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Research Article

A Simplified Calculation Method of Length Adjustment of Datum Strand for the Main Cable with Small Sag

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In order to overcome the complicated iterative process of the cable length adjustment based on catenary theory and large error of length adjustment for cable with a small sag based on the parabola theory, this paper firstly develops a direct and simply calculation method based on parabola theory, considering the influence of elastic elongation on the cable unstressed length, which can apply for datum strand of the main cable and catwalk bearing rope with a small sag to improve the construction accuracy of the datum strand of suspension bridges. Then, the applicability of the proposed cable length adjustment formula under different conditions of the sag-span ratio is analyzed and compared with other calculation methods based on the theory of catenary or parabola. Finally, numerical examples are presented and discussed to illustrate the accuracy and efficiency of the proposed analytical method.

1. Introduction

Cable-supported structures such as suspension bridges have been recognized as the most appealing structures due to their aesthetic appearance as well as the structural advantages of the cables [1–6]. It is well known that cables cannot behave as structural members until large tensioning forces are induced. Therefore, in order to design a cable-supported structure economically and efficiently, it is extremely important to determine the optimized initial cable tensions or unstrained lengths.

Generally, designers cannot determine the initial shape arbitrarily when cable structures are considered. The initial shape is determined to satisfy the equilibrium condition between dead loads and internal member forces including cable tensions in the preliminary design stage, because cable members display strongly geometric nonlinear behaviors and the configuration of a cable system cannot be defined in the stress-free state. The process determining the initial state of cable structures is referred to as "shape finding," "form finding," or "initial shape or initial configuration" [7–13].

Cables in cable-supported structures present highly nonlinear behavior, so there have been various studies of the cables. Cable analysis can be separated into two categories: parabolic approach and catenary approach.

The catenary approach aims at obtaining the exact solutions of cable behavior. This approach was originally presented by O'Brien and Francis [14]. Irvine et al. subsequently derived the flexibility matrix of a two-dimensional inclined cable [15, 16]. For the analysis of suspension bridges, three-dimensional catenary cable elements were later developed by several researchers [11, 17]. In particular, there are two catenary-type analytical elements available which can be used to model the cables with a large sag in suspension bridges: (1) *inextensible catenary elements*: the cable elements adopted are infinitely stiff in the axial direction and cannot experience any increment of the length; (2) *elastic catenary elements*: an elastic catenary curve is defined as the curve formed by a perfectly elastic cable, which obeys Hooke's law and has negligible bending resistance when suspended from its ends and subjected to gravity.

In contrast, the parabola approach provides an approximate solution. To account for a cable's sag effect, Ernst proposed the equivalent modulus of elasticity for a parabolic cable [18]. The simplicity of Ernst's formula has made it widely used not only in the research field but also for practical designs of cable-supported structures such as suspension bridges. Later, Ren et al. [19] proposed a two-dimensional horizontal parabolic cable element that includes the vertical stiffness as well as the horizontal stiffness determined by Ernst's formula.

Generally, the main cable should be erected before the installation of the main girder for earth-anchored suspension bridges; the configuration of the main cable under construction stage is very important. In order to reach the initial configuration of the main cable, the configuration of the datum strand should be controlled precisely. The key to precise control is how to calculate the cable length adjustment and what is the relationship between cable length adjustment ΔS and sag adjustment Δf [20].

Based on the abovementioned cable analysis approaches, the cable length adjustment can be calculated using a parabolic or catenary approach. The calculation of cable length adjustment based on catenary theory can provide exact results, but a complicated iterative method must be used [20]. The calculation formula of cable length adjustment based on quasi-catenary theory (adopt inextensible catenary elements) can represent explicitly by the ratio (c) of applied distribution load to the horizontal component of cable force, but the solution of c also need to use a complicated iterative method. The cable length adjustment based on parabola theory is a direct method, but the error of the adjustment amount is large when the sag is small (generally, the sag-span ratio is less than 1/30), since the effect of elastic elongation on the unstressed cable length of the cable strand is not considered.

In order to overcome the complicated iterative process of the cable length adjustment based on catenary theory and large error of length adjustment for cable with small sag based on the parabola theory, we aim to find a simple and direct calculation method having both high search efficiency and accuracy. This paper starts from the basic principle of parabolic theory, considering the influence of elastic elongation on the unstressed cable length, and establishes a simplified and direct calculation method which can apply for datum strand of the main cable and catwalk bearing rope with a small sag. Then, the applicability of cable length adjustment formula based on parabolic theory is analyzed under different conditions of sag-span ratios. Finally, numerical examples are presented and discussed to illustrate the accuracy and efficiency of the proposed analytical method.

2. The Complete Solution of Unstressed Cable Length Based on Parabolic Theory

2.1. Basic Equations. Following the theory of Irvine [15] and on the basis of the assumption of a parabolic cable, the self-weight is distributed uniformly along the horizontal direction, and the ratio of the sag at the midpoint to the horizontal length is kept relatively small, that is, 1/8 or less. Furthermore, the cable's cross section, elastic modulus, and density are considered to be constant along its length, and the cable is under a small strain. The geometry of an inclined parabolic cable considered in this study is shown in Figure 1; a uniform cable is suspended between two rigid supports with a horizontal distance of l, a vertical height difference of h, a sag of f at midspan, and subjected to a uniform distributed load q along the horizontal length of the cable; the value of q can be calculated as follows:

$$q \approx \gamma \cdot A \cdot \frac{\sqrt{h^2 + l^2}}{l} = \frac{\gamma A}{\cos\theta}, \tag{1}$$

where E is the elastic modulus of the suspension cable; A and γ are the cross-sectional area and the density of the suspension cable, respectively; and θ is the horizontal angle of the connecting line between two ends of the suspension cable.

Based on a parabolic configuration, the cable ordinate with respect to the x-axis, shown in Figure 1, is expressed as follows:

$$y = \frac{4f}{l^2} x(l - x) + \frac{h}{l} x. \tag{2}$$

The sag f is calculated from $ql^2/8H$, in which H is the horizontal component of the cable force, because $q \cdot l$ represents the total weight, and its value does not vary as the shape changes and it can be replaced by the self-weight $w (w = \gamma \cdot A)$ per unit unstressed length multiplied by the unstressed length S_0. Thus, the sag f can be expressed as follows:

$$f = \frac{ql^2}{8H} = \frac{(wS_0)l}{8H}. \tag{3}$$

The total length S and its elastic elongation ΔS_q of a parabolic cable (stressed cable length) can be presented as equations (4) and (5), along with equations (2) and (3):

$$S = \int_0^l \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \frac{l^2}{16f}\left(D_1\sqrt{1 + D_1^2} - D_2\sqrt{1 + D_2^2} + \ln\frac{D_1 + \sqrt{1 + D_1^2}}{D_2 + \sqrt{1 + D_2^2}}\right), \tag{4}$$

$$\begin{aligned}\Delta S_q &= \int_0^l \varepsilon \cdot ds = \int_0^l \frac{H}{EA}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ &= \frac{H}{EA}\int_0^l \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \frac{H}{EA}\left(l + \frac{16f^2}{3l} + \frac{h^2}{l}\right) \\ &= \frac{\gamma l^2}{8E \cdot f \cdot \cos\theta}\left(l + \frac{16f^2}{3l} + \frac{h^2}{l}\right),\end{aligned} \tag{5}$$

where $D_1 = h + 4f/l$ and $D_2 = h - 4f/l$.

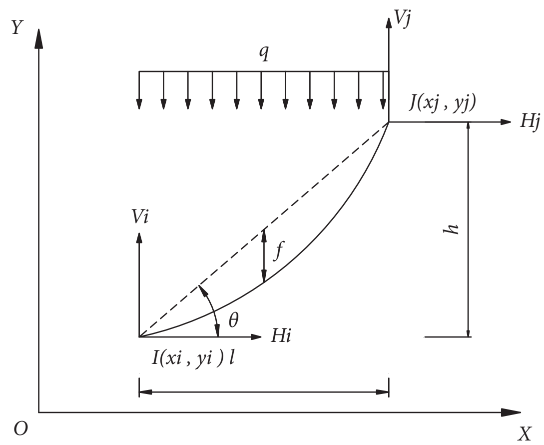

Figure 1: A parabolic cable segment.

Therefore, the unstressed length S_0 can be determined as follows:

$$\begin{aligned} S_0 &= S - \Delta S_q, \\ &= \frac{l^2}{16f}\left(D_1\sqrt{1+D_1^2} - D_2\sqrt{1+D_2^2} + \ln\frac{D_1+\sqrt{1+D_1^2}}{D_2+\sqrt{1+D_2^2}}\right) \\ &\quad - \frac{\gamma l^2}{8E\cdot f\cdot\cos\theta}\left(l + \frac{16f^2}{3l} + \frac{h^2}{l}\right). \end{aligned} \tag{6}$$

2.2. Relationship between Cable Length Adjustment and Sag Variation. We consider the unstressed length S_0 in equation (6) as a function of cable sag f; differentiating equation (6) with respect to f, we get

$$\begin{aligned} \frac{dS_0}{df} &= \frac{l^2}{16f}\left[D_2D_4 - D_1D_3 + \ln(D_6) - \ln(D_5) + D_7\right] \\ &\quad + \frac{\gamma l}{8E\cdot f^2\cdot\cos\theta}\left(h^2 + l^2\right) - \frac{2\gamma l}{3E\cdot\cos\theta}, \end{aligned} \tag{7}$$

where $D_3 = \sqrt{1+D_1^2}$, $D_4 = \sqrt{1+D_2^2}$, $D_5 = D_1 + D_3$, $D_6 = D_2 + D_4$, and $D_7 = [((D_3 + D_4)\ (1 + 1/(D_3D_4))) + (D_1^2/D_3) + (D_2^2/D_4)]\cdot(4f/l)$.

3. Simplified Calculation of Cable Length Adjustment Based on Parabola Theory

3.1. Traditional Simplified Cable Length Adjustment Formula without considering the Effect of Elastic Elongation. The total length S of parabolic cable in equation (4) can be expanded as series of (f/l) in the following equation, and only the first two items are adopted [21]:

$$S = l\left[\frac{1}{\cos\theta} + \frac{8\cos^3\theta}{3}\cdot\left(\frac{f}{l}\right)^2\right]. \tag{8}$$

Equation (8) is called the traditional simplified cable length formula without considering the effect of elastic elongation.

Differentiating equation (8) with respect to f, we obtain

$$\frac{dS}{df} = \frac{16\cos^3\theta}{3}\cdot\frac{f}{l}. \tag{9}$$

Equation (9) is the traditional simplified cable length adjustment formula without considering the effect of elastic elongation, only the first two items of the expansion series are considered, and the value of f/l is required small enough (less than 1/8) to ignore the high-order terms. The literature [22] pointed out that the error is relatively large when calculating the cable length adjustment of the side span (the sag is small) using equation (9).

3.2. Improved Cable Length Formula considering the Effect of Elastic Elongation. Similarly, expanding the first item of the right side in equation (6) as series of (f/l) and considering the effect of elastic elongation, adopting the first two items, we get

$$\begin{aligned} S_0 &= l\left[\frac{1}{\cos\theta} + \frac{8\cos^3\theta}{3}\cdot\left(\frac{f}{l}\right)^2\right] \\ &\quad - \frac{\gamma l^2}{8E\cdot f\cdot\cos\theta}\left(l + \frac{16f^2}{3l} + \frac{h^2}{l}\right). \end{aligned} \tag{10}$$

Equation (10) is called the improved cable length formula considering the effect of elastic elongation.

Differentiating equation (10) with respect to f, we obtain

$$\frac{dS_0}{df} = \frac{16\cos^3\theta}{3}\cdot\frac{f}{l} + \frac{\gamma l^3}{8E\cdot f^2\cdot\cos\theta}\left(1+\tan^2\theta\right) - \frac{2\gamma l}{3E\cdot\cos\theta}. \tag{11}$$

Equation (11) is the simplified cable length adjustment formula considering the effect of elastic elongation; we call equation (11) as *simplified cable length adjustment formula I*. The latter two items at the right side of equation (11) consider the effect of elastic elongation.

3.3. Further Improved Cable Length Formula considering the Effect of Elastic Elongation. Generally, the density γ and elastic modulus E of parallel steel wires (or steel stands) are 80 kN/m^3 and 2.0×10^5 MPa, respectively; substituting them into equation (11), we get

$$\frac{dS_0}{df} = \frac{16\cos^3\theta}{3}\cdot\frac{f}{l} + \frac{5\times10^{-8}l^3}{f^2\cdot\cos\theta}\left(1+\tan^2\theta\right) - \frac{2.6667\times10^{-7}l}{\cos\theta}, \tag{12}$$

where the unit of the span and height is meter.

Let cable length adjustment tolerance $(\Delta(\Delta S))$ be less than 1 mm. For the midspan of a suspension bridge, if we set Δf = 200 mm, $\cos\theta\approx1$, $l\le3000$ m, the effect of the third item in the right side of equation (12) on the cable length adjustment is $\Delta S_3 = (2.6667\times10^{-7}l/\cos\theta)\Delta f\le0.16$ mm, which is much smaller than $(\Delta(\Delta S))$; therefore, the third item can be ignored in this condition. For the side span, if we set Δf = 100 mm, $l\le1000$ m, and $\cos\theta\ge1/3$ (generally, suspension bridges satisfied these conditions), then the effect of the third item in the right side of equation (12) on the cable length adjustment is $\Delta S_3 = (2.6667 \times 10^{-7} l / \cos\theta)\Delta f \leq 0.08$ mm, which is also much smaller than $(\Delta(\Delta S))$, thus, the third item can be ignored.

Therefore, for stay cables in a cable-stayed bridge or main cable in both middle and side span of a suspension bridge, the third item in the right side of equation (12) or equation (11) can be ignored.

Thus, equation (11) can be further simplified as

$$\frac{dS_0}{df} = \frac{16\cos^3\theta}{3} \cdot \frac{f}{l} + \frac{\gamma l^3}{8E \cdot f^2 \cdot \cos\theta}\left(1 + \tan^2\theta\right). \tag{13}$$

Equation (13) is the further simplified cable length adjustment formula considering the effect of elastic elongation; we call equation (13) as *simplified cable length adjustment formula II*.

From equation (13), when the first item in the right side plays a leading role, it can be found that the smaller the f/l, the faster the variation for the sag at midspan. However, when the first item in the right side can be ignored, the second item plays a leading role and the sag at midspan changes more slowly with the cable length if the value of f/l is small enough.

Similarly, equation (12) is further simplified to the following equation for a suspension cable which is made of parallel steel wires (or steel stands):

$$\frac{dS_0}{df} = \frac{16\cos^3\theta}{3} \cdot \frac{f}{l} + \frac{5 \times 10^{-8} l^3}{f^2 \cdot \cos\theta}\left(1 + \tan^2\theta\right). \tag{14}$$

3.4. Calculation of Cable Length Adjustment. The cable length adjustment amount (ΔS) is expressed as follows if the sag difference (Δf) is known:

$$\Delta S = \overline{\frac{dS_0}{df}}\Delta f \text{ or } \qquad \Delta S = \overline{\frac{dS}{df}}\Delta f, \tag{15}$$

where $\overline{dS_0}/df$ (or $\overline{dS}/df$) is the average value of dS_0/df (or dS/df) when f is changed from f_0 to $f0 + \Delta f$ (i.e. $f = f_0 \sim f_0 + \Delta f$), in which f_0 is the sag before adjustment and $f0 + \Delta f$ is the target sag adjustment. Since the average value is not easily obtained, it usually is replaced by the value of dS_0/df at $f = f_0$ [19, 21]. If sag difference (Δf) is relatively large, the value of dS_0/df changes obviously when the sag changes from f_0 to $f_0 + \Delta f$; thus, the value of $\overline{dS_0}/df$ is taken as the value of dS_0/df at the midpoint of the interval $[f_0,\ f_0 + \Delta f]$, i.e., at $f = f0 + (\Delta f/2)$ [19].

4. Applicability of Traditional Simplified Cable Length Adjustment Formula for Small Sag-Span Ratio

4.1. Cable in the Middle Span. Let $\Delta f = 200$ mm and $\cos\theta = 1$ and in equation (14) $((5 \times 10^{-8} l^3 / f^2 \cdot \cos\theta)(1 + \tan^2\theta)) \times 0.200 \leq [\Delta(\Delta S)]$, then

$$\frac{f}{l} \geq \left[\frac{f}{l}\right] = \sqrt{\frac{l}{10^8[\Delta(\Delta S)]}}. \tag{16}$$

The unit of l in equation (16) is meter.

Equation (16) shows the applicable range of traditional simplified cable length adjustment for midspan. When the ratio of sag to span in midspan is larger than the critical value $[f/l]$, the traditional simplified equation for cable length adjustment meets the accuracy requirements. And $[f/l]$ is proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{[\Delta(\Delta S)]}$.

As long as the requirement of $\sqrt{[\Delta(\Delta S)]}$ is not too strict, the length adjustment of datum strand cable for most suspension bridges can meet the requirement of equation (16), so the traditional simplified cable length adjustment equation can be adopted in midspan. Take the datum strand cable at midspan in Huangpu Suspension Bridge as an example, the horizontal distance $l = 1105.662$ m, the sag at midspan $f = 99.221$ m, and let $[\Delta(\Delta S)] = 0.002$ m; substituting them into equation (16), we get $f/l = 99.221/1105.622 = \quad 0.08974 \geq \sqrt{l/(10^8[\Delta(\Delta S)])} = 0.0743$. Thus, equation (16) can be satisfied.

4.2. Cable in the Side Span (or the Main Cable of the Suspension Bridge with a Single Tower). Let $\Delta f = 100$ mm and in equation (14), $((5 \times 10^{-8} l^3 / f^2 \cdot \cos\theta)(1 + \tan^2\theta)) \times 0.100 \leq [\Delta(\Delta S)]$, then

$$\frac{f}{l} \geq \left[\frac{f}{l}\right] = \sqrt{\frac{5 \cdot l}{10^9\cos^3\theta[\Delta(\Delta S)]}}. \tag{17}$$

The unit of l in equation (17) is meter. Equation (17) shows the applicable range of traditional simplified cable length adjustment for side span. When the ratio of sag to span in the side span is larger than the critical value $[f/l]$, the traditional simplified equation for cable length adjustment meets the accuracy requirements. And $[f/l]$ is proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{[\Delta(\Delta S)]}$ and $\sqrt{\cos^3\theta}$.

When the value of $[\Delta(\Delta S)]$ adopts an acceptable value satisfying engineering accuracy, the length adjustment of datum strand cable at side span can be calculated by equation (13) and the calculated value by the second item in equation (13) will be smaller or larger than the value of $[\Delta(\Delta S)]$; therefore, the length adjustment of datum strand cable at side span should be calculated considering the influence of elastic elongation.

From equations (16) and (17), it can be found that the sag-span ratio $[f/l]$ should not be too small when traditional simplified cable adjustment formulas are used without considering the influence of elastic elongation.

5. Numerical Examples

5.1. Example 1. A flexible cable in References [23, 24] was adopted as an example; the geometrical and material parameters are as follows: $l = 210.925$ m; $h = 110.485$ m; $E = 2.0 \times 10^5$ MPa; $f_0 = 1.082276$ m; $A = 0.011$ m^2; $\gamma = 72.5$ kN/m^3; and $\Delta f = 67.357$ mm. The unstressed cable length S_0 and cable length adjustment ΔS will be solved. The calculation results by a perfect solution of parabola theory, improved simplified formula, traditional simplified formula, quasi-catenary theory, and the theory of catenary are shown in Table 1.

It can be found that for the suspension cable with $f/l = 1/194.5$, $h/l = 0.524$, and $l = 210.925$ m, unstressed cable length calculation using perfect solution of parabola theory has enough accuracy, as well as the improved simplified formula (but with the increase of sag-span ratio, the calculation error for unstressed cable length gradually increases since improved simplified method only adopts the first two items in the expansion series); cable length adjustment amount calculation using perfect solution of parabola theory, improved simplified formulas I and II, has sufficient accuracy; the absolute error does not exceed 1 mm, while the relative error does not exceed 2.5%; but the calculation absolute error is large using traditional simplified method, especially the error of cable length adjustment is too large to be accepted.

5.2. Example 2. The calculation error of length adjustment for datum strand in the middle span using traditional and improved simplified formula was compared in this example.

The known conditions for a datum strand in the middle span of the Guangzhou Huangpu suspension bridge [20] (Figure 2) are as follows: $l = 1105.622$ m; $h \approx 0$; $E = 2.02 \times 10^5$ MPa; and $\gamma = 78.495\,\mathrm{kN/m^2}$. Comparison of length adjustment results of datum strand in the middle span with different sag to span ratio under the condition that the sag f was reduced 20 cm ($\Delta f = -20$ cm) as shown in Table 2.

5.3. Example 3. The calculation error of length adjustment for datum strand in the side span using traditional and improved simplified formula was compared in this example.

The known conditions for a datum strand in the side span of the Guangzhou Humen suspension bridge [22] are as follows: $l = 298$ m; $h = 96.798$; $E = 2.0 \times 10^5$ MPa; and $\gamma = 78.358\,\mathrm{kN/m^2}$. Comparison of the calculation results of length adjustment for datum strand in the side span with different sag to span ratio under the condition that the sag f was reduced 8.7 cm ($\Delta f = -8.7$ cm) as shown in Table 3.

From Tables 2 and 3, the following can be found:

(1) The applicable range of sag to span ratio for four solutions based on parabola theory is different. There is no lower limit of sag to span ratio for the methods of the perfect solution of parabola theory, improved simplified formulas I and II, but the lower limit for traditional simplified formula is about 1/30. The upper limit for the perfect solution of parabola theory is the highest for all four solutions; the upper limit for other three simplified formula is about 1/8, since those three methods ignore the effect of high-order items of expansion series. The calculation error of length adjustment for datum strand by the methods of perfect solution of parabola theory, improved simplified formulas I and II, is less than 4% for the suspension cable whose sag-span ratio is less than 1/8.

(2) From the calculation results by the methods of the perfect solution of parabola theory, the improved simplified formula I and II, the finding “the smaller the sag-span ratio, the faster the sag variation in midspan with the change of cable length” is right only in a certain range of sag-span ratio.

6. Engineering Application

6.1. Bridge Overview. Lishui Bridge on the expressway from the city of Zhangjiajie to Huayuan is a suspension bridge with a single span, two towers (no hangers at two side spans), and steel truss girders [24]. The deck system uses a composite section with steel-stiffened girder and concrete slab. The main cable is arranged with dimensions (200 + 856 + 190) m; the ratio of sag to span is 1/10 at the main span; the bridge uses 69 pairs of hangers; the standard spacing of these hangers is 12 m; and the distance from the end hanger to the tower is 20 m. The bridge arrangement is shown in Figure 3.

6.2. Cable Length Adjustment. The sag of the datum strand and catwalk bearing rope at both main span and side span were corrected according to the cable length adjustment calculated using the modified simplified formula II to get the target sag. The elevation error of the erection of datum strand and catwalk bearing rope is less than 1 cm, much higher than the required accuracy in construction specifications.

Table 4 lists target geometry parameters of the datum strand and catwalk bearing rope at the main span and side span in Zhangjiajie direction. In addition, the main mechanical parameters of catwalk bearing rope and datum strand are listed in the following: the density γ of catwalk bearing rope and datum strand are $123.9\,\mathrm{kN/m^3}$ and $77\,\mathrm{kN/m^3}$, respectively, and the elastic modulus of catwalk bearing rope and datum strand are 1.21×10^5 MPa and 1.96×10^5 MPa, respectively.

Table 5 shows cable length adjustment process of the datum strand and catwalk bearing rope at the main span and the side span in Zhangjiajie direction from the beginning to the target state using different methods. The calculated results indicate that the cable length adjustment can quickly reach the goal with high accuracy using the improved simplified formula II.

7. Conclusions

(1) The improved simplified formulas considering the influence of elastic elongation on the cable unstressed length was derived for a suspension cable with small sag to develop parabola theory of cable analysis.

(2) In comparison with the catenary theory (exact method) and quasi-catenary theory, three proposed cable length adjustment formulas considering the effect of elastic elongation are direct

Table 1: Comparison of the calculation results of the unstressed length and cable length adjustment.

Item	Theory of catenary	Quasi-catenary theory	Perfect solution of parabola theory	Traditional simplified formula	The improved simplified formula I	The improved simplified formula II
Unstressed cable length (m)	237.56707	237.56706	237.57255	238.12012	237.55485	/
Cable length error (m)	0	−0.00001	0.00548	0.55305	−0.01222	/
Cable length adjustment (mm)	33.60	35.95	34.4279	13.21	34.4278	33.9906
Relative error of adjustment	0%	7.0%	2.5%	−60.7%	2.5%	1.2%

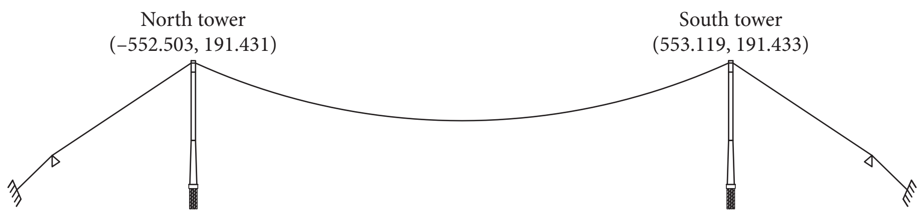

Figure 2: Main cable system of Huangpu Suspension Bridge (unit: m).

Table 2: Comparison of length adjustment results for datum strand in midspan with different sag-span ratios.

Sag to span ratio	Sag f_0 (m)	Exact solution ΔS (mm)	Perfect solution of parabola theory		Traditional simplified formula		The improved simplified formula I		The improved simplified formula II	
			ΔS (mm)	Error (%)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)
1/3	368.541	263.2710	252.8554	−4.0	355.5655	35.1	355.6049	35.1	355.6622	35.1
1/5	221.124	187.5098	182.6118	−2.6	213.3426	13.8	213.5538	13.9	213.6111	13.9
1/8	138.203	126.8748	125.1264	−1.4	133.3432	5.1	133.9733	5.6	134.0305	5.6
1/11.14	99.221	94.3145	93.5452	−0.8	95.7347	1.5	97.0108	2.9	97.0681	2.9
1/15	73.708	72.3814	72.0170	−0.5	71.1206	−1.7	73.4794	1.5	73.5367	1.6
1/20	55.281	57.1265	56.9534	−0.3	53.3429	−6.6	57.5804	0.8	57.6377	0.9
1/50	22.112	47.6561	48.0732	0.9	21.3426	−55.2	48.1141	1.0	48.1714	1.1
1/70	15.7946	66.457	67.2319	1.2	15.3346	−76.9	67.2471	1.2	67.3044	1.3

Table 3: Comparison of length adjustment results for datum strand in side span with different sag-span ratios.

Sag to span ratio	Sag f_0 (m)	Exact solution ΔS (mm)	Perfect solution of parabola theory ΔS (mm)	Traditional simplified formula		The improved simplified formula I		The improved simplified formula II	
				ΔS (mm)	Error (%)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)
1/8	37.25	48.2564	48.1414	49.8407	3.5	49.9282	3.7	49.9354	3.7
1/10	29.8	/	39.0885	39.8610	2.0	40.0019	2.3	40.0090	2.4
1/20	14.9	/	20.3755	19.9017	−2.3	20.4884	0.6	20.4955	0.6
1/35.98	8.2830	12.9359	12.9419	11.0379	−14.7	12.9611	0.1	12.9682	0.2
1/50	5.9600	/	11.6553	7.9261	−32.0	11.6624	0.1	11.6696	0.1
1/70	4.2571	/	13.0155	5.6450	−56.6	13.0180	0.0	13.0251	0.1
1/100	2.9800	19.0276	19.1201	3.9343	−79.4	19.1209	0.0	19.1280	0.0

calculation methods which do not need iteration and programming. Especially for improved simplified formula II, the calculation work is further reduced.

(3) The applicability and accuracy of three proposed cable length adjustment formulas at both middle and side span for suspension bridges were verified through numerical examples and practical engineering projects.

(4) The sag variation rate with the change of cable length depends on sag-span ratio, when the sag-span ratio is greater than a certain value (such as 1/60); the

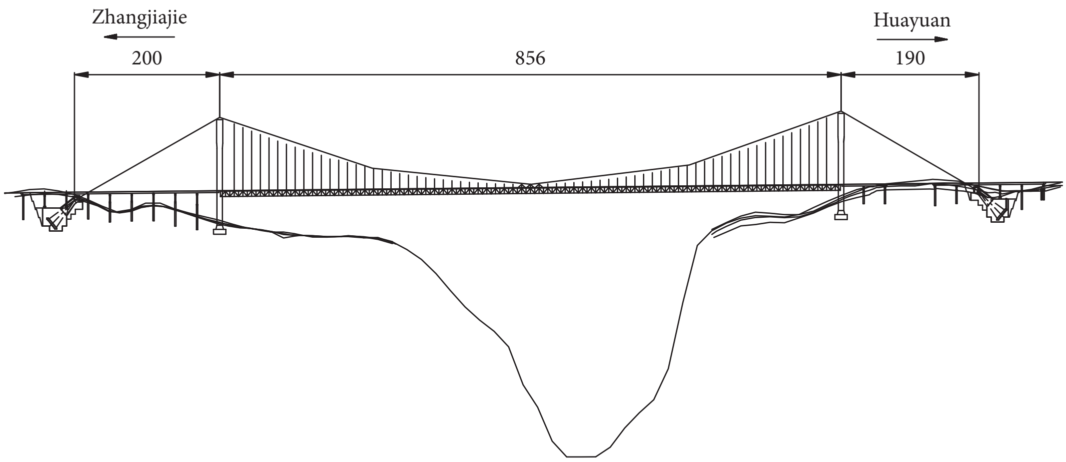

Figure 3: Bridge layout of Lishui Bridge on expressway from Zhangjiajie to Huayuan (unit: m).

Table 4: The target geometry parameters of the datum strand and catwalk bearing rope under erection stage.

Item	Target unstressed length (m)	Position	Mileage (m)	Elevation (m)	l (m)	h (m)	f (m)
Catwalk bearing rope at main span	863.989	Anchorage point at Zhangjiajie side	2545.782	592.072	848.406	7.601	73.322
		Middle span	2970.000	522.551			
		Anchorage point at Huayuan side	3394.188	599.673			
Catwalk bearing rope at side span	213.937	Anchorage point at girder end	2347.093	494.047	190.859	97.991	1.950
		Middle point at side span in Zhangjiajie	2442.523	541.093			
		Anchorage point at tower end	2537.952	592.038			
Datum strand at main span	869.797	Tangent point at tower end in Zhangjiajie	2543.967	593.374	851.937	7.614	76.914
		Middle span	2969.920	520.267			
		Tangent point at tower end in Huayuan	3395.904	600.988			
Datum strand at side span	218.338	Tangent point at girder end	2343.368	493.770	194.505	98.986	4.599
		Middle span	2440.621	538.664			
		Tangent point at tower internal end	2537.873	592.755			

Table 5: The length adjustment of the datum strand and catwalk bearing rope.

Item	Target sag to span ratio	Target sag f_0 (m)	Sag before adjusted f_0 (m)	Exact solution	Perfect solution of parabola theory		Traditional simplified formula		The improved simplified formula I		The improved simplified formula II	
				ΔS (mm)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)	ΔS (mm)	Error (%)
Catwalk bearing rope at main span	1/11.57	73.322	73.508	86.11	85.43	−0.79	85.72	−0.45	88.32	2.56	88.43	2.69
Catwalk bearing rope at side span	1/97.87	1.950	2.136	68.89	68.94	0.07	7.14	−89.64	68.94	0.07	68.96	0.11
Datum strand at main span	1/11.08	76.914	77.147	109.91	109.22	−0.63	112.18	2.07	113.32	3.11	113.37	3.15
Datum strand at side span	1/42.29	4.599	4.821	25.17	25.16	−0.02	19.82	−21.27	25.16	−0.03	25.18	0.02

smaller the sag-span ratio, the faster the sag variation at the middle span with the change of cable length; however, when the sag-span ratio is less than a certain value (such as 1/60), the smaller the sag-span ratio, the slower the sag variation at middle span with the change of cable length.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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