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Research Article

Calculation of Natural Frequencies of Retaining Walls Using the Transfer Matrix Method

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The dynamic response magnitudes of retaining walls under seismic loadings, such as earthquakes, are influenced by their natural frequencies. Resonances can occur when the natural frequency of a wall is close to the loading frequency, which could result in serious damage or collapse. Although field percussion tests are usually used to study the health state of retaining walls, they are complicated and time consuming. A natural frequency equation for retaining walls with tapered wall facings is established in this paper using the transfer matrix method (TMM). The proposed method is validated against the results of numerical simulations and field tests. Results show that fundamental frequencies decrease gradually with wall height; soil elastic modulus exerts a great influence on the fundamental frequency for walls with smaller facing stiffness; fundamental frequencies are smaller for a hinged toe than a fixed toe condition, and this difference is smaller in taller walls.

1. Introduction

Retaining walls are widely used in civil engineering (e.g., roads and railways) due to their advantages, such as simple structural forms and convenient constructions. Earthquakes have occurred frequently in recent years, and the dynamic performances of retaining walls under seismic loadings have received increasing attention. Tatsuoka et al. [1] and Fang et al. [2] found that many gravity retaining walls damaged and even collapsed during the Kobe earthquake and the Chuetsu earthquake. Nakamura [3] carried out centrifuge model tests to evaluate the rationality of the M-O theory and found that it could not explain the dynamic responses accurately under earthquake loadings. To overcome limitations of the pseudostatic method [4], Choudhury and Nimbalkar [5, 6] proposed a new pseudodynamic method and used it to analyze earth pressures and displacements. Using a modified pseudodynamic method, Pain et al. [7] proposed an equation of critical seismic acceleration for gravity retaining walls.

As an inherent structural characteristic, natural frequencies have a significant influence on dynamic displacements under seismic loadings [8]. When the natural frequency of retaining walls is close to the loading frequency, larger displacement can occur, resulting in serious damage and significant financial losses. To evaluate the dynamic performances of structures directly, percussion tests have been carried out by many researchers [9, 10]. In this method, walls are subjected to a weak excitation induced by an iron ball to avoid generating large displacement, and response accelerations are collected to obtain the natural frequency. Similarly, white noise waves with small amplitude are also used in shaking table tests to obtain the natural frequencies before loadings [11]. However, these tests are costly and their accuracies are influenced by the sensors used. To calculate the natural frequencies of retaining walls, Scott [12] and Wu and Finn [13] proposed equations using the linear elastic theory. However, the influence of wall facing is not considered in these methods. Ghanbari et al. [14] modeled the wall-soil interaction as a series of springs and then calculated the fundamental frequency using the Rayleigh method. This method can only yield an approximate value for a structure with a constant cross section shape. In addition to the analytical methods above, the finite element method (FEM) is also used to analyze the dynamic performance of structures [15–17]. Although the FEM provides a possible approach to study the fundamental frequency, many material parameters need to be determined before using it. For example, strength parameters need to be determined by direct shear tests or triaxial tests, which often takes a lot of time; the parameters of interface between soils and structures (e.g., stiffness and strength) usually cannot be obtained by conventional laboratory tests. Besides, when conducting a dynamic simulation, boundary conditions and constitutive models also have a great influence on the final results.

For retaining walls in practical engineering, tapered concrete facings are often used to increase their external stabilities. Therefore, the above methods [12, 13] are limited in applications for such structures with a variable cross section shape. In addition, the Rayleigh method [14] can only predict the first-order natural frequency. However, the high-order values also can affect the structure's stability, especially when the structure is relatively tall or it is subjected to a high frequency load.

The natural frequencies of retaining walls have a significant effect on the dynamic responses of structures. Although field percussion tests and existing analytical methods have been carried out and proposed to study the natural frequencies, they are either costly, time consuming, or limited in structures with constant cross section shapes. To predict the natural frequencies of retaining walls with tapered facings which are generally used in civil engineering, a new method is proposed in this paper. Additionally, examples are used to evaluate the accuracy of the proposed method, and a parametric analysis is carried out to analyze the influences of wall facing, wall height, and soil on its natural frequencies.

2. Calculation Model and Solution

2.1. Calculation Assumption. The retaining wall model used for analysis is shown in Figure 1. As weak excitations and white noise waves with small amplitudes are used in field percussion tests [9, 10] and shaking table tests [11] to obtain the natural frequencies with nearly no displacement generated, the retaining wall is assumed as a homogeneous elastic body, which is also adopted by many researchers [12–14]. Besides, for a cantilever retaining wall, the width of the concrete facing is usually less than the height. When the ratio of the width to the height is larger than 5, the facing can be treated as a beam.

2.2. Model Solution. When the bottom width of the concrete wall facing w_b is equal to the top width w_f in Figure 2, the classical vibration equation can be used to solve this problem [18], but it cannot be applied directly to calculate the natural frequency of a wall with an irregular shape (e.g., the general trapezoidal cross section).

In this paper, the TMM is proposed to solve the problem above. Dividing the wall height h into N equal parts, the length of the ith part l_i is h/N. If N is large enough, each section can be considered as a uniform cross section beam, and its bending stiffness $(EI)_i$ and unit mass $(\rho A)_i$ can be expressed as follows:

$$\begin{aligned}(EI)_i &= \frac{1}{l_i}\int_{x_i}^{x_{i+1}} E(x)I(x)\,dx,\\(\rho A)_i &= \frac{1}{l_i}\int_{x_i}^{x_{i+1}} \rho(x)A(x)\,dx.\end{aligned} \tag{1}$$

The displacement of the ith beam in Figure 2 is $w(x,t)$, and the following equilibrium equation is obtained:

$$(EI)_i\frac{\partial^4 w_i(x,t)}{\partial x^4} + (\rho A)_i\frac{\partial^2 w_i(x,t)}{\partial t^2} + k_s w_i(x,t) = 0. \tag{2}$$

If the displacement, $w(x,t)$ can be assumed as a product of $Y_i(x)$ and $P_i(t)$, the mode shape function, $Y_i(x)$ can be expressed as

$$Y_i(x) = A_i\sin\beta_i x + B_i\cos\beta_i x + C_i\sinh\beta_i x + D_i\cosh\beta_i x. \tag{3}$$

The boundary conditions of the ith beam can be used to solve the four unknowns (A_i, B_i, C_i, and D_i) in equation (3).

The natural frequency of the ith beam is expressed by the following equation:

$$\beta_i^4 = \frac{(\rho A)_i\omega^2 - k_s}{(EI)_i}, \tag{4}$$

where β_i is an unknown constant, ω is the unknown angular frequency to be obtained, and k_s is the spring stiffness of the backfill that can be given by Scott [12] as

$$k_s = \frac{4G_s(1-\nu_s)}{5h(1-2\nu_s)}, \tag{5}$$

where G_s and ν_s are the shear modulus and Poisson ratio of the backfill, respectively. When the tensile strength of the backfill is considered, the value of k_s is set as 0.

Similarly, the $i+1$th part's mode shape function is

$$\begin{aligned}Y_{i+1}(x) &= A_{(i+1)}\sin\beta_{i+1}x + B_{(i+1)}\cos\beta_{i+1}x\\&\quad + C_{(i+1)}\sinh\beta_{i+1}x + D_{(i+1)}\cosh\beta_{i+1}x.\end{aligned} \tag{6}$$

The boundary conditions of the two adjacent beams is expressed by the following equation:

$$\begin{cases}Y_i(x_i) = Y_{i+1}(x_i),\\Y_i'(x_i) = Y_{i+1}'(x_i),\\(EI)_iY_i''(x_i) = (EI)_{i+1}Y_{i+1}''(x_i),\\(EI)_iY_i'''(x_i) = (EI)_{i+1}Y_{i+1}'''(x_i).\end{cases} \tag{7}$$

Let $S_i = \left[\,A_i\ \ B_i\ \ C_i\ \ D_i\,\right]^T$, $S_{i+1} = \left[\,A_{i+1}\ \ B_{i+1}C_{i+1}\ D_{i+1}\right]^T$, then, by using equations (3), (6), and (7), the following equation can be obtained:

$$M_iS_i = M_{i+1}S_{i+1}, \tag{8}$$

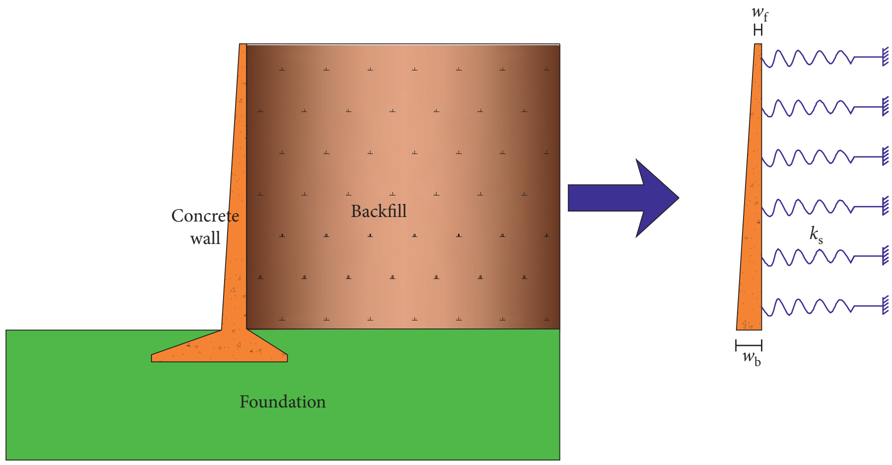


FIGURE 1: Cross section of a cantilever retaining wall and the mechanical model.

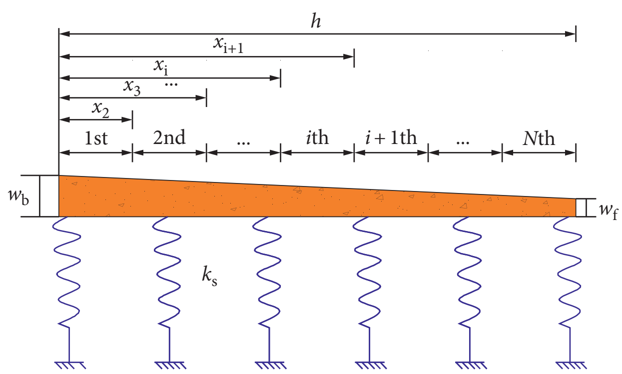


FIGURE 2: Calculation models.

where

$$M_i = \begin{bmatrix} \sin\beta_i & \cos\beta_i & \sinh\beta_i & \cosh\beta_i \\ K\cos\beta_i & -K\sin\beta_i & K\cosh\beta_i & K\sinh\beta_i \\ -SK^2\sin\beta_i & -SK^2\cos\beta_i & SK^2\sinh\beta_i & SK^2\cosh\beta_i \\ -SK^3\cos\beta_i & SK^3\sin\beta_i & SK^3\cosh\beta_i & SK^3\sinh\beta_i \end{bmatrix},$$

$$M_{i+1} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ k & 0 & k & 0 \\ 0 & -k^2 s & 0 & k^2 s \\ -k^3 s & 0 & k^3 s & 0 \end{bmatrix}, \tag{9}$$

where $K = \beta_i$, $S = (EI)_i$, $k = \beta_{i+1}$, and $s = (EI)_{i+1}$. When the matrix M_i in equation (8) is invertible, it is rewritten as

$$S_{i+1} = M_{i+1}^{-1} M_i S_i = P_i S_i. \tag{10}$$

The relationship between each two adjacent beams is established by equation (10). Using this equation, the relationship between the first and the last beam can be expressed as

$$S_N = P_{N-1} \cdots P_2 P_1 S_1. \tag{11}$$

In analytical calculations, numerical simulations, and even model tests, ideal bottom constraint conditions, either a fixed boundary or a hinged boundary [8, 14, 19], are usually considered. In this study, the same boundary condition adopted by Ghanbari et al. [14] is used. The boundary conditions are expressed as

$$\begin{cases} Y_1(0) = 0, \\ Y_1'(0) = 0, \\ (EI)_N Y_N''(h) = 0, \\ (EI)_N Y_N'''(h) = 0. \end{cases} \tag{12}$$

Simplifying the above equation, we obtain

$$\begin{cases} A_1 = C_1, \\ B_1 = D_1, \\ \psi S_N = 0, \end{cases} \tag{13}$$

where

$$\psi = \begin{bmatrix} -\sin\beta_N l_N & \cos\beta_N l_N \\ -\cos\beta_N l_N & \sin\beta_N l_N \\ \sinh\beta_N l_N & \cos\beta_N l_N \\ \cosh\beta_N l_N & \sinh\beta_N l_N \end{bmatrix}^T. \tag{14}$$

Judging from equations (4), (11), and (13), the angular frequency ω can be derived for each order. To obtain a quick solution and also easy its application, the equations above can be programmed using the software of Mathematica.

3. Model Validation

As introduced in Section 1, using a 1D shear beam model, Scott [12] proposed the following formula to predict the fundamental frequency of retaining walls, f:

$$f = \frac{v_s}{4h} \mathrm{GF}, \tag{15}$$

where v_s is the shear wave speed and GF is a geometric factor.

To evaluate the accuracy of the method proposed in this paper, the results predicted are compared with those given by Scott [12] and Ghanbari et al. [14]. Ghanbari et al. [14] carried out a numerical simulation to analyze the fundamental frequencies of retaining walls with different heights. Table 1 lists the parameters used in this calculation.

Table 2 shows the results obtained using the proposed method and other methods. As shown in Table 2, results show that the natural frequencies decrease with wall height. Besides, the proposed method in this paper shows a better agreement with the finite element method [14] as compared to Scott's method [12].

A field vibration test was conducted by Klymenkov et al. [20] to examine the health state of retaining walls. Besides, they also used the software of LIRA 9.6 to further study their dynamic characteristics. In this paper, the following data were used to calculate the fundamental frequency: $h = 5$ m, $E = 25$ GPa, $\rho = 2300$ kg/m^3, $w_b = 0.4$ m, $E_s = 19$ MPa, and $v_s = 0.15$. The fundamental frequencies reported by Klymenkov et al. [20] and those predicted from the proposed method and Scott's analytical method [12] are shown in Table 3. Results in Table 3 indicate that although the predicted fundamental frequency from the proposed method in this paper is slightly larger than that measured from the field test, it is closer to the measured value as compared to the numerical simulation result and that predicted from Scott's analytical method [12].

Xu [21] carried out a full-scale model test to investigate the damage intensity of retaining walls. Comparison was conducted for the wall with $h = 2.2$ m, $E = 21.1$ GPa, $\rho = 2500$ kg/m^3, $w_b = 0.2$ m, $E_s = 15.4$ MPa, and $v_s = 0.3$. As shown in Table 4, the fundamental frequency reported by Xu [21] can be well predicted by the proposed method.

Table 1: Calculation parameters.

Parameter	Value
h (m)	6/8/10
E (GPa)	26
w_b (m)	1.0
w_f (m)	0.5
ρ (kg/m^3)	2320
k_s (MN/m^2)	1.55/1.16/0.93

Table 2: Calculated results.

H (m)	Proposed method	Scott [12]	FEM [14]
6	14.6	10.6	16.0
8	9.0	6.5	9.6
10	6.5	3.2	6.4

Table 3: Predicted results using different methods.

Method	Fundamental frequency (Hz)
Field test [20]	8.00
Numerical simulation [20]	9.46
Scott [12]	10.16
Proposed method	8.83

Table 4: Comparison of the fundamental frequency obtained from a full-scale test and the proposed method.

Method	Fundamental frequency (Hz)
Xu [21]	22.41
Proposed method	21.37

4. Parametric Analysis

To study the influence of different parameters on the natural frequencies of retaining walls under the stability precondition, the following basic parameters are defined: $h = 6.0$ m, $E = 25$ GPa, $w_b = w_f = 0.5$ m, $\rho = 2500$ kg/m^3, $v_s = 0.3$, $G_s = 20$ MPa, and $\gamma_s = 17$ kN/m^3. In this section, the tensile strength of the backfill is not considered.

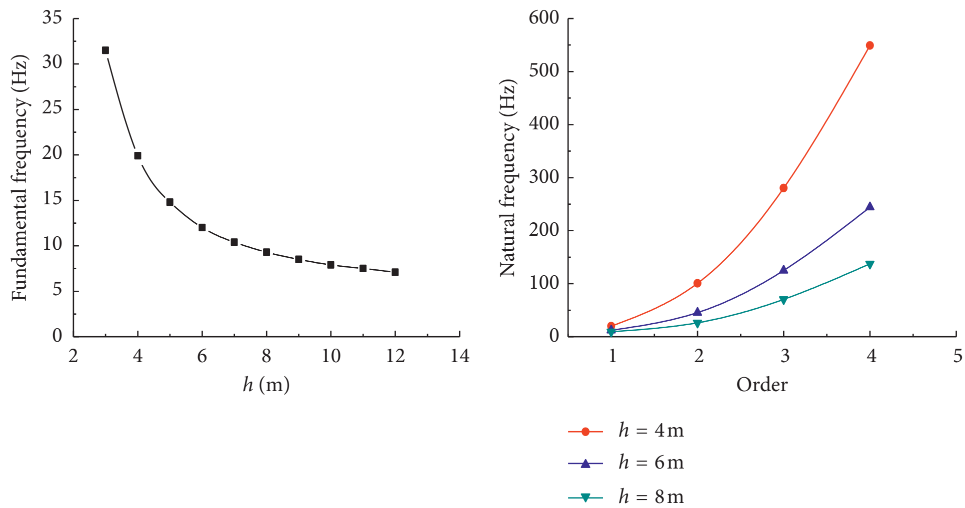
4.1. Wall Height. Figure 3 shows the relationship between the natural frequency predicted and wall height. As shown in Figure 3(a), the predicted values decrease nonlinearly with wall height and the influences of wall height on fundamental frequencies also weaken gradually, which is consistent with the previous research studies [12–14]. The minimum fundamental frequency value is 7.1 Hz for a 12 m high wall in Figure 3(a). Since the predominant frequency of earthquakes varies within the range between 2 and 3 Hz [22], resonances will not occur for practical retaining walls which are usually less than 10 m under earthquake loadings based on the results obtained.

Figure 3(b) shows the relationship between the first four frequencies and wall height. Results in Figure 3(b) show that the wall height examined in this paper has a greater influence on higher-order natural frequencies than on the first-order value.

Since retaining walls are usually considered as short-period structures [22], the following parametric analysis was carried out only for the minimum natural frequency (i.e., fundamental frequency).

4.2. Wall Width. Although a larger wall width can result in a larger mass, it also increases the flexural rigidity of beams, so the relationships between wall width and fundamental frequencies are nonlinear. Results in Figure 4 indicate that when the wall facing shape is rectangular, the fundamental frequency decreases firstly and then increases with wall facing width. The minimum fundamental frequency is obtained when the ratio of w to h is 0.1. Besides, according to the vibration theory of beams on elastic foundation, as the fundamental frequency is more affected by the flexural rigidity for a shorter wall [18], the increasing trend is more significant for the 4 m high wall in Figure 4.

The cross section facing shapes of retaining walls are usually trapezoidal. To study the effect of facing shape on



(a) (b)

Figure 3: (a) Effects of wall height on fundamental frequencies; (b) effects of wall height on first four natural frequencies.

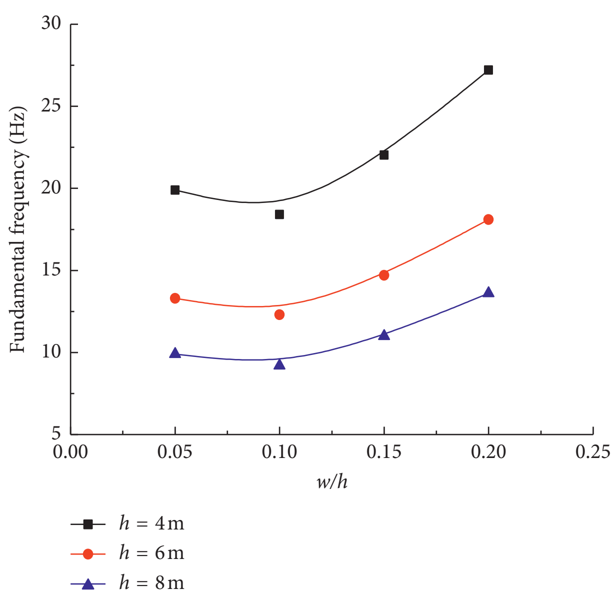


Figure 4: Effects of wall width on fundamental frequencies for walls with rectangular facings.

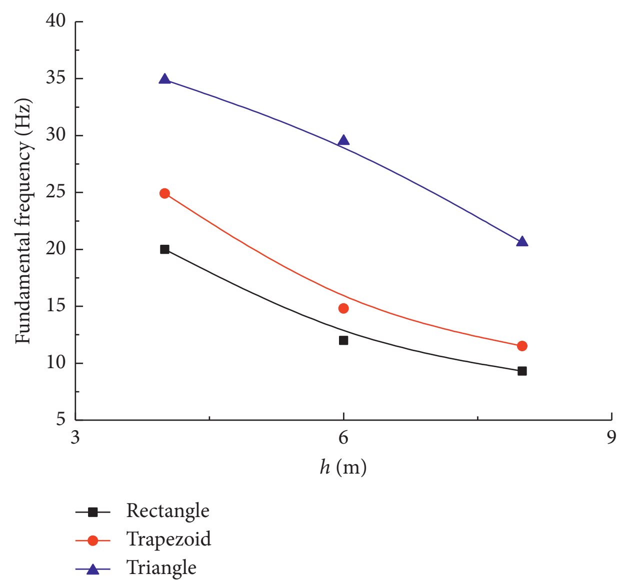


Figure 5: Effects of cross section shape on fundamental frequencies.

the fundamental frequencies of retaining walls, three shapes (i.e., triangular, trapezoidal, and rectangular) are examined in the case of a constant facing cross-sectional area. Figure 5 shows the variation of fundamental frequencies against facing shapes for different values of h. Although the facing cross-sectional area is a constant value, the fundamental frequencies differ with wall shapes, and they increase with the wall facing slope. The reason can be attributed to that the fundamental frequency in equation (11) is determined by the maximum width. Additionally, as shown in Figure 4, the fundamental frequency for the wall with a lager width is usually larger than that with a smaller width. Therefore, the largest fundamental frequency occurs for retaining walls with triangular facing shapes.

4.3. Wall Modulus and Soil Modulus. The relationships between the fundamental frequencies and wall facing modulus are shown in Figure 6. The predicted fundamental frequency from the proposed method increases gradually with wall facing modulus. Similarly, the effect of the wall facing modulus on the fundamental frequency is more significant when walls are shorter. The above analysis suggests that walls with a full-height rigid facing in the practical project can avoid resonances more effectively under dynamic loadings as compared to those with a blocked facing or a smaller facing stiffness.

Figure 7 shows the relationship between the fundamental frequency and the soil modulus for walls with different heights. As the fundamental frequency of beams on elastic foundation is affected by the beam and the foundation

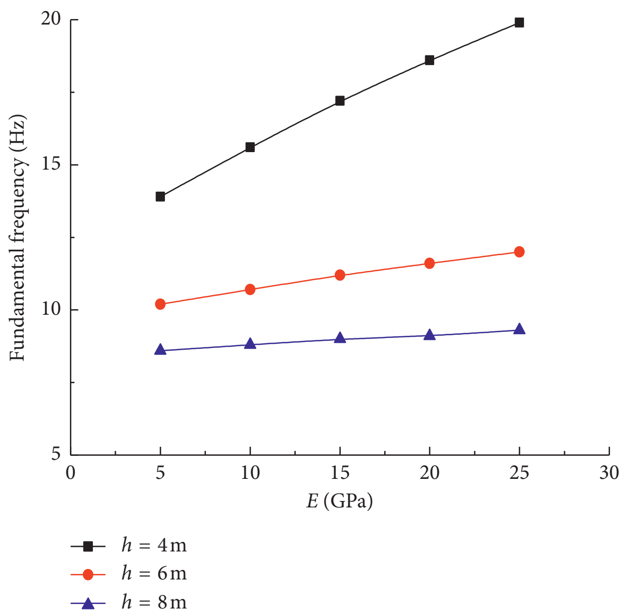


FIGURE 6: Effects of wall modulus on fundamental frequencies.

stiffness [18], the fundamental frequency gradually increases with soil modulus, especially for shorter walls. The increasing trend is more obvious in Figure 7(b) than that in Figure 7(a). Therefore, we can conclude that the influence of soil modulus on the fundamental frequency is more significant for the wall with a wrapped facing or a blocked facing.

4.4. Boundary Condition of the Wall. Results in Figure 8 indicate that the fundamental frequencies for walls with a fixed bottom condition are larger than those with a hinged bottom condition, which means that the fundamental frequencies can be increased by compacting the backfills close to wall facings in practical engineering. Besides, the difference between the two curves in Figure 8 is negligible in taller walls. The reason can be attributed to that the magnitude of the fundamental frequency of a beam on elastic foundation is more influenced by the wall height [18]. Therefore, the influence of the toe constraint on the fundamental frequency decreases gradually with wall height. It should be noted that although ideal hinged or fixed boundary conditions are often used in analytical methods [14] and numerical simulations [8], the actual fundamental frequency will more likely be somewhere in between.

5. Conclusions

The dynamic response of a retaining wall is influenced by the natural frequency of the structure. Although the natural frequency can be obtained by field tests, these percussion tests are complicated and time consuming. Besides, the existing analytical methods are usually limited to structures with constant cross sections. However, the shape of the retaining wall facing is usually trapezoidal in practical engineering. To improve the accuracy and solve the natural frequency of retaining walls with tapered facings, the transfer matrix method (TMM) is employed. The following conclusions are drawn:

(1) The results predicted from the proposed method are closer to the FEM results as compared to those obtained using the existing analytical method. Besides, the accuracy of the proposed method is also validated against the results of two full-scale model tests.

(2) The fundamental frequencies decrease with wall height. The influence of wall height on fundamental frequencies is more significant on taller walls as compared to the shorter walls.

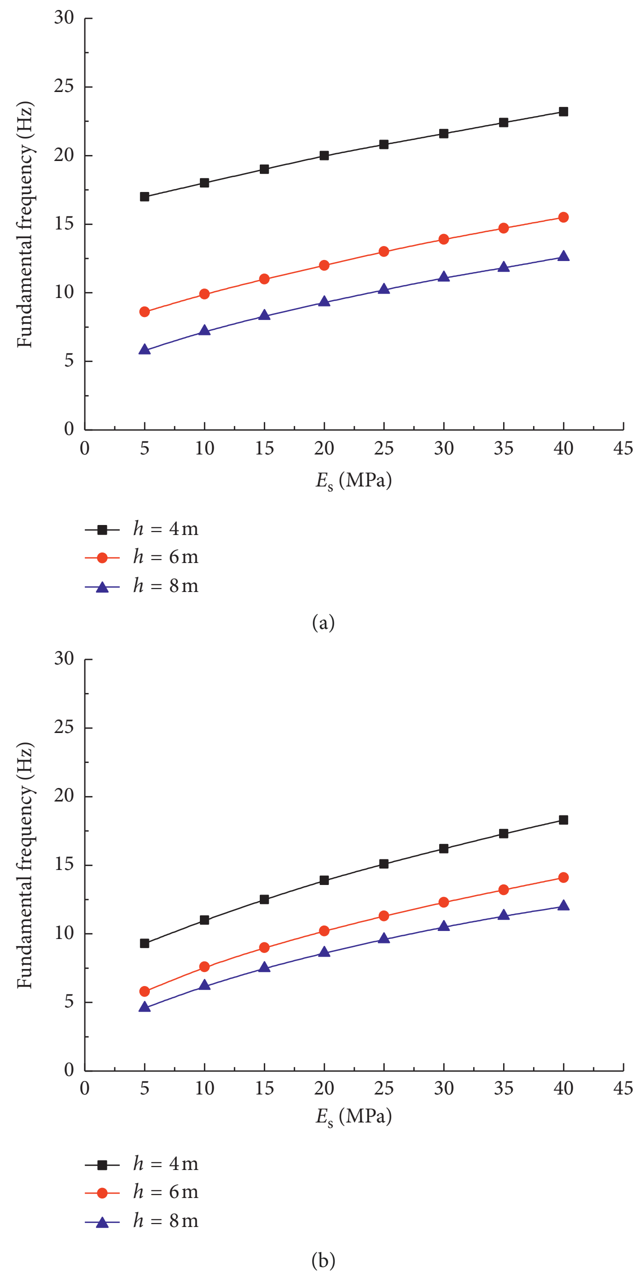


FIGURE 7: Effects of soil modulus on fundamental frequencies: (a) $E = 25$ GPa; (b) $E = 5$ GPa.

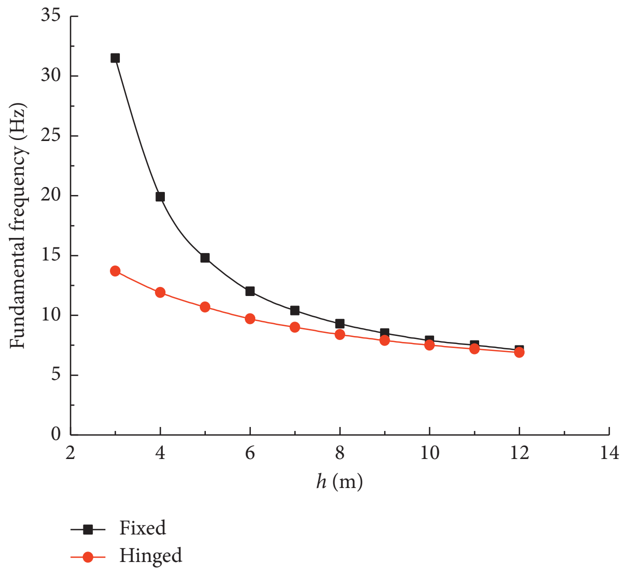


Figure 8: Effects of boundary condition of wall facing on fundamental frequencies.

(3) The relationship between fundamental frequencies and wall width is nonlinear because wall mass and flexural rigidity are both influenced by the wall width. In addition, the minimum fundamental frequency is obtained when the value of w/h is 0.1 for walls with rectangular facing shapes. The maximum natural frequency is obtained for walls having a triangular facing shape compared with other shapes examined in this paper.

(4) The fundamental frequencies increase with facing modulus and soil modulus, and the increasing trends are more obvious for shorter walls.

(5) The fundamental frequency is smaller for a hinged boundary than a fixed boundary condition, and this difference is negligible with wall height.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] F. Tatsuoka, M. Tateyama, J. Koseki, and T. Yonezawa, "Geosynthetic-reinforced soil structures for railways in Japan," *Transportation Infrastructure Geotechnology*, vol. 1, no. 1, pp. 3–53, 2014.

[2] Y. S. Fang, Y. C. Yang, and T. J. Chen, "Retaining walls damaged in the Chi-Chi earthquake," *Canadian Geotechnical Journal*, vol. 40, no. 6, pp. 1142–1153, 2003.

[3] S. Nakamura, "Reexamination of Mononobe-Okabe theory of gravity retaining walls using centrifuge model tests," *Soils and Foundations*, vol. 46, no. 2, pp. 135–146, 2006.

[4] R. S. Steedman and X. Zeng, "The influence of phase on the calculation of pseudo-static earth pressure on a retaining wall," *Géotechnique*, vol. 40, no. 1, pp. 103–112, 1990.

[5] D. Choudhury and S. Nimbalkar, "Seismic passive resistance by pseudo-dynamic method," *Géotechnique*, vol. 55, no. 9, pp. 699–702, 2005.

[6] S. Nimbalkar and D. Choudhury, "Sliding stability and seismic design of retaining wall by pseudo-dynamic method for passive case," *Soil Dynamics and Earthquake Engineering*, vol. 27, no. 6, pp. 497–505, 2007.

[7] A. Pain, D. Choudhury, and S. K. Bhattacharyya, "Seismic stability of retaining wall-soil sliding interaction using modified pseudo-dynamic method," *Géotechnique Letters*, vol. 5, no. 1, pp. 56–61, 2015.

[8] K. Hatami and R. J. Bathurst, "Effect of structural design on fundamental frequency of reinforced-soil retaining walls," *Soil Dynamics and Earthquake Engineering*, vol. 19, no. 3, pp. 137–157, 2000.

[9] M. Shinoda, S. Nakajima, K. Abe, T. Ehara, and Y. Kubota, "Stability inspection method for existing retaining walls," *Quarterly Report of RTRI*, vol. 54, no. 3, pp. 159–165, 2013.

[10] M. Shinoda, T. Mai, T. Ehara et al., "Vibration-based inspection of railway bridge substructures with small vibration exciter and fibre optic sensor," *Journal of Japan Society of Civil Engineers, Ser. A1 (Structural Engineering ↑ Earthquake Engineering (SE/EE))*, vol. 69, no. 1, pp. 40–56, 2013.

[11] C.-C. Huang, J.-C. Horng, W.-J. Chang, J.-S. Chiou, and C.-H. Chen, "Dynamic behavior of reinforced walls—horizontal displacement response," *Geotextiles and Geomembranes*, vol. 29, no. 3, pp. 257–267, 2011.

[12] R. F. Scott, "Earthquake-induced pressures on retaining walls," in *Proceedings of the 5th World Conference on Earthquake Engineering*, pp. 1611–2160, Rome, Italy, June 1973.

[13] G. Wu and W. D. L. Finn, "Seismic pressures against rigid walls," in *Proceedings of the ASCE Specialty Conference on Analysis and Design of Retaining Structures against Earthquakes*, pp. 1–18, Washington, DC, USA, November 1996.

[14] A. Ghanbari, E. Hoomaan, and M. Mojallal, "An analytical method for calculating the natural frequency of retaining walls," *International Journal of Civil Engineering*, vol. 11, no. 1, pp. 1–9, 2013.

[15] S. Dey, T. Mukhopadhyay, H. H. Khodaparast, and S. Adhikari, "Stochastic natural frequency of composite conical shells," *Acta Mechanica*, vol. 226, no. 8, pp. 2537–2553, 2015.

[16] B. K. Maheshwari, K. Z. Truman, P. L. Gould, and M. H. El Naggar, "Three-dimensional nonlinear seismic analysis of single piles using finite element model: effects of plasticity of soil," *International Journal of Geomechanics*, vol. 5, no. 1, pp. 35–44, 2005.

[17] T. Real, C. Zamorano, F. Ribes, and J. I. Real, “Train-induced vibration prediction in tunnels using 2D and 3D FEM models in time domain,” *Tunnelling and Underground Space Technology*, vol. 49, pp. 376–383, 2015.

[18] Z.-H. Ni, *Vibration Mechanics*, Xi’an Jiaotong University Press, Xi’an, China, 1989.

[19] M. M. El-Emam and R. J. Bathurst, “Experimental design, instrumentation and interpretation of reinforced soil wall response using a shaking table,” *International Journal of Physical Modelling in Geotechnics*, vol. 4, no. 4, pp. 13–32, 2004.

[20] O. A. Klymenkov, A. H. Trofymchuk, K. A. Khavkin, and I. A. Berchun, “Experimental diagnostics and mathematical modelling of stress-strain state of a railway retaining wall,” *Bulletin of the Belarusian-Russian University*, vol. 1, no. 50, pp. 140–148, 2016.

[21] Q. Xu, *Damage Identification and Alarming for Pile Plate Retaining Wall*, Chongqing University, Chongqing, China, 2015.

[22] M. Sabermahani, A. Ghalandarzadeh, and A. Fakher, “Experimental study on seismic deformation modes of reinforced-soil walls,” *Geotextiles and Geomembranes*, vol. 27, no. 2, pp. 121–136, 2009.

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