

Research Article

Comparative Study on the Test Method for Tensile Elastic Modulus of Rock Materials

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Rock material has different mechanical behaviors under compressive and tensile loading. Correspondingly, there are two types of elastic modulus: compressive elastic modulus E_c and tensile elastic modulus E_t , respectively. To distinguish which indirect test methodology, including three-points bending test and Brazilian disc test, is more suitable to measure the tensile elastic modulus E_t of rock materials, a series of uniaxial compressive test (UCT), direct tensile test (DTT), three-points bending test, and Brazilian disc test are performed for three typical types of rock: marble, granite, and sandstone. Comparative investigation on the reliability of measurement results of tensile elastic modulus E_t is systematically conducted. Finally, it is found that Brazilian disc test could be a suitable method to measure tensile elastic modulus of rock materials, due to the excellent agreement with that measured by DTT and the simplicity of sample preparation, as well as test operation.

1. Introduction

Elastic modulus of rock materials is an inherent parameter, describing the deformation characteristics of rock material under loading. It plays an important role regardless of quantitative numerical modelling, or in the qualitative evaluation system of engineering rock mass. It has been well known that rock material has different mechanical behaviors under compressive and tensile loading [1, 2]. Consequently, elastic modulus of rock material is classified into compressive elastic modulus and tensile elastic modulus. Generally, rock material has much more excellent performance under compression than that under tension. Therefore, it is a design goal to let the vast majority of rock material to be in the compression state in rock engineering. However, there still is tensile stress inevitably appearing at some special parts in the engineering practice, for example, the back edge area of a large-scale slope as illustrated in Figure 1(a) and the surrounding rock of a large-scale underground cavern in the zone with high geostress [3, 4] as illustrated in Figure 1(b).

In previous investigation, little attention was specially paid on the tensile deformation and induced tensile failures in rock materials. The behavior of rock materials was treated as the same under compression and tension states. As a result, the deformation of rock in tensile zones is significantly underestimated. It could bring great risk to the safe service of engineering infrastructures. Until recent years, the mechanism of tensile deformation and induced tensile failures increasingly attract more and more attention from scientists and engineers in the field of rock engineering [5, 6]. As an important inherent parameter estimating the tensile deformation, it is meaningful to develop test methods to reliably measure the tensile elastic modulus of rock materials. In the estimation of deformation in rock engineering, compressive elastic modulus and tensile elastic modulus of rock should both be used.

For the measurement of tensile elastic modulus E_t of rock materials, there generally are three kinds of test methods: (1) direct tensile test (DTT), (2) three-point bending test, and (3) Brazilian disc test. The DTT method is a kind of direct method, while three-point bending test and

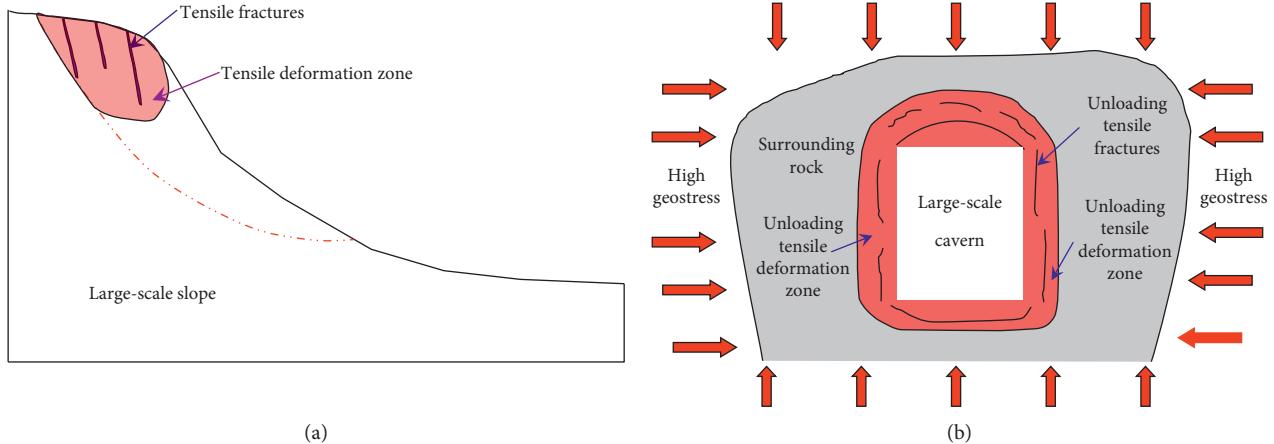


FIGURE 1: Typical rock engineering where tensile stress appears in special zones. (a) Large-scale slope. (b) Large-scale underground cavern with high geostress.

Brazilian disc test are two indirect methods. Undoubtedly, DTT method is the best method to measure E_t . However, its success rate is very low due to the complexity of sample preparation and testing operation. Generally, indirect methods are popular for measuring the tensile elastic parameters of rock materials due to the fact that relatively high success rate could be guaranteed. Three-point bending test was initially used to only measure the tensile strength of rock material. It was further improved to estimate the tensile elastic modulus by sticking strain gauges on the sample of rock beam [7]. However, three-point bending test also has the shortage that the preparation of the sample of rock beam is relatively complex. Brazilian disc test would be the most popular test method for the measurement of the tensile strength of rock materials [8, 9]. There were also several test methods proposed to measure the elastic modulus of rock by Brazilian disc test, for example, Hondros [10], Yu and Wang et al. [11] and Wang et al. [12]. However, the elastic modulus of compression and tension is not distinguished in the above test methods. In 2009, Ye et al. [13] proposed a theory perfect test method to estimate the tensile elastic modulus of rock materials adopting Brazilian disc test. This test method has attracted much attention not only from the field of rock engineering [14, 15], but also from the field of material science, such as composite materials [16–18] and ceramics [19]. After that, Ye et al. [20] further improved the method to measure the tensile and compressive elastic modulus of rock materials synchronously by adopting the digital image correction (DIC) technique. The usage of DIC technique to observe the deformation of the disc in Brazilian disc test becomes more and more popular thereafter [21–24]. Until recently, Patel and Martin [25] further demonstrates the feasibility of DIC technique for the measurement of tensile elastic modulus of rock materials adopting Brazilian disc test. However, due to the fact that the measurement accuracy of strain of DIC technique is difficult to reach the order of $O(10^{-6})$, the method proposed by Ye et al. [20] is not suitable for hard brittle rock.

Although there have been two kinds of indirect test methods for the tensile elastic modulus of rock materials, to

the authors' knowledge so far, there still is no systematic comparative investigation for the three-point bending test and Brazilian disc test on the tensile elastic modulus measurement. Namely, there is no investigation so far to ask the following questions: (1) which test method gives out the measurement results more close to that measured by DTT and (2) which test method is more worth to be suggested to measure the tensile elastic modulus of rock materials, in terms of simplicity of testing operation and reliability of measured results.

In this study, taking three typical types of rock (marble, granite, and sandstone) as the representatives, a series of uniaxial compressive test (UCT), DTT, three-point bending test, and Brazilian disc test are performed. Comparative investigation on the reliability of measurement results is systematically conducted to ask the above mentioned two questions. Finally, it is found that Brazilian disc test could be a suitable method to measure the tensile elastic modulus of rock materials, due to the excellent agreement with that measured by DTT and the simplicity of sample preparation, as well as test operation.

2. Experimental Methods

2.1. Uniaxial Compressive Test (UCT). Uniaxial compressive test (UCT) is a kind of simple and conventional test in rock mechanics. Traditionally, only one set of stain gauge is glued on the lateral side of rock sample to record the axial and radial deformation. However, four sets of strain rosettes are glued symmetrically on the lateral side of samples in this study, as illustrated in Figure 2, to record more deformation information due to the possible inhomogeneity. As a result, four sets of curves of stress-axial strain, as well as stress-radial strain, can be obtained for each sample. For each set of curve of stress-strain, the compressive elastic modulus E_c of the rock sample is determined as the slope of stress-axial strain curve; Poisson's ratio is the ratio of radial strain to axial strain where the stress is half of its peak value. Finally, four compressive elastic modulus E_c and four Poisson's ratio ν can be measured in only one test procedure for each rock

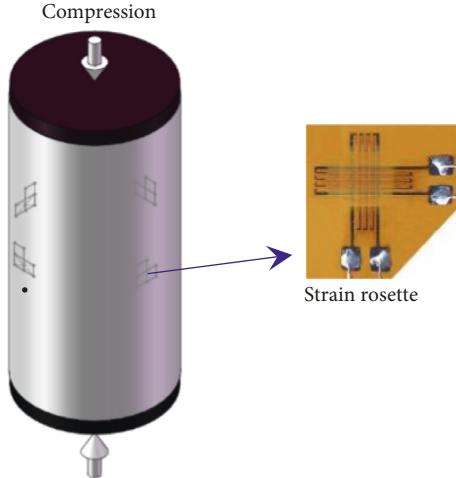


FIGURE 2: Sketch of the sample for uniaxial compression test. Four strain rosettes are glued on the lateral side of samples to record the axial and radical deformation.

sample. In this study, UCT are conducted by the way of displacement controlling. The loading rate applied by test apparatus is set as 0.15 mm/min.

2.2. Direct Tensile Test (DTT). Direct tensile test (DTT) is an unconventional test in rock mechanics due to the fact that it is apparently difficult to apply tensile load on rock samples. In the previous testing, the rock samples were generally made as the shape of dog bone with loading hole at ends. This sample preparation procedure was very cumbersome. Consequently, direct tensile test generally was not conducted. Alternatively, the Brazilian disc test was widely used to measure the tensile strength of rock materials proposed by ISRM [9] and ASTM [26]. Recently, in the past ten years, a kind of cementing material called epoxy adhesive tube kit 2216B/A with high bonding strength for rock is used in the direct tensile test. As a result, rock sample nowadays can be cylindrical, rather than dog bone shape. The cementing glue sticks the rock sample and two tensile loading heads together, as illustrated in Figure 3. Before starting the direct tensile test, the cementing glue sticking rock sample and loading heads must be placed for 24 hours, making sure the sticking strength between rock sample and loading heads is higher than that of rock sample. Furthermore, it must be guaranteed that the axes of rock sample and tensile loading heads coincide, avoiding the failure resulting from the existence of eccentricity.

Like that in the uniaxial compressive test, four sets of strain rosettes are glued symmetrically on the lateral side of the samples in the direct tensile test, to record four sets of axial tensile strain and radial strain. Also, the tensile elastic modulus E_t of rock sample is directly determined as the slope of tensile stress-axial strain curve; Poisson's ratio in tensile status is the ratio of radial strain to axial strain where the tensile stress is half of its peak value. In this study, the loading rate in the direct tensile test is set as 0.15 mm/min.

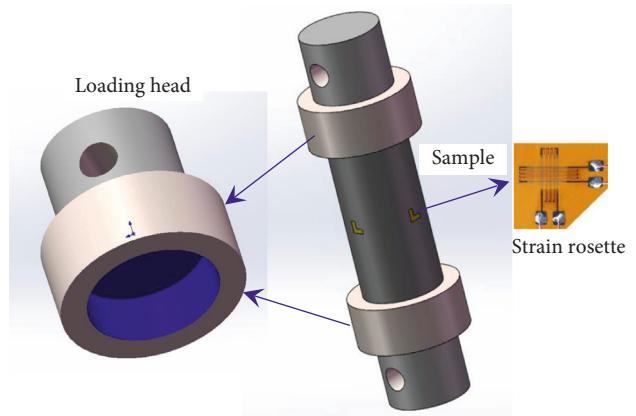


FIGURE 3: Sketch of the samples for direct tension test. Four strain rosettes are glued on the lateral side of samples.

2.3. Three-Point Bending Test. The three-point bending test is also a kind of alternate test method to measure the tensile strength of rock materials. Some improvements have been made attempting to synchronously measure the compressive elastic modulus and tensile elastic modulus adopting three-point bending test [7] because the upper part of beam sample is in compressive state; meanwhile, the lower part is in the tensile state, as illustrated in Figure 4. An obvious priority for measuring E_c and E_t of rock materials at the same time is that the stress status in the beam sample is very simple, and its analytical solution is available for this three-point bending test. For the sake of clarity, the basic theory to measure E_c and E_t of rock materials is briefly summarized as following.

As illustrated in Figure 4, a beam with height h and thickness b is applied by a force P on the symmetrical plane. Two fulcrums with a distance L support the beam at the bottom. Taking the symmetrical plane as the focus, the stress on it in x direction must be in equilibrium firstly:

$$\frac{1}{2}\sigma_c bh_1 = \frac{1}{2}\sigma_t bh_2, \quad (1)$$

where σ_c and σ_t are the maximum compressive stress at upper boundary and the maximum tensile stress at the bottom of beam. Secondly, the moment around z axis also must be in equilibrium:

$$\frac{1}{4}PL = \frac{1}{3}\sigma_c bh_1^2 + \frac{1}{3}\sigma_t bh_2^2, \quad (2)$$

where h_1 and h_2 ($h = h_1 + h_2$) are the height of compressive and tensile zones, respectively. Finally, the deformation coordination relation on the symmetrical plane satisfies [7]

$$\frac{h_1}{h_2} = \frac{\varepsilon_c}{\varepsilon_t}, \quad (3)$$

where ε_c and ε_t are the maximum compressive strain at upper boundary and the maximum tensile strain at the bottom of beam. At elastic deformation stage of rock material, applying Hooke's law $\sigma_c = E_c \varepsilon_c$, $\sigma_t = E_t \varepsilon_t$, the following formulation is established:

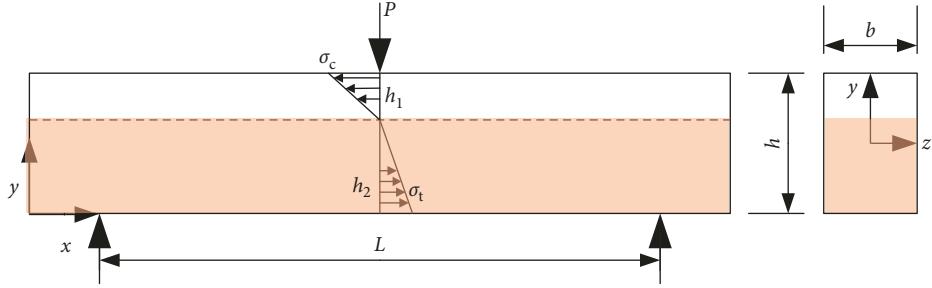


FIGURE 4: Diagram of compressive and tensile stress in three-points bending test.

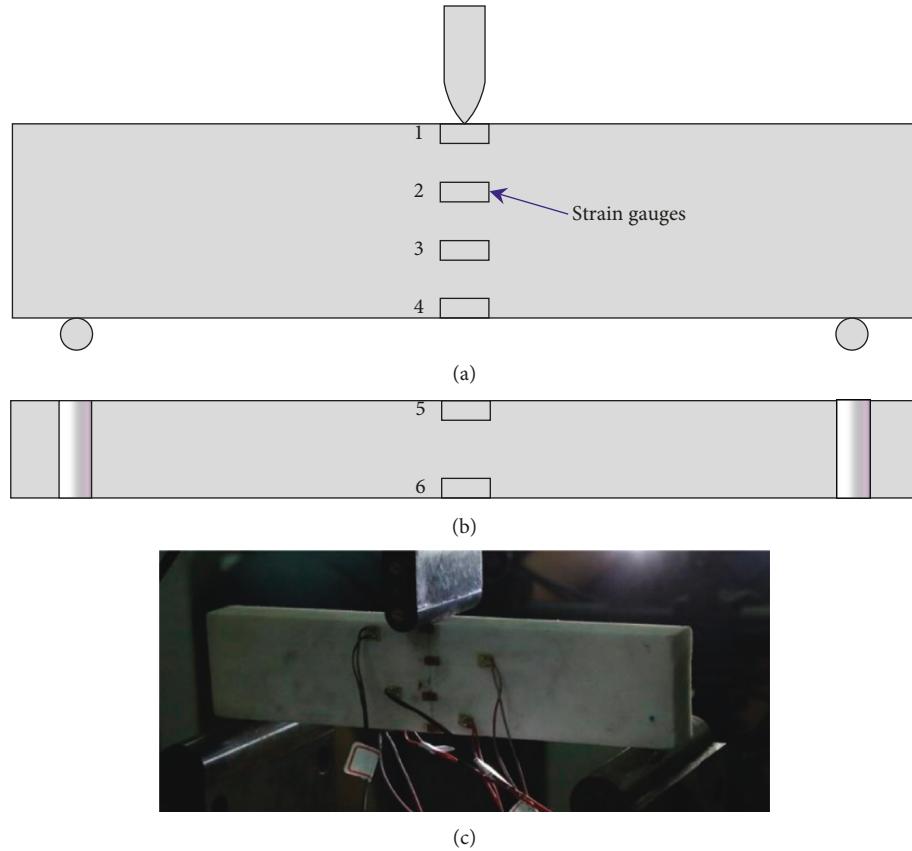


FIGURE 5: Sketch of the rock sample of three-point bending test. Four strain gauges are glued on the front lateral side and two are glued on the bottom of sample beam. (a) Front view. (b) Bottom view. (c) A real view.

$$E_c = \frac{3PL}{4h^2b} \left(1 + \frac{\varepsilon_t}{\varepsilon_c} \right) \frac{1}{\varepsilon_c}, \quad (4)$$

$$E_t = \frac{3PL}{4h^2b} \left(1 + \frac{\varepsilon_t}{\varepsilon_c} \right) \frac{\varepsilon_c}{\varepsilon_t^2}. \quad (5)$$

Based on equations (4) and (5), it is expected to measure the compressive elastic modulus E_c and tensile elastic modulus E_t of rock materials, so long as the applied loading P , and maximum compressive strain ε_c at upper boundary and the maximum tensile strain ε_t at the bottom of beam are recorded in elastic deformation stage.

In this study, totally six strain gauges are glued on the beam sample to record the deformation. Among them, four are glued on the front lateral side (Nos. 1 to 4), and two (Nos. 5 and 6) are on the bottom of beam, as illustrated in Figure 5. Strain gauge No. 1 can record the maximum compressive strain ε_c at the upper boundary; strain gauge Nos. 4, 5, and 6 record the maximum tensile strain ε_t at the bottom. The applied loading P is recorded by a force sensor in testing.

2.4. Brazilian Disc Test. Brazilian disc test is a kind of popular test method to measure the tensile strength of materials in the field of rock mechanics proposed by ISRM [9] and ASTM [26]. Its advantages include the following: (1)

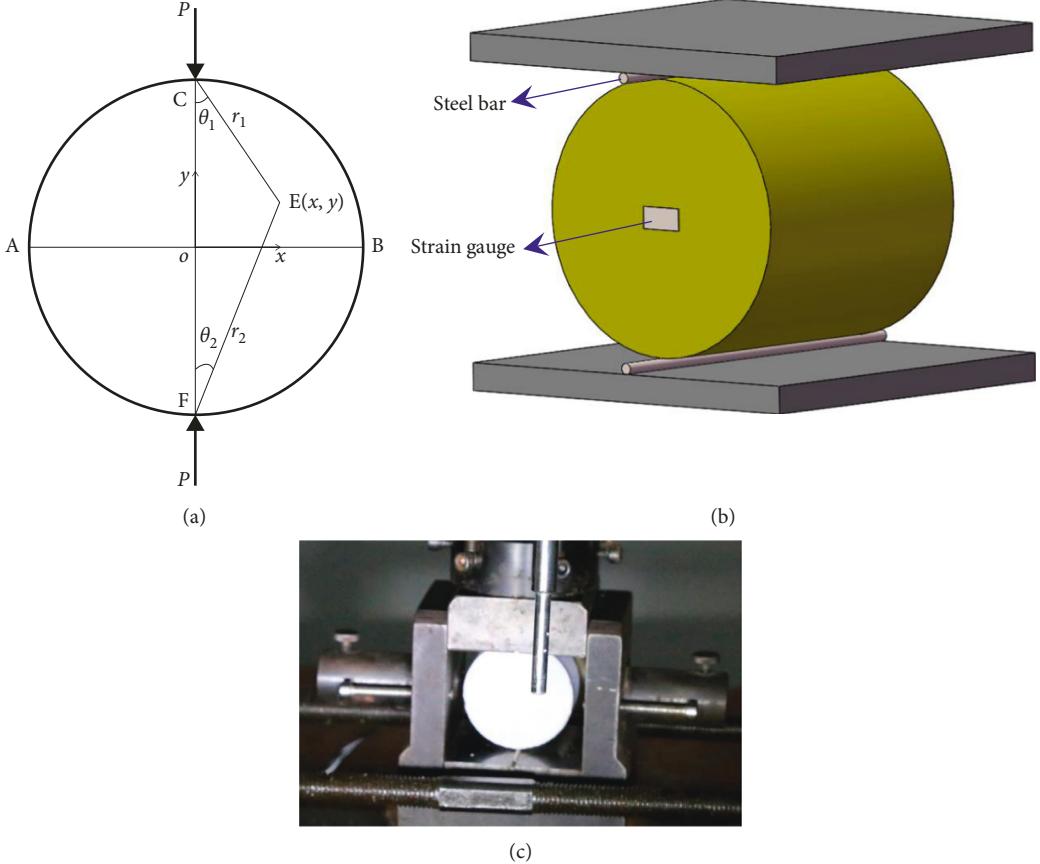


FIGURE 6: Sketch of the Brazilian disc test. A strain gauge is horizontally glued at the center on the front lateral side of disc to record the tensile deformation. (a) Diagram for analytical solution. (b) Sketch of the test. (c) A real view.

sample preparation is easy; (2) testing procedure and operation is relatively simple; and (3) the closed form analytical solution of stress in disc is available, making the testing theory rigorous. Ye et al. [13] gave out the analytical solution of stress in disc applied by two concentrated point loading (Figure 6(a)) as

$$\begin{aligned} \sigma_x &= \frac{2P}{\pi l} \left\{ \frac{(D/2 - y)x^2}{((D/2 - y)^2 + x^2)^2} + \frac{(D/2 + y)x^2}{((D/2 + y)^2 + x^2)^2} - \frac{1}{D} \right\}, \\ \sigma_y &= \frac{2P}{\pi l} \left\{ \frac{(D/2 - y)^3}{((D/2 - y)^2 + x^2)^2} + \frac{(D/2 + y)^3}{((D/2 + y)^2 + x^2)^2} - \frac{1}{D} \right\}, \\ \tau_{xy} &= \frac{2P}{\pi l} \left\{ \frac{(D/2 - y)^2 x}{((D/2 - y)^2 + x^2)^2} - \frac{(D/2 + y)^2 x}{((D/2 + y)^2 + x^2)^2} \right\}, \end{aligned} \quad (6)$$

where D is the diameter of disc, P is the applied force, and l is the thickness of disc. Based the above analytical solution of disc, Ye et al. [13] further proposed a theoretic formulation to measure the tensile elastic modulus E_t of rock materials (Figure 6(b)):

$$\begin{aligned} E_t(t) &= \frac{2P(t)}{\pi D l \varepsilon_t(t)} \left\{ \left(1 - \frac{D}{L} \arctan \frac{2L}{D} \right) (1 - \nu) + \frac{2D^2 (1 + \nu)}{4L^2 + D^2} \right\} \\ &= A \times E_s, \end{aligned} \quad (7)$$

in which

$$E_s = \frac{2P}{\pi D l \varepsilon_t}, \quad (8)$$

$$A = \left(1 - \frac{D}{L} \arctan \frac{2L}{D} \right) (1 - \nu) + \frac{2D^2 (1 + \nu)}{4L^2 + D^2}. \quad (9)$$

ε_t is the strain recorded by the gauge horizontally glued at the center of disc. E_s is named as splitting elastic modulus in Ye et al. [13]. A is a correction coefficient related to the size of disc and strain gauge, as well as Poisson's ratio ν . It must be determined under the state of compression. This is the reason why the uniaxial compression test needs to be conducted in this study. t in equation (7) represents time. It means that tensile elastic modulus E_t of rock materials could be variable with time. In the real test procedure (Figure 6(c)), once the applied force P and strain ε_t are recorded synchronously, the tensile elastic modulus E_t of rock materials could be measured adopting equations (7)–(9).

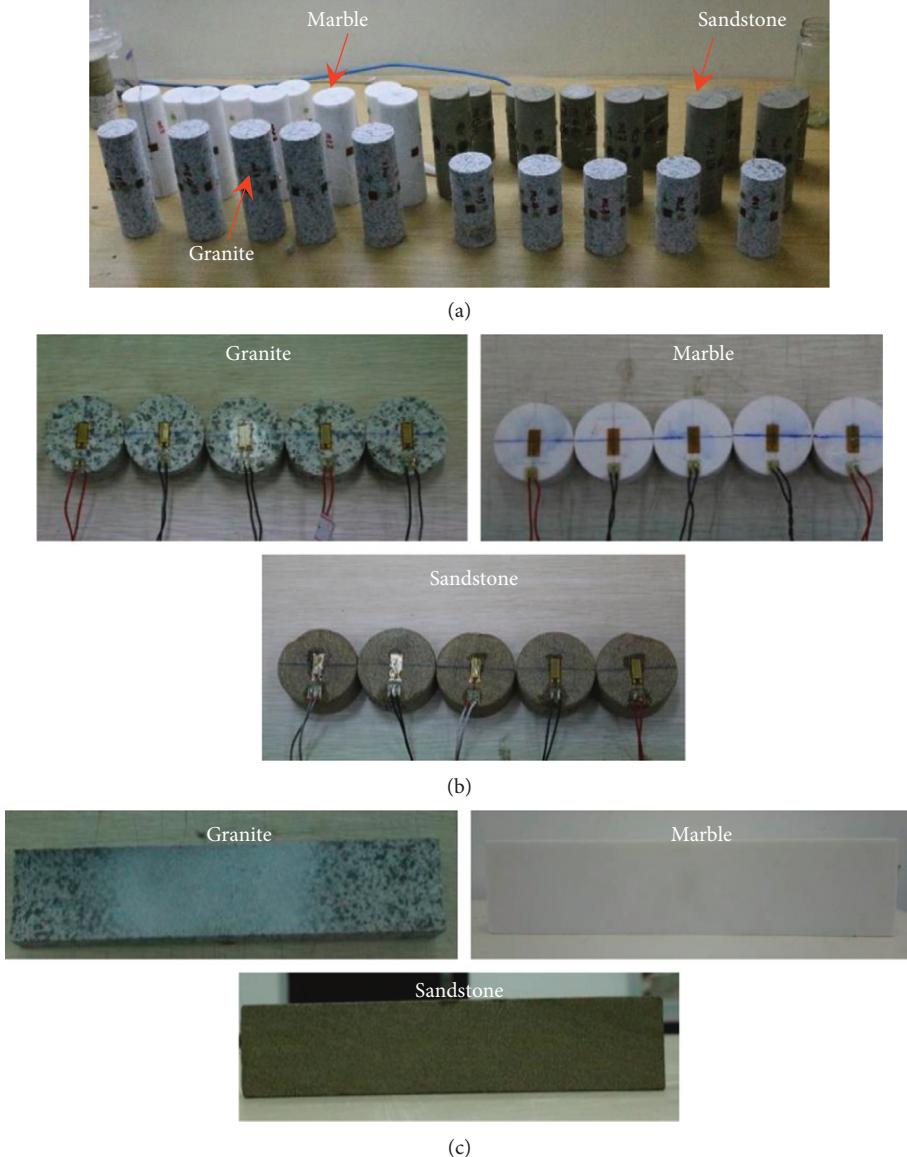


FIGURE 7: A view of rock samples for (a) UCT and DTT, (b) Brazilian disc test, and (c) three-point bending test.

3. Experimental Results

3.1. Test Samples. In this study, three types of typical rock are selected as the representatives. They are granite, white marble, and sandstone. All test samples are prepared according to related Chinese specifications [27] or international standards [9, 26, 28, 29] in the field of rock mechanics. In order to guarantee the reliability of test results, five sets of parallel testing are conducted for each type of test and each type of rock. Totally 60 rock samples are prepared (Figure 7). The size of all rock samples is listed in Tables 1 and 2. Numbering the rock samples follows the following rules. (1) The first character represents the rock type: "M" is marble, "G" is granite, and "S" is sandstone. (2) The second character represents the test type: "C" is the compression test, "T" is the tension test, "W" is the three-point bending test, and "P" is the

Brazilian disc test. (3) The third represents the sequence in the five parallel testing.

3.2. UCT and DTT Results. A real view of the unaxial compressive test (UCT) and direct tensile test (DTT) for rock samples is shown in Figures 8 and 9, respectively. The typical failure model of rock samples in UCT and DTT is shown in Figures 10 and 11, respectively. Following, the detailed test results of marble, granite, and sandstone are analyzed one by one.

3.2.1. Marble's Results. The typical stress-strain curves obtained from UCT and DTT for marble are shown in Figure 12. In previous study, generally only one set of strain gauge is glued on the lateral side of samples. Here, four sets of

TABLE 1: Size of rock samples for UCT and DTT.

Rock	Serial number	D (mm)	H (mm)	Serial number	D (mm)	H (mm)	ρ (g/cm ³)
Marble	MC1	49.76	100.23	MT1	50.21	139.82	2.72
	MC2	49.8	100.31	MT2	50.23	139.8	
	MC3	49.86	100.25	MT3	50.28	139.78	
	MC4	49.75	100.21	MT4	50.24	139.84	
	MC5	49.83	100.19	MT5	50.2	139.76	
Granite	GC1	50.21	100.03	GT1	49.82	138.95	2.45
	GC2	50.16	100.11	GT2	49.76	139.81	
	GC3	50.32	100.25	GT3	49.85	138.93	
	GC4	50.12	100.11	GT4	49.73	139.81	
	GC5	50.22	100.14	GT5	49.89	139.65	
Sandstone	SC1	50.11	99.73	ST1	49.86	139.25	2.14
	SC2	50.36	99.86	ST2	49.86	138.91	
	SC3	50.25	99.83	ST3	49.75	139.21	
	SC4	50.23	99.69	ST4	49.74	138.84	
	SC5	50.21	99.78	ST5	49.92	138.65	

TABLE 2: Size of rock samples for the three-point bending test and Brazilian disc test.

Rock	Serial number	$L \times H \times B$ (mm)	Serial number	D (mm)	l (mm)	ρ (g/cm ³)
Marble	MW1	278.21 × 60.34 × 30.12	MP1	49.82	25.21	2.72
	MW2	278.43 × 60.24 × 30.23	MP2	49.8	24.87	
	MW3	278.54 × 60.31 × 30.21	MP3	49.87	25.13	
	MW4	278.26 × 60.25 × 30.15	MP4	49.82	25.25	
	MW5	278.36 × 60.23 × 30.16	MP5	49.85	25.13	
Granite	GW1	279.51 × 60.24 × 30.21	GP1	49.73	25.32	2.45
	GW2	278.83 × 60.22 × 30.33	GP2	49.72	24.75	
	GW3	278.84 × 60.12 × 30.18	GP3	49.76	25.26	
	GW4	278.56 × 60.30 × 30.16	GP4	49.86	25.21	
	GW5	279.16 × 60.13 × 30.26	GP5	49.81	25.32	
Sandstone	SW1	279.51 × 60.34 × 30.27	SP1	49.75	25.22	2.14
	SW2	278.83 × 60.14 × 30.12	SP2	49.74	24.85	
	SW3	278.84 × 60.23 × 30.31	SP3	49.8	24.83	
	SW4	278.56 × 60.19 × 30.23	SP4	49.76	24.76	
	SW5	279.16 × 60.31 × 30.06	SP5	49.71	24.86	

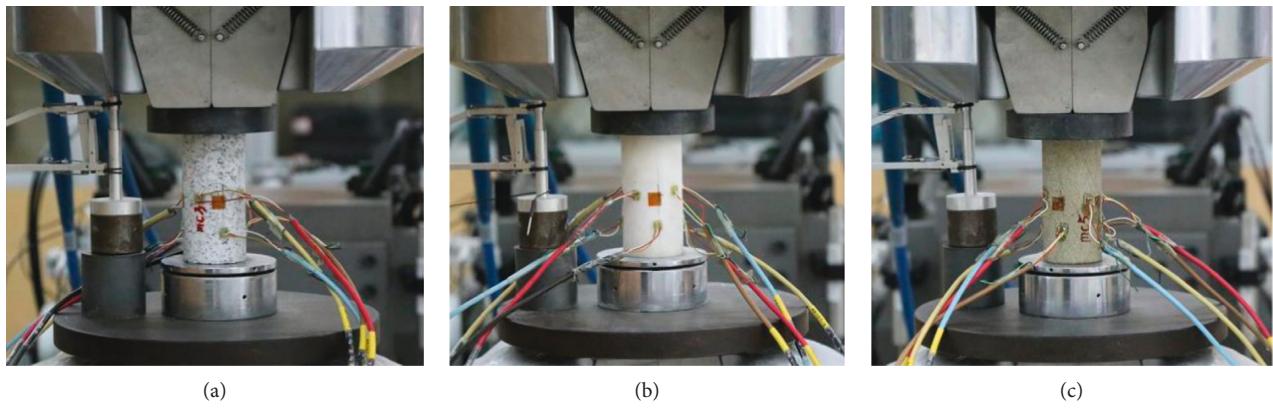


FIGURE 8: A real view of UCT. (a) Granite. (b) Marble. (c) Sandstone.

stress-strain are obtained because four sets of strain gauges are used in tests. As illustrated in Figure 12, the recorded strain by different strain gauge is not the same, indicating that the

deformation is not homogeneous in the whole rock sample. Due to the existence of various errors resulting from, for example, test apparatus itself, sensors, artificial operation, and

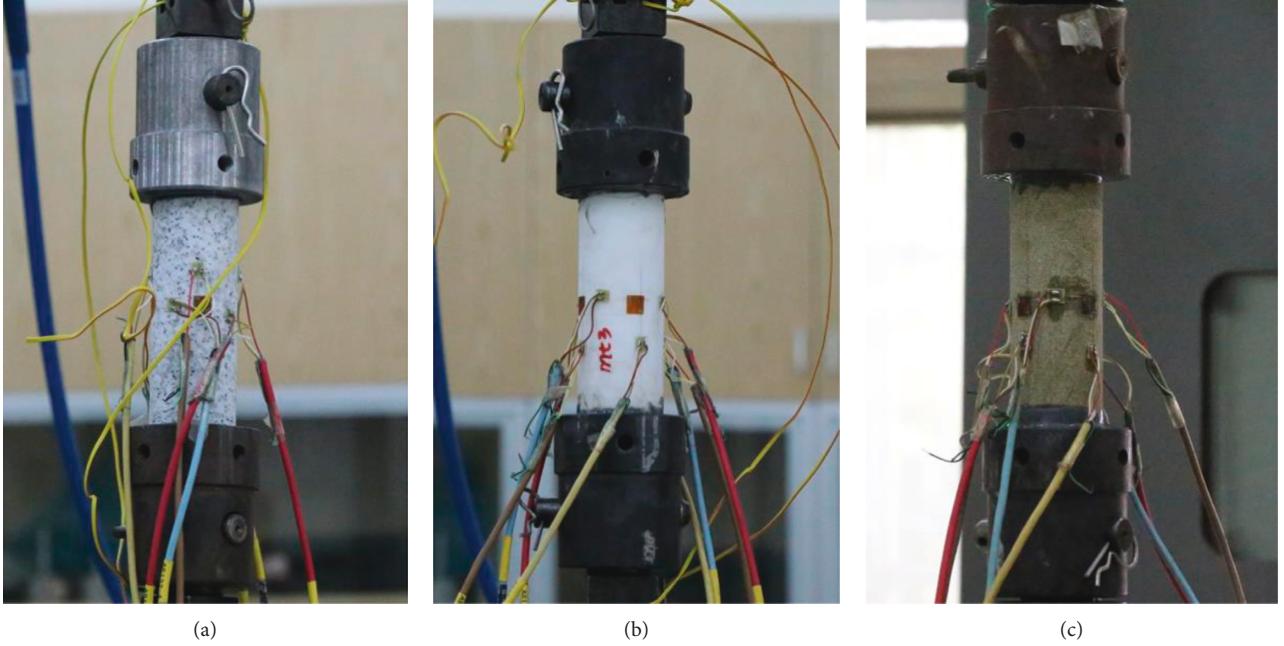


FIGURE 9: A real view of DTT. (a) Granite. (b) Marble. (c) Sandstone.

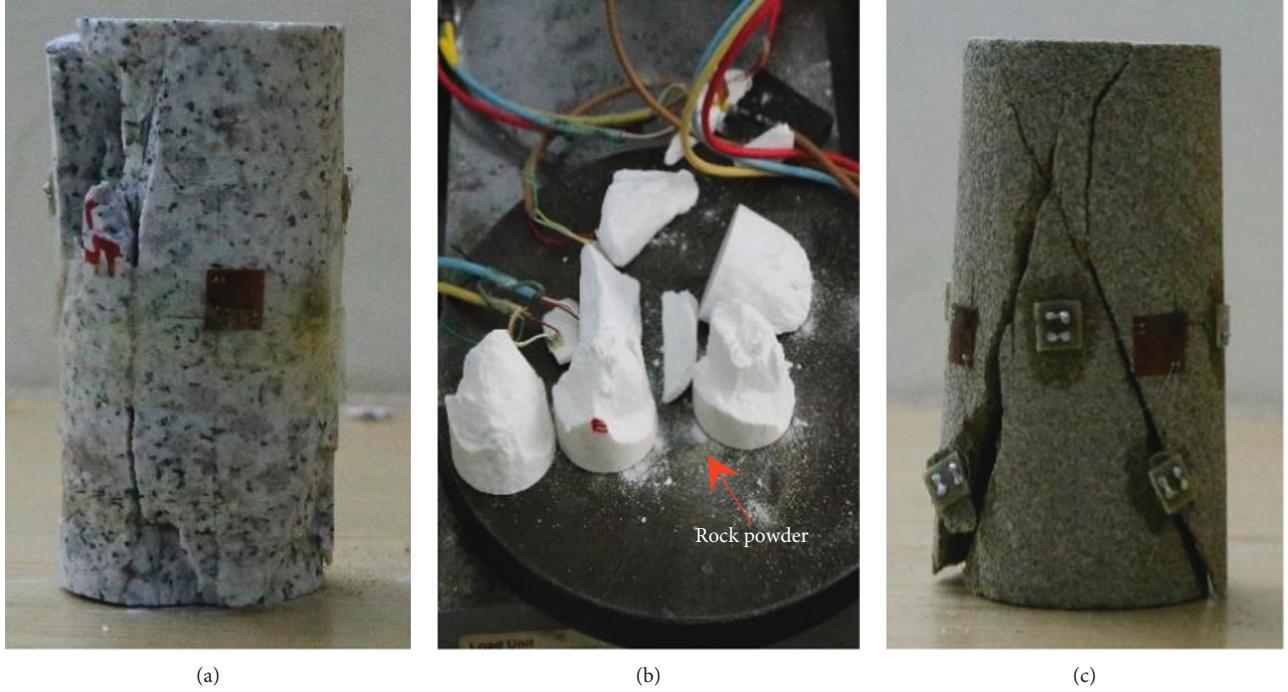


FIGURE 10: Typical failure model of rock samples after UCT. (a) Granite. (b) Marble. (c) Sandstone.

some other unknown random factors, some strain measurement could be not reasonable, for example, the strain recorded by gauge 8 shown in Figure 12(a). Therefore, results of deformation measurement could be not representative or accurate if only one strain gauge is used.

According to the testing method presented in Sections 2.1 and 2.2, the compressive elastic modulus E_c , tensile elastic modulus E_t , and Poisson's ratio ν of marble are

determined. All results are listed in Table 3. In case a recorded strain is obviously not reasonable, it will not be used when processing data. “—” is placed at the related position in Table 3.

As shown in Table 3, the compressive strength of marble used in this study is huge, reaching up to 126.2 MPa, averaging 107.6 MPa. However, its tensile strength from DTT is apparently small, being only 0.7–1.4 MPa. This huge

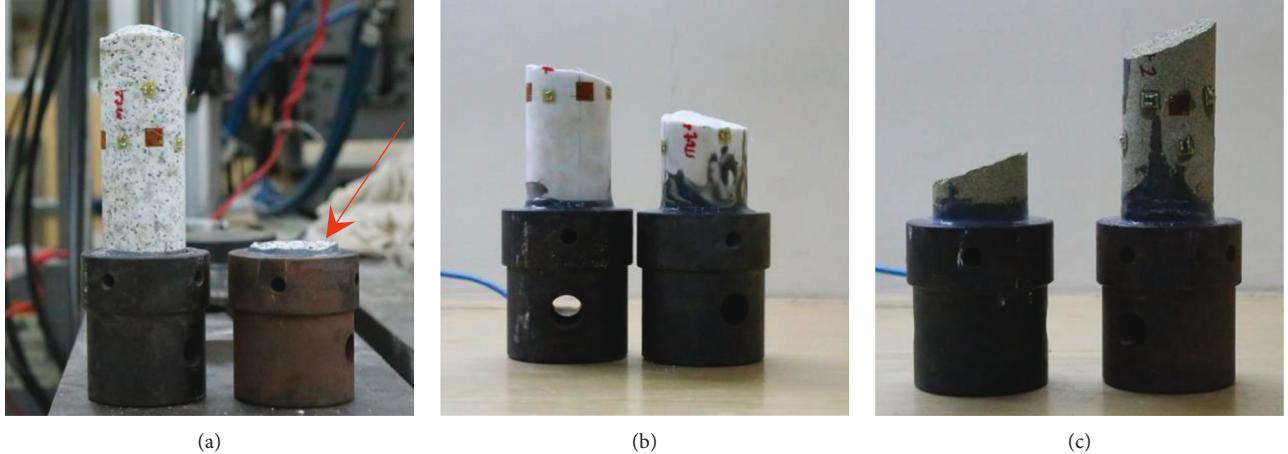


FIGURE 11: Typical failure model of rock samples after DTT. (a) Granite. (b) Marble. (c) Sandstone.

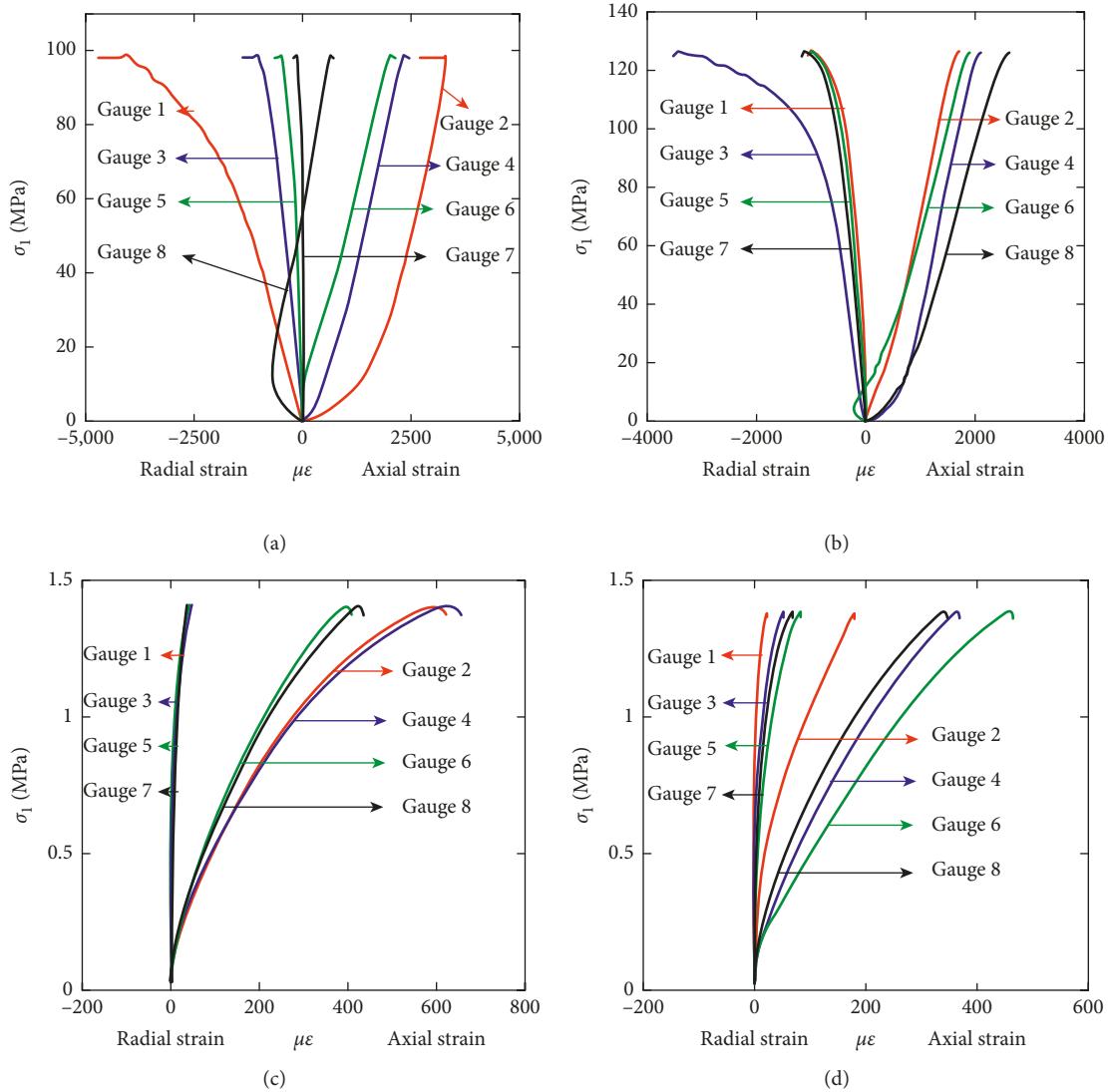


FIGURE 12: Typical stress-strain curves obtained from UCT and DTT for marble ($1\mu\epsilon = 1.0 \times 10^{-6}$). (a) MC2. (b) MC4. (c) MT2. (d) MT3.

TABLE 3: Test results of UCT and DTT for marble.

SN	Gauge number	E_c (GPa)	μ	σ_c (MPa)	SN	Gauge number	E_t (GPa)	μ	σ_t (MPa)		
MC1	1, 2	—	0.24	123.7	MT1	1, 2	2.0	—	1.4		
	3, 4	—	0.18			3, 4	—	—			
	5, 6	69.8	0.38			5, 6	4.7	0.11			
	7, 8	62.2	0.32			7, 8	2.0	—			
	Mean	66.0	0.28			Mean	2.2	0.11			
MC2	1, 2	50.7	—	98.8	MT2	1, 2	2.9	—	1.4		
	3, 4	55.2	0.27			3, 4	2.6	—			
	5, 6	49.2	—			5, 6	3.8	—			
	7, 8	—	—			7, 8	3.6	—			
	Mean	51.7	0.27			Mean	3.2	—			
MC4	1, 2	—	0.16	126.2	MT3	1, 2	6.6	0.05	1.4		
	3, 4	—	0.38			3, 4	3.7	—			
	5, 6	65.7	0.21			5, 6	3.1	—			
	7, 8	65.7	0.19			7, 8	3.9	—			
	Mean	65.7	0.23			Mean	4.3	0.05			
MC5	1, 2	81.2	—	81.8	MT4	1, 2	5.1	—	0.7		
	3, 4	61.2	0.35			3, 4	2.4	—			
	5, 6	—	—			5, 6	9.4	—			
	7, 8	—	0.27			7, 8	3.7	—			
	Mean	71.2	0.31			Mean	3.7	—			
				MT5		1, 2	2.3	—	1.4		
						3, 4	4.8	—			
						5, 6	—	0.15			
						7, 8	3.5	—			
						Mean	2.7	—			
Total mean		63.6	0.27	107.6		Total mean	3.7	0.1	1.3		

Note. The test for MC3 is not successful.

difference between compressive and tensile behavior can be attributed to the microstructure of this kind of marble. As shown in Figures 10(b) and 11(b), the rock blocks cannot get together, and some rock powder is generated after compression failure. SEM scanning (not shown here) indicates that the cemented minerals between particles make the marble own excellent compression capability, but weak tensile capacity.

It is observed in Figure 11 that the samples of marble and sandstone are pulled off at the middle part of samples, indicating that the DTT for marble and sandstone in this study is successful. However, the granite sample fails at the end. In this case, the measurement of tensile strength is not effective. However, the tensile elastic modulus E_t can still be effectively determined because there still is an elastic deformation stage before failure occurrence, regardless of breaking at the middle part or at end of rock samples. In this study, 60% of the direct tensile test (DTT) is successful; only a few samples break at end. From Table 3, it is known that the average of compressive elastic modulus E_c is 63.6 GPa, while that of the tensile elastic modulus E_t is 3.7 GPa.

3.2.2. Granite's Results. The typical stress-strain curves obtained from UCT and DTT for granite are shown in Figure 13. It is also shown that the strain is different recorded in one sample test. Taking the same data processing method with marble, the compressive elastic modulus E_c , tensile elastic modulus E_t , and Poisson's ratio ν of granite are determined and listed in Table 4.

It is known from Table 4 that the compressive strength of the granite is also relatively high, ranging from 61.5 MPa to 85.9 MPa, averaging 74.7 MPa, while its tensile strength is only 2.2 MPa to 4.0 MPa. The difference is considerable. The compressive elastic modulus of granite ranges from 23.1 GPa to 63.4 GPa, averaging 37.3 GPa. The tensile elastic modulus ranges from 9.1 GPa to 23.6 GPa, averaging 14.8 GPa.

3.2.3. Sandstone's Results. The typical stress-strain curves obtained from UCT and DTT for sandstone are shown in Figure 14. Taking the same data processing method with marble, the compressive elastic modulus E_c , tensile elastic modulus E_t , and Poisson's ratio ν of sandstone are determined and listed in Table 5.

It is known from Table 4 that the compressive strength of the sandstone is also relatively high, ranging from 20.5 MPa to 24.7 MPa, averaging 22.4 MPa, while its tensile strength is only 0.27 MPa to 0.99 MPa. The difference also is considerable. The compressive elastic modulus of sandstone ranges from 2.5 GPa to 5.2 GPa, averaging 3.4 GPa. The tensile elastic modulus ranges from 1.8 GPa to 4.3 GPa, averaging 3.0 GPa. Although the strength under compression and tension state is also significantly different for sandstone, the elastic modulus under compression and tension state basically is close. Overall, the tensile elastic modulus is still less than the compressive elastic modulus.

3.3. Three-Point Bending Test Results. Typical compressive and tensile stress-strain curves of marble, granite, and

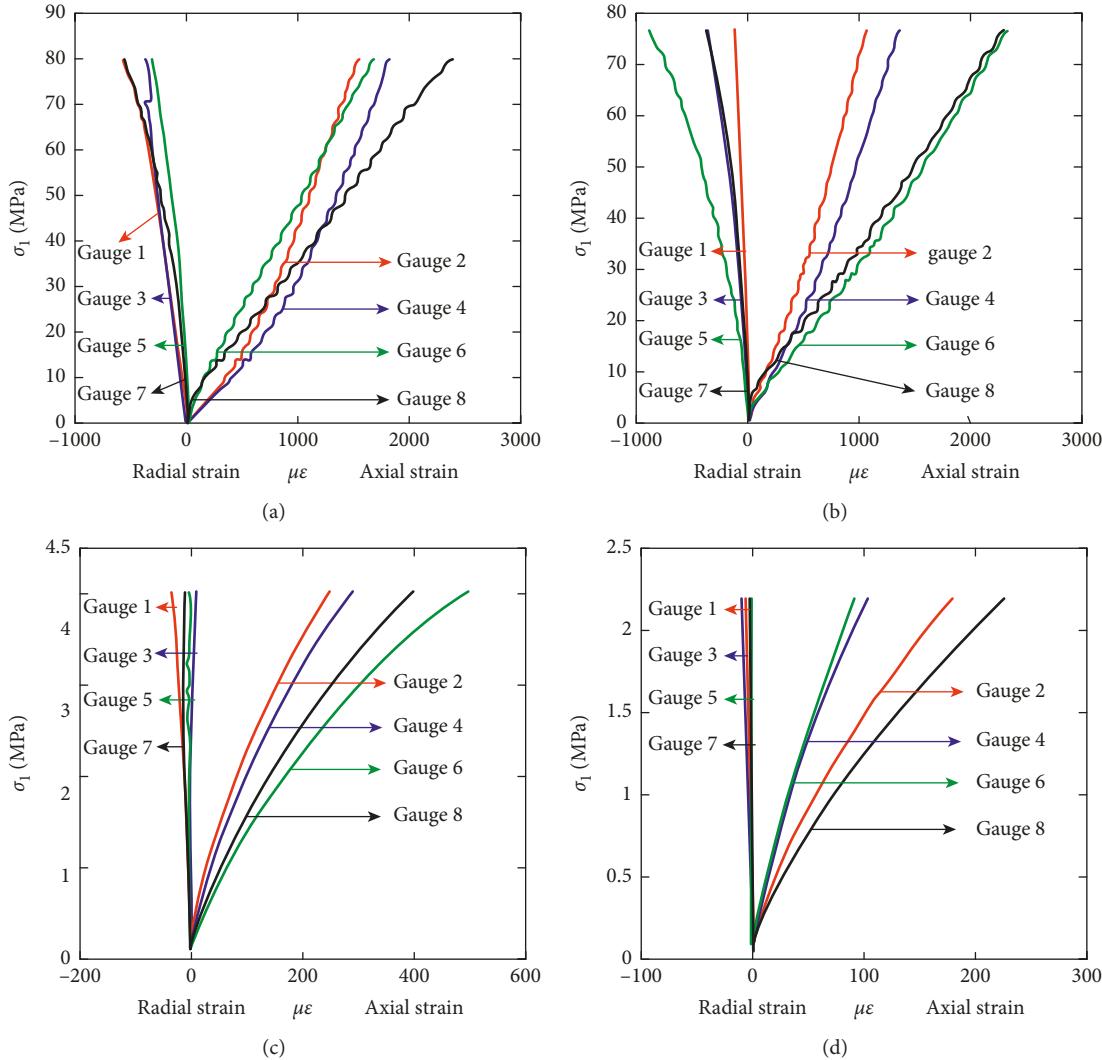
FIGURE 13: Typical stress-strain curves obtained from UCT and DTT for granite ($1\mu\epsilon = 1.0 \times 10^{-6}$). (a) GC2. (b) GC4. (c) GT1. (d) GT3.

TABLE 4: Test results of UCT and DTT for granite.

SN	Gauge number	E_c (GPa)	μ	σ_c (MPa)	SN	Gauge number	E_t (GPa)	μ	σ_t (MPa)
GC1	1, 2	—	—			1, 2	16.2	0.15	4.0
	3, 4	57.0	0.25			3, 4	13.7	—	
	5, 6	39.1	0.13	70.6	GT1	5, 6	8.5	0.02	
	7, 8	23.1	0.18			7, 8	10.5	0.08	
	Mean	39.7	0.19			Mean	12.2	0.08	
GC2	1, 2	63.4	0.23			1, 2	11.9	0.06	2.2
	3, 4	53.2	0.19			3, 4	21.3	0.10	
	5, 6	45.2	0.11	85.9	GT3	5, 6	22.2	—	
	7, 8	32.6	0.15			7, 8	9.1	0.01	
	Mean	48.6	0.17			Mean	16.1	0.04	
GC3	1, 2	24.2	—			1, 2	10.1	—	3.8
	3, 4	26.4	—			3, 4	15.5	0.06	
	5, 6	—	—	61.5	GT4	5, 6	15.7	0.02	
	7, 8	—	—			7, 8	23.6	—	
	Mean	25.3				Mean	16.2	0.04	

TABLE 4: Continued.

SN	Gauge number	E_c (GPa)	μ	σ_c (MPa)	SN	Gauge number	E_t (GPa)	μ	σ_t (MPa)
GC4	1, 2	—	—						
	3, 4	—	0.16						
	5, 6	36.3	0.24	84.8					
	7, 8	34.4	—						
Mean		35.4	0.20						
GC5	1, 2	35.1	0.19						
	3, 4	55.5	—						
	5, 6	26.3	0.21	70.5					
	7, 8	33.9	—						
Mean		37.7	0.20						
Total mean		37.34	0.19	74.7	Total mean		14.8	0.05	3.3

Note. The test for GT2 and GT5 is not successful.

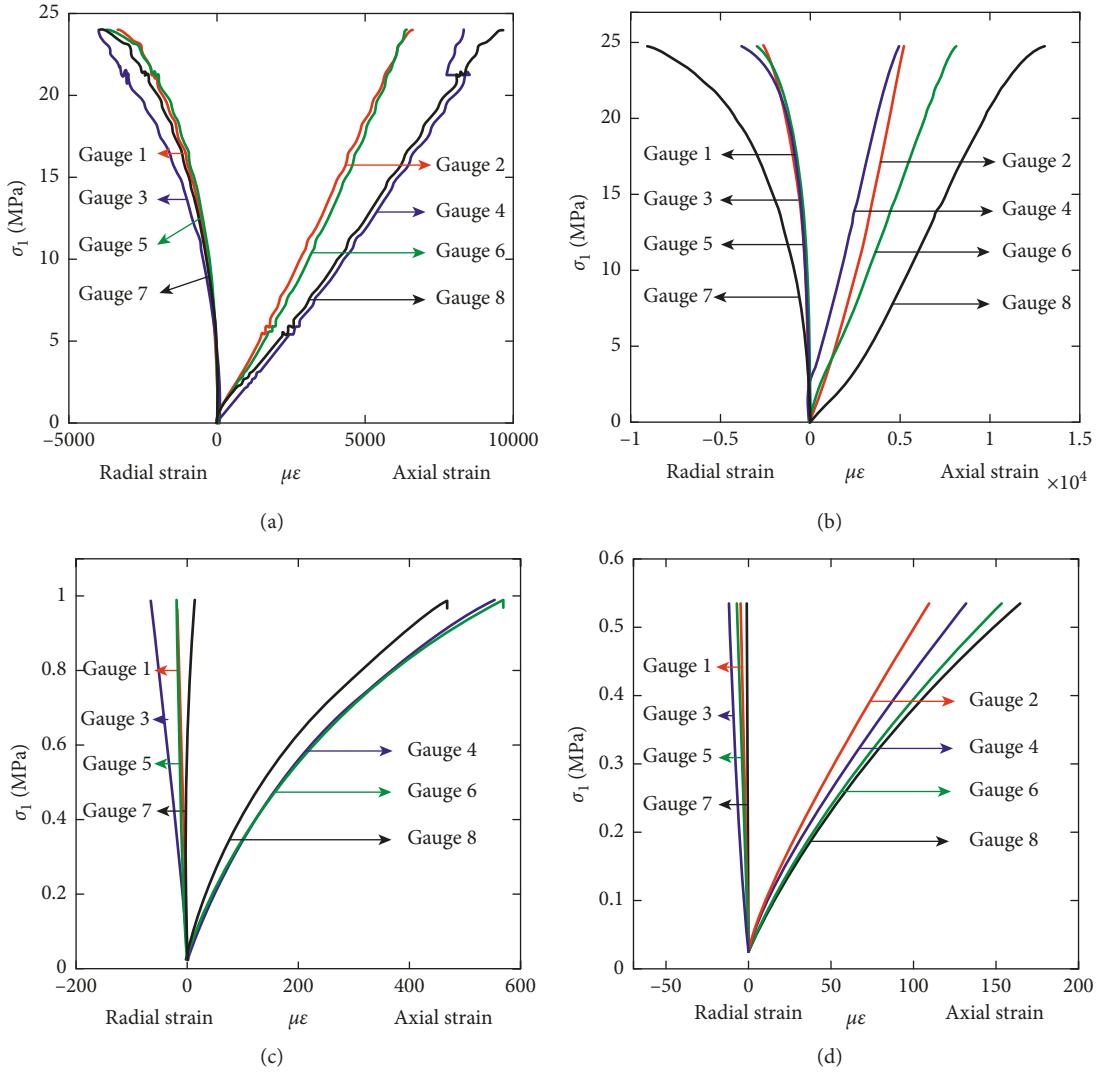


FIGURE 14: Typical stress-strain curves obtained from UCT and DTT for sandstone ($1\mu\epsilon = 1.0 \times 10^{-6}$). (a) SC1. (b) SC4. (c) ST3. (d) ST4.

sandstone obtained from the three-point bending test are shown in Figure 15. Due to the complexity of testing operation, and some other uncontrollable factors, the success rate of the three-point bending test is relatively low in this

study. Even through 15 rock beam samples are prepared, only 6 tests are successful. Among them, marble has three successful tests, granite has two, and sandstone only has one. All results coming from successful tests are plotted in

TABLE 5: Test results of UCT and DTT for sandstone.

SN	Gauge number	E_c (GPa)	μ	σ_c (MPa)	SN	Gauge number	E_t (GPa)	μ	σ_t (MPa)
SC1	1, 2	3.9	0.16			1, 2	2.4	0.21	
	3, 4	2.8	0.15			3, 4	2.2	0.17	
	5, 6	4	0.12	24.0	ST2	5, 6	3.6	0.30	0.44
	7, 8	2.8	0.12			7, 8	—	—	
	Mean	3.4	0.14			Mean	2.8	0.23	
SC2	1, 2	4.7	—			1, 2	—	—	
	3, 4	5.2	—			3, 4	1.9	0.17	
	5, 6	2.5	—	20.5	ST3	5, 6	1.8	0.08	0.99
	7, 8	2.9	0.23			7, 8	2.3	0.00	
	Mean	3.8	0.23			Mean	2.0	0.08	
SC4	1, 2	5.2	0.15			1, 2	4.3	0.06	
	3, 4	5	0.13			3, 4	3.5	0.14	
	5, 6	3.2	—	24.7	ST4	5, 6	3.1	0.06	0.54
	7, 8	2.5	0.21			7, 8	3.1	—	
	Mean	4	0.16			Mean	3.5	0.09	
SC5	1, 2	3.9	0.09			1, 2	3.8	0.21	
	3, 4	3.2	0.12			3, 4	3.8	0.30	
	5, 6	2.6	0.11	21.4	ST5	5, 6	3.5	0.10	0.27
	7, 8	2.9	0.09			7, 8	--	0.40	
	Mean	3.2	0.10			Mean	3.7	0.25	
SC6	1, 2	2.6	0.10						
	3, 4	2.5	0.08						
	5, 6	2.5	0.24	21.6					
	7, 8	3	0.08						
	Mean	2.7	0.13						
Total mean		3.4	0.15	22.4	Total mean		3.0	0.16	0.56

Note. The test for ST1 is not successful.

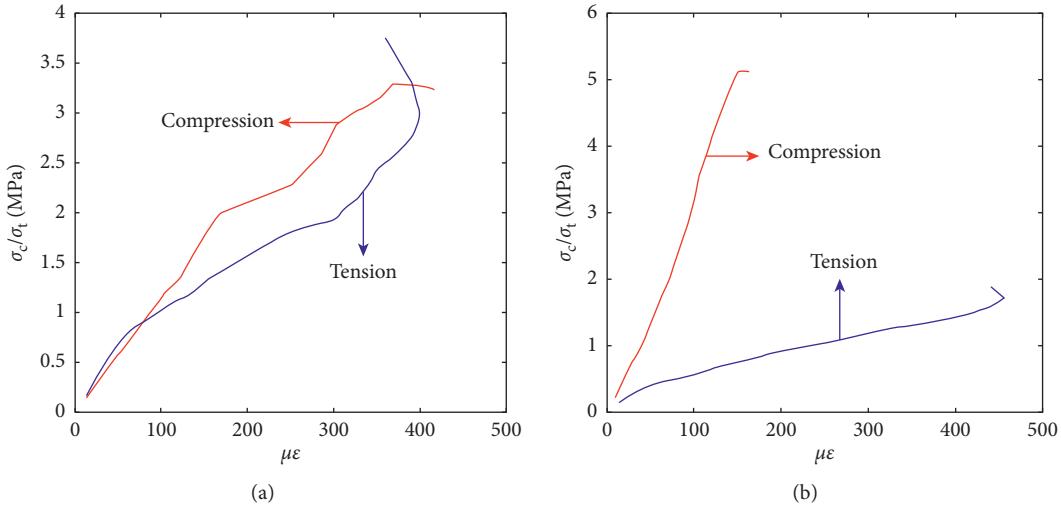


FIGURE 15: Continued.

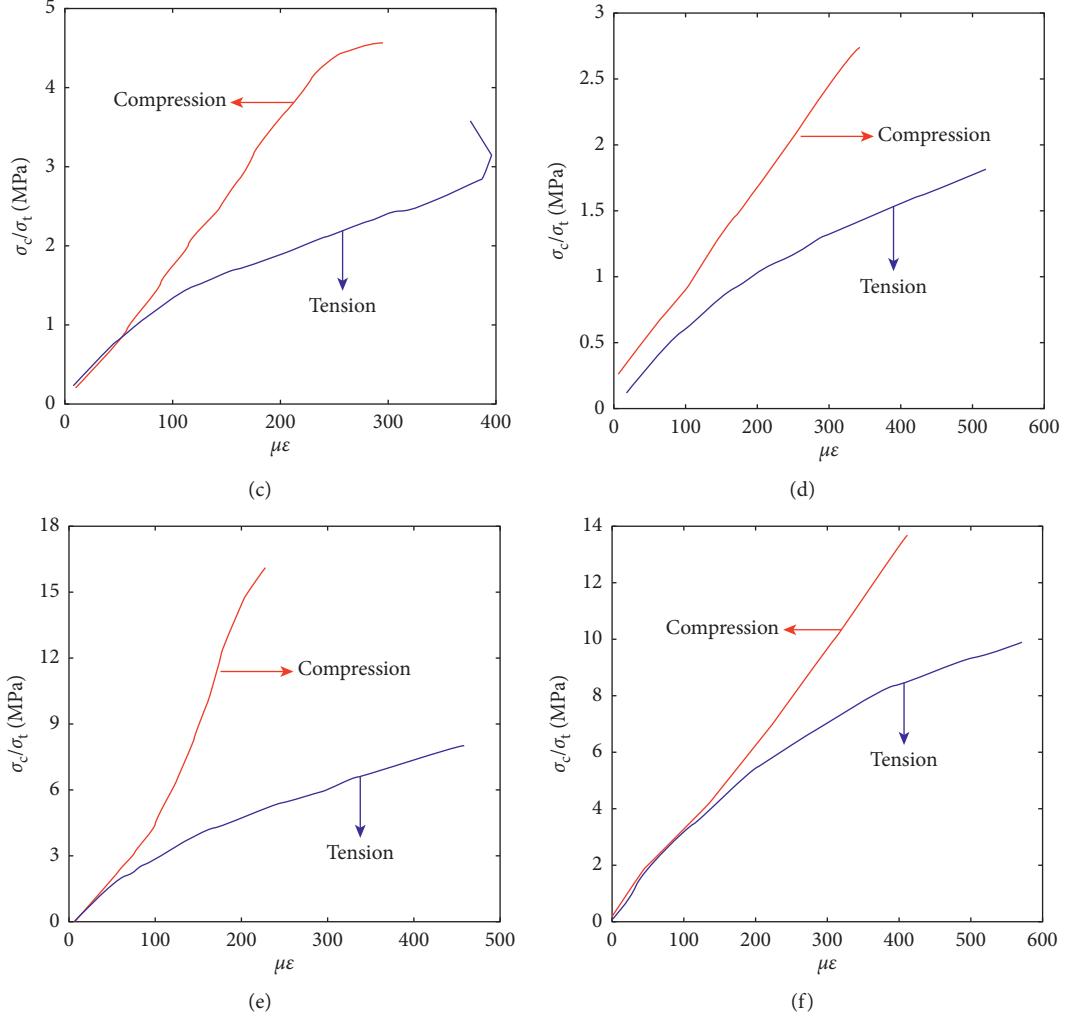


FIGURE 15: Typical stress-strain curves of marble, granite, and sandstone obtained from three-point bending test ($1\mu\epsilon = 1.0 \times 10^{-6}$). (a) MW1. (b) MW2. (c) MW4. (d) SW3. (e) GW1. (f) GW5.

Figure 15. The typical failure model of rock samples after three-points bending test is shown in Figure 16. It is observed that there is only a thick fracture in the middle part of sample beams, indicating that the failure of sample beam is due to tension at the bottom.

In Figure 15, the tensile strain is the average recorded by strain gauge 4, 5, and 6; while the compressive strain is recorded by strain gauge 1, as illustrated in Figure 5. The maximum compressive and tensile stress σ_c and σ_t at upper boundary and bottom of sample beam are determined according to Equations (1)–(3). It is clearly observed that tensile elastic modulus E_t is less than compressive elastic modulus E_c for all three types of rock. Parameters E_c , E_t , and tensile strength of rock samples from the three-point bending test are all listed in Table 6. Additionally, the tensile strength of rock samples measured by three-point bending test is also listed in Table 6.

3.4. Brazilian Disc Test Results. Typical tensile stress-strain curves at the center of disc for marble, granite, and

sandstone obtained from Brazilian disc test are shown in Figure 17. Due to the fact that the sample preparation and test operation of Brazilian disc test are relatively simple, the success rate of Brazilian disc test is high in this study, and the testing results are also relatively stable. Typical failure model of rock samples after Brazilian disc test is shown in Figure 18. It is observed that the failure fractures basically go along the vertical diameter of disc, meeting the requirement of success for Brazilian disc test.

According to the theoretic formulation (equations (7)–(9)) proposed by Ye et al. [13], the tensile elastic modulus E_t of rock samples are determined and listed in Table 7. Meanwhile, the tensile strength from Brazilian disc test also is listed in Table 7. Poisson's ratios are all determined by the preceding UCT. From Table 7, it is known that the tensile elastic modulus of marble measured by Brazilian disc test is in the range of 5.1–7.9 GPa, averaging 6.2 GPa, that of granite is 12.4–14.7 GPa, averaging 13.5 GPa, and that of sandstone is 2.4–2.6 GPa, averaging 2.5 GPa. The tensile strength of marble, granite, and sandstone measured

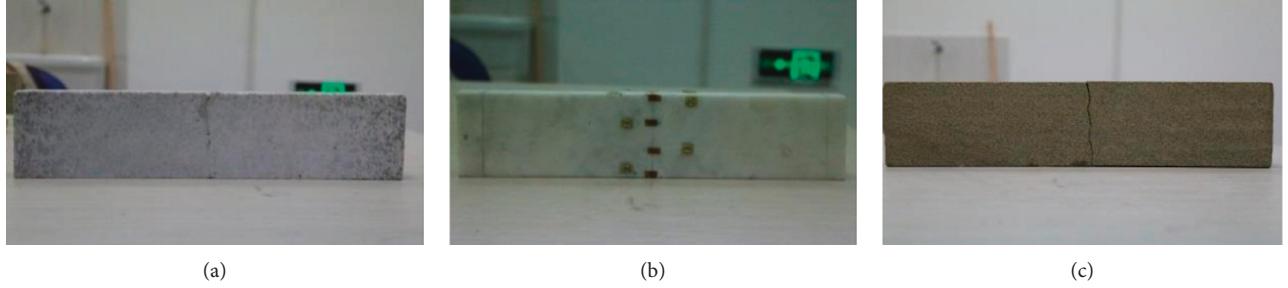


FIGURE 16: Typical failure model of rock samples after three-point bending test. (a) Granite. (b) Marble. (c) Sandstone.

TABLE 6: Test results of three-point bending test for marble, granite, and sandstone.

Rock	E_c (GPa)	E_t (GPa)	Tensile strength (MPa)
MW1	11.70	8.00	3.8
MW2	31.43	3.98	1.8
MW4	17.47	10.38	3.0
Mean	20.20	7.45	2.87
GW1	53.42	24.12	8.2
GW5	31.16	26.27	10.0
Mean	42.29	25.20	9.1
SW3	8.82	5.16	1.8

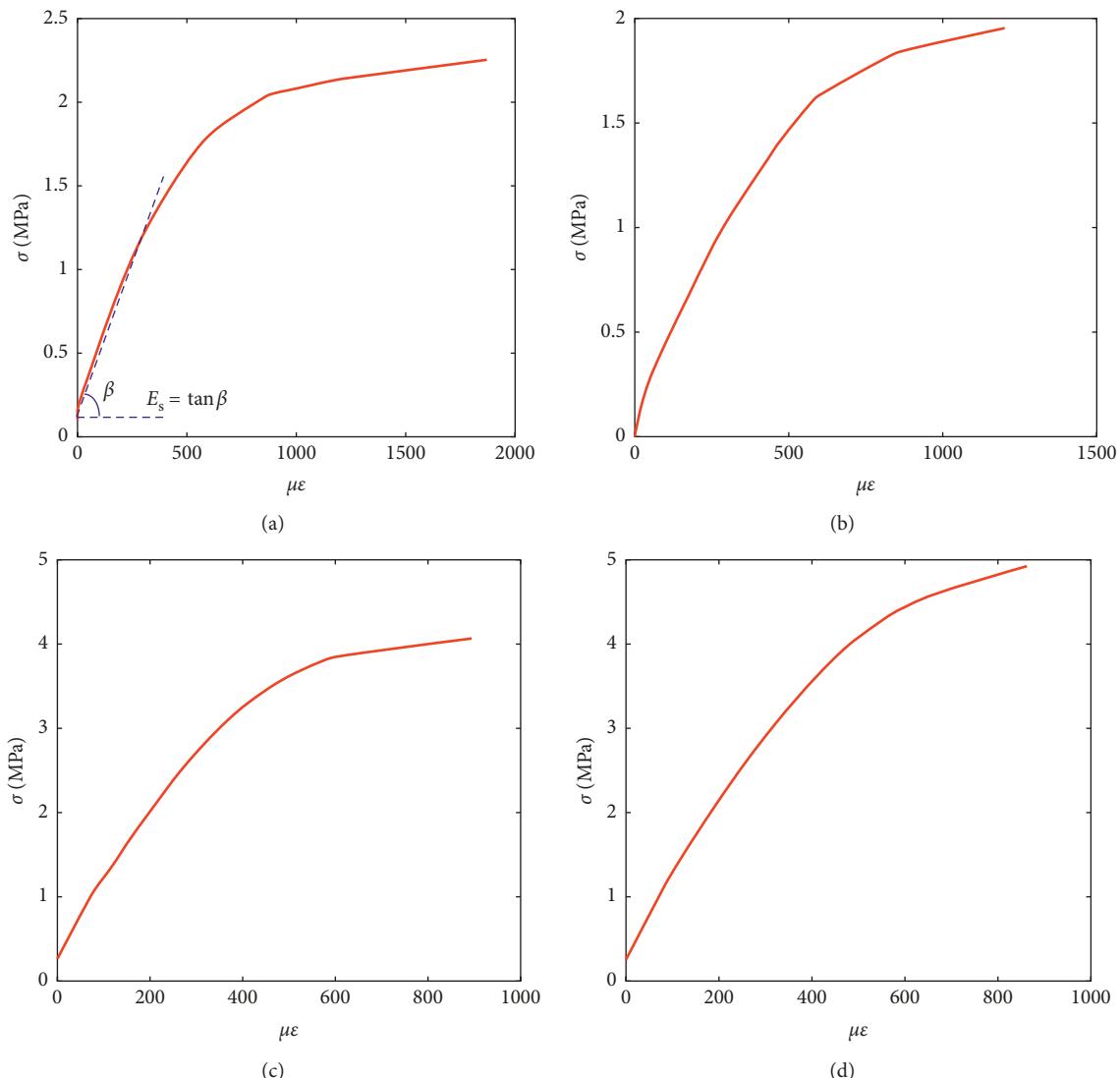


FIGURE 17: Continued.

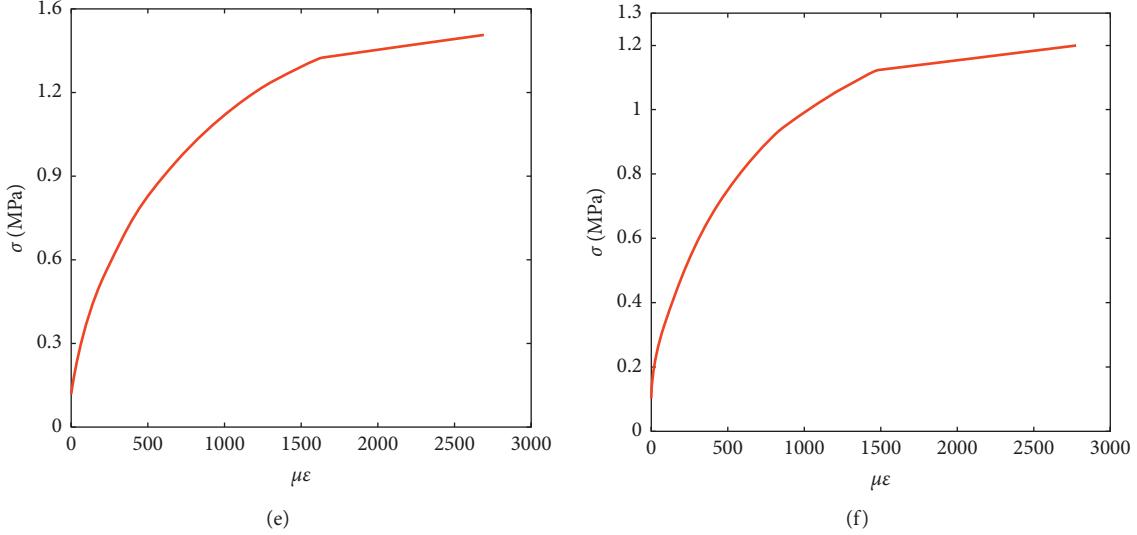


FIGURE 17: Typical tensile stress-strain curves at disc center of marble, granite, and sandstone obtained from Brazilian disc test ($1\mu\epsilon = 1.0 \times 10^{-6}$). (a) MP2. (b) MP5. (c) GP2. (d) GP4. (e) SP1. (f) SP4.

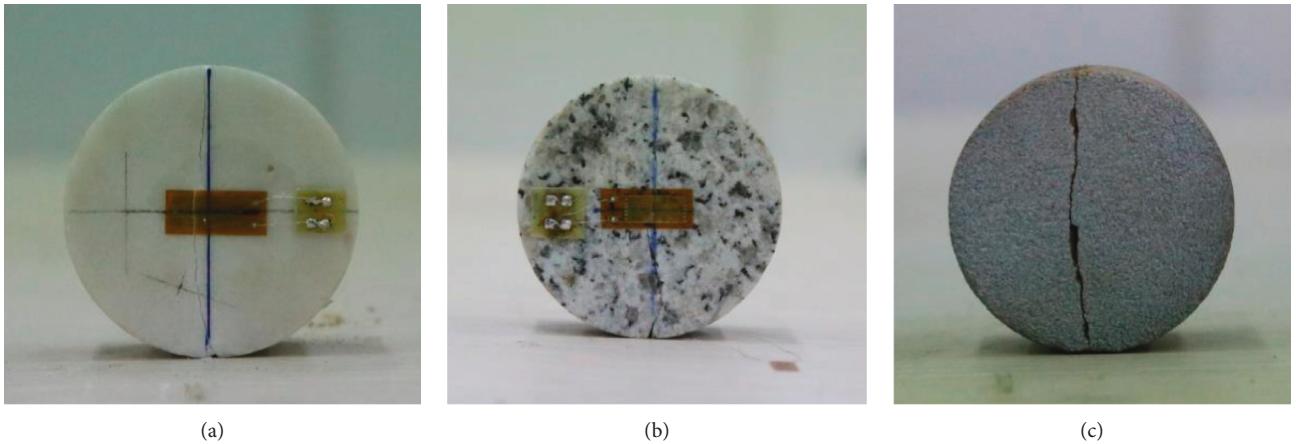


FIGURE 18: Typical failure model of rock samples after Brazilian disc test. (a) Marble. (b) Granite. (c) Sandstone.

by Brazilian disc test is averagely 2.05, 4.40, and 1.11 MPa, respectively.

3.5. Comparative Study. The basic purpose of this study is to screen which indirect test method, the three-point bending test or Brazilian disc test, is the better method to measure the tensile elastic modulus of rock materials. To implement this, all test results are summarized in Table 8. In Table 8, it is found that E_t and tensile strength measured by three-point bending test are much greater than that measured by DTT. The ratio is in the range of 170% to 203% for E_t , and it is 221% to 321% for tensile strength. While E_t and tensile strength of rock materials measured by Brazilian disc test are close to that measured by DTT. For granite and sandstone, the E_t measured by Brazilian disc test is slightly less than that measured by DTT, while it is much greater than that measured by DTT for marble. Due to the particularity of the

marble used in this study, as stated in Section 3.2, this result of marble could not be representative. Therefore, we would prefer to make the following suggestions. (1) Tensile elastic modulus E_t of rock materials measured by the three-point bending test is much greater than its real value (here E_t measured by DTT is taken as the real value), and E_t measured by Brazilian disc test is slightly less than its real value, but it is more close to real value. (2) Tensile strength measured by indirect test method, regardless of three-points bending test or Brazilian disc test are both greater than that measured by DTT; however, the one measured by Brazilian disc test are more close to real value. (3) Due to the complexity of sample preparation and testing operation, the three-point bending test is not easy to be implemented. Additionally, due to the excellent test results, the Brazilian disc test can be suggested to be a suitable method to measure tensile elastic modulus of rock materials.

TABLE 7: Test results of Brazilian disc test for marble, granite, and sandstone.

Rock materials	SN	E_s (GPa)	ν	D (mm)	L (mm)	A	E_t (GPa)	σ_t (MPa)
Marble	MP1	3.0	0.27	49.82	5	1.74	5.1	1.88
	MP2	3.6		49.8			6.2	2.25
	MP3	4.6		49.87			7.9	2.13
	MP4	3.5		49.82			6.0	1.94
	MP5	3.3		49.85			5.7	2.04
	Mean	3.6					6.2	2.05
Granite	GP1	9.8	0.19	49.73	5	1.49	14.7	4.46
	GP2	8.3		49.72			12.4	4.07
	GP3	8.9		49.76			13.3	4.14
	GP4	9.1		49.86			13.6	4.92
	GP5	—		—			—	—
	Mean	9.0					13.5	4.40
Sandstone	SP1	1.7	0.15	49.75	5	1.38	2.4	1.41
	SP2	—		49.74			—	1.02
	SP3	1.8		49.8			2.5	1.02
	SP4	1.9		49.76			2.6	1.20
	SP5	—		49.71			—	0.92
	Mean	1.4					2.5	1.11

Note. The test on GP5, SP2, and SP5 is not successful.

TABLE 8: Summarized test results of E_c , CS, E_t , and TS determined by four types of test methods.

Test type	UCT		DTT		Three-point bending test			Brazilian disc test	
Parameter	E_c (GPa)	CS (MPa)	E_t (GPa)	TS (MPa)	E_c (GPa)	E_t (GPa)	TS (MPa)	E_t (GPa)	TS (MPa)
Marble	63.6	107.6	3.7	1.3	20.2	7.5	2.87	6.2	2.05
Granite	37.3	74.7	14.8	3.3	42.3	25.2	9.1	13.5	4.40
Sandstone	3.4	22.4	3.0	0.56	8.8	5.2	1.8	2.5	1.11

Note. CS, compression strength; TS, tension strength.

4. Conclusion

In this study, a series of experiment tests, including uniaxial compressive test (UCT), direct tensile test (DTT), three-points bending test, and Brazilian disc test, are conducted for three typical types of rock materials: marble, granite, and sandstone. Compressive/tensile strength and elastic modulus E_c , E_t , and so on of rock materials are widely measured. Based on these measured results and comparative analysis, the following recognitions can be obtained. (1) The recorded strain value would be not representative if only one set of strain gauge is glued on rock samples. It is suggested that several sets of strain gauges are glued symmetrically on rock samples in UCT, DTT, and three-point bending test and that strain gauge should be glued, respectively, at the center of the two lateral sides of disc in the Brazilian disc test. (2) Due to the usage of the cementing material with high bonding strength (epoxy adhesive tube kit 2216B/A), performing the direct tensile test with cylinder samples is much easier than that if the shape of rock sample is dog bone. However, the success rate of DTT is still low. (3) Due to the complexity of sample preparation and testing operation, the success rate of three-point bending test is apparently low. It is only 40% in this study. (4) It is proposed that the Brazilian disc test could be a suitable method to measure the tensile elastic modulus of rock materials, due to the excellent agreement with the results measured by DTT and the simplicity of sample preparation, as well as testing operation.

Data Availability

The original data of stress-strain curves shown in Figures 12–18 used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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