the mathematical model is worthy as it is less time consuming due to no requirement of hectic modelling procedure.

The comparison of load-deflection curves and strain profile, obtained from the ATENA 3D program as a result of finite element analysis, with the experimental results is shown and discussed in the forthcoming section.

9.1. Comparison of the Load-Deflection Curves and Tensile Strain. Figure 8 shows the comparison of the simulated versus experimental load-deflection curves for the specimens of series “P” (plain concrete). Figures 9 and 10 show the comparison of the simulated versus experimental load-deflection curves for the specimens of series “S” (silica fume concrete) and series “M” (metakaolin concrete).

The goodness of fit of the load-deflection curves shown in Figures 8–10 of the tested beams of series “P,” series “S,” and series “M” containing PVA and basalt fibres was determined by calculating the root mean square “RMS” error and absolute fraction of variance “V” manually, which showed that load-deflection responses and strain results obtained as a result of FEA are not only closed but also represent the experimental load-deflection response and strain results. The procedure for calculating “RMS” and “V” manually was followed from Khan et al. [28].

10. Conclusion

In the present paper, a flexural capacity model based on traditional equivalent rectangular stress block concept for FRC beams reinforced with PVA and basalt fibres is assessed through the experimental results of 21 beams. The model parameters for both compressive and tensile stresses were defined by allowing the conversion of the stress-strain curve models, which were suggested by the authors in earlier studies, into equivalent rectangular stress blocks, similar to Eurocode 2 and ACI 318-08.

Furthermore, the successful applicability and suitability of the analytical compressive and tensile stress-strain curve models of PVA and basalt fibre-reinforced concrete have been investigated through FEA approach in 3D nonlinear finite element analysis program ATENA 3D. The results of FEA showed approximately similar stiffness as exhibited by the tested FRC beams reinforced with 0 to 3% of PVA and basalt fibres. The results of the parameters were also close to the one obtained as a result of experimental testing.

The comparison of the results of load-deflection curves, strain profiles, and failure modes of FRC beams obtained as

— Beam M-0 (experimental)
— Beam M-0 (simulation)

(a)

— Beam MP-1 (experimental)
— Beam MP-1 (simulation)

(b)

— Beam MB-1 (experimental)
— Beam MB-1 (simulation)

(c)

— Beam MP-2 (experimental)
— Beam MP-2 (simulation)

(d)

— Beam MB-2 (experimental)
— Beam MB-2 (simulation)

(e)

— Beam MP-3 (experimental)
— Beam MP-3 (simulation)

(f)

FIGURE 10: Continued.

FIGURE 10: Experimental versus simulation load-deflection curve for series “M.” (a) Beam “S-0” (without fibres). (b) Beam “SP-1” (1% PVA fibres). (c) Beam “SB-1” (1% basalt fibres). (d) Beam “SP-2” (2% PVA fibres). (e) Beam “SB-2” (2% basalt fibres). (f) Beam “SP-3” (3% PVA fibres). (g) Beam “SB-3” (3% basalt fibres).

a result of finite element results using “ATENA 3D program” showed close agreement with the experimental results.

Notations

E_o or E_{it} :	Initial modulus of elasticity calculated from compressive stress-strain curve
E_c :	Secant modulus of elasticity calculated from compressive stress-strain curve
E_{cf} :	Modulus of elasticity of FRC
E_{ct} :	Modulus of elasticity of the composite
E_m :	Modulus of elasticity of the concrete matrix without fibre
E_{cf} :	Modulus of elasticity of FRC
E_{of} :	Secant modulus of elasticity
E_f :	Modulus of elasticity of fibre
ε :	Strain
ε_0 :	Strains corresponding to f'_c
ε_{0f} :	Compressive strain corresponding to the maximum stress (f'_{cf}) of FRC
ε_c :	Strain corresponding to the effective peak compressive strength of concrete, f'^{ef}_c
$\varepsilon_{c,lim}$:	Compressive strain corresponding to the limiting stress ($f_{c,lim}$) in the post-peak branch of the stress-strain curves of FRC
ε_{cr} or ε'_t :	Concrete cracking strain
ε_{cu} :	Ultimate compressive strain
ε_{tu} :	Ultimate tensile strain corresponding to σ_{pc}
ε_{cu} :	Ultimate compressive strain assumed as 0.0035
$F_{t(pc)}$:	Flexural load at ultimate
F_t :	Flexural load at cracking
f'_c :	Maximum compressive stress of plain concrete
f'_{cf} :	Maximum compressive stress of FRC
f'_t :	Maximum tensile stress of plain concrete

f'_{tf} or σ_{cc} :	Maximum tensile strength/stress of FRC
f_{pc} or σ_{pc} :	Tensile stress after cracking
M_{cr} :	Flexural capacity or moment capacity of FRC at stress
n :	Material parameter in compressive stress-strain curve modelling
P_{cr} :	Load corresponding to cracking
RMS :	Root mean square errors
V :	Absolute fraction of variance
V_f :	Volume fraction of the fibres
ω :	Material parameter used for predicting tensile stress-strain curves.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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