

## Research Article

# Seismic Fragility Analysis of Bridge System Based on Fuzzy Failure Criteria

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In the traditional bridge seismic fragility analysis, the criterion for judging the structural damage state is clear. That is to say, when the damage index exceeds a specific value, the structure is judged to enter the new damage state. However, the actual condition is that the boundary of structural damage is not clear but fuzzy. Taking a three-span V-shaped continuous girder bridge as an example, the damage process of the structure is described by fuzzy mathematics. Considering the uncertainties of ground motion and structure itself, a seismic fragility analysis method is established, which can consider the randomness of bridge itself, seismic load, and structural failure fuzziness simultaneously. Finally, the improved product of conditional marginal (I-PCM) method for fragility analysis of bridge system is further optimized and improved. The new improved method is used to form the seismic fragility curves of bridge structure system. The results show that it is possible to underestimate the potential seismic fragility of bridge components and system without considering the structural fuzzy failure criteria; the fragility curves formed by different membership functions are obviously different; the new system fragility analysis method can significantly improve the analysis accuracy.

## 1. Introduction

Bridge is an important part of traffic lifeline engineering and plays a key role in earthquake relief. In the past investigation of earthquake damage, it was found that the bridge often suffered partial damage, such as beam falling, pier bending damage, and bearing failure, or even overall collapse, which delayed the development of earthquake relief work and aggravated casualties and economic losses. With the rapid development of nonlinear finite element analysis method and the continuous improvement of performance-based seismic design ideas and concepts, bridge seismic fragility analysis method based on probabilistic seismic demand analysis has been widely used [1]. Through the fragility analysis of bridge components and system, the probability of different damage states of bridges under earthquake can be obtained, and the seismic performance evaluation of bridge structure system can be realized, which provides an effective basis for bridge reinforcement and maintenance.

In the seismic damage analysis of bridge structures, many uncertain factors are often accompanied. From the

perspective of stochastic statistics, the uncertainties include randomness and fuzziness. Randomness means that the conditions of the event cannot be strictly controlled, resulting in some accidental factors that make the test results uncertain. The influence of randomness on structure has been widely recognized by scholars. Considering the material and geometric uncertainties, Pang et al. [2] established the seismic fragility analysis method of cable-stayed bridges by using the analysis samples generated by uniform design method. Wu et al. [3] found out the potential fragile components by analyzing the seismic fragility of midspan concrete cable-stayed bridges. Liu and Zhang [4] analyzed the fragility of steel frames by using artificial neural network considering the uncertainties of steel frame materials and geometry. Mangalathu et al. [5] proposed the relative importance of using artificial neural network to identify each uncertainty parameter (including seismic intensity, span, and longitudinal reinforcement ratio of bridge piers) affecting seismic fragility curve of skew box girder bridges. Pan et al. [6] had done parameter analysis on the sensitivity of various factors affecting structural uncertainty. Compared

with abutment wall-soil stiffness, friction coefficient of expansion bearings, and expansion-joint gap size, it was considered that the bulk density of superstructure, yield strength of steel bar, and compressive strength of concrete had greater effects.

However, fuzziness refers to the uncertainty caused by the impossibility of defining and evaluating certain events clearly. In the field of bridge structural system failure, fuzziness often manifests itself in the diversification of failure modes, while in the field of damage, fuzziness often manifests in the gradual change of damage grade and the overlap of damage grade. In the fragility analysis of bridge structures, the given damage index is usually fixed, which represents the sudden change of structure from reliable state to failure state, which is often inconsistent with the actual situation. The value of actual structure damage and damage grade shows gradual change. In this case, the boundary of the structural damage domain is not clear. At present, most of the studies on seismic fragility of bridges focus on the random effects of structures and seismic loads, while the research on fuzziness is rare.

In this paper, a three-span V-shaped continuous beam bridge was taken as an example. Firstly, the fuzzy stochastic theory was introduced, and the membership function was used to simplify the consideration of the gradual change of damage boundary. Then, the corresponding fuzzy failure criteria were deduced by using the classical reliability theory. Secondly, the uncertainties of bridge structure itself and seismic load were considered. Thirdly the improved product of conditional marginal (I-PCM) method for fragility analysis of bridge system was further optimized and improved. Finally, an earthquake fragility analysis method considering both randomness and fuzziness is established. In addition, the applicability of membership function is also studied in order to explore the influence of selection of membership function on seismic fragility analysis of bridge system.

## 2. Fuzzy Failure Theory

*2.1. Fuzzy Set.* According to the classical reliability theory, for the function  $Z = g(X)$ , the failure domain is  $\{x \mid g_X(x) \leq 0\}$  and the failure probability is

$$P_f = \Pr(Z \leq 0) = \int_{-\infty}^0 f_Z(z) dz. \quad (1)$$

In classical mathematics, ordinary sets can be represented by  $A$ , and in fuzzy mathematics, the corresponding fuzzy sets are represented by  $\underline{A}$ . Set to the deterministic domain  $U$  to any mapping  $u_{\underline{A}}$  on the closed interval  $[0, 1]$ :

$$\begin{aligned} u_{\underline{A}}: U &\longrightarrow [0, 1], \\ u &\longrightarrow u_{\underline{A}}(u), \end{aligned} \quad (2)$$

where  $\underline{A}$  is a fuzzy set of  $U$ ;  $u_{\underline{A}}$  is membership function of fuzzy set  $\underline{A}$ ; and  $u$  is the membership of  $\underline{A}$ . The probability of fuzzy random event  $\underline{A}$  is  $\Pr(\underline{A})$ ,  $\underline{A} = \{(x, u_{\underline{A}}(x)) \mid x \in U\}$ , and its probability density function is  $f(x)$ :

$$\Pr(\underline{A}) = \int_{-\infty}^{+\infty} u_{\underline{A}}(x) f(x) dx. \quad (3)$$

*2.2. Membership Function.* In fuzzy mathematics, the establishment of membership functions is the foundation of establishing fuzzy sets. Membership function is to describe the membership of fuzzy sets to fuzzy sets. The determination of membership function has certain objective regularity.

The membership function is generally determined through the following procedures: (i) determine the domain, analyze the background of the problem, and summarize it abstractly; (ii) determine the overall structure of the membership function and analyze the type of the membership function; and (iii) determine the interval of the membership function.

## 3. System Fragility Analysis Method Based on Fuzzy Failure Criterion

### 3.1. Fragility Analysis of Components Based on Membership Functions

*3.1.1. Cloud Approach.* Seismic fragility analysis of structures refers to the conditional probability that the demand of structures exceeds their capacity under given ground motion intensity. Seismic fragility analysis of structures describes the relationship between damage probability of structures and seismic intensity. The research object can be a single component or structure or an area (urban area, traffic network, etc.).

Cloud approach is the most commonly used method to establish seismic fragility function. The seismic wave selected in this method should not only reflect the uncertainty of seismic wave (spectrum characteristics, magnitude and epicentral distance, etc.) but also be consistent with the site type of the structure studied. The method of choosing seismic waves by cloud approach was originally proposed by Shome et al. [7]. They suggested that the selected seismic waves should be classified according to epicentral distance, magnitude, and soil condition at bridge site to form seismic wave database. After selecting the seismic wave database and determining the intensity measures (IM), the finite element model of the bridge is established and a series of nonlinear time history analysis is carried out to obtain the discrete points between engineering demand (D) and intensity measures (IM). According to the proposal of Cornell et al., it can be assumed that the seismic demand of structure follows the lognormal distribution, and the median value of the engineering demand parameter ( $S_d$ ) satisfies the following relationship [8]:

$$\begin{aligned} S_d &= a \cdot \text{IM}^b, \\ \text{or } \ln(S_d) &= b \cdot \ln(\text{IM}) + \ln(a), \end{aligned} \quad (4)$$

where  $a$  and  $b$  are estimated parameters which can be obtained by least square regression analysis. The equation

(4) is called the probabilistic seismic demand model (PSDM), which can better reflect the relationship between  $S_d$  and IM.

Many scholars assume that the limit state of components is lognormal [9, 10]. Therefore, the fragility curve can be deduced from the probabilistic demand model. The fragility curve is a lognormal cumulative distribution function, and the fragility function can be expressed as

$$P_f = P[D \geq C | IM] = \Phi \left[ \frac{\ln(S_d) - \ln(S_C)}{\sqrt{\beta_{D|IM}^2 + \beta_C^2}} \right], \quad (5)$$

where  $S_d$  is the median structural demand,  $S_C$  is the average value of structural capacity,  $\beta_C$  is the logarithmic standard deviation of structural capacity, and  $\beta_{D|IM}$  is the dispersion of the seismic demand. According to HAZUS99, when the independent variable is PGA,  $\sqrt{\beta_{D|IM}^2 + \beta_C^2} = 0.5$ .

**3.1.2. Fuzzy Failure Probability Based on Membership Function.** In traditional fragility analysis methods, the damage index is usually fixed, as shown in Table 1.

In Table 1,  $C_i$  corresponds to the seismic capacity under different damage levels. In the table, the damage index of bridge components is quantified; that is, when the damage index exceeds a specific value, components are assessed to enter a new damage state. This is not consistent with the actual situation. The change of component damage state is a gradual process, and there is a fuzzy state between two adjacent damage states. For example, when the bridge pier is in transition from no damaged state to slightly damaged state, the state of the component is a fuzzy event  $\tilde{A}$ , which can be defined as

$$\tilde{A} = \left\{ \ln D \lesssim \ln C_i \right\}, \quad (6)$$

when the uncertainties of structural seismic capacity are not taken into account, the probability that structural demand exceeds structural seismic capacity can be calculated by the following equation:

$$\begin{aligned} P_f &= P[D \geq C_i | IM] = 1 - P[D < C_i | IM] \\ &= 1 - \int_{-\infty}^{\ln C_i} f(\ln D) d \ln D, \end{aligned} \quad (7)$$

where  $f(\ln D)$  is the demand probability distribution function.

Therefore, the structural fuzzy failure calculation probability equation is

$$\begin{aligned} P_f &= 1 - P(\ln D \lesssim \ln C_i) \\ &= 1 - \int_{-\infty}^{\ln C_i} u_{\tilde{A}}(\ln D) \cdot f_{\ln D}(\ln D) d \ln D. \end{aligned} \quad (8)$$

The membership function  $u_{\tilde{A}}(\ln D)$  is used to describe event  $\tilde{A}$ :

TABLE 1: Relationship between damage state and damage index.

Damage state	Damage index
No damage	$0 < D < C_1$
Slight (SL) damage	$C_1 < D < C_2$
Moderate (MO) damage	$C_2 < D < C_3$
Extensive (EX) damage	$C_3 < D < C_4$
Complete (CO) damage	$D > C_4$

$$u_{\tilde{A}}(\ln D) = \begin{cases} 1, & \text{Reliable state,} \\ \in (0, 1), & \text{Fuzzy limit state,} \\ 0, & \text{Failure state,} \end{cases} \quad (9)$$

where taking into account the uncertainties associated with seismic capacity,  $f_{\ln D}(\ln D)$  is the probability density function of the state variable  $\ln D$  and its expression is

$$f_{\ln D}(\ln D) = \frac{1}{\sqrt{2\pi} \sqrt{\beta_{D|IM}^2 + \beta_C^2}} \exp \left[ -\frac{(\ln D - \ln S_C)^2}{2(\beta_{D|IM}^2 + \beta_C^2)} \right]. \quad (10)$$

**3.2. System Fragility Analysis.** Bridge structure is composed of different components. The existing research results [11] show that bridge system is more fragile than any other component. The bridge system is actually a very complex series parallel system. However, when analyzing series parallel system, the failure process cannot be predicted. From the perspective of safety, the bridge system can be regarded as a series system for fragility analysis [12]. Therefore, the failure probability of the bridge system can be calculated by the following equations:

$$\begin{aligned} P_{\text{sys}} &= P(F_1 \cup F_2 \cup \dots \cup F_m) \\ &= \int \int \dots \int_D f(x_1, x_2, \dots, x_m) dx_1, dx_2, \dots, dx_m, \end{aligned} \quad (11)$$

where  $P_{\text{sys}}$  denotes the system failure probability;  $m$  is the number of failure modes of the bridge system;  $F_i$  ( $i = 1, 2, 3, \dots, m$ ) is the structure's  $i$ -th failure event and  $D$  is the failure domain; and  $f(x_1, x_2, \dots, x_m)$  is the multivariate joint probability density function. The whole analysis process is carried out in normal space or equivalent normal space, and the failure probability of each failure mode and the correlation coefficient between failure modes need to be used in the calculation. Therefore, for the series system, the failure probability of the bridge structure system can also be expressed by the following equation [12]:

$$\begin{aligned} P_{\text{sys}} &= 1 - \Phi_m(\beta; \rho) = 1 - \int_{-\infty}^{\beta_1} \dots \int_{-\infty}^{\beta_m} (2\pi)^{-(m/2)} |\rho|^{-(1/2)} \\ &\quad \cdot \exp\left(-\frac{1}{2} X^T \rho^{-1} X\right) dx_1, \dots, dx_m, \end{aligned} \quad (12)$$

where  $m$  is the number of failure modes for the bridge system,  $\beta = (\beta_1, \dots, \beta_m)^T$  is the reliability index corresponding to the  $i$  failure mode  $i = (1, \dots, m)$ ,  $\rho = [\rho_{ij}]_{m \times m}$  is the matrix of the correlation model for the failure mode, and  $X(x_1, x_2, \dots, x_m)$  is the standard normal random vector for the  $m$  dimension. It can be seen from equation (12) that the failure probability of the structure system is a high dimensional integral. There are three methods: direct numerical integration, boundary, and approximation. The direct numerical integration method is an exact solution, but in the case, the calculation efficiency is very low [13]. The boundary method is simple in principle and easy to operate, but when there are many failure modes and the correlation coefficient is large, the range given will be obviously wider [14–16]. In comparison, approximation method is widely used because of its high calculation accuracy, operability, and engineering requirements.

The approximate calculation method of  $\Phi_m(\beta; \rho)$  mainly includes the first-order multinormal (FOMN) method and the product of conditional marginal (PCM) method. PCM [14] was proposed by Pandey to simplify the calculation process of FOMN and improve the calculation efficiency. At the same time, Yuan and Pandey [14] analyzed the deficiency of PCM method in the calculation of reliability of series system and made improvement, thus forming the improved PCM (I-PCM) method.

In this paper, the I-PCM method is improved to provide a more efficient and accurate approximation method for calculating the failure probability of structural system. Therefore, in the next section, a detailed process of forming a system fragility analysis method based on the new improved method is presented.

Pandey proposed a PCM method based on the basic idea of conditional probability theory which is denoted as follows [17]:

$$\begin{aligned} \Phi_m(\beta, \rho) &= P\left[X_m \leq \beta_m \mid \bigcap_{k=1}^{m-1} (X_k \leq \beta_k)\right] \\ &\times P\left[X_{m-1} \leq \beta_{m-1} \mid \bigcap_{k=1}^{m-2} (X_k \leq \beta_k)\right] \times \dots \times P(X_1 \leq \beta_1) \\ &\approx \Phi[\beta_{m|(m-1)}] \times \Phi[\beta_{(m-1)|(m-2)}] \times \dots \times \Phi[\beta_{2|1}] \times \Phi(\beta_1) \\ &= \prod_{k=1}^m \Phi[\beta_{k|(k-1)}], \end{aligned} \quad (13)$$

where  $\beta_{k|(k-1)}$  is the conditional normal fractile. In the calculation process, each time a new conditional normal quintile and correlation coefficient matrix is obtained; it can also be used as the input value for the next step. The formula is as shown in equations (14) and (15).

$$\beta_{i|k} = \frac{\beta_{i|(k-1)} + \rho_{ik|(k-1)} A_{k|(k-1)}}{\sqrt{1 - \rho_{ik|(k-1)}^2} B_{k|(k-1)}}, \quad (14)$$

$$\rho_{ij|k} = \frac{\rho_{ij|(k-1)} - \rho_{jk|(k-1)} \rho_{ik|(k-1)} B_{k|(k-1)}}{\sqrt{1 - \rho_{jk|(k-1)}^2} B_{k|(k-1)} \sqrt{1 - \rho_{ik|(k-1)}^2} B_{k|(k-1)}}, \quad (15)$$

where  $k = 1, \dots, m-1; i, j = k+1, \dots, m$

$$A_{k|(k-1)} = \frac{\phi(\beta_{k|(k-1)})}{\Phi(\beta_{k|(k-1)})}, \quad (16)$$

$$B_{k|(k-1)} = A_{k|(k-1)} [\beta_{k|(k-1)} + A_{k|(k-1)}].$$

Equations (13)–(15) describe the calculation process of PCM in detail. However, Yuan and Pandey [14] used the numerical integration method to analyze the error of the PCM method and found that this method over quantizes the failure probability of the complex series system. The following formula is used to improve the PCM:

$$\beta_{i|k} = \Phi^{-1} \left[ 1 - \frac{\Phi[-\beta_{i|(k-1)}] - [-\beta_{i|(k-1)}, -\beta_{k|(k-1)}; \rho_{ik|(k-1)}]}{\Phi[\beta_{k|(k-1)}]} \right], \quad (17)$$

$$\Phi[-\beta_{i|(k-1)}, -\beta_{k|(k-1)}; \rho_{ik|(k-1)}] \approx \Phi(c_{i|k}) \Phi(c_{k|(k-1)}), \quad (18)$$

$$c_{i|k} = \frac{[-\beta_{i|(k-1)} + \rho_{ik|(k-1)} D_{k|(k-1)}]}{\sqrt{1 - \rho_{ik|(k-1)}^2} D_{k|(k-1)} [-\beta_{i|(k-1)} + D_{k|(k-1)}]}, \quad (19)$$

$$D_{k|(k-1)} = \frac{\phi[c_{k|(k-1)}]}{\Phi[c_{k|(k-1)}]}, \quad (20)$$

$$c_{k|(k-1)} = -\beta_{k|(k-1)}. \quad (21)$$

Equations (17)–(21) is the calculation process of the I-PCM method. Compared with the PCM method, the I-PCM method uses different expressions to calculate the two-dimensional normal distribution function, thus improving the accuracy of failure probability calculation of series system. However, both methods use the same approximation method to calculate the two-dimensional normal distribution function. If more accurate methods are used to calculate the two-dimensional normal distribution function, then the calculation accuracy of PCM and I-PCM can be fundamentally improved.

According to the research results of Kotz et al. [18], the two-dimensional normal distribution function  $\Phi(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}; \rho_{ik|(k-1)})$  can be transformed into the following one-dimensional integral expression:

$$\begin{aligned} \Phi(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}; \rho_{ik|(k-1)}) &= \Phi(-\beta_{i|(k-1)}) \Phi(-\beta_{k|(k-1)}) \\ &+ \int_0^{\rho_{ik|(k-1)}} \phi(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}, t) dt. \end{aligned} \quad (22)$$

Therefore, equation (17) can be rewritten as

$$\begin{aligned}\beta_{i|k} &= \Phi^{-1} \left[ \frac{\Phi(\beta_{i|(k-1)}) + \Phi(\beta_{k|(k-1)}) + \Phi_2(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}, \rho_{ik|(k-1)}) - 1}{\Phi(\beta_{k|(k-1)})} \right] \\ &= \Phi^{-1} \left[ \frac{\Phi(\beta_{i|(k-1)}) + \Phi(\beta_{k|(k-1)}) + \Phi(-\beta_{i|(k-1)})\Phi(-\beta_{k|(k-1)})}{\Phi(\beta_{k|(k-1)})} + \frac{\int_0^{\rho_{ik|(k-1)}} \phi(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}, t) dt - 1}{\Phi(\beta_{k|(k-1)})} \right],\end{aligned}\quad (23)$$

where  $\int_0^{\rho_{ik|(k-1)}} \phi(-\beta_{i|(k-1)}, -\beta_{k|(k-1)}, t) dt$  can be calculated by Simpson numerical integration method.

The accuracy of conditional normal fractile obtained by equation (23) is higher than that obtained by approximate expression of equations (14) and (17). At the same time,  $B_{k|(k-1)}$  can be revised by the following equation [19]:

$$B_{k|(k-1)} = \frac{1}{\rho_{ik|(k-1)}^2} - \frac{(\beta_{i|(k-1)} + \rho_{ik|(k-1)} A_{k|(k-1)})^2}{\rho_{ik|(k-1)}^2 \beta_{i|k}^2}. \quad (24)$$

Substituting equation (24) into equation (15) yields a modified matrix of correlation coefficients. Compared with the PCM method and I-PCM method, the new improved PCM method improves the calculation of conditional normal fractile and correlation coefficient, which can further improve the calculation accuracy.

The flow chart of system fragility analysis is illustrated in Figure 1. Figure 2 summarizes the detailed steps of bridge seismic fragility analysis method based on fuzzy failure criteria.

## 4. Case Study

**4.1. Description.** In this study, a typical three-span V-shaped continuous girder bridge is analyzed. As illustrated in Figure 1, the total length of the three-span bridge is 200 meters, consisting of two sides span (55 meters) and a middle span (90 m). The 22.5 m wide deck is supported by V-shaped beams, and the height of these beams from the bottom to the top is 22.7 m. Hollow thin-walled box girder is adopted in the section of V beam V-shaped beams to reduce weight and improve stress. The middle beam connects the top of the V-shaped beam with the main beam and the V-shaped beams intersect at the top of the main pier to form an arch abutment. The V-angle is about 80°. The support occurs underneath the arch abutment. Figure 3 shows the overall layout and structural form of the bridge.

**4.2. Finite Element Analysis Model.** The 3D finite element model used in this study was established in OpenSees. It is assumed that the box girder and slab maintained elasticity under the earthquake in the longitudinal direction, though nonlinearities of the V-shaped beams and transitional piers are considered. Thus, the box girder and slab were simulated by linear beam-column elements and the V-shaped beams and transitional piers were simulated by nonlinear fiber elements. It is suggested that bilinear elastic-plastic spring elements can be used to model the pot bearings [20]. Rigid links were also used to connect the V-shaped beams

to the box girder. In OpenSees, the behaviour of the pot bearings is represented using a zero-length element with a bilinear model for material behaviour. Figure 4 show the force-deformation skeleton curves of the pot bearings in this study.

This investigation uses the constitutive model of confined and unconfined concrete used in the Kent-Scott-Park model [21]. The steel bar adopts the Giuffr -Menegotto-Pinto constitutive model [22]. The longitudinal reinforcement ratio of the V-shaped beams is 0.89% to 1.08%, and the longitudinal reinforcement ratio of the transitional piers is 0.8%.

As shown in Figure 3, finite element software analysis was carried out to find out the locations of demonstrating nonlinear behaviours in time history analysis. These positions (section 1-1 to 5-5) were simulated by the pivot model. The areas outside the plastic-hinge length were assumed to be linear elastic under seismic action. In this paper, the bilinear moment-rotation ( $M - \theta$ ) relationship can be used to approximate the skeleton curve of the pivot model. Assuming a constant curvature over the length of the plastic hinge ( $L_p$ ), the angle of rotation can be calculated as  $\theta = \phi L_p$ , which can be calculated by [23].

$$L_p = 0.08L + 0.022f_y d_{bl} \geq 0.044f_y d_{bl}, \quad (25)$$

where  $L$ =length from the point of contra-flexure to the section of maximum moment and  $d$ =diameter of a longitudinal reinforcing bar.

### 4.3. Uncertainty

**4.3.1. Uncertainty of Ground Motion.** Ground motion has strong randomness, and different seismic excitation will have a unique impact on seismic response. The seismic responses of structures under different seismic waves will also be different [24–26]. Therefore, in order to ensure the rationality of the nonlinear time history analysis results, the input ground motion data should be chosen reasonably.

According to the ‘‘Guidelines for Seismic Design of Highway Bridges in China’’ [26], the site type of the bridge in this example is Class III. The seismic fortification intensity of the proposed site is 8 degrees.

This paper selected strong ground motions from the Pacific Earthquake Engineering Research Center Ground Motion Database (PEER) of the United States, which were consistent with the site conditions of the case bridge and the magnitude  $M > 5.5$  and the epicentral distance  $R > 20$  km. Considering the code difference between China and the United States, equations [27] are applied to produce VS30 (representing the shear

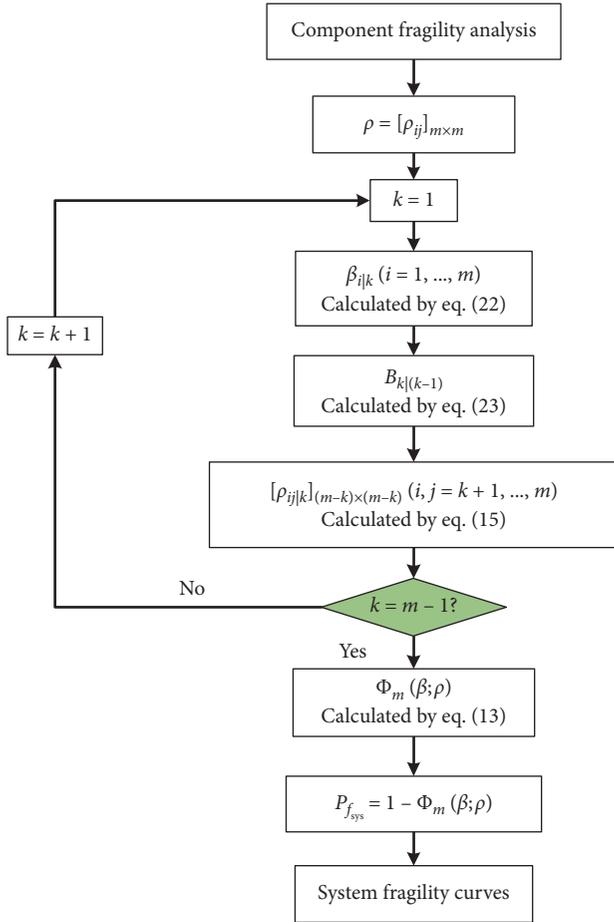


FIGURE 1: Flow chart of structural system fragility curves based on the improved method.

wave velocity of the 30 m deep soil layer) which accords with regional site conditions of the case bridge.

Ground motions can be described by such intensity indices as peak ground acceleration (PGA), peak ground velocity (PGV), and response spectrum  $S_a(T, 5\%)$  corresponding to the period. Padgett et al. [28] evaluated the efficiency, applicability, and computability of hazards of IM and showed that PGA is a more suitable index of ground motion intensity. Therefore, PGA is chosen as the intensity index of ground motion. For the sake of simplification, this paper only studies the response of bridges under longitudinal seismic action.

Figure 5 shows the spectral acceleration curves for each seismic wave with an amplitude-modulated damping ratio of 5%. Figure 6 shows the distribution of the ground motions at various peak ground acceleration levels.

**4.3.2. Uncertainty of Structural Parameters.** According to the structural characteristics of a V-shaped continuous girder bridge, the research of Pan et al. [6], and domestic construction technology, this paper establishes bridge samples from factors such as the compressive strength of concrete, yield strength of steel, and the unit weight of concrete. The probability distribution of variables [2] is presented in Table 2.

**4.3.3. Establishment of Bridge-Ground Motion Analysis Samples.** After determining the uncertain variables of bridge structure, these four parameters were randomly combined by the Latin hypercube sampling method, and 10 groups of bridge sample parameters were obtained. Then 10 samples are randomly selected from 100 selected ground motions and assigned to each group of bridge samples. Finally, the bridge seismic analysis software OpenSees is used to build the nonlinear time history analysis model.

**4.4. Seismic Demand Analysis.** The program OpenSees is used to analyze 100 pairs of ground motion-structure samples. The seismic response of the bridge structure can be measured based on the curvature ductility or displacement of each component. For space reasons, the logarithmic regression analysis of demand response value and peak acceleration PGA of transition pier and P1 bearing is given, as shown in Figure 7. Seismic demand can be calculated using the fitting function in Table 3.

## 5. Damage Index

Damage index is a dimensionless index to evaluate the damage state of a structure or component after receiving earthquake action. It is an important theoretical basis for evaluating the seismic risk loss of a bridge and making decision. HAZUS and ATC-40 [29] are graded to assess the damage. The macroscopical descriptions of the damage at all levels are given, as shown in Table 4.

**5.1. Damage Index of Concrete Structure.** According to the description in Table 4, the damage grade of the structure is divided into no damage, slight damage, moderate damage, extensive damage, and complete damage. Since the curvature ductility index can be applied to both conventional bridges and thin-walled long-span structures, it has wider applicability than displacement ductility index. In this paper, moment curvature analysis of V-shaped beams and transition piers is carried out. Curvature ductility coefficient is used to define damage state. The curvature ductility coefficient is the ratio of the maximum curvature of the section to its first yield curvature under earthquake action. The damage index of V-shaped beams and transition piers is shown in Table 5.

**5.2. Damage Index of Bearings.** Bridge bearings are the weak parts of bridges. It is necessary to study the seismic fragility of bridge bearings. Previous studies [30] have shown that the damage degree of the bearing is directly related to its deformation, so this paper takes the deformation of the bearing as an index to measure its damage degree. The bridge in this paper adopts pot bearings. From the perspective of bridge design, the allowable longitudinal bridge displacement is 0.2 m. Due to the lack of research on damage index of pot bearings, the relative displacement is taken as shown in Table 5 with reference to the relevant literature [3].

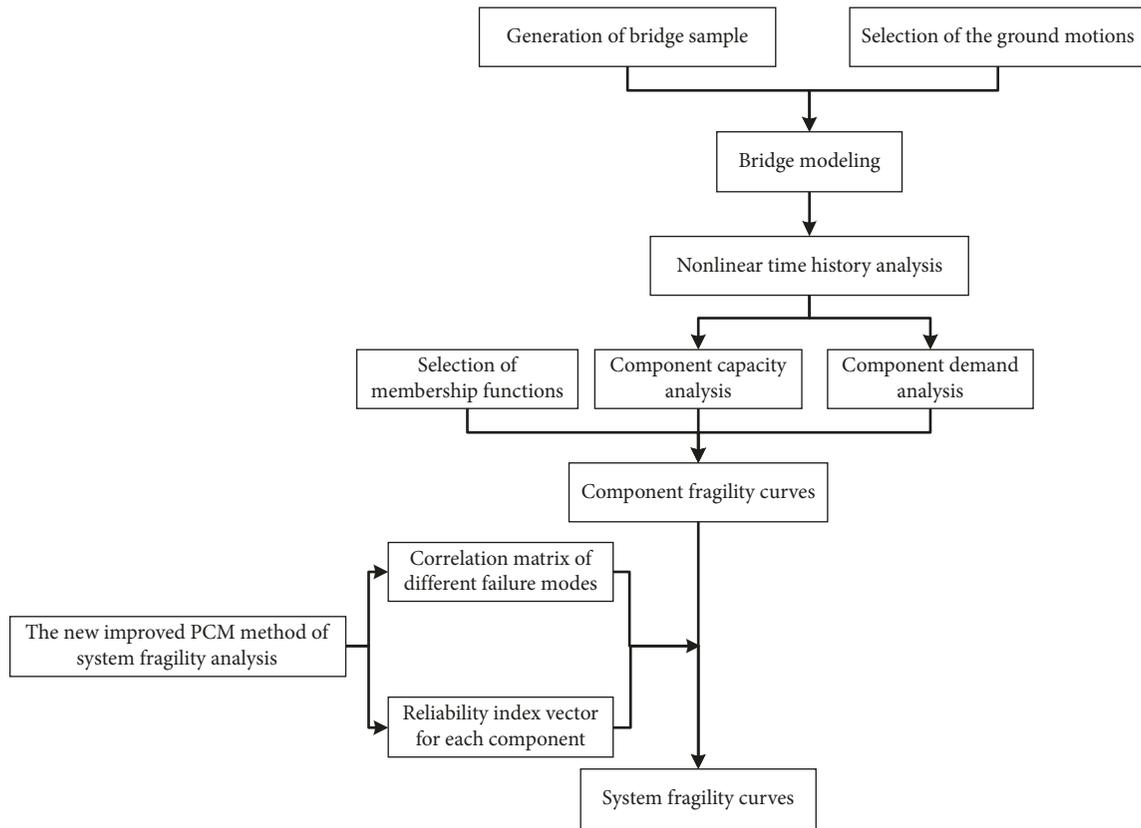


FIGURE 2: Flow chart of developing system fragility curves.

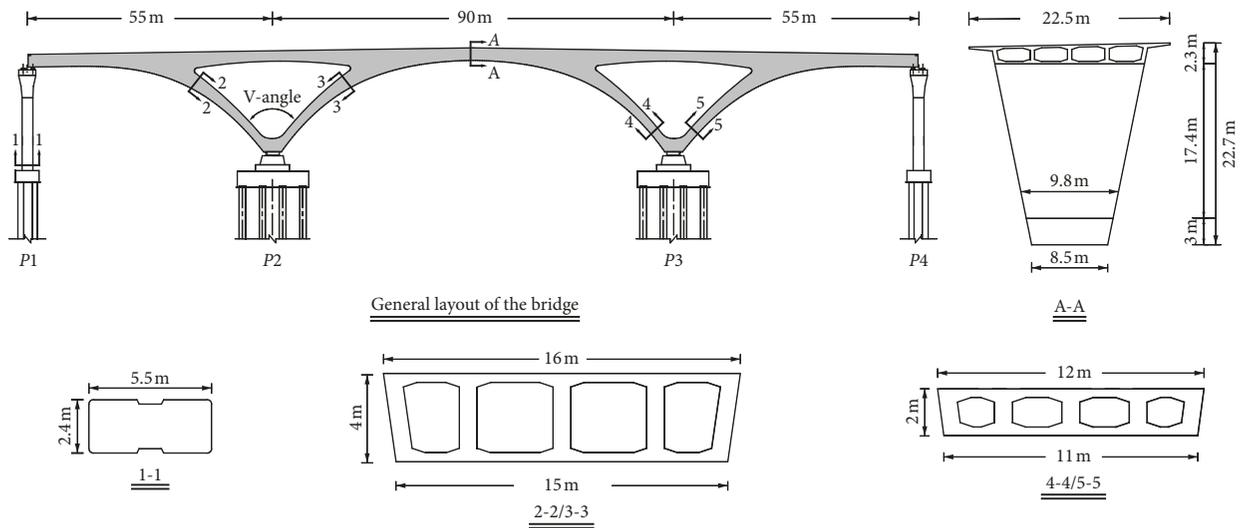


FIGURE 3: Configuration of the three-span V-shaped continuous girder bridge.

## 6. Seismic Fragility Analysis Based on Fuzzy Failure Criteria

6.1. *Selection of Membership Functions.* The choice of membership function is very important for fragility analysis of bridges. The membership functions are mainly divided into two aspects: the determination of the

function formula and the determination of the membership interval.

The failure probability of bridge seismic fragility study is considered from the aspect of structural reliability. The membership function describes the membership degree of security in a certain interval. The initial state of components is entirely safe. With the increase of structural

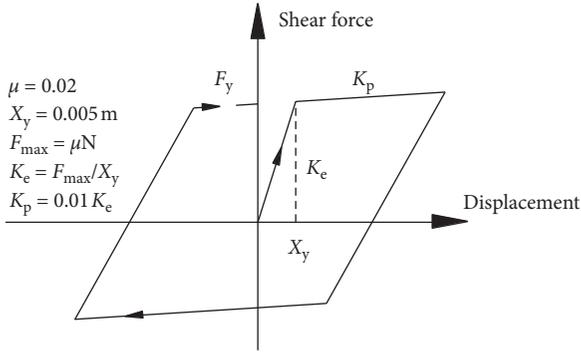


FIGURE 4: Mechanical model of the pot bearings.

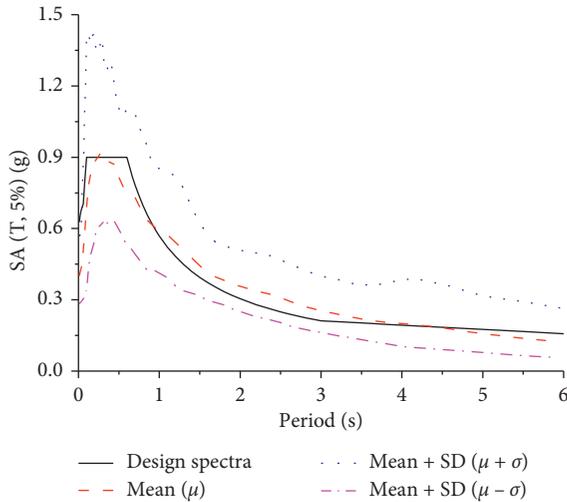


FIGURE 5: Response spectra for selected records and the standard design spectra.

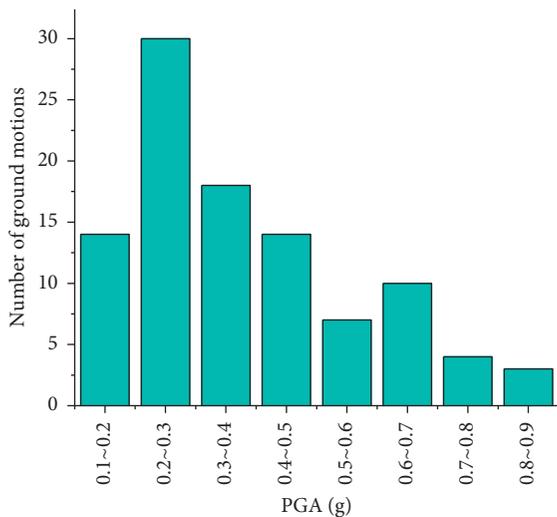


FIGURE 6: Distribution of PGA of the 100 ground motions.

requirements, the degree of membership for safety is becoming smaller and smaller. Therefore, in the seismic fragility analysis, the type of membership function is

deflection minor type. The deflection minor membership functions commonly used in engineering are as follows: seminormal distribution, semitrapezoidal distribution, and semiridged distribution. The deflection minor membership functions used in engineering are shown in Table 6. This paper will compare the effects of different membership functions on fragility curves.

There are many ways to determine the membership interval, and the amplification coefficient method is used in this paper. Based on the accumulated experience of conventional design specifications, the amplification coefficient method determines the upper and lower bounds of the transition interval by introducing the amplification coefficient based on the allowable values given by the design specifications. The amplification coefficient is usually taken as 0.05 to 0.3 times the allowable value [27]. This paper takes 0.3, that is,  $a_1 = 0.7C_i$ ;  $a_2 = 1.3C_i$ .

**6.2. Fragility Analysis of Components Based on Fuzzy Failure Criteria.** In this paper, three forms of membership functions are selected, and the membership interval is determined by the amplification coefficient method commonly used in engineering. In this section, based on the deterministic failure criterion and the fuzzy failure criterion, the seismic peak ground acceleration (PGA) is selected as the intensity measure to analyze the fragility of the bridge member under the action of the longitudinal bridge. After the analysis of the fragility of the bridge, the upper structure of the bridge and the V-shaped beams remain elastic during the earthquake, and no curvature ductile damage occurs. V-shaped beams are the least fragile components, while transition piers and bearings are the most fragile to earthquakes. Therefore, this paper gives the fragility curves of bridge members when using three membership functions, as shown in Figure 8.

It can be seen from Figure 8 that the fragility curves of the piers without considering the fuzzy failure criterion are obviously different from those with considering the fuzzy failure criterion. Without considering the fuzzy criterion, the probability of seismic damage exceedance of the structure is sometimes greater than that of the structure considering the fuzzy criterion. This may be due to the fact that under the condition of the fuzzy damage criterion, the original damage sample data are not damaged under the fuzzy definition. At the same time, seismic fragility analysis without considering the fuzziness of damage boundary often overestimates the probability of certain damage level of structure under earthquake action. This is because (1) the membership function changes the probability distribution of structural demand and (2) for example, incomplete damage, critical state, and incomplete slight damage state exist between the no damaged state and slightly damaged state [31]. The existence of these states leads to the increase of failure probability of bridge components.

At this time, the higher value of the fuzzy failure probability more objectively shows the actual failure probability of components and more exposes the potential

TABLE 2: Parameter of variable statistics.

Parameters	Distribution	Mean	COV (%)
Compressive strength of the transitional piers, $f_{c,piers}$ (MPa)	Normal	27.31	20
Compressive strength of V-shaped beams, $f_y$ (MPa)	Normal	31.08	20
Steel yield stress, $f_{c,beams}$ (MPa)	Lognormal	433.96	7
Unite weight of concrete, $W$ (kN/m <sup>3</sup> )	Normal	26.25	10

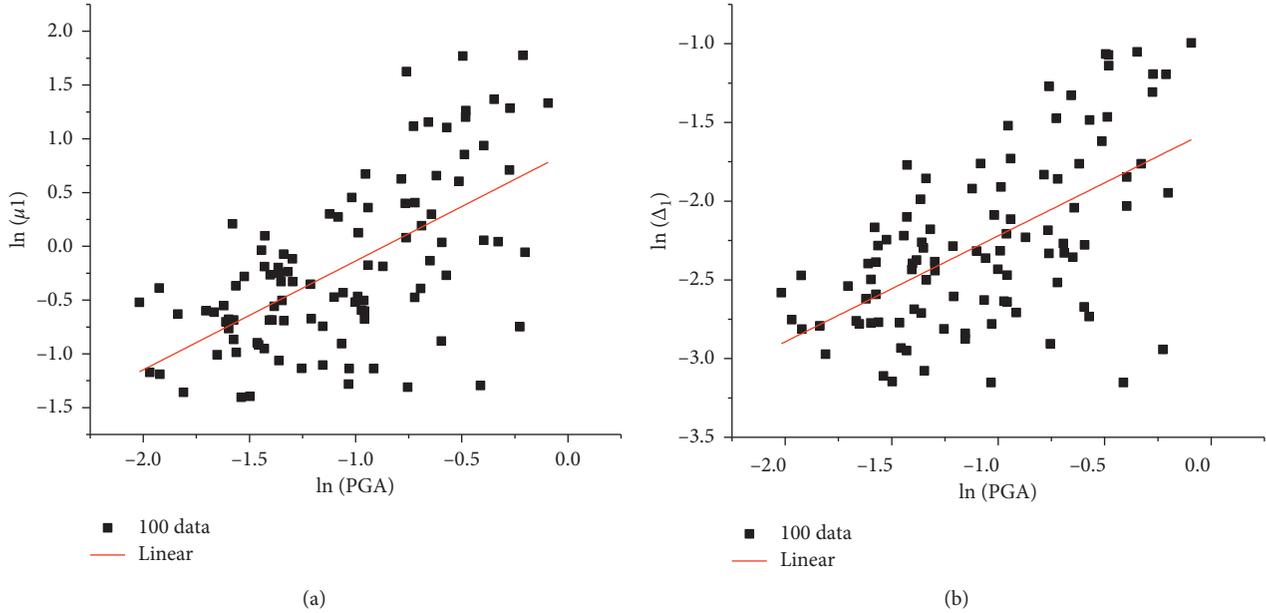


FIGURE 7: Regression analysis of different component response under earthquake excitation. (a) Section 1-1. (b) Bearing\_P1.

TABLE 3: Probabilistic demand model for seismic response at each critical component location.

Component	Fitting function	$R^2$	$\beta_{D IM}$
Section 1-1	$\ln(\mu_1) = 1.0110 \ln(PGA) + 0.8749$	0.3709	0.6148
Section 2-2	$\ln(\mu_2) = 1.1395 \ln(PGA) - 2.4829$	0.3596	0.7098
Section 3-3	$\ln(\mu_3) = 0.3966 \ln(PGA) - 3.1566$	0.4899	0.6552
Section 4-4	$\ln(\mu_4) = 0.8594 \ln(PGA) - 2.1799$	0.3080	0.5999
Section 5-5	$\ln(\mu_5) = 0.7104 \ln(PGA) - 1.5051$	0.2883	0.5192
Bearing_P1	$\ln(\Delta_1) = 0.6731 \ln(PGA) - 1.5469$	0.3306	0.4465
Bearing_P2	$\ln(\Delta_2) = 0.6826 \ln(PGA) - 1.1654$	0.3263	0.4572

TABLE 4: Description of bridge damage.

Damage state	Damage index
No damage	—
Slight (SL) damage	The pier and abutment are slightly cracked and need not be repaired
Moderate (MO) damage	The pier is cracked but still safe, with displacement at the top or bottom
Extensive (EX) damage	The large area of concrete spalling, the joint damage, and displacement
Complete (CO) damage	Pier collapse and girder fall

TABLE 5: Damage index under different damage states.

Component	Damage index (unit)	Damage state			
		SL	MO	EX	CO
V-shaped beams/ transition piers	Curvature ductility ratio	1	2	4	7
Bearings	Displacement (m)	0.2	0.4	0.6	0.8

safety hazards of bridge structures. Therefore, it is necessary to consider the influence of fuzzy failure criterion on seismic fragility analysis of structures.

6.3. *Fragility Analysis of Bridge System Based on Fuzzy Failure Criteria.* The bridge structure is a complex system consisting of superstructures, bearings, piers, abutments, and other interacting components. In the previous section, it is not enough to obtain the fragility curves of components. The fragility curves can be a good indicator of the overall seismic performance of the bridge. Therefore, this section

TABLE 6: Deflection minor membership function.

Category	Expression
Seminormal	$u(x) = \begin{cases} 1, & x \leq a, \\ e^{-((x-a)/b)^2}, & x > a, b > 0, \end{cases}$
Semitrapezoidal	$u(x) = \begin{cases} 1, & x \leq a_1, \\ (x - a_2)/(a_1 - a_2), & a_1 < x \leq a_2, \\ 0, & x > a_2, \end{cases}$
Semiridged	$u(x) = \begin{cases} 1, & x \leq a_1, \\ (1/2) - (1/2)\sin(\pi/(a_2 - a_1)) \cdot (x - ((a_2 + a_1)/2)), & a_1 < x \leq a_2, \\ 0, & x > a_2, \end{cases}$

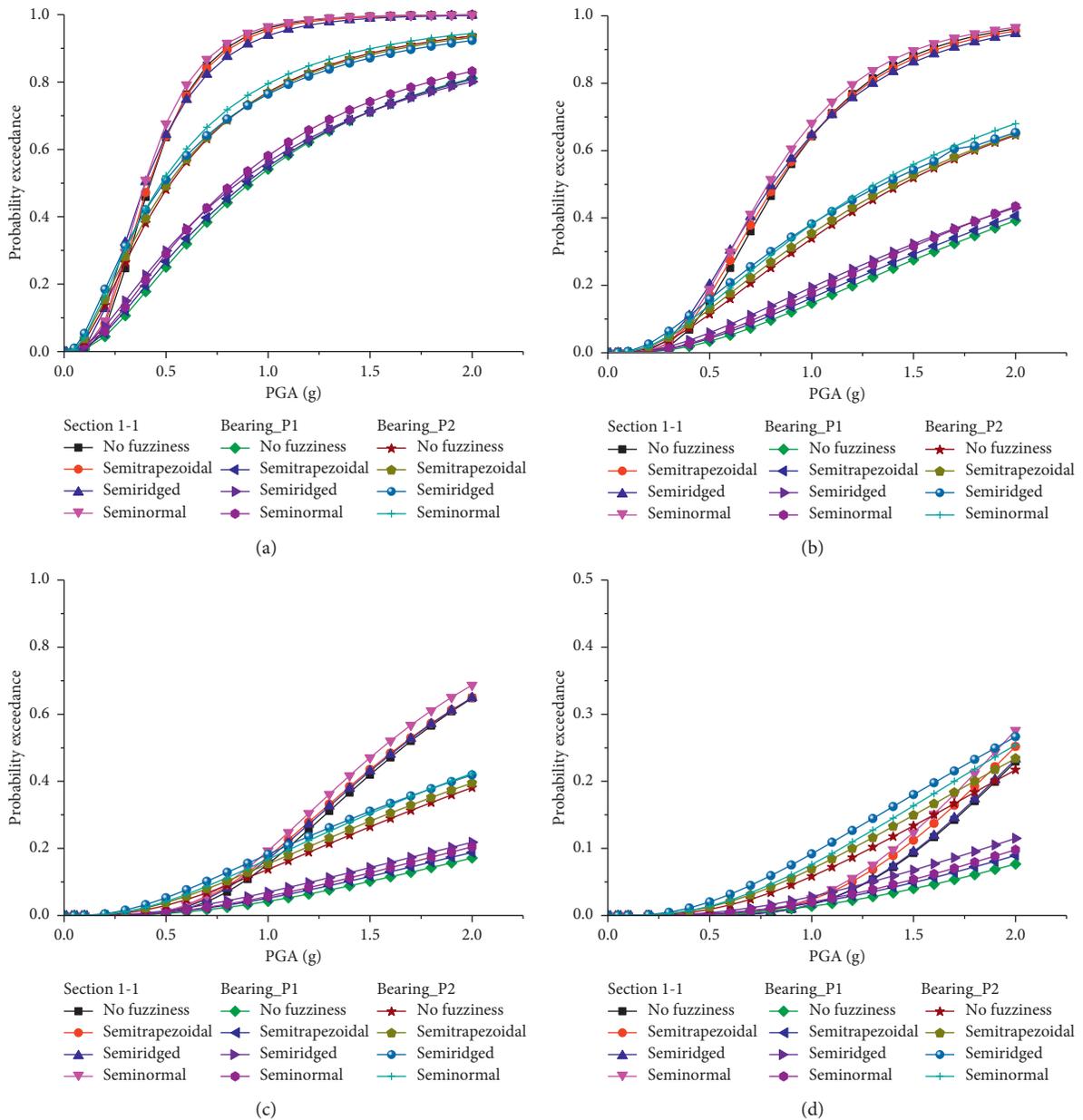


FIGURE 8: Fragility curves based on fuzzy failure criteria. (a) Slight damage. (b) Moderate damage. (c) Extensive damage. (d) Complete damage.

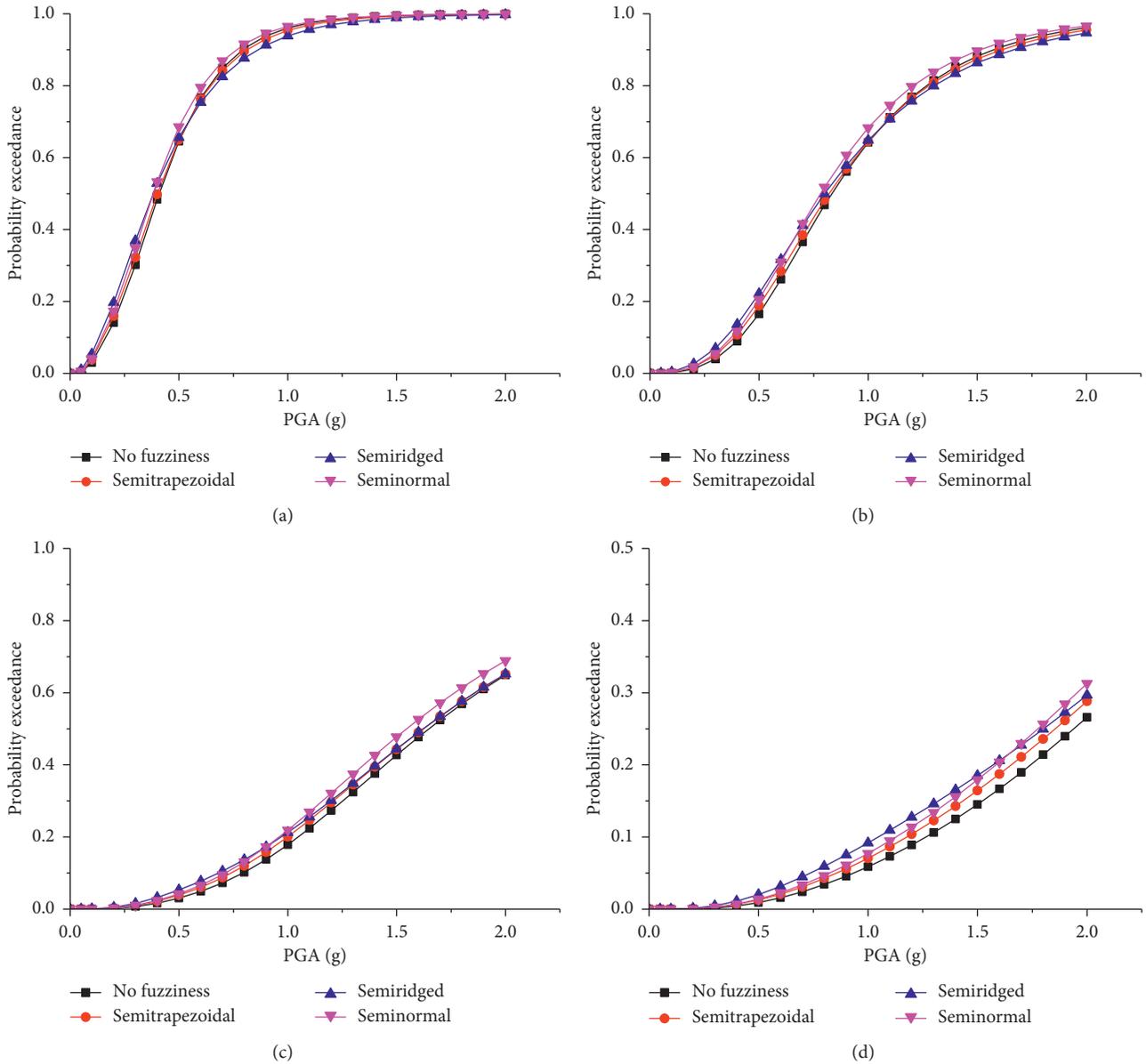


FIGURE 9: System fragility curves based on fuzzy failure criteria. (a) Slight damage. (b) Moderate damage. (c) Extensive damage. (d) Complete damage.

uses the new improved system fragility calculation method, which gives the bridge system fragility curves when using three membership functions, as shown in Figure 9.

It can be seen from Figure 9 that the fragility analysis of bridge system without considering the fuzzy failure criterion tends to overestimate the seismic performance of the bridge. In the slight damage state,  $PGA \geq 0.5$  g, and in the moderate damage state,  $PGA \geq 1.0$  g; the system failure probability without considering fuzzy failure is higher than the system failure probability considering fuzzy failure, but in the extensive damage and complete damage state, the failure probability is greater than the traditional fragility analysis method. Therefore, it is very necessary to comprehensively consider the

fuzzy failure criterion in the fragility analysis of bridge system.

## 7. Conclusion

Aiming at the characteristics of traditional fragility analysis methods for bridge system, this paper firstly introduces the fuzzy random theory and simplifies the gradual change of damage boundary with membership function. Then, the corresponding fuzzy failure criteria are derived from the classical reliability theory. Finally, taking a three-span V-shaped continuous girder bridge as an example, considering the uncertainties of bridge structure and seismic load, the fragility analysis of the bridge system is carried out by the new improved method. The following conclusions are obtained:

- (1) This paper presents an alternative system fragility framework based on a new improved PCM method. Compared with PCM and I-PCM, this framework can provide an efficient and accurate approach to developing system fragility function.
- (2) It is possible to underestimate the potential seismic fragility of structures without considering the fuzzy criteria in the fragility analysis of bridge components and system. Therefore, it is necessary to consider the influence of the fuzzy failure criteria on the failure probability of structures in the seismic fragility analysis.
- (3) This paper compares the fragility curves based on different membership functions. The results show that when the seismic demand parameters are the curvature and displacement of the structural section, the fragility curve risks of the components with different membership functions are different. By comparing the fragility curves of the bridge system under four damage states, the failure probability of the bridge system obtained by the seminormal membership function is higher than that obtained without considering the fuzzy failure criterion.

### Data Availability

The data supporting the conclusions of the study are included within the article.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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