

Research Article

Development of a Nonlinear Integer Optimization Model for Tenant Mix Layout in a Shopping Centre

Hongyue Lv and Ting-Kwei Wang 

BIM Research Centre, School of Management Science and Real Estate, Chongqing University, Chongqing 400045, China

Correspondence should be addressed to Ting-Kwei Wang; tingkwei@cqu.edu.cn

Received 13 September 2019; Revised 3 February 2020; Accepted 10 February 2020; Published 19 March 2020

Guest Editor: Maksym Grzywinski

Copyright © 2020 Hongyue Lv and Ting-Kwei Wang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The tenant mix layout of shopping malls affects shopper consumption behaviour and the performance of malls. The main function of the tenant mix layout is to increase store sales by increasing footfall. However, although existing studies have shown the importance of the spatial clustering effect and the physical information about tenants, the authors of those studies did not properly consider both the spatial clustering effect and the physical information about tenants at the meantime. Through this study, we aimed to maximize the spillover effect of the stores in the shopping centre while considering both the spatial clustering effect and physical information about tenants. Therefore, we present a problem called *the tenant mix problem*, which is to determine the optimal tenant configuration scheme for existing shopping centre space segmentation to maximize the rental income of a shopping centre. To solve this problem, a nonlinear integer optimization model with defined characteristics was proposed and solved using a genetic algorithm. A shopping centre case study is also presented to verify the performance of the model.

1. Introduction

The retail industry is facing great challenges and transformation opportunities [1]. Traditional retailers are facing difficulties as retail spending shifts from offline to online shopping, and a large proportion of retail spending is covered by service spending. Online retailing increased by 10% a year from 2009 to 2014 while spending at big box and department stores declined by about 4% a year [2], followed by a spate of retail property closures in China and the United States [3].

Shopping centres are faced with escalating retail challenges, forcing owners to optimize rental pricing, store layout, tenant mix, and product selection and continuously utilize other strategies [4]. A shopping centre's lease form gives the owner enough motivation to create and improve the retail environment for retailers during the operating period [5, 6]. Previous studies have shown that tenant mix is one of the determinants of the success of shopping centres [7–11]. The physical environment of a store is a trigger that can significantly affect the shopping attitudes and behaviours of consumers. The reasonable

distribution of anchor stores and nonanchor stores in a mall, as well as the accumulation of homogeneous and heterogeneous retail types, enables each area of the shopping centre to have a specific dynamic atmosphere and guide the flow of customers, thus reducing gross shopping time and increasing the frequency at which shops on both sides are patronized [12, 13]. An excellent tenant mix layout can make the customer flow in a shopping centre orderly, strengthen customers' impressions of the stores, encourage shopping behaviour, improve shopping efficiency, and benefit both retailers and developers. Therefore, both scientific researchers and retail practitioners have a great interest in the mix of tenants in a shopping centre, the effective use of shopping space, and the optimization of a retail layout [14].

Researchers have summarized many general rules for the spatial arrangement and selection of tenants in shopping centres [15]. In practice, however, those theories are generally single stores and lack practicability and effectiveness in the whole mall. Hence, developers and owners have to come up with preliminary tenant mix layouts based on experience or existing shopping centre layouts. The searching for an

optimal tenant mix layout is a complex decision problem called the tenant mix problem (TMP) [16]. Different from the classic facility layout problem (FLP) and location allocation problem (LAP) in the field of spatial layout, first, the goal of the TMP is to maximize the benefits of shopping centre owners rather than minimizing the travel time of customers or maximizing area utilization. Second, although consumer paths are guided by their shopping purposes and shopping mall movement lanes, they still choose random routes instead of flowing among certain facilities. Third, the spatial layout of tenants needs to involve taking the competition and cooperation between different retail stores into account. The relationship between material flow and fixed facilities is often not complex.

Unfortunately, there exists limited research on the TMP. Early TMP researchers only calculated the rental income of specific shopping centres under the constraints of several indicators, including total leasable area, the upper and lower limits of each type of business area, the upper and lower limits of each size of a shop, and the maximum amount of interior decoration allowance [16]. Nevertheless, in shopping centre retail theories, the layout of tenant mix is also affected by the interaction between shops, namely, the agglomeration of the same retail types and the retail externality of the anchor store. The reason for these influences is the consumer's psychology during shopping, which means the tenant mix is not determined based on the physical constraints of the shopping centre. Yim Yiu and Xu [17] compare a shopping centre to an ecosystem, so the tenant mix of a mature and stable shopping centre can be regarded as the product of evolution after the selection of consumers. Shopping centre operators often take consumer psychology into consideration as much as possible at the beginning of considering tenant layout to balance the proportion and spatial layout among various retail types. However, due to the dilemma of quantifying interstore interaction, it is difficult to solve this problem using qualitative analysis and empirical rules alone. Therefore, the problem of how to arrange tenants for each floor of a shopping centre so that merchants can get the maximum patronage from consumers remains to be solved.

Aimed at solving this problem, we propose a tenant mix layout model. This model is a solution to the TMP that involves determining the optimal tenant spatial layout scheme for each floor of a shopping centre to maximize customer flow past shops. In this way, scattered customers can be converted into actual sales to maximize the rent of the shopping centre. Considering the complexity involved in solving an integer nonlinear programming model, we adopted a genetic algorithm (GA). We also analysed the sensitivity of the layout generated by the GA by changing parameters representing different consumer types.

In Section 2, we present the existing tenant mix layout literature. Then, in Section 3, we propose the tenant composite space layout model, and, in Section 4, we define the model parameters and procedures for a GA. In Section 5, a case study is presented and the program execution results are discussed. Finally, in Section 6, we present the conclusions and prospects of our research.

2. Literature Review

The idea of optimizing a shopping centre space has existed for a long time. Scholars have conducted many empirical studies on the tenant allocation of retail space, but the literature on the TMP is very limited. According to Nie, generally, the TMP includes three levels of decisions, namely, the division of physical space [18–20], the selection of retail types, and the brand level of stores [18]. In the development process of shopping malls, tenants are arranged into empty shops through these three procedures. Spatial division was the common subject of literature research on retail layout in the past. However, spatial allocation was calculated without considering the retail type and brand level of stores. Therefore, if researchers only consider the division of physical space in the TMP problem, the estimated value of shopping centre obtained from the conclusion is biased. However, the actual mall development process is one in which the operations team only needs to decide quickly how to allocate retail types and brand levels to vacant stores so that owners can negotiate with potential tenants rather than partition the mall's interior. In other words, operators are more concerned about matching suitable tenants to existing empty spaces than designing the interior of the mall. The concerns of our research are consistent with those of operators. However, the literature on tenant mix layout is insufficient. We refer to the limited related literature [6, 16, 21–24] to explain how we selected variables in the proposed model.

The tenant mix describes the number, size, sales, or service categories of stores, as well as their locations in a shopping centre [11, 25]. It plays an important role in gross rental income, retail sales income, tourist attraction, shopping centre image, and user experience of the shopping centre [26]. It is an internal factor that determines the attractiveness of the shopping centre [15, 27] and one of the decisive factors in the success of a shopping centre [5, 11, 28]. In shopping centres, retail leases usually include both basic and excess rents, also known as percentage rents, which take effect when store sales reach a certain threshold [29, 30]. Rent paid by stores is often affected by brand level, retail type, location, size, and other factors [11, 31, 32]. Therefore, rent is adjusted for each store according to these influencing factors in our model.

Seagle [33] first tried to model the layout for commercial real estate tenants. He established a linear programming model to allocate an area for each tenant to maximize the present value of shopping malls. This model took into account the limitations of usable area, investment, and other developer resources. Jensen and Arthur [34] proposed a mixed integer programming model for the TMP, which solved the problems of space segmentation and the number of tenants in different size classes. Bean et al.'s nonlinear programming model [16] inspired our research. Bean's team planned the number of tenants of each tenant type for a new shopping centre and arranged the size and location of each tenant. They tried to give a simple description of the interaction effects among each tenant class, but, due to the complexity of computing, the final model simplified the

interaction. In summary, past models have involved too little consideration of customer behaviour in shopping centres. The interaction between the same and different types of retail stores has not been taken into account, and the roles of specific retail stores in a shopping centre have not been defined.

In other cases, the tenant mix layout uses a set of general rules. For example, (1) the classic “barbell” shopping centre model advocates placing anchor shops and dining areas at both ends and setting smaller tenants on both sides of the corridor connecting the two ends; (2) nonanchor stores should not be clustered together but scattered in the mall; (3) the layout of a shopping centre should enable consumers to pass through as many stores as possible [35, 36]. Scholars help architects envision possible layouts through surveys and questionnaires [27]. Borgers et al. [11] used a virtual reality tool to generate a virtual shopping centre that enabled respondents to select the major categories, subcategories and specific stores to measure their preferences and help owners select optimal layout strategies.

Consumer behaviour in a shopping centre can create retail externalities in stores due to shopping attitudes, such as multipurpose shopping, comparison shopping, and reducing search costs, and those externalities have been widely identified [6, 21, 22]. These externalities have been widely observed; for example, when customers make multipurpose purchases, they require shopping malls to have rich retail types. In addition, the presence of the anchor stores creates customer spillover effect for other stores. At the same time, the external economy is reflected in the gathering of stores. Clustering retail stores of the same type reduces search times and uncertainty for customers, which benefits stores while generating competition [37, 38]. Yuo and Lizieri [39] focused on the decentralization and agglomeration strategies of stores in a multilayer shopping centre and considered the search cost of consumers to achieve the purpose of shopping. They believe that the dispersion of low-floor nonanchor stores can minimize total search distances for shoppers and enhance retail externality, while in a shopping centre with a large crowd flow or vertical structure, the same type of clustering enables consumers to identify their destinations, creating spillover effects and increasing flow effectiveness.

Peter et al. [23] linked store sales to whether the store can be seen. In other words, products that repeatedly appear in consumers’ sight will prompt consumers to buy them on impulse. Sorensen [24] proposed a visibility-based attractiveness evaluation, believing that the size of a store and the distance between a store and a shopper affect a shopper’s attention. Lu and Seo [40] measured visibility and exposure based on a GIS and confirmed the two-way influence between store layout and shoppers through a set of experiments in bookstores. In a shopping centre, the degrees of exposure of the stores are complicated and related to the vision of consumers, the floor space of the store, the distance of the store from consumers, and other indoor things, such as elevators and toilets.

Some researchers simulate consumer behaviour to layout the tenant mix. Hirsch et al. [41] analysed the customer density and retail type concentration in a

shopping centre with a GIS. They coupled the retail cluster using category aggregation and variable agglomeration methods and then visualized the passenger flow of the shopping centre through the core density estimator (KDE), thus reflecting retail agglomeration externalities more clearly through customer trajectories. In the exploration of retail correlation, GIS has incomparable advantages, but the clustering method in GIS can only tell us which tenants are related. Because the customer trajectories simulated by GIS are affected by each different tenant layout, it is difficult to determine how to optimize tenant placement. In addition, the tenant mix layout obtained from the above research does not involve consideration of the maximum profit of the owner; that is, the researchers did not attempt to maximize the size of stores in the eyes of customers in their search paths. However, if customers are aware of the larger size of the store, tenants can get more sales, which is the purpose of mall operators.

Based on the research gaps in the TMP, our contribution focuses on proposing a nonlinear programming model that takes into account both the spatial clustering effect and physical information about tenants. The model describes the impacts of consumer behaviour on store externalities which were not presented in previous TMP models. Then we use a GA to solve this complex problem. Finally, a case study of a realistic shopping centre is used to verify the model.

3. Nonlinear Integer Optimization Model for the TMP

3.1. Assumptions. Early versions of the TMP were similar to the FLP, which was designed to arrange rectangular objects (empty stores, machines, or departments) in a defined entire space without overlapping.

The TMP discussed in this paper is similar to the LAP, whose purpose is to explore how to best arrange the retail type and brand level of each empty shop in a vacant shopping centre to maximize the exposure of the shops on the floor to customers, increase the sales of the shops, and maximize the total rental income of a shopping centre.

We made the following assumptions when developing our optimization model:

- (i) Customers go shopping for multiple purposes, so one customer can be a potential consumer for various types of business.
- (ii) The attraction of a store is directly proportional to its size.
- (iii) Anchor store generates retail externalities but is not affected by the externalities of other stores. Because of the large size of the anchor store, customers need to spend a lot of time shopping in it. To save time, customers will not wander in without the intention to buy products in the anchor store [42].
- (iv) Each floor has a main type of retail; for example, the 3rd floor of a shopping mall mainly sells clothes, and the 4th floor mainly sells electrical appliances. This

main type of retail is called floor theme, and floor theme retail stores are only distributed on this floor.

- (v) To simplify the problem, the potential relationship between business types is not considered (e.g., the potential relationship between beer and diapers is not considered).

Table 1 lists the coefficients and decision variables. To improve tenant mix layout, we propose the use of the following optimization model.

3.2. The Objective Function for Calculating Rent. Our rental estimation equation is based on the widely accepted shopping centre lease framework [5, 42–44]. That is, total rent usually consists of basic rent and percentage rent:

$$\text{Rent} = \text{Basic rent} + \text{Excess rent.} \quad (1)$$

First, basic rent is the fixed rent per square meter of the leasable area of a shopping centre. It constitutes the fixed income of the owner. Excess rent is also called percentage rent. When the sales of the shop reach the agreed threshold, the percentage will take effect, and the tenants need to pay the excess rent.

$$c_{po} = \frac{1}{\sum_{\text{entr}} \sqrt{(x_{\text{store}} - x_{\text{entr}})^2 + (y_{\text{store}} - y_{\text{entr}})^2} + \sum_{\text{elev}} \sqrt{(x_{\text{store}} - x_{\text{elev}})^2 + (y_{\text{store}} - y_{\text{elev}})^2}} \quad (3)$$

That means the further away the store is from the entrance or elevator, the less likely it is to be seen by customers and, thus, the lower the basic rent that those tenants must pay.

To eliminate the problems of dimensional inconsistency and numerical incomparability between different coefficients, the adjustment coefficients were normalized. Thus, the basic rental equation of the store located on the f floor with the central coordinate of (x, y) :

$$\text{Basic rent} = S_{(f,x,y)} r_B c_f c_h c_{po}. \quad (4)$$

Excess Rent. Sales above the threshold generate excess rent, contributing to total rent. There is a significantly positive correlation between the rent paid by a single store and its retail sales. Therefore, when discussing the influencing factors of excess rent, we tend to pay attention to factors that affect store sales.

Obviously, the sales of a store are directly related to its type of retail and brand level, as well as the customer flow that the store can get. In our previous Assumptions, the number of customers in each location is equal at the initial state of the mall. However, this situation is not realistic. To make it more realistic, we revised the original customer distribution. Shopping centre customers tend to be concentrated at the elevator entrance. In addition, due to retail externalities, customer flow is affected by the concentration of anchor stores and similar retail types.

Basic rent. The authors of many empirical studies have proved that, among the micro factors that affect shopping central rent, the basic rent discrimination of stores mainly comes from the influence of store area (AREA), the brand rank of the store tenant (LEVEL), store floor (FLOOR), and location on the floor (POSITION). Then we use the functional form of equation (2) to describe the basic rent:

$$\text{Basic rent} = f(\text{AREA, FLOOR, LEVEL, POSITION, TYPE}). \quad (2)$$

In equation (2), AREA is determined in a given shopping centre, and LEVEL is generated by a GA. Moreover, the coefficient c_f was used to quantify FLOOR's impact on rent. We used empirical data in this case.

When measuring the coefficient c_{po} of POSITION, in addition to considering how the distribution of customers is attracted by different types of stores, the necessary entrance and elevator on the floor also lead to their uneven distribution in space. Because the shopping mall is convex, we can use the Euclidean distance equation to measure the distance between the centre point of each store and the entrance or elevator centre point and define the store position adjustment coefficient c_{po} as follows:

Generally, consumers come to a shopping centre for two reasons, to hang out or to buy certain items, or a combination of the two.

As a general rule for tenant layout, the owner usually sets the anchor stores at both ends of the shopping central corridor to guide consumers to pass other shops on both sides of the corridor. The anchor store is regarded as the most important tenant that guides customer flow. The closer a store is to the anchor store, the more likely that store is to be exposed to the view of consumers. Secondly, to save time, consumers are inclined to conduct shopping in areas with many similar retail stores. The externalities of the anchor store and similar agglomeration externalities are referred to as retail externalities. Based on the above analysis, we propose the function of retail sales per unit area of the store as

$$\text{SALES} = f(\text{TYPE, LEVEL, EXTERNALITY, POSITION}). \quad (5)$$

According to the second assumption, we use the store area to represent the attraction of the store. We defined the externality of the anchor store as w_1 and adopted the gravity model to calculate the attraction level of the anchor store in relation to the other stores around it:

$$w_1 = \frac{S_{\text{anchor}} S_{(f,x,y)}}{d_{\text{anchor}}^2}. \quad (6)$$

TABLE 1: Parameters used in the model.

Notation	Definition
r_B	Basic rent for the first floor (US\$/m ²)
c_{f_i}	Adjustment coefficient of shop grade (for basic rent)
c_f	Adjustment coefficient of the floor (for basic rent)
F	The floor number
f_1	Floor threshold coefficient
c_{po}	Regulation coefficient of the entrance and exit and elevator (for basic rent)
(x_{store}, y_{store})	The (x, y) coordinates of the centre of the store
(x_{entr}, y_{entr})	The (x, y) coordinates of the centre of entry and exit
(x_{elev}, y_{elev})	The (x, y) coordinates of the centre of the elevator
S_{anchor}	The size of the anchor store (m ²)
$S_{(f,x,y)}$	The store's area of coordinates (f, x, y) ; f is the floor of the store
d_{anchor}	European distance between other stores and the anchor store
w_1	Externality coefficient of the anchor store
n_i	The number of potential consumers of retail type i
$cost_i$	The annual consumption capacity of consumers for goods of type i
S_i	The total area of stores with retail type i
p_i	Unit area sales when the total area of the retail type i store is S_i
$S_{trav(i)}$	The maximum size of a retail type, which potential consumers are willing to search to find products that meet their needs
$S_{(f,i)}$	The total area of the i retail type stores on floor f
α	The proportion of customers entering other stores of the same type of retail for the purpose of comparison shopping
β	The proportion of real consumers that individual stores lose in competition
$S_{thre(i)}$	The total area of a store with retail type i when the retail sales of per unit area reach the maximum
p_m	The threshold that stores with retail type i should pay a percentage rent
$S_{max(i)}, S_{min(i)}$	The total area of stores with retail type i when the sales are p_m (two possibilities)

Compared to the retail externalities of anchor stores, similar agglomeration externalities are more complex. Clustering stores of the same retail type can effectively increase customers by enabling comparison shopping, thus improving the sales of stores [22]. In practice, owners are also aware of this externality and enhance the shopping experience by setting a specific retail theme for each floor. However, it is difficult to control the limits of store agglomeration, and the competition caused by the increase in the number of stores of the same type will reduce the sales of stores. Therefore, under the premise of a fixed number of potential consumers in the shopping centre, economies of scale gradually shift to diseconomies of scale as the area of stores increases [13]. In our model, a floor theme control matrix is built-in, and the theme of each floor is set by the owner. On each floor, more tenants will be arranged in the corresponding theme retail type, so as to realize the spatial agglomeration of theme retail type.

Next, we deduced the mathematical expression of the agglomeration externality of the same type of retail stores in shopping malls.

Suppose the following:

- (i) The number of potential consumers in a shopping centre with retail type i is n_i
- (ii) The annual consumption capacity of consumers for commodities with type i is $cost_i$
- (iii) The total area of retail type i in a shopping centre is S_i
- (iv) The sales of unit area is p_i when the total area of type i shops is S_i

Then

$$S_i p_i \leq n_i \text{ cost}_i, \quad (7)$$

S_i and p_i are both variables, and p_i can be written as a function of S_i . Due to the influence of agglomeration and competition, the relationship between p_i and S_i is not simply linear. This relationship is typical of scale effects, and Henderson [45] made a similar point in his model of urban systems. With the continuous expansion of the city scale, the utility generated by diseconomy will gradually offset the scale benefit of industrial concentration within the city. Therefore, the synergy between the external economy and diseconomy determines the optimal size of a city. There is an inverted U-shaped relationship between the size of a city and the utility of a typical resident. Based on the above analysis, we assume that there is an inverted U-shaped relationship between the sales per unit area of a typical store in a shopping centre and the sum area of the same type of retail stores, as shown in Figure 1.

In Figure 1, S_{thre} is the total area of a typical retail store of type i with the largest retail sales per unit area in the shopping centre. S_{max} and S_{min} are the maximum total area and minimum area of the stores that do not need to pay percentage rent, respectively. S_{thre} , S_{max} , and S_{min} are all variables affected by typical stores.

The constant S_{trav} is defined as the maximum i -type store area that potential consumers are willing to search for to find desired goods. When the store area of this type reaches S_{trav} , it can meet the needs of consumers for comparison shopping and control the time consumers spend on shopping.

Therefore, in the theme floor, we built a sales function for a typical theme store:

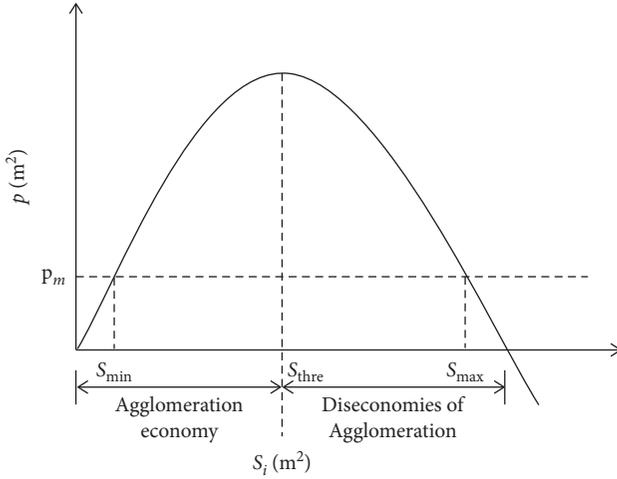


FIGURE 1: The relationship between p_i and S_i .

$$g = \frac{S_i}{S_{trav}} \cdot \frac{S_{(f,i)}}{S_i} \cdot n_i \cdot \frac{S_{(f,x,y)} + \alpha(S_{(f,i)} - S_{(f,x,y)}) - \beta(S_{(f,i)} - S_{(f,x,y)})}{S_{trav}} \quad (8)$$

In equation (8), S_i is the total area of shops of type i in the shopping centre, $S_{(f,i)}$ is the total store area of type i on floor f , α refers to the proportion of customers entering a store (f, x, y) from other stores of the same retail type on the same floor for the purpose of comparison shopping, and β is the proportion of real consumers lost by store (f, x, y) due to competition.

Under the assumption that all shops of type i in the shopping centre are clustered on floor f , namely, $S_{(f,i)} = S_i$, we differentiate the variable S_i of function g :

$$\frac{dg}{d(S_i)} = \frac{2(\alpha - \beta)S_i + (1 - \alpha + \beta)S_{(f,x,y)}}{S_{trav}^2} n_i \text{cost}_i = 0. \quad (9)$$

Hence,

$$S_i = \frac{(1 - \alpha + \beta)S_{(f,x,y)}}{2(\beta - \alpha)} = S_{thre}. \quad (10)$$

It is easy to know that, for any single store in the shopping centre, when the sales per unit area reaches the maximum, the corresponding S has nothing to do with the purchasing ability of potential customers and the store area that customers are willing to search for but does have to do with the demand for comparison shopping and store competition. Given that $\beta - \alpha \neq 0$ and $\alpha - \beta \neq 1$, the agglomeration area that a store can afford is related to its size. Compared to a small store, a large store needs a larger agglomeration to obtain the maximum sales per unit area. After the agglomeration area exceeds S_{thre} , the sales per unit area of a single store decreases as competition gradually becomes dominant in synergy (see Figure 2).

Total rental income is the combination of excess rent and basic rent, and the opportunity cost is the basic rent of the

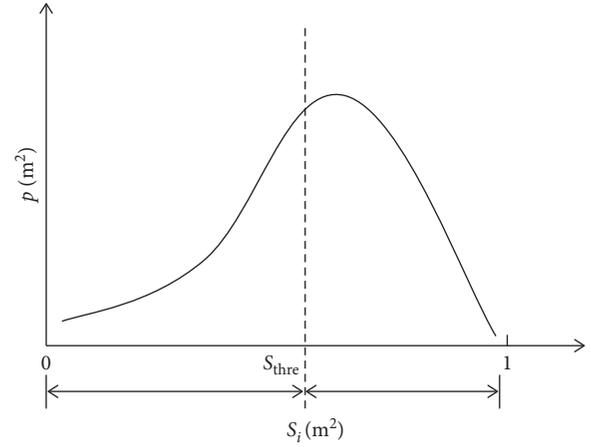


FIGURE 2: The relationship between the total store area of type i and the total sales of one store of this type.

occupied area. When the income equals the opportunity cost, the maximum S_i can be obtained:

$$S_{max} p_i \cdot a\% + S_{max} r_B c_f c_h c_{po} = S_{max} r_B c_f c_h c_{po}. \quad (11)$$

Therefore, $S_{max} p_i \cdot a\% = 0$

At this point, p_i equals p_m of threshold sales, and the rental income of a store of type i is the basic rent.

$$\frac{S_i}{S_{trav}} n_i \text{cost}_i \frac{S_{(f,x,y)} + (\alpha - \beta)(S_i - S_{(f,x,y)})}{S_{trav}} = p_m, \quad (12)$$

$$\frac{(\alpha - \beta)S_i^2 + (1 - \alpha + \beta)S_{(f,x,y)}S_i}{S_{trav}^2} n_i \text{cost}_i = p_m. \quad (13)$$

According to the above equation, S_{max} and S_{min} of a typical theme store in a shopping centre can be obtained.

For the nonthemed retail shops on themed floors, according to the third assumption, the spillover effect of retail externalities is spread from the dominant shop in a shopping centre to nondominant shops. The anchor store is not affected by the externalities of other shops in a shopping centre, and its customers are only attracted by its own gathering ability. Hence, similarly, due to the multiobjective shopping demand of consumers, the nontheme store is affected by the spillover effect of theme store aggregation, increasing visitor flow. Therefore, the sales function of a typical nontheme store on the floor is calculated as follows:

$$u = \frac{S_j}{S_{trav}} \cdot \frac{S_{(f,j)}}{j} \cdot n_j \cdot \frac{S_{(f,m,n)} + (\alpha - \beta)(S_{(f,j)} - S_{(f,m,n)}) + \varphi S_{(f,j)}}{S_{trav}} \text{cost}_j, \quad (14)$$

where φ is the spillover effect coefficient of the theme retail type to the nontheme retail type. Thus, the sales of nontheme retail type stores are restricted by the ability of theme stores to gather customers.

3.3. *Model for the TMP.* Based on the above analysis, for a particular floor, the function of sales per unit area of a single store with a thematic retail type is calculated:

$$P_{(f,x,y)} = \begin{cases} 0, S \notin U, \\ \frac{S_i}{S_{\text{trav}}} \cdot \frac{S_{(f,i)}}{S_i} \cdot n_i \cdot \frac{S_{(f,x,y)} + (\alpha - \beta)(S_{(f,i)} - S_{(f,x,y)})}{S_{\text{trav}}} \text{cost}_i c_f c_{p0}, & S_i \in U. \end{cases} \quad U = [S_{\min}, S_{\max}], \quad (15)$$

The function of sales per unit area of a store with nonthematic retail type is calculated as follows:

$$P_{(f,m,n)} = \begin{cases} 0, S \notin U, \\ \frac{S_i}{S_{\text{trav}}} \cdot \frac{S_{(f,i)}}{S_i} \cdot n_i \cdot \frac{S_{(f,m,n)} + (\alpha - \beta)(S_{(f,i)} - S_{(f,m,n)}) + \varphi S_{(f,i)}}{S_{\text{trav}}} \text{cost}_i c_f c_{p0}, & S_i \in U. \end{cases} \quad U = [S_{\min}, S_{\max}], \quad (16)$$

The form of excess rent for a single store is as follows, where $a\%$ is the payment ratio when the store reaches the sales threshold:

$$\text{Excess rent} = f(\text{AREA}, \text{SALE}, \text{PERCENT}), \quad (17)$$

$$\text{Excess rent} = \begin{cases} P_{(f,x,y)} S_{(f,x,y)} w_1 a_{\text{type}} \%, & \text{type} = i, \\ P_{(f,m,n)} S_{(f,m,n)} w_1 a_{\text{type}} \%, & \text{type} \neq i. \end{cases} \quad (18)$$

Meanwhile, according to the actual business conditions of shops in a shopping centre, there are two stages of rent:

$$\text{rent} = \begin{cases} \text{Basic rent}, & P_{(f,x,y)} \leq P_m, \\ \text{Basic rent} + \text{Excess rent}, & P_{(f,x,y)} > P_m. \end{cases} \quad (19)$$

The complete model can be simply expressed as follows:

$$\text{Maximum grossrent} : \sum \text{Rent}, \quad (20)$$

S.t.

$$\sum_{f=\min}^{\max f} \sum_{x=x_1}^{x_n} \sum_{y=y_1}^{y_n} S_{(f,x,y)} \leq G, \quad (21)$$

$$\sum_{x=x_1}^{x_n} \sum_{y=y_1}^{y_n} S_{(F,x,y)} \in G_F, \quad F = \min f, \dots, 0, \dots, \max f, \quad (22)$$

$$S_{(F,i_F)} \geq 0.5G_F, \quad F = \min f, \dots, 0, \dots, \max f, \quad (23)$$

$$S_{\{(F,X,Y),i_F,h\}} = \max(s), \quad F = \min f, \dots, 0, \dots, \max f, \quad (24)$$

$$h \in \{1, 2\},$$

$$\begin{aligned} \beta - \alpha &\neq 0, \\ \alpha - \beta &\neq 1. \end{aligned} \quad (25)$$

The purpose of constraints (21) and (22) is to make full use of the leasable area in the shopping centre. Constraint (23) makes the floor area of theme shops dominant on the floor, constraint (24) indicates that the retail type of the largest store on each floor is the floor themed retail type, and constraint (25) guarantees the uniqueness of S_{thre} .

In addition, it is easy to be troubled by the following problem: according to the lease agreement of shopping centre, different retail types of stores pay different rent proportions ($a\%$); for example, the rent proportion of food stores is higher than that of movie theatres. If all stores are placed with a retail type requiring high rent, then the rent of the shopping centre will undoubtedly reach the maximum, but this obviously violates reality and the principle of shopping centre diversity. Therefore, there is a potential constraint between the tenant mix and the available space in the shopping centre. The general rules are detailed in Section 4.2.

4. Genetic Algorithm Optimization for the Objective Optimization Problem

A GA is an adaptive optimization technique based on a biological genetic and evolutionary mechanism first proposed by Holland [46]. It first generates a set of candidate populations, each of which represents a solution. Individual fitness is calculated by simulating adaptive conditions. According to the idea of natural selection, the algorithm automatically retains excellent individuals and eliminates others. Under the continuous evolution of the population, the surviving individuals gradually converge as the optimal solution to the problem.

To apply a GA to solve a model, we improve the original algorithm. The improved GA makes the population quickly

converge to the optimal solution through three key subroutines:

- (1) Layout generation: when initializing the tenant layout of shops on all floors of a shopping centre, the relationship between store level, retail type, and store area is stipulated to ensure that the randomly generated tenant mix is feasible and in line with the practical logic.
- (2) Rent calculation: by simulating the shopping behaviours of different types of consumers, the algorithm calculates the basic rent and excess rent for each store in the shopping centre.
- (3) Layout update: brand level and retail type in the same individual are updated simultaneously and correspondingly. The population is then updated for the next iteration. The algorithm structure is shown in Table 2.

Next, we discuss how to represent the spatial layout of the tenant mix in the GA and explain the key settings in the algorithm.

4.1. Representation of Tenant Mix Scheme in the GA. Aickelin and Dowsland [47] proposed the GA of direct coding and indirect coding for the problems of mall layout and tenant combination. In our experiment, a clear and straightforward coding method was adopted to determine the retail type and brand grade of stores. A single individual represents a solution in the search space and represents the tenant mix layout of a certain floor in a shopping centre. That means each individual's chromosome is represented by a row of an integer array, which is composed of $2N$ elements, where N is the number of shops on the floor, the first N elements correspond to the retail types, and the last N elements represent the brand level.

For example, if a shopping centre has five floors in total, then the final result of a feasible tenant mix scheme is a cell containing five arrays that represent the retail type and brand level of each tenant on one floor of a shopping centre.

4.2. Subroutine: Layout Generation and Layout Update. As mentioned above, because there are many shops in a shopping mall, the solution space is very complex. Most randomly generated sample populations are not feasible, and it is difficult for the algorithm to converge to the optimal feasible solution in the solution space without restricting the relationship between the vacant shop area, retail type, and brand level.

The following is the general logic of the tenant mix layout:

- (1) Brand level setting: the brand level is divided into five levels, the anchor store ($h = 1$), the secondary anchor store ($h = 2$), secondary anchor stores that contribute to increased rent ($h = 3$), nonanchor stores that make rental contributions ($h = 4$), and nonanchor stores that enrich the retail type of the shopping mall ($h = 5$).

TABLE 2: Algorithm flow.

Input information about the shopping centre to be laid out
Initialize the tenant layout population under logical constraints
Set parameters of the GA
Do
For each iteration
Calculate the fitness of all individuals in the population
Find and then save the best solution and elite individuals in this generation into the next generation population
Use the roulette method to select the individuals who can enter the next iteration
The selected individual genes are crossed and mutated under logical constraints
Update the population
End
Until default number of iterations

- (2) Area setting: the store is mainly divided into three levels. G represents the leasable area of the store floor, and $S_{(f,x,y)}$ represents the area of a store.

Small store, $S_{(f,x,y)} < 0.05G$.

Medium store, $0.05G \leq S(f, x, y) < 0.1G$.

Large stores are divided into three levels:

- (i) Third level: third class is a large area store that is not the largest store on a floor. That is, $S_{(f,x,y)} > 0.1G$ and $S_{(f,x,y)}$, and the store (f, x, y) is not the largest store on the floor.
- (ii) Second class: the store area is the largest on the floor, but it is not big enough to accommodate an anchor store; $0.1G \leq S(f, x, y) < 0.25G$.
- (iii) First class: it is a store that can accommodate the anchor store; $S(f, x, y) \geq 0.25G$.

- (3) Setting of brand level matching area: stores with brand levels of 3, 4, and 5 can be placed in small stores. Stores with brand levels of 2, 3, 4, and 5 can be placed in medium stores. The second class of large stores can arrange tenants with brand levels of 2, 3, and 4. The first class of large store can be placed in stores with brand levels of 1 and 2. Table 3 shows matching principles of area and level more clearly.

- (4) Agreement on the expression of retail types:

We classified and numbered the retail types of a shopping centre with the Guidance of the Shopping Centre Tenant Mix Strategy (SB/T 10813-2012), which is the business industry standard published by the ministry of commerce of China:

Beauty salon (1), photo studio (2), training and education (3), the cinema (4), the gym (5), children's park (6), KTV (7), skating rink or video games city (8), and desserts (9), beverages (10), Chinese/western fast food (11), Chinese style dinner (12), western food (13), gourmet street (14), supermarket (15), department store (16), women's clothing (17), men's clothing (18), children's clothing (19), sports equipment (20), personal daily (21), and jewelry (22).

TABLE 3: Matching principle of area and level.

Area setting	Area range	Brand level of store
Small stores	$S_{(f,x,y)} < 0.05G$	3, 4, 5
Medium stores	$0.05G \leq S_{(f,x,y)} < 0.1G$	2, 3, 4, 5
Large stores	$S_{(f,x,y)} \geq 0.1G$ and $S_{(f,x,y)} \neq \max(s)$	2, 3, 4
	$0.1G \leq S_{(f,x,y)} < 0.25G$ and $S_{(f,x,y)}$ is $\neq \max(s)$	2, 3, 4
	$S_{(f,x,y)} \geq 0.25G$ and $S_{(f,x,y)} = \max(s)$	1, 2

*S is the area vector of a single floor.

Particularly, when a store is vacant, its retail type shall be expressed as (0).

(5) Level and retail type matching setting:

The corresponding relationship between store area and brand level is shown in Table 4.

In layout design and update, the randomly generated scheme must strictly follow settings 3 and 5 to ensure that the scheme is feasible and complies with reality.

4.3. Subroutine: Rent Calculation. The second key subroutine is calculating the rental income of each feasible scheme according to the fitness function. In our program, the adaptability function is a nonclosed subroutine that is divided into a coefficient definition module, basic rent calculation module, excess rent calculation module, and fitness calculation module. In the program, the excess rent module includes the anchor store retail externality coefficient calculation module, the cluster externality coefficient calculation module, and the store sales per unit area calculation module. In the last module, to obtain the sales per unit area of the store, we simulate the shopping behaviour of consumers.

4.4. Population Regeneration. During each iteration, the new population will enter the next-generation population after crossover and mutation by the individuals selected from the previous population. To preserve the genes of excellent individuals from being destroyed and accelerate the rate of convergence, we skip the crossover and mutation operation to retain an elite individual with the best fitness from the previous generation for the new population.

4.5. Optimal Solution. Each individual (corresponding to a feasible tenant combination layout plan) can determine the optimal solution in the population according to the rent calculation module. Such an individual is called an elite individual.

In the GA, the approximate global optimum is determined based on whether the average fitness is close enough to the maximum fitness. When the average fitness of the population is close to the maximum fitness and tends to be stable, it is generally regarded that the algorithm has found an optimal solution, which is the termination condition of the whole algorithm. When solving high-dimensional optimization problems, GAs are prone to ‘‘prematurity’’; that

TABLE 4: Matching principle of level and retail type.

Brand level	Area rules	Retail types
1	$S_{(f,x,y)} \geq 0.25G$ and $S_{(f,x,y)} = \max(s)$	15, 16
2	$S_{(f,x,y)} < 0.25G$ and $S_{(f,x,y)} = \max(s)$, or $S_{(f,x,y)} > 0.05G$ and $S_{(f,x,y)} \neq \max(s)$	5, 6, 7, 8, 9
	$S_{(f,x,y)} \geq 0.25G$ and $S_{(f,x,y)} = \max(s)$	4
3	$S_{(f,x,y)} > 0.1G$ and $S_{(f,x,y)} \neq \max(s)$, or $S_{(f,x,y)} < 0.25G$ and $S_{(f,x,y)} = \max(s)$	17, 19
	$S_{(f,x,y)} > 20$ and $S_{(f,x,y)} \leq 0.1G$	17, 18, 19
	$S_{(f,x,y)} < 0.1G$	20, 21
4	$S_{(f,x,y)} > 20$ and $S_{(f,x,y)} \neq \max(s)$, or $S_{(f,x,y)} < 0.25G$ and $S_{(f,x,y)} = \max(s)$	11, 12, 13, 14
	$S_{(f,x,y)} < 20$	10
5	$S_{(f,x,y)} < 0.1G$	1, 2, 3, 22

*S is the area vector of a single floor.

is, the algorithm does not reach the end conditions mentioned above and only finds a local optimum within a small search range. In this case, the results of each operation of the algorithm are different, showing the characteristics of instability. According to our test, the crossover and mutation operators in the algorithm will slow down the convergence rate of the population. Therefore, adding new individuals to the population and improving the probability of finding the global optimal solution are two methods to deal with premature convergence.

5. Case Study

We used the indoor map data of a shopping centre in Chongqing, China, to test the optimization model. The mall has six floors: five above ground and one below ground, and it also has 158 shops for rent. After a simple estimation, the feasible solution space was found to be large. Through multiple tests of the algorithm coefficients, we chose to randomly generate a population of 5000 individuals according to the constraints. A total of 400 iterations were carried out and repeated three times to observe the stability of the algorithm and obtain multiple feasible solutions. The crossover rate and mutation rate of the algorithm were 0.8 and 0.003, respectively.

5.1. Operation Results. Figure 3 shows three tenant layouts after the program runs three times (Figure 3 only shows the first floor plan, while the complete results are stored in the appendix), the three-tenant layout is expected to help operators to obtain rental income 5.025×10^4 , 5.007×10^4 , and 5.028×10^4 thousand USD, respectively, and the average total rental income is 5.021×10^4 thousand USD. According to the maximum and average fitness curves (see Figure 4), we find that the maximum rent fluctuates by 0.4%, thus confirming the stability of the algorithm.

Figure 5 shows the original tenant layout, and the calculated rental income is 2.191×10^4 thousand USD. In Table 5, the number of stores and the leasable area of each floor are shown.

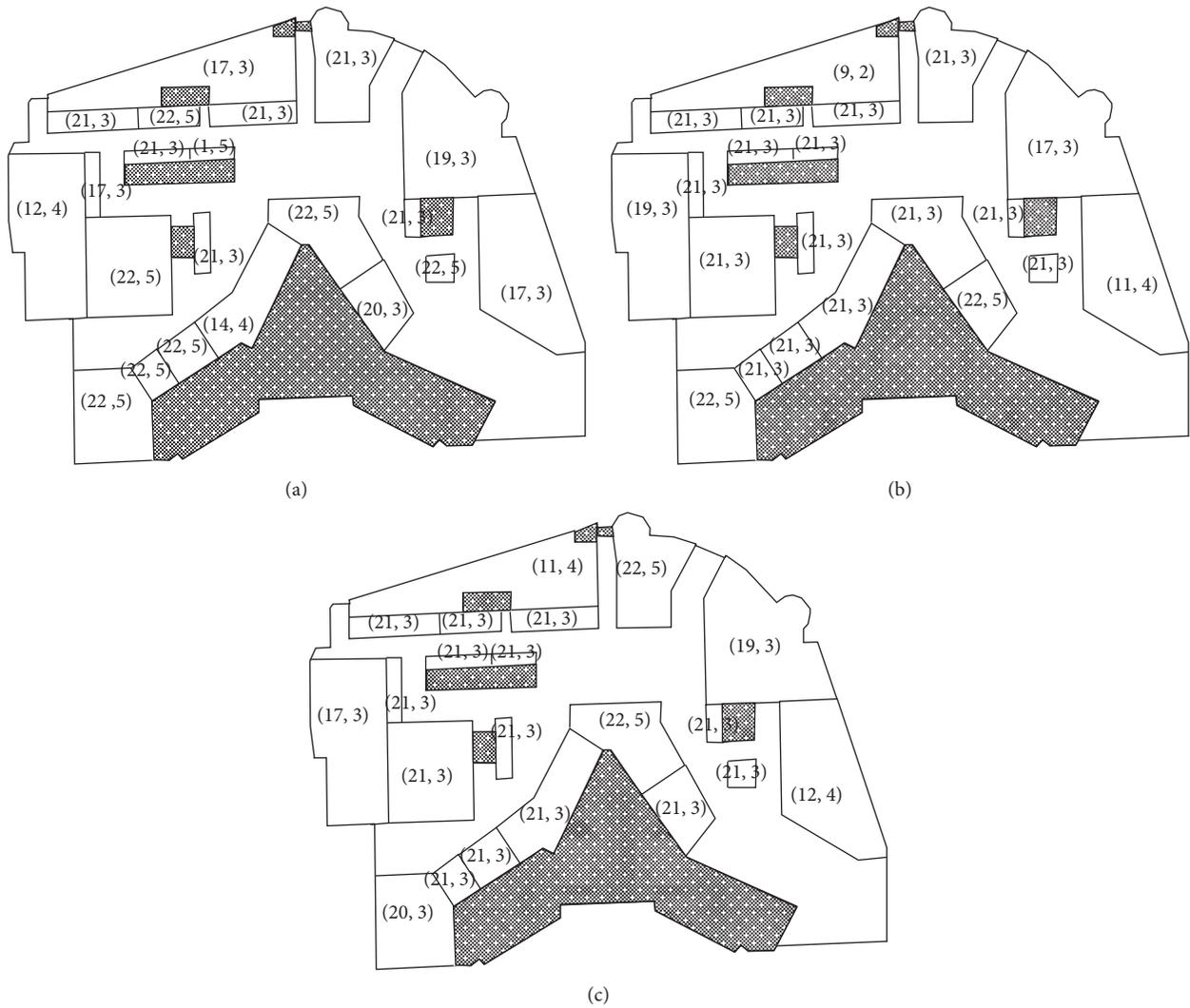


FIGURE 3: The optimized tenant mix layout. (a) Tenant mix layout, operation 1. (b) Tenant mix layout, operation 2. (c) Tenant mix layout, operation 3. *In (i, h) , i represents the retail type of the tenant, and h represents the brand level of the store. The shaded area represents the nonrental area.

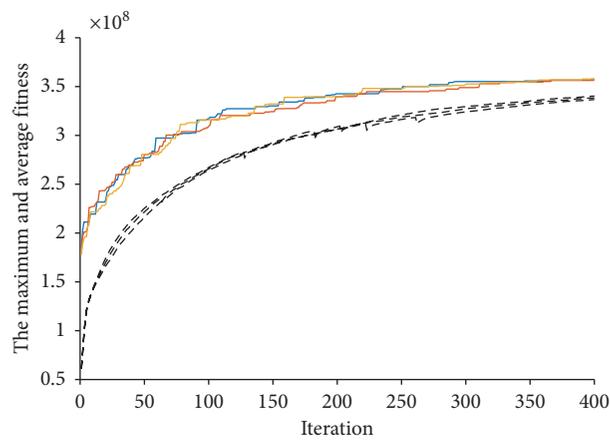


FIGURE 4: The fitness curve of GA.

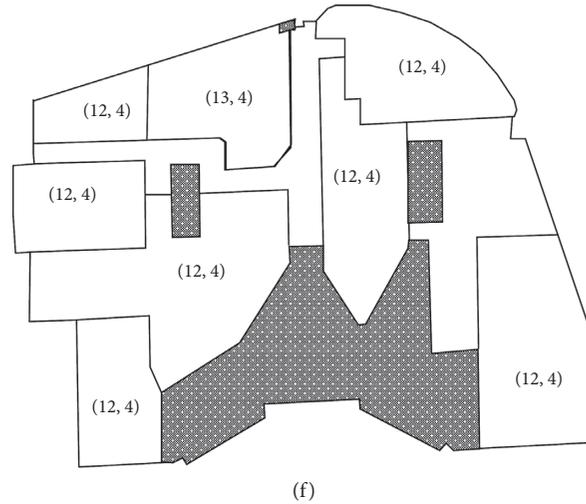


FIGURE 5: Original tenant mix layout. (a) Original tenant mix layout, B1. (b) Original tenant mix layout, F1. (c) Original tenant mix layout, F2. (d) Original tenant mix layout, F3. (e) Original tenant mix layout, F4. (f) Original tenant mix layout, F5. *In (i, h) , i represents the retail type of the tenant, and h represents the brand level of the store. The shaded area represents the nonrental area.

TABLE 5: Physical information.

Floor	Leasable area (m ²)	The number of stores
B1	8750.59	46
F1	2192.24	21
F2	2843.75	29
F3	2834.47	28
F4	2853.80	26
F5	2788.61	8

5.2. Model Stability Test. The fitness curve shows the stability of the improved algorithm. To verify whether the coefficients have an effect on the stability of the model results, we tested the effects of partial coefficients on the stability of model results. The coefficients in the model are mainly divided into three categories: (1) physical information coefficients of the shopping centre, such as the position coefficient (c_{po}) and floor coefficient (c_f); (2) coefficients of rental agreement, such as the deduction point ($a\%$) and store grade coefficient (c_h); and (3) assumptions coefficients obtained from the survey, such as the store area (S_{trav}) that customers are willing to search for, consumer consumption capacity ($n_i \cdot cost_i$), and consumer preference coefficient (α, β, φ).

According to the conclusion deduced using equation (10), S_{thre} , which leads to the largest sales per unit area of a store, is only related to the demand of comparison shopping and the competition among stores and has nothing to do with the purchasing capacity of potential consumers and the store area consumers are willing to search for products in. Therefore, we discuss the theme store through a simplified model.

In the simplified model, the consumer's consumption capacity, $n_i \cdot cost_i$, is a fixed value. We set a store with an initial area of 350 m² and gradually increase the area of the same type of retail stores in the shopping centre with a growth step of 1 m². To test whether the consumption preference coefficient will affect the results of the model, we tested four groups of different (α, β) combinations, $\beta -$

$\alpha \in \{0.2, 0.3, 0.4, 0.5\}$. The results are shown in Figure 6. Figure 6(a) shows the impact of the consumer preference coefficient on the sales of the store per unit area, and Figure 6(b) shows the impact of the consumer preference coefficient on the total sales of all stores in the retail type of shopping centre. Similarly, by fixing (α, β) and changing the consumption power $n_i \cdot cost_i \in \{2.4 \times 10^6, 3.4 \times 10^6, 4.4 \times 10^6\}$ (USD), which means the three groups of consumers, we discussed the influence of consumption power on the total sales of a single retail type. The conclusion is shown in Figure 7.

According to Figure 7, the consumption power is the same as the physical coefficient of the shopping centre, and its influence on the total sales does not change with the accumulation of similar stores. In the interview with the owner of the shopping centre, we compared the rent of the shopping centre calculated using the model to the actual rent, and the owner thought it could be used as an auxiliary tool for tenant layout. As shown in Figure 6, the difference in consumer preference coefficient causes the change in sales volume because it affects the store agglomeration of certain retail types in the shopping centre, so the difference in its value may have some impact on our conclusion. However, when we modify the consumer preference coefficient in the program's rent calculation module several times within a reasonable range, the rental performance of the optimized tenant mix is still better than that in the original tenant mix. Thus, the validity of the model is proved.

6. Conclusions

Tenant mix influences shopper behaviour and store performance. One of its essential purposes is to create a shopping environment that stimulates consumption and meets shoppers' preferences and needs to improve the overall sales of a shopping centre. The study on tenant mix layout has solved the problem of how to arrange the location,

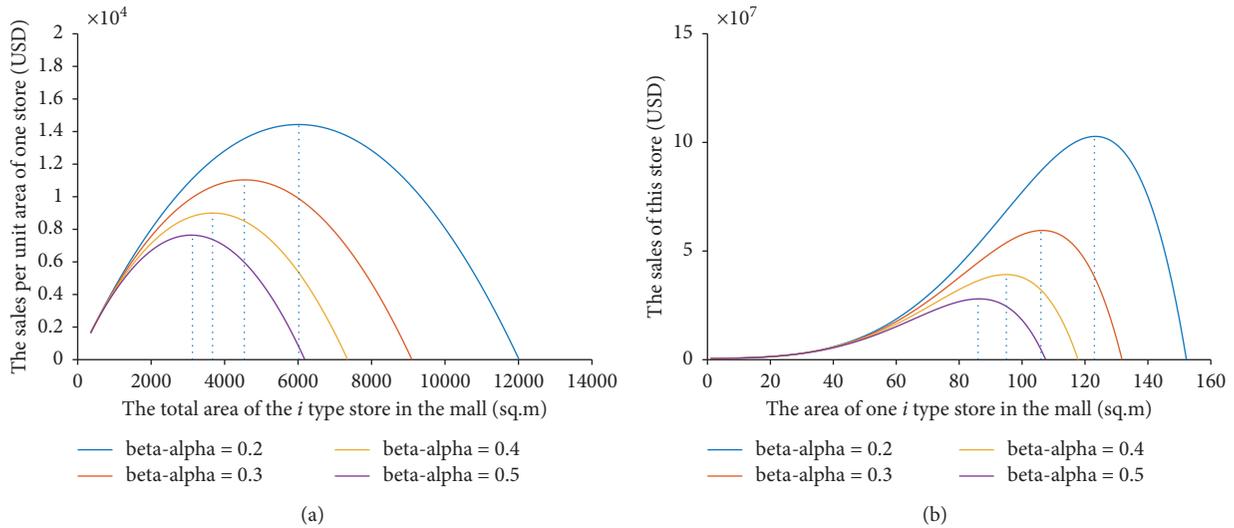


FIGURE 6: (a) Alternative preference coefficients' effect on sales of per unit area. (b) Alternative preference coefficients' effect on one store's total sales.

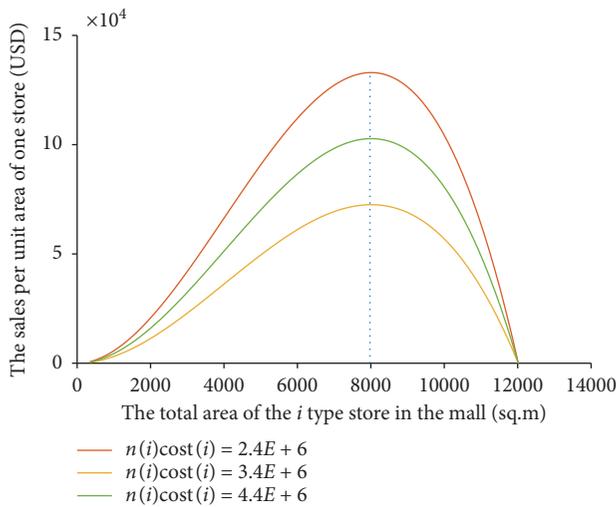


FIGURE 7: Alternative consumption ability.

retail type, and brand level of shops in shopping centre. The observation of the actual tenant mix of many local shopping centres shows that the owners used some previous research conclusions in their decisions about the placement of tenants, such as clustering certain types of retail shops on a certain floor and arranging many small shops near large supermarkets. However, the general rules they used are too complex and trivial to be applied to an entire shopping mall to determine the ideal tenant mix.

To solve this problem, the main contribution of our study is that it led to the development of a mathematical model that helps the owners of shopping centres optimize tenant mix layout and increase rental income. In the proposed model, we considered store externalities caused by consumer behaviour, which makes the operation results of the model closer to the actual needs. However, the proposed model is not intended to replace the decision-making

process of shopping centre management. It is a powerful tool that can be used to assist owners in quickly generating preliminary tenant mix layout plans, measuring the financial performance of the established tenant mix, and updating the layout plan according to the actual sales data in the operation process. We hope that the model proposed in this paper can be incorporated into software tools in the future to provide better decision-making information for shopping centre retail space planning.

This paper considers only the two main externalities that affect the layout of tenant mix, namely, the anchor store retail externalities and the similar agglomeration externalities. In future research, it is worth exploring to extend the model proposed in this paper. For example, researchers can discuss the agglomeration externalities of different types of retail in a mall, match shoppers' consumption habits with store impressions, or use more specific methods to simulate the exposure of the store in the vision.

Data Availability

The data of the real shopping centre used to support the findings of this study are included in the article, while other data and code are included within the supplementary information file(s).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This paper was vitally supported by Industry-University-Research Cooperation Project (no. 4418a4c49307416 78058b6cb12ca860c) of Ministry of Education of the People's Republic of China and Shenzhen THS Hi-Tech

Corporation. The authors greatly appreciate the support from these institutions.

Supplementary Materials

The data of the real shopping centre used to support the findings of this study are included in the article, while other data and code are included within the supplementary information file(s). (*Supplementary Materials*)

References

- [1] V. Couture, B. Faber, Y. Gu, and L. Liu, *E-Commerce Integration and Economic Development: Evidence from China*, CEPR, Washington, DC, USA, 2018.
- [2] POPAI, "Mass merchant shopper engagement study executive summary report," 2014, <http://www.shopassociation.org/a-re-popai-mass-merchant-shopper-study/>.
- [3] D. Zhang, P. Zhu, and Y. Ye, "The effects of E-commerce on the demand for commercial real estate," *Cities*, vol. 51, pp. 106–120, 2016.
- [4] V. Kumar, A. Anand, and H. Song, "Future of retailer profitability: an organizing framework," *Journal of Retailing*, vol. 93, no. 1, pp. 96–119, 2017.
- [5] W. C. Wheaton, "Percentage rent in retail leasing: the alignment of landlord-tenant interests," *Real Estate Economics*, vol. 28, no. 2, pp. 185–204, 2000.
- [6] J. K. Brueckner, "Inter-store externalities and space allocation in shopping centers," *The Journal of Real Estate Finance and Economics*, vol. 7, no. 1, pp. 5–16, 1993.
- [7] M. Gerbich, "Shopping center rentals: an empirical analysis of the retail tenant mix," *Journal of Real Estate Research*, vol. 15, no. 3, p. 283, 1998.
- [8] C. Teller, "Shopping streets versus shopping malls—determinants of agglomeration format attractiveness from the consumers' point of view," *The International Review of Retail, Distribution and Consumer Research*, vol. 18, no. 4, pp. 381–403, 2008.
- [9] J. Kunc, F. Križan, K. Bilková, P. Barlík, and J. Maryáš, "Are there differences in the attractiveness of shopping centres? Experiences from the Czech and Slovak Republics," *Moravian Geographical Reports*, vol. 24, no. 1, pp. 27–41, 2016.
- [10] J. D. Benjamin, *Megatrends in Retail Real Estate*, vol. 3, Springer, Dordrecht, Netherlands, 1996.
- [11] A. Borgers, M. Brouwer, T. Kunen, J. Jessurun, and I. Janssen, "A virtual reality tool to measure shoppers' tenant mix preferences," *Computers, Environment and Urban Systems*, vol. 34, no. 5, pp. 377–388, 2010.
- [12] V. Shankar, M. S. Yadav, M. Mantrala, E. Kelley, and R. Rizley, "Innovations in shopper marketing: current insights and future research issues," *Journal of Retailing*, vol. 87, no. 1, pp. S29–S42, 2011.
- [13] L. C. Mejia and M. J. Eppli, "Inter-center retail externalities," *The Journal of Real Estate Finance and Economics*, vol. 27, no. 3, pp. 321–333, 2003.
- [14] J. A. Dawson, *Shopping Center Development*, Longman Inc., New York, NY, USA, 1983.
- [15] S. Brown, "Tenant mix, tenant placement and shopper behaviour in a planned shopping centre," *The Service Industries Journal*, vol. 12, no. 3, pp. 384–403, 1992.
- [16] J. C. Bean, C. E. Noon, S. M. Ryan, and G. J. Salton, "Selecting tenants in a shopping mall," *Interfaces*, vol. 18, no. 2, pp. 1–9, 1988.
- [17] C. Yim Yiu and S. Y. S. Xu, "A tenant-mix model for shopping malls," *European Journal of Marketing*, vol. 46, no. 3/4, pp. 524–541, 2012.
- [18] N. Chong, *Empirical Study on Micro Determinants of Retail Rent and Tenant Mix in Shopping Center*, Economic Sciences Press, Cambridge, MA, USA, 2008.
- [19] M. A. Vitorino, "Empirical entry games with complementarities: an application to the shopping center industry," *Journal of Marketing Research*, vol. 49, no. 2, pp. 175–191, 2012.
- [20] H. Cho and J. D. Shilling, "Valuing retail shopping center lease contracts," *Real Estate Economics*, vol. 35, no. 4, pp. 623–649, 2007.
- [21] D. H. Gatzlaff, G. T. Sirmans, and B. A. Diskin, "The effect of anchor tenant loss on shopping center rents," *Journal of Real Estate Research*, vol. 9, no. 1, pp. 99–110, 1994.
- [22] M. J. Eppli and J. D. Shilling, "Changing economic perspectives on the theory of retail location," in *Megatrends in Retail Real Estate*, J. D. Benjamin, Ed., vol. 3, Berlin, Germany, Springer, 1996, Research Issues in Real Estate.
- [23] B. A. Peters, G.-A. Klutke, and A. R. Botsali, "Research issues in retail facility layout design progress," in *Progress in Material Handling Research*, R. D. Meller, Ed., pp. 399–414, Material Handling Institute, Charlotte, NC, USA, 2004.
- [24] H. Sorensen, "Long tail media in the store," *Journal of Advertising Research*, vol. 48, no. 3, pp. 329–338, 2008.
- [25] M. Kirkup and M. Rafiq, "Managing tenant mix in new shopping centres," *International Journal of Retail & Distribution Management*, vol. 22, no. 6, pp. 29–37, 1994.
- [26] H. Hamzah, "The tenant mix of shopping malls: a customers' perspective," in *Proceedings of the Pacific Rim Real Estate Society Conference*, Kuala Lumpur, Malaysia, January 2015.
- [27] C. Teller and T. Reutterer, "The evolving concept of retail attractiveness: what makes retail agglomerations attractive when customers shop at them?" *Journal of Retailing and Consumer Services*, vol. 15, no. 3, pp. 127–143, 2008.
- [28] C.-Y. Yiu, S. Y. Xu, and H. C. Ng, "Space allocation and tenant placement at high-rise shopping malls," *Journal of Retail & Leisure Property*, vol. 7, no. 4, pp. 315–324, 2008.
- [29] J. D. Benjamin, G. W. Boyle, and C. F. Sirmans, "Retail leasing: the determinants of shopping center rents," *Real Estate Economics*, vol. 18, no. 3, pp. 302–312, 1990.
- [30] M. J. Eppli and J. D. Shilling, "What's a shopping center worth?" in *Proceedings of the American Real Estate and Urban Economics Association Annual Meeting*, Key West, FL, USA, January 1993.
- [31] F. Rosiers, M. Thériault, and L. Ménétrier, "Spatial versus non-spatial determinants of shopping center rents: modeling location and neighborhood-related factors," *Journal of Real Estate Research*, vol. 27, no. 3, pp. 293–320, 2005.
- [32] C. C. Cater and K. D. Vandell, "Store location in shopping center: theory and estimates," *Journal of Real Estate Research*, vol. 27, no. 3, pp. 237–265, 2005.
- [33] J. P. Seagle, *An Allocation Model for Commercial Real Estate*, Graduate School of Business, Stanford University, Stanford, CA, USA, 1967.
- [34] Jensen and N. Arthur, *A Mixed Integer Programming Approach to Shopping Center Planning*, Arizona State University, Tempe, AZ, USA, 1980.
- [35] Urban Land Institute, *Shopping Center Development Handbook*, Urban Land Institute, Washington, DC, USA, 1985.
- [36] J. A. Dawson, *Shopping Centre Development*, Longman, London, UK, 1983.

- [37] J. J. Gabszewicz and J. F. Thisse, "On the nature of competition with differentiated products," *The Economic Journal*, vol. 96, no. 381, pp. 160–172, 1986.
- [38] B. P. Pashigian and E. D. Gould, "Internalizing externalities: the pricing of space in shopping malls," *The Journal of Law and Economics*, vol. 41, no. 1, pp. 115–142, 1998.
- [39] S. T. Yuo and C. Lizieri, "Tenant placement strategies within multi-level large-scale shopping centers," *Journal of Real Estate Research*, vol. 35, no. 1, pp. 25–51, 2013.
- [40] Y. Lu and H.-B. Seo, "Developing visibility analysis for a retail store: a pilot study in a bookstore," *Environment and Planning B: Planning and Design*, vol. 42, no. 1, pp. 95–109, 2015.
- [41] J. Hirsch, M. Segerer, K. Klein, and T. Wiegmann, "The analysis of customer density, tenant placement and coupling inside a shopping centre with GIS," *Journal of Property Research*, vol. 33, no. 1, pp. 37–63, 2016.
- [42] C. L. Mallows and R. D. Luce, "Individual choice behaviour," *Biometrika*, vol. 48, no. 1/2, p. 234, 1961.
- [43] T. J. Miceli and C. F. Sirmans, "Contracting with spatial externalities and agency problems the case of retail leases," *Regional Science and Urban Economics*, vol. 25, no. 3, pp. 355–372, 1995.
- [44] J. D. Benjamin and P. Chinloy, "The structure of a retail lease," *Journal of Real Estate Research*, vol. 26, no. 2, pp. 223–236, 2004.
- [45] J. V. Henderson, "The sizes and types of cities," *American Economic Review*, vol. 64, no. 4, pp. 640–656, 1974.
- [46] J. H. Holland, "Genetic algorithms and the optimal allocation of trials," *SIAM Journal on Computing*, vol. 2, no. 2, pp. 88–105, 1973.
- [47] U. Aickelin and K. A. Dowsland, "Enhanced direct and indirect genetic algorithm approaches for a mall layout and tenant selection problem," *Journal of Heuristics*, vol. 8, no. 5, pp. 503–514, 2002.