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Research Article

Prediction of the Shear Failure of Opened Rock Fractures and Implications for Rock Slope Stability Evaluation

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Rock slope commonly fails due to the shear failure of rock fractures. Shear strength of rock fractures are reduced substantially once the fracture surfaces are mismatched or opened. We propose a new criterion to predict the shear strength of rock fractures in different opening states. The degree of interlocking representing the true asperity contact area is incorporated into the modified model of Saeb and Amadei. The effect of fracture opening on asperity dilation and degradation is separately considered. The transitional stress that is a critical parameter involved in the model is analytically determined based on energy consideration. The new model is validated with experimental results from direct shear tests on synthetic fractures with regular-shaped asperities. Good agreement between the analytical solution and the experimental data confirms the capacity of the proposed model. Therefore, the model has great potential for assessing the stability of rock slopes where fractures are often opened due to stress relief and engineering disturbances.

1. Introduction

Understanding of rock slope failure mechanisms has increased substantially during the last several decades in response to continuing development of urban populations in mountainous areas and to the challenges faced by engineers and scientists in mining, civil, and geoscience fields. Rock slopes commonly fail along the rock discontinuities or fractures which offer weak planes to slide (Figure 1). Due to long-time stress relief or natural and engineered disturbances such as earthquakes, nearby excavation, and blasting [1–4], rock fractures are dislocated or opened. Opened rock fractures exhibit substantially lower shear resistance than the tightly mated ones since asperity interlocking is weakened because of surface dislocation [3, 5]. To predict the stability of rock slopes, the shear strength of rock fractures under opened states should be quantified.

For decades, the shear behaviour of closed rock fractures has drawn tremendous attention mainly through experimental and theoretical approaches [2, 6–10]. By conducting direct shear tests on synthetic fractures with regular-shaped asperities, Patton [10] observed that the fractures subjected to shear under a low normal stress mainly fail in the mode of asperity dilation. When the normal stress is sufficiently high, the asperities of rock fractures are seriously damaged or sheared off. Actually, dilation and shear-off occur simultaneously for rock fractures submitted to shear under non-extreme normal stress confinement [1, 2, 11]. When the rock fractures are opened, the contact area between asperities is reduced [2, 5]. Compared with the closed rock fractures, the asperities of opened rock fractures are more easily to be sheared off rather than to dilate due to shear stress concentration [5]. Consequently, the shear resistance of opened rock fractures decreases and the strength reduction depends on the degree of opening [12, 13]. However, existing models predicting the shear strength of rock fractures have rarely considered the effect of opening on dilation and degradation of asperities [4, 14–16].

In this study, we propose a new shear strength model that considers the influence of opening on asperity dilation and degradation. The opening state is represented by the degree of interlocking which is a function of measurable physical parameters. The dilation and degradation are

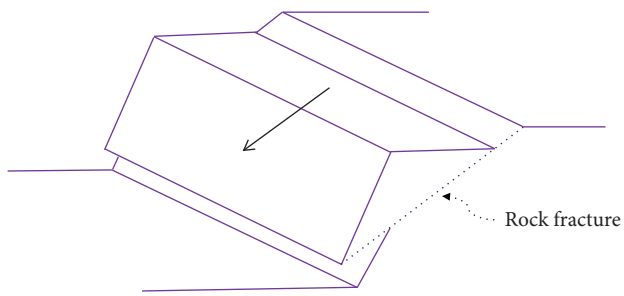

Figure 1: Illustration of plane failure along rock fracture of rock slope.

predicted by considering the true asperity area involved in shear. Direct shear tests on synthetic fractures under varying opening degrees are conducted. The close agreement between analytical predictions and measured data demonstrates the credibility of the proposed model.

2. Analytical Modelling

The shear resistance of a rock fracture comprises three shear strength components, i.e., the strength to overcome basic friction, dilation, and shear-off, respectively [17]. Saeb and Amadei [1] proposed that the shear strength of a tightly closed rock fracture is

$$\tau = \sigma_{\mathrm{n}} \tan\left(\phi_{\mathrm{b}} + i\right)\left(1 - a_{\mathrm{s}}\right) + a_{\mathrm{s}} s_{\mathrm{r}}, \tag{1}$$

where a_{s} denotes the ratio between shear-off area and the total asperity area of the rock fracture, s_{r} denotes the shear strength of the asperities, and i represents the dilation angle at the peak shear stress. Ladanyi and Archambault [2] suggested that both s_{r} and i depend on the ratio between normal stress and transitional stress (σ_{T}) which refers to the normal stress under which asperity dilation is completely constrained:

$$a_{\mathrm{s}} = 1 - \left(1 - \frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{T}}}\right)^{k_1}, \tag{2}$$

$$\tan i = \left(1 - \frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{T}}}\right)^{k_2} \tan i_0, \tag{3}$$

where k_1 and k_2 possess suggested values of 1.5 and 4.0, respectively. Equations (2) and (3) indicate that a_{s} increases from 0 to 1.0 when σ_{n} grows from 0 to σ_{T}, whereas $\tan i$ decreases from $\tan i_0$ to zero. σ_{T} was estimated at the uniaxial strength of the intact rock [6]. However, experimental studies by Grasselli and Egger [18] suggested that the asperity dilation is entirely replaced by asperity shearing at a high normal stress of around $0.2\sigma_{\mathrm{c}}$. Actually, the magnitude of transitional stress (σ_{T}) relies heavily on a number of variables including rock mineralogy, surface roughness, and porosity [19]. The most reliable way to determine transitional stress is to conduct direct shear tests under a series of sufficiently high normal stresses, which is technically challenging and time-consuming. In the following, we propose an analytical approach to predict transitional stress through energy consideration. We assume that in the shear process of a rock fracture, the asperities are sheared progressively without the occurrence of sudden breakage and substantial energy dissipation. Based on the principle of energy equilibrium, the work required to overcome joint dilatancy should compensate for the energy to shear off the asperities, i.e.,

$$\sigma_{\mathrm{n1}} dy_1 S + GS_1 = \sigma_{\mathrm{n2}} dy_2 S + GS_2, \tag{4}$$

where σ_{n1} is a low normal stress under which the peak dilation angle is i_1 and S_1 is the shear-off area, σ_{n2} is a high normal stress under which the peak dilation angle is i_2 and S_2 is the shear-off area (Figure 2), G is the surface energy that represents the work required to fracture the unit-area asperity, S is the total projected asperity area, and dy_1 and dy_2 denote the increments of normal displacements under normal stresses of σ_{n1} and σ_{n2}, respectively.

Rearranging equation (4), we have

$$\sigma_{\mathrm{n2}} dy_2 S - \sigma_{\mathrm{n1}} dy_1 S = G\left(S_1 - S_2\right) = G\Delta S. \tag{5}$$

Dividing equation (5) by the shear displacement increment (dx), we get

$$S\left(\sigma_{\mathrm{n2}} \tan i_2 - \sigma_{\mathrm{n1}} \tan i_1\right) = \frac{G}{2}\left(\Delta y_2 - \Delta y_1\right). \tag{6}$$

Considering that the asperities are completely sheared off under the transitional stress (σ_{T}), we have

$$\sigma_{\mathrm{T}} AS = GS_0 = \frac{G}{2} dx\, A, \tag{7}$$

where A is the asperity amplitude and S_0 is the area of a single asperity.

Rearranging equation (7), we have

$$G = \frac{2\sigma_{\mathrm{T}} S}{dx}. \tag{8}$$

Introducing equation (8) into equation (5), we get

$$\sigma_T = \frac{\sigma_{\mathrm{n2}} \tan i_2 - \sigma_{\mathrm{n1}} \tan i_1}{\tan i_2 - \tan i_1}. \tag{9}$$

To model the shear behaviour of opened rock fractures, the degree of interlocking quantifying the opening state of a rock fracture proposed by Ladanyi and Archambault [2] is introduced. The degree of interlocking of rock joint is as follows (Figure 3):

$$\eta = 1 - \frac{2\Delta x}{\lambda}, \tag{10}$$

where Δx is the initial dislocation displacement and λ is the asperity wavelength.

Oh and Kim [3] related the degree of interlocking η to the measurable parameters fracture opening (δ) and asperity amplitude (A):

$$\begin{aligned} \delta &= \Delta x \cdot \tan i_0, \\ A &= \frac{\lambda}{2} \cdot \tan i_0. \end{aligned} \tag{11}$$

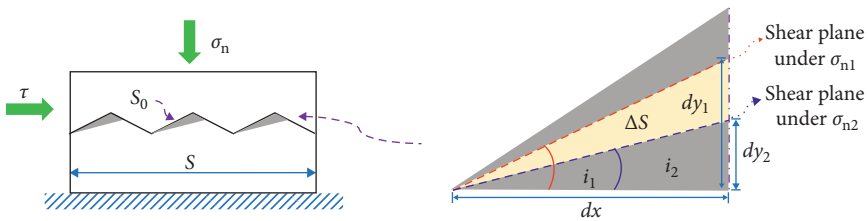

Figure 2: Dilation angles and shear-off areas of fracture asperities under low and high normal stresses, respectively. τ and σ_n are normal stress and shear stress, respectively. S and S_0 are the total projected asperity area and the area of a single asperity, respectively. σ_{n1} is a low normal stress under which the peak dilation angle is i_1, σ_{n2} is a high normal stress under which the peak dilation angle is i_2, and ΔS denotes the difference of shear-off area under normal stresses σ_{n1} and σ_{n2}, respectively. dy_1 and dy_2 represent the normal displacement increments under normal stresses of σ_{n1} and σ_{n2}, respectively. dx is the shear displacement increment.

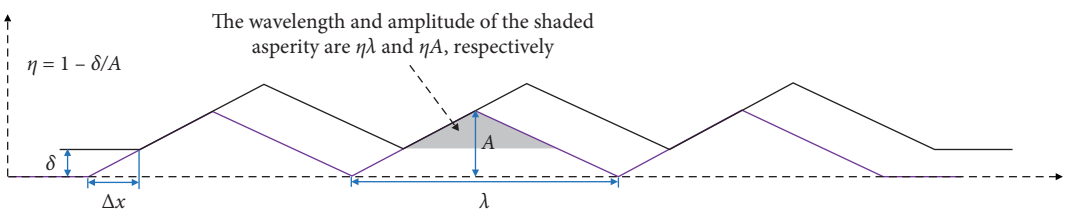

Figure 3: The definition of degree of interlocking (after Ladanyi and Archambault [2]). η is the degree of interlocking, δ is the fracture opening, and A and λ are the asperity amplitude and wavelength, respectively.

The degree of interlocking is rewritten as

$$\eta = 1 - \frac{2\delta}{\lambda \cdot \tan i_0} = 1 - \frac{\delta}{A}. \tag{12}$$

When a rock fracture is opened, asperity contact area is reduced and the true normal stress acting on the asperity is increased proportionally (Figure 3). Consequently, the asperity is more easily to be sheared off with less dilatancy and lower transitional stress (Figure 4). Therefore, the true normal stress and transitional stress of an opened rock fracture are, respectively, as follows:

$$\sigma_n^o = \frac{\sigma_n}{\eta}, \tag{13}$$

$$\sigma_T^o = \eta\sigma_T. \tag{14}$$

The shear strength of opened rock fractures is

$$\tau = \sigma_n \tan\left(\phi_b + i^o\right)\left(1 - a_s^o\right) + a_s^o s_r^o, \tag{15}$$

where

$$a_s^o = 1 - \left(1 - \frac{\sigma_n/\eta}{\eta\sigma_T}\right)^{k_1} = 1 - \left(1 - \frac{\sigma_n}{\eta^2\sigma_T}\right)^{k_1}, \tag{16}$$

$$\tan i^o = \left(1 - \frac{\sigma_n}{\eta^2\sigma_T}\right)^{k_2} \tan i_0, \tag{17}$$

$$s_r^o = \sigma_T^o \tan\phi_b = \eta\sigma_T \tan\phi_b. \tag{18}$$

Note that s_r should be the shear strength of an opened rock fracture, which is a multiplication of transitional stress (σ_T^o) and tangent of the basic friction angle of the rock surface ($\tan\phi_b$).

3. Experimental Validation

A series of direct shear tests were conducted on artificial fractures with triangular asperities at 20° and 30°. Hydrostone TB Gypsum Cement ($CaSO_4{\cdot}1/2H_2O > 95\%$, Portland cement <5%) was used to replicate the fracture samples. Hydrostone and water were mixed in the ratio of 1 : 0.35 by weight. The samples were cured at a constant temperature of 40°C in a curing oven for 14 days. Following the standard experimental procedures of ISRM [20], 10 cylindrical specimens of 41 mm in diameter and 102 mm in height were tested to determine the uniaxial compressive strength, and 10 cylinder specimens with diameter to thickness ratio at around 2.0 were experimented under the Brazilian test. The uniaxial compressive strength of the mixture is 46.3 MPa, and the tensile strength is 2.5 MPa. The basic friction angle of the synthetic fracture is determined by conducting direct shear test on flat surfaces and was found to be 42.4°. Casted fractures exhibit two triangular profiles, i.e., 20° and 30° inclination angles. The length of a fracture is 100 mm. The asperity wavelength of the two samples is the same at 25 mm. The asperity amplitudes are 4.55 mm and 7.22 mm for 20° and 30° profiles, respectively (Figure 5).

A Geotechnical Consulting and Testing System (GCTS) servo-hydraulic testing machine RDS-300 was used to carry out the experiments [5] (Figure 6). The servo-hydraulic testing system comprises a 500-kN compression frame and a direct shear apparatus. The electrohydraulic shear and normal load actuators, respectively, have the load capacities of 300 and 500 kN. The maximum stroke is 100 mm in the vertical direction and ±50 mm in the horizontal direction. The normal and shear displacements are measured by several linear variable differential transducers (LVDTs). The vertical

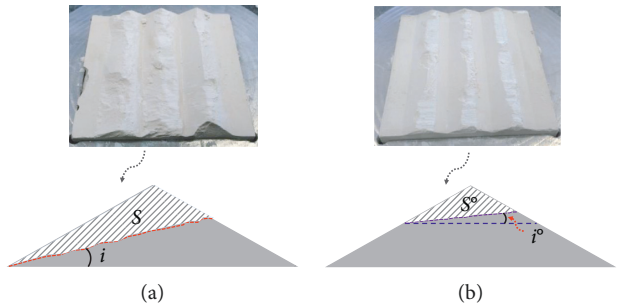

Figure 4: Comparison of dilation and shear-off of closed and opened rock fractures under direct shear. (a) Closed rock fracture; (b) opened rock fracture. S and S^o are shear-off area of closed and opened rock fractures under direct shear, respectively, and i and i^o are dilation angles at peak shear stress of closed and opened rock fractures, respectively (Li et al. [5]).

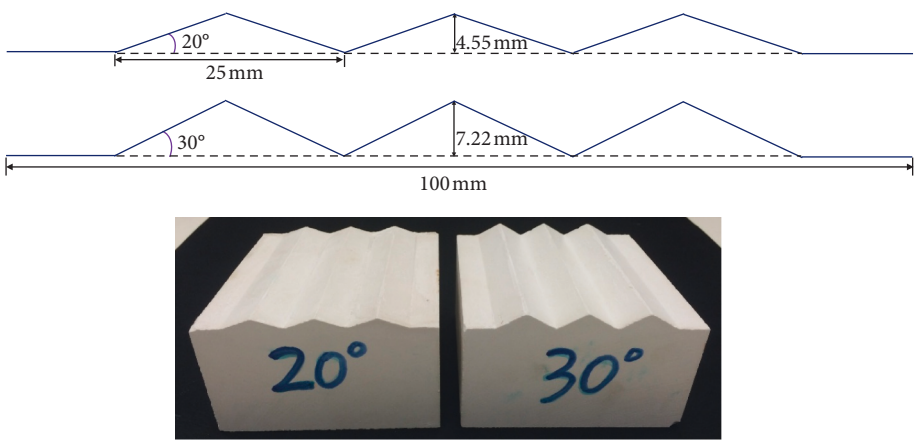

Figure 5: Profile geometry of rock fracture replicas and experimental samples (after Li et al. [5]).

displacement between the two shear boxes is measured by four LVDTs that are amounted square around the sample, with one in each corner (Figure 6(a)). Each LVDT has a measuring range of 12 mm. The normal displacement is calculated as the average values of the four LVDTs. The relative displacement of the two shear boxes in the horizontal direction is measured by one LVDT, which has a range of 100 mm (Figure 6(b)). The precisions of the LVDTs are 0.025 mm for shear displacement and 0.0025 mm for normal displacement, respectively. Experimental outputs including normal and shear stresses, shear displacement, and normal displacement are acquired automatically by the accompanied data-collection software.

Direct shear tests were performed on both closed and opened rock fractures with 20° and 30° asperity inclination angles, respectively (Figure 7). A constant shear velocity was set at 0.5 mm/min with a maximum shear displacement of 10 mm. For closed rock fractures, the degree of interlocking η before the test was 1.0. Two initial shear displacements were prescribed in the opened rock fracture tests, i.e., $\Delta x = 4.0$ mm and 8.0 mm. The corresponding degrees of interlocking were 0.68 and 0.36. Direct shear tests under a large range of normal stresses from 0.5 MPa ($\sigma_n \approx 0.01\sigma_c$) to 5.0 MPa ($\sigma_n \approx 0.1\sigma_c$) were carried out to measure the shear resistance of closed and opened rock fracture under low to high normal stress levels.

Figure 8 compares the analytical predictions to the experimental data from direct shear tests on replicated fractures with 20° and 30° asperity inclination angles, respectively. The analytical solutions agree with the experimentally-measured shear strength. However, the analytical model seemingly overestimates the shear resistance of rock fractures under varying degrees of interlocking. The differences possibly resulted from the empirical parameters k_1 and k_2 that are actually dependent of surface roughness and rock strength [21]. However, for convenience, the suggested values by Ladanyi and Archambault [2] are adopted in this work.

4. Implications for Rock Slope Stability Analysis

In the stability assessment of rock slope, constitutive models of rock fractures are required to adequately represent the shear behaviour of rock fractures, such as the Mohr–Coulomb model, the JRC-JCS model [22], and the continuously-yielding model [23]. However, these models tacitly assumed that the rock fractures are initially tightly closed. Our experimental findings showed that the shear resistance of rock fractures can be appreciably decreased due to the opening of rock fracture walls. Therefore, accurate evaluation of rock slope stability should incorporate the effect of opening on the shear behaviour of rock fractures.

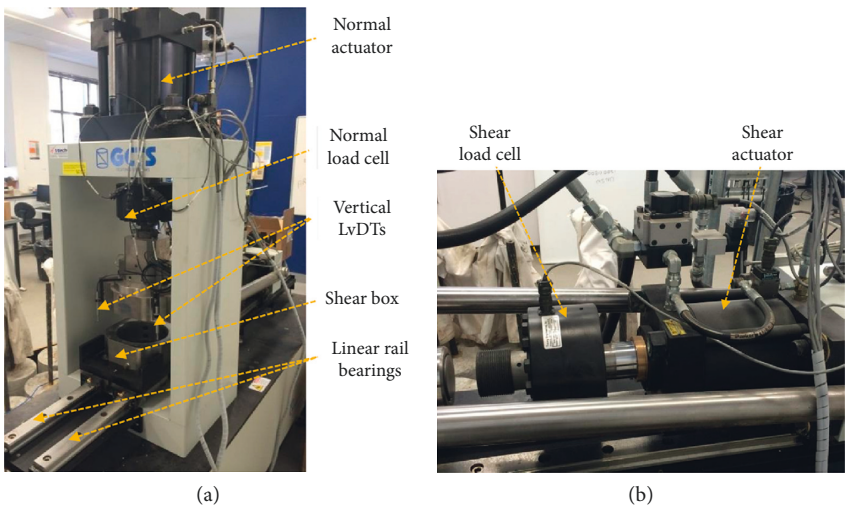

Figure 6: Frame of the GCTS direct shear test system RDS-300. (a) Front view; (b) shear actuator component (after Li et al. [5]).

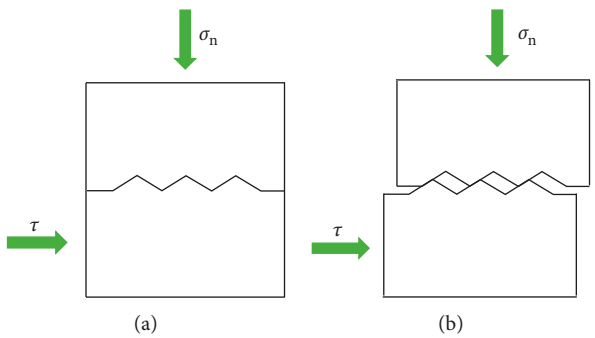

Figure 7: Configuration of closed and opened rock fractures under direct shear. (a) Closed rock fracture; (b) opened rock fracture (after Li et al. [5]). τ and σ_n are normal stress and shear stress, respectively.

In this work, we proposed an analytical model for predicting the shear strength of rock fractures with varying degrees of fracture opening. The model can be encapsulated into the commonly used finite element and discrete element codes for large-scale rock slope stability estimation. Since the degree of interlocking is measurable using the fracture opening and other physical descriptors of the rock fracture surface (equation (12)), in the model implementation, the degree of opening can be calculated automatically by continuously monitoring the opening or aperture of rock fractures. Then, equations (12) to (18) are used to predict the shear resistance of the opened rock fracture. By doing this, the stability of the rock mass with opened rock fractures can be numerically assessed. On the other hand, the normal deformation of rock fractures also depends highly on the opening state of the fracture surface [5]. The shear behaviour of rock fracture in turn is affected by its normal properties [24]. That is to say, the mechanical properties of rock fractures are coupled by the opening degree of the rock fracture. Future investigation will focus on the dependence and coupling of normal and shear deformation under varying openings.

5. Discussion and Conclusions

We proposed an analytical formulation to estimate the shear strength of opened rock fractures by modifying the classic model of Saeb and Amadei [1]. For simplicity, triangular-shaped fractures have been used in this study, both experimentally and mathematically. Saw-toothed asperities may not ideally describe the irregular type of fracture surfaces observed in the field. However, they still provide a simplified basis for understanding the fundamentals of the effect of opening on the shear behaviour of rock fractures. Zhao [12] reported that the shear resistance of a natural rock fracture also decreases as the fracture opening increases, indicating the possibility of the proposed model for predicting the shear strength of rock fractures with natural profiles. Rock fractures occur in broad scales from millimetres to kilometres. The shear properties of rock fractures are strongly scale-dependent, e.g., the shear strength varies as the size of rock fracture changes [25–28]. Several studies reported that the shear strength of a rock fracture decreased as its scale increased, termed positive scale effect [15, 25]. However, some

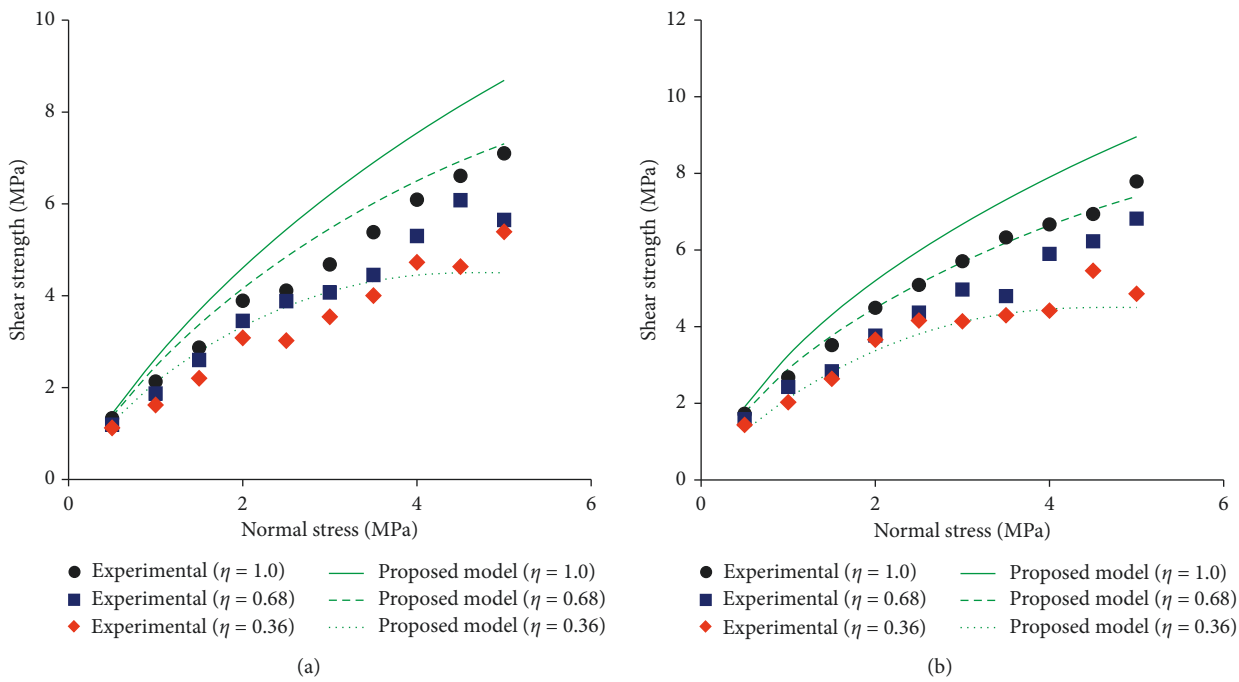

Figure 8: Model validation. (a) Comparison between proposed model and experimental data from fractures samples with 20° triangular asperities; (b) comparison between the proposed model and experimental data from fractures samples with 30° triangular asperities.

investigations observed opposite results, i.e., negative scale effect and no scale effect [26–28]. The shear behaviour of a rock fracture is complicated by the interplay of scale and opening degree. Therefore, application of the proposed model for large-scale rock-engineering should simultaneously consider the influence of scale on the shear behaviour of rock fractures. However, how the scale influences the effect of opening degree on the shear resistance of a rock fracture requires further study.

The proposed model employed the degree of interlocking to quantify the contact state of a rock fracture. The degree of interlocking was initially proposed to quantify the contact state of a regular-shaped rock fracture [2]. Oh and Kim [3] and Li et al. [15] showed that the degree of interlocking is also applicable for estimating the contact state of a naturally profiled rock fracture through equation (12). Alternatively, Zhao [12] proposed JMC (joint matching coefficient) to describe the contact state of a rock fracture. JMC is an independent parameter, having no correlation with JRC (joint roughness coefficient). Determination of JMC is purely qualitative. When the joint is smooth or perfectly matched, JMC is 1, whereas JMC = 0 indicates that the joint is totally dislocated. Drawbacks of JMC mainly arise from visualised JMC profiles. First, the comparison profiles cover incomplete mismatching cases. Additionally, subjective observation leads to remarkable deviation. The proposed model was derived by analysing the dilation and degradation of a regular-shaped rock fracture. Natural rock fractures are rough and irregular, displaying different orders of roughness [29, 30]. The dilation and degradation of natural rock fractures are much more complicated than those of the regular-profiled ones. The performance of the proposed model in predicting the shear resistance of natural rock fractures requires further examination.

This paper presents an analytical model for predicting the shear resistance of rock fractures with varying initial openings. The opening state of a rock fracture is quantified by the degree of interlocking that represents the true asperity area involved in shear. The influence of opening on asperity dilation and shear-off has been separately considered. The transitional stress, a key coefficient in the analytical formulation, is analytically determined by considering the energy balance between dilatancy and shear-off. The proposed model has been validated against experimental data from direct shear tests on two types of triangle-shaped fractures. The close match between analytical and experimental results demonstrates the ability of the new model. Rock fractures often become opened when rock slopes are loosened due to gradual stress relief and disturbances from earthquakes and nearby excavation. All the parameters involved in the proposed model are measurable with clear physical meanings. Therefore, the proposed model is practicable for appraising the rock slope stability once it is implemented into finite element or discrete element codes.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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