Research Article

Bayesian Approach for Sequential Probabilistic Back Analysis of Uncertain Geomechanical Parameters and Reliability Updating of Tunneling-Induced Ground Settlements

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This paper proposes a new sequential probabilistic back analysis approach for probabilistically determining the uncertain geomechanical parameters of shield tunnels by using time-series monitoring data. The approach is proposed based on the recently developed Bayesian updating with subset simulation. Within the framework of the proposed approach, a complex Bayesian back analysis problem is transformed into an equivalent structural reliability problem based on subset simulation. Hermite polynomial chaos expansion-based surrogate models are constructed to improve the computational efficiency of probabilistic back analysis. The reliability of tunneling-induced ground settlements is updated in the process of sequential back analyses. A real shield tunnel project of No. 1 Nanchang Metro Line in China is investigated to assess the effectiveness of the approach. The proposed approach is able to infer the posterior distributions of uncertain geomechanical parameters (i.e., Young's moduli of surrounding soil layers and ground vehicle load). The reliability of tunneling-induced ground settlements can be updated in a real-time manner by fully utilizing the time-series monitoring data. The results show good agreement with the variation trend of field monitoring data of ground settlement and the post-event investigations.

1. Introduction

In order to alleviate the pressure of urban traffic, constructing underground rail transit network has become a main development of urban transportation system. Urban metro shield tunnel construction often leads to surrounding ground movements and further endangers adjacent structures, which, in turn, poses great threat to the tunnel itself (e.g., [1, 2]). To mitigate the adverse impact of shield tunnel construction on the surrounding environment, the geomechanical parameters that can reflect the performance of a specific construction site and help to analyze the shield tunneling-induced ground subsidence shall be rationally determined (e.g., [3, 4]). Typically, only limited amount of field observation data (including geological survey data, test

data, and monitoring data) can be acquired for a specific construction site due to the restrictions on the construction cost, time, and project sites (e.g., [5, 6]). Therefore, how to obtain the geomechanical parameters of the shield tunnels with low uncertainty efficiently based on the limited field observation data remains an unsolved problem.

To make up for the deficiency of field observation data, many researches estimated the geomechanical parameters of shield tunnels through back analysis. For example, Chi et al. [7] applied an optimization technique for the back analysis of tunneling-induced ground movement based on monitoring data. Zhu et al. [8] proposed an artificial bee colony algorithm-based displacement back analysis method to determine the subsoil parameters during shield tunneling process. Gao and Ge [4] employed evolutionary neural

networks to back analyze the soil parameters and the initial geo-stress of Longtan tunnel and evaluate the stability of the tunnel. Although significant advances have been made in the back analysis of shield tunnel parameters, the above-mentioned studies do not account for the influence of the inherent variability of geomechanical parameters (e.g., [4, 9]). It is widely accepted that the geomechanical parameters vary spatially and temporally because of the concealment of underground space and complexity of underground environment (e.g., [10–16]). To obtain the shield tunnel parameters in line with the engineering practice, the uncertainties of the geomechanical parameters should be taken into account in the back analysis.

Several studies have investigated probabilistic back analysis problems of uncertain parameters for the shield tunnels by combining probability theory and statistics. For example, Haas and Einstein [17] employed a Markov chain Monte Carlo (MCMC) method to update the posterior distributions of surrounding rock mass parameters of tunnels based on the monitoring data. Špačková and Straub [18] proposed a dynamic Bayesian network based-tunneling process model to update the probability of tunnel failure by utilizing the observation data from geological survey and construction stages. Park and Park [19] conducted a probabilistic back analysis of uncertain parameters for tunnel surrounding rock masses by adopting a response surface method. Liu et al. [20] also employed the MCMC method for the probabilistic back analysis of subsoil cohesion and internal friction angle around a tunnel below river. Although many investigations on the probabilistic back analysis of shield tunnel parameters have been carried out, the time-series monitoring data that are collected during the tunnel advancement were rarely incorporated in the back analysis. To the best of our knowledge, only Miro et al. [21] applied a MCMC method in the probabilistic back analysis of uncertain parameters for surrounding rock masses by utilizing the tunnel-induced ground movements in a sequential manner. However, the MCMC method adopted by Miro et al. [21] is not effective for high dimensional problems, and the reliability of shield tunnels with the probabilistic back analysis results was not investigated. The updated statistics (i.e., means, standard deviations, probability distributions) of geomechanical parameters via the probabilistic back analyses can provide an important basis for reliability analysis of shield tunnels. Based on the updated probability distributions of the uncertain parameters, more realistic reliability assessment of the tunnel face stability or serviceability limit state can be achieved.

In this paper, a Bayesian Updating with Subset simulation (BUS)-based sequential probabilistic back analysis approach is proposed for estimating uncertain geomechanical parameters and updating the reliability of tunneling-induced ground settlements. To improve computational efficiency, the proposed approach transforms a complex Bayesian back analysis problem into an equivalent structural reliability problem which is then solved by subset simulation. The Hermite polynomial chaos expansion-based surrogate models are constructed. In Section 2, the proposed

approach comprising the construction of surrogate models and likelihood functions, inference of posterior distribution using BUS approach, and reliability updating of tunneling-induced ground settlements is presented. In Section 3, the No. 1 Nanchang Metro Line tunnel is treated as a typical case for probabilistically determining the uncertain geomechanical parameters based on the time-series monitoring data of ground settlement. Finally, the reliability of tunneling-induced ground settlements is updated on the basis of the probabilistic back analysis results.

2. BUS-Based Sequential Probabilistic Back Analysis Approach

Shield tunneling is often fraught with inevitable uncertainties that include, but not limited to, (1) the inherent variability of soils, (2) the measurement errors in quantifying the soil properties by field and laboratory tests, and (3) the limited knowledge on geotechnical conditions and simplified geotechnical models in predicting the ground settlements (e.g., [1, 21]). These uncertainties can be reduced by incorporating the field observation data into a Bayesian back analysis by estimating the posterior joint probability density function (PDF) of geomechanical parameters (e.g., [9, 22]). In this study, a BUS-based sequential probabilistic back analysis approach is proposed to estimate the posterior PDF, and its implementation procedure is briefly introduced as follows:

- (1) Determine the prior statistics (i.e., mean, coefficient of variation (COV), and distribution) based on the knowledge (e.g., geological survey reports, engineering experience, and existing data in the literature) about the uncertain input parameters.
- (2) Construct Hermite polynomial chaos expansion (HPCE)-based surrogate models to facilitate the calculation of tunneling-induced ground settlements for each excavation step.
- (3) Establish the likelihood function using the monitoring data of ground settlement obtained from the first excavation step.
- (4) Infer the posterior PDFs of uncertain input parameters using the BUS approach. Then, treat the posterior PDFs as the prior PDFs, and build a new likelihood function using the monitoring data of ground settlement obtained from the next excavation step until the last regarded excavation stage.
- (5) Update the reliability of tunneling-induced ground settlements for each excavation step based on the obtained posterior statistics of uncertain input parameters.

The flowchart of the BUS-based sequential probabilistic back analysis approach is presented in Figure 1. The involved construction of surrogate models and likelihood functions, inference of posterior distribution using the BUS approach, and reliability updating of tunneling-induced ground settlements are presented in the following subsections.

FIGURE 1: Flowchart of BUS-based sequential probabilistic back analysis and reliability updating approach.

2.1. Construction of Surrogate Models and Likelihood Functions. The ground settlements due to shield tunneling and the corresponding limit state functions are commonly the nonlinear implicit functions of the geomechanical parameters, which need to be evaluated through deterministic finite element or finite difference analyses. Generally, tens of thousands of finite element or finite difference analyses are required in a Bayesian back analysis [23], which is computationally demanding for a complicated shield tunnel model. To avoid the high computational burden in the probabilistic back analysis, the polynomial chaos expansion or multivariate adaptive regression splines-based response surface method can be adopted to construct the surrogate models of the ground settlements (e.g., [24, 25]). The response surface method can provide a powerful tool to approximate the nonlinear implicit limit state functions and has many successful applications (e.g., [26, 27]). This study applies the HPCE to construct the surrogates of deterministic numerical models for facilitating the calculation of ground settlements for each excavation step. A surrogate model of the ground settlement u_j at the j^{th} excavation step that involves the uncertain input parameters can be constructed as follows (e.g., [11, 28, 29]):

$$\begin{aligned}
 u_j = & a_0 \Gamma_0 + \sum_{i_1=1}^n a_{i_1} \Gamma_1(\xi_{i_1}) + \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} a_{i_1 i_2} \Gamma_2(\xi_{i_1}, \xi_{i_2}) \\
 & + \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3} \Gamma_3(\xi_{i_1}, \xi_{i_2}, \xi_{i_3}) + \dots,
 \end{aligned} \quad (1)$$

where $j = 1, 2, \dots, t$ is the total number of excavation steps; n is the number of random variables; $a_0, a_{i_1}, a_{i_1 i_2}, a_{i_1 i_2 i_3}, \dots$ are the unknown coefficients; $\Gamma_{j_p}(\cdot)$, $j_p = 1, 2, 3, \dots$ are Hermit polynomials with j_p degrees of freedom [30]; and $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ are a set of independent standard normal random variables. For the n_{HPCE} -th order HPCE, there are a total of $M = (n + n_{\text{HPCE}})! / (n! \times n_{\text{HPCE}}!)$ unknown coefficients (i.e., $a_0, a_{i_1}, a_{i_1 i_2}, a_{i_1 i_2 i_3}, \dots$) in equation (1). The unknown coefficients in the HPCE can be evaluated by solving a series of linear equations given by equation (1). The left side of the linear equations is u_j evaluated via deterministic finite element or finite difference analysis based on N realizations of the random variables that are generated by Latin Hypercube Sampling (LHS) technique (e.g., [31], in which $N \geq M$).

Having obtained the explicit and approximate expressions for calculating the ground settlements at different shield excavation steps in equation (1), the likelihood functions with the consideration of the uncertainties can be constructed. The likelihood function reflects the model fit with the field observation data for given input parameters $\mathbf{X} = (X_1, X_2, \dots, X_n)^T$. Typically, the difference (i.e., ε_{m_j} measurement error) between a measured and a simulated ground settlement for the given values $\mathbf{x}$ of random variables $\mathbf{X}$ can be expressed as

$$\varepsilon_{m_j} = u_j^m - u_j(\mathbf{x}), \quad (2)$$

where u_j^m is the measurement of the ground settlement which is made at the j^{th} excavation step and $u_j(\mathbf{x})$ is

evaluated through the deterministic analysis. Note that a mathematical transformation between $\mathbf{x}$ and ξ in equation (1) can be done by a Nataf transformation procedure [29]. Following Miro et al. [21], ε_{m_j} , $j = 1, 2, \dots, t$, are assumed to be independent and obey normal distributions with zero mean and constant standard deviations of $\sigma_{\varepsilon_{m_j}}$. Based on these, the likelihood functions at the j^{th} excavation step can be established as follows [32]:

$$L_j(\mathbf{x}) = \frac{\phi\left\{\left[u_j^m - u_j(\mathbf{x})\right]/\sigma_{\varepsilon_{m_j}}\right\}}{\sigma_{\varepsilon_{m_j}}}, \quad (3)$$

where $\phi(\cdot)$ is the PDF of a standard normal variable. With the constructed surrogate models using equation (1), the computational cost taken on the evaluations of likelihood functions can be substantially reduced, and so are the total computational costs of the sequential probabilistic back analysis.

2.2. Inference of Posterior Distribution. BUS approach is adopted herein to infer the posterior distributions of the uncertain input parameters, which defines the Bayesian back analysis problem as an equivalent structural reliability problem [23]. Subset simulation (SS) is then employed to solve the structural reliability problem to obtain samples from $f_{\mathbf{x}}''(\mathbf{x})$ (e.g., [33, 34]). The field observation information collected at the j^{th} excavation step can be described by a likelihood function $L_j(\mathbf{x})$ which is utilized to define a failure domain $\Omega_{\mathbf{x}}$ in an augmented outcome space $\mathbf{x}_+ = [\mathbf{x}; p]$:

$$\Omega_{\mathbf{x}} = \{H_j(\mathbf{x}_+) \leq 0\}, \quad (4)$$

where $H_j(\mathbf{x}_+)$ is the limit state function which is given by [35]

$$H_j(\mathbf{x}_+) = \ln p - \ln[cL_j(\mathbf{x})], \quad (5)$$

where p is the realization of a standard uniform random variable in $[0, 1]$ that is independent with $\mathbf{x}$ and c is a likelihood multiplier that satisfies the following inequality for all $\mathbf{x}$ [23]:

$$cL(\mathbf{x}) \leq 1.0. \quad (6)$$

It can be noted that sampling the posterior distribution of $\mathbf{x}$ is equivalent to finding the samples generated from the prior distribution of $\mathbf{X}$ and falling in the domain $\Omega_{\mathbf{x}}$ when determining the probability of information event Z , $P(Z)$ [36, 37]. As the subset simulation does, the BUS approach can also express the $P(Z)$ as a product of larger conditional probabilities of a series of nested intermediate events:

$$P(Z) = P[H_j(\mathbf{x}_+) \leq 0] = P(Z_1) \prod_{i=2}^m \Pr(Z_i|Z_{i-1}), \quad (7)$$

where $Z_1, Z_2, \dots, Z_{m-1}, Z_m$ are intermediate events defined as $Z_i = \{H(\mathbf{x}_+) < g_i\}$, in which g_i , $i = 1, 2, \dots, m$, are threshold values satisfying $g_1 > g_2 > \dots > g_{m-1} > 0 \geq g_m$; m is the number of subset levels required to reach the domain $\Omega_{\mathbf{x}}$; $P(Z_1)$ is the

probability of the first subset level; and $P(Z_i|Z_{i-1})$ is the conditional probability of Z_i given Z_{i-1} . Specifically, a modified Metropolis algorithm proposed by Au and Beck [33] is adopted for the computation of the conditional probabilities of $P(Z_i|Z_{i-1})$, $i = 2, 3, \dots, m$. The threshold values g_i , $i = 1, 2, \dots, m$, are determined adaptively such that the intermediate conditional probabilities take a target value p_0 .

Once the failure region $\Omega_{\mathbf{x}}$ is reached, the failure samples $\mathbf{x}_f$ in the final subset level are extracted and utilized to estimate the posterior statistics of the uncertain input parameters and compute $P(Z)$. The computational effort of the BUS approach decreases significantly with the logarithm of $P(Z)$, which in turn is proportional to the value of the constant c . To this end, the value of c is usually selected as large as possible such that equation (6) holds. Following Betz et al. [35] and Jiang et al. [36, 37], c is adaptively estimated as the reciprocal of the maximum of the likelihood function values over the samples at the current subset level, i.e.,

$$c_i = \frac{1}{\max\left[c_{i-1}^{-1}, \left\{L(\mathbf{x}_{i,k}), k = 1, 2, \dots, N_i\right\}\right]}, \quad (8)$$

($i = 1, 2, \dots, m$),

where N_i is the number of samples at each subset level; $c_1 \geq c_2 \geq \dots \geq c_m$ used in different subset levels should guarantee that the intermediate failure domain Z_i is entirely contained in the Z_{i-1} , $i = 2, 3, \dots, m$.

2.3. Reliability Updating of Tunneling-Induced Ground Settlements. Once the posterior statistics of the uncertain geomechanical parameters are obtained via the probabilistic back analysis, the reliability of tunneling-induced ground settlements for each excavation step can be updated. The limit state function expressing the maximal ground settlement exceeding an admissible threshold can be defined as

$$g_j(\mathbf{x}) = u_{\max} - u_j(\mathbf{x}), \quad (9)$$

where $u_{\max}$ is the admissible threshold of ground settlement. The posterior probability ($P_{f,j}$) of tunneling-induced ground collapse can be estimated using direct Monte Carlo simulation (MCS) as follows:

$$P_{f,j} = P(F_j|Z) = \frac{P(F_j \cap Z)}{P(Z)} = \frac{\sum_{k=1}^{N_f} I\left[u_{\max} \leq u_j(\mathbf{x}_f^k)\right]}{N_f}, \quad (10)$$

where F denotes the ground collapse event, $F_j = \{\mathbf{x} \in \Omega_{F_j}\}$, in which $\Omega_{F_j} = \{g_j(\mathbf{x}) \leq 0\}$; N_f is the number of posterior samples $\mathbf{x}_f$; and $I(\cdot)$ is the indicator function. In most cases, the joint probability $P(F_j \cap Z)$ is very small; thus the estimate of $P_{f,j}$ using the direct MCS will become rather time consuming. Alternatively, Jiang et al. [36] proposed to conduct a new SS operation following the BUS operation for calculating $P_{f,j}$. Interested readers can refer to Jiang et al. [36] for detailed procedures for estimating $P_{f,j}$.

3. Project Background

3.1. Shield Tunnel Overview. The shield tunnel project of No. 1 Nanchang Metro Line is located in Jiangxi province, China. It is a single-line and double-tunnel structure and about 28 km in length. The shield tunneling interval originates from the Aixi lake west station, passes through the front square of China telecom of Nanchang branch and Beijing east road, and finally arrives at the Gaoxin avenue station. The Beijing east road is the main traffic artery in Nanchang city. The ground traffic volume is large particularly on the holiday. The underground pipelines are complex, and thus the tunneling-induced ground subsidence is relatively easy to occur. The shield tunnel in this interval has a diameter of $D = 6.0$ m and a buried depth of $H = 14.0$ m, which is constructed with a single-circle shield machine. Figure 2 shows the profile of soil layers surrounding the shield tunnel. For the concerned tunnel interval, the design of five excavation steps is listed in Table 1. The total length of shield tunnel excavation is 1.12 km.

During the shield tunnel construction from the Gaoxin avenue station to the Aixi lake west station, a ground collapse accident occurred at the 827th ring on October 2, 2012. Post-event investigations of this accident found the leakage of the underground water pipes in the silty clay layer and the variation of subsoil property caused by the shield excavation disturbance were the main reasons of the collapse of the 827th ring. The Beijing east road above the shield tunnel was the main traffic artery, so the increase in the vehicle loads during the Chinese National day in 2012 was another main cause. In addition, the shield tunneling construction intensively occurred in the gravel layer (see Figure 2). The monitoring data of ground settlement obtained from the point Ds826 is utilized for sequential probabilistic back analysis of uncertain geomechanical parameters and reliability updating. Note that the point Ds826 is installed on the 826th ring and close to the collapsed 827th ring. Figure 3 presents the monitoring data of ground settlement collected from the point Ds826 at the five excavation steps.

3.2. Numerical Model and Parameters. The shield tunneling interval between the 821th and 845th rings is selected to establish the numerical model using the finite difference programme FLAC^{3D}. Figure 4 illustrates a three-dimensional (3D) finite difference model of the tunnel, which has a length of 42 m in the Y-axis direction, a width of 30 m in the X-axis direction, and a depth of 35 m in the Z-axis direction. With regard to the boundary conditions, the normal movements on the all sides of the 3D model are restrained, whereas the bottom of the model is not allowed to move in the three directions. The excavation face of the model is free, but the nodes around the excavated tunnel have a fixed radial displacement. To simulate the influence of the ground moving vehicle loads on shield tunnel excavation, a uniform vehicle load of $q = 10$ kPa is applied to the top of the model in accordance to Yang et al. [38].

The commonly used elastic and perfectly plastic model based on the Mohr-Coulomb failure criterion is utilized to

represent the stress-strain behavior of the subsoil masses. The initial stress is generated by applying gravitational acceleration to the model. The subsoil masses surrounding the tunnel are modeled using the cylinder elements, while the rest is modeled using the hexahedral elements. The excavated tunnel is lined with a linear elastic material with a Young's modulus of 3.45 GPa, a Poisson's ratio of 0.2, and a density of 2450 kg/m³. The lining segment is prefabricated with C50 concrete with a thickness of 0.3 m and a width of 1.2 m, which is modeled using a shell element. A discretization of the model with a total number of 34,400 elements and 37,154 nodes is adopted after a preliminary study of the influence of mesh size. According to Mollon et al. [3], the considered tunnel may result in large ground settlements since it corresponds to a shallow tunnel with the overburden depth being about 2.33 times the outer diameter. For simplicity, the groundwater table is not considered in this study.

The geomechanical parameters of different soil layers are determined based on the geological survey reports [39], as summarized in Table 2. According to the post-event investigations as mentioned in Section 3.1, Young's modulus E_1 of the gravel layer, Young's modulus E_2 of the silty clay layer, and ground vehicle load q that are closely related to the ground collapse are identified as random variables after a simple parametric sensitivity study. The prior information of the three random variables (i.e., E_1 , E_2 , and q) is determined on the basis of the field observation data and existing data in the literature (e.g., [28, 38, 40–42]). The prior statistics of three random variables are summarized in Table 3.

3.3. Construction and Validation of Surrogate Models. Typically, 3D deterministic finite difference analysis of tunneling-induced ground settlements suffers from excessive computational effort. To improve the computational efficiency of the probabilistic back analysis, the 4th order HPCE-based surrogate models of ground settlements are constructed for different excavation steps in advance. For each excavation step, the number of unknown coefficients of the 4th order HPCE is $M = 35$. $N = 70$ random samples are generated according to the prior statistics of three random variables by the LHS technique to establish the linear equations and determine the unknown coefficients. The expansion terms and the corresponding coefficients of the 4th order HPCE for the 1st excavation step are listed in Table 4.

To balance the computational accuracy and efficiency, 100 direct MCS random samples are generated to verify the surrogate models. Based on these 100 random samples, the probability distributions of the uncertain geomechanical parameters and ground settlements can be inferred with acceptable accuracy. Figures 5(a)–5(f) compare the tunneling-induced ground settlements (i.e., u_1 , u_2 , and u_5) and their PDFs for three representative excavation steps (i.e., 1, 2, and 5) determined from the 4th order HPCE-based surrogate models and original deterministic finite difference analyses using these 100 random samples, respectively. As observed from Figure 5, the ground settlements and their PDFs

FIGURE 2: Profile of soil layers surrounding the shield tunnel.

TABLE 1: Excavation design for the concerned shield tunneling interval.

Step no.	Excavation time	Ring no.
1	2012/9/30 15:00	836
2	2012/10/1 07:00	838
3	2012/10/1 15:00	839
4	2012/10/2 07:00	840
5	2012/10/2 15:00	841

obtained from these two methods are in good agreement. It indicates the 4th order HPCE-based surrogate models can well approximate the 3D numerical models and replace the deterministic finite difference analyses to accurately calculate the u_j at each excavation step in this example.

3.4. Sequential Probabilistic Back Analysis Results. In this section, the BUS approach is employed to infer the posterior distributions of E_1 , E_2 , and q via the sequential probabilistic back analysis using the time-series monitoring data of ground settlement as shown in Figure 3. Based on the trade-off between the computational accuracy and efficiency, the number of samples at each subset level $N_j = 5000$ and conditional probability $p_0 = 0.1$ are chosen. Following Miro et al. [21], the standard deviations of measurement errors $\sigma_{\varepsilon_{m_j}} = 2.0$ mm are used. Figures 6–8 compare the posterior PDFs of E_1 , E_2 , and q estimated from the five different excavation steps, respectively. The prior PDFs of E_1 , E_2 , and

FIGURE 3: Time-series monitoring data of tunneling-induced ground settlement.

q are also plotted in Figures 6–8, respectively, for comparison. As observed from Figures 6–8, the posterior PDF curves of E_1 , E_2 , and q get steeper and narrower as the shield tunnel advances, and are much more peaked than the corresponding prior PDFs. The posterior means of E_1 and E_2 become smaller and smaller, while that of q becomes larger and larger as the shield tunnel progresses. This is consistent

FIGURE 4: 3D finite difference model for the No. 1 Nanchang Metro Line tunnel.

TABLE 2: Geomechanical parameters for different soil layers.

Soil layers	Density (kg/m ³)	Young's modulus (MPa)	Poisson's ratio	Cohesion (kPa)	Friction angle (°)
Plain fill	1813	15	0.42	5	10
Silty clay	1933	15	0.35	44.7	19.3
Fine sand	1913	16	0.4	1	30
Gravel	1893	28	0.39	1	36
Highly weathered silty mudstone	2050	120	0.3	60	37
Moderately weathered silty mudstone	2390	450	0.39	120	32

TABLE 3: Prior statistics of three random variables.

Random variable	Mean (MPa)	Standard deviation (MPa)	COV	Distribution
Young's modulus of gravel layer, E_1	28	4.2	0.15	Lognormal
Young's modulus of silty clay layer, E_2	15	2.25	0.15	Lognormal
Uniform vehicle load, q	10	1.0	0.1	Lognormal

with the post-event investigations and the common sense that the increase in the ground deformation is usually caused by the reduction of soil stiffness or the increase of external loads. Significant changes can be observed on the posterior PDFs of E_1 , E_2 , and q when the time-series monitoring data are sequentially incorporated in the probabilistic back analysis. This lies in the fact that the occurrence position of ground collapse is close to the 827th ring and the ground settlement collected from the monitoring point Ds826 sharply increases at the 5th excavation step (see Figure 3). It indicates the proposed approach not only can make full use of the time-series monitoring data to effectively update the statistics and reduce the uncertainties of geomechanical parameters, but also can well characterize the realistic change trends of surrounding subsoil properties.

Additionally, the COVs of E_1 , E_2 , and q decrease successively from the prior COVs as the monitoring data are sequentially used in the probabilistic back analysis, as

shown in Figure 9. The prior COVs of E_1 , E_2 , and q are 0.15, 0.15, and 0.1, respectively, which are reduced to 0.1, 0.12, and 0.085 at the 3rd excavation step, and to 0.07, 0.11, and 0.08 at the 5th excavation step. It is interesting to note that the uncertainty of E_1 is reduced the most, which implies the gravel layer affects the ground subsidence the most. The uncertainties of the geomechanical parameters associated with the shield tunnel have been significantly reduced through a Bayesian back analysis in a sequential manner.

3.5. Reliability Updating Results of Ground Settlements.

Based on the obtained posterior distributions of the uncertain geomechanical parameters for each excavation step, the reliability of tunneling-induced ground settlements can be updated using equations (9) and (10). An admissible threshold of ground settlement $u_{\max} = 30$ mm is selected for

TABLE 4: Expansion terms and coefficients of the 4th order HPCE for the 1st excavation step.

No.	Term	Coefficient	Value	No.	Term	Coefficient	Value
1	1	a_0	12.08	19	$\xi_2^2 \xi_3 - \xi_3$	a_{18}	-0.03
2	ξ_1	a_1	-1.38	20	$\xi_1 \xi_2 \xi_3$	a_{19}	-0.02
3	ξ_2	a_2	-0.32	21	$\xi_1^4 - 6\xi_1^2 + 3$	a_{20}	0.01
4	ξ_3	a_3	0.57	22	$\xi_2^4 - 6\xi_2^2 + 3$	a_{21}	-0.05
5	$\xi_1^2 - 1$	a_4	0.27	23	$\xi_3^4 - 6\xi_3^2 + 3$	a_{22}	-0.01
6	$\xi_2^2 - 1$	a_5	-0.16	24	$\xi_1 \xi_2^3 - \xi_1 \xi_2$	a_{23}	0.04
7	$\xi_3^2 - 1$	a_6	0.01	25	$\xi_1 \xi_3^3 - \xi_1 \xi_3$	a_{24}	0
8	$\xi_1 \xi_2$	a_7	0.02	26	$\xi_1^3 \xi_2 - \xi_1 \xi_2$	a_{25}	0
9	$\xi_1 \xi_3$	a_8	-0.15	27	$\xi_2^3 \xi_3 - \xi_2 \xi_3$	a_{26}	-0.03
10	$\xi_2 \xi_3$	a_9	-0.01	28	$\xi_1^3 \xi_3 - \xi_1 \xi_3$	a_{27}	-0.04
11	$\xi_1^3 - 3\xi_1$	a_{10}	-0.10	29	$\xi_2^3 \xi_3 - \xi_2 \xi_3$	a_{28}	0.04
12	$\xi_2^3 - 3\xi_2$	a_{11}	-0.01	30	$\xi_1^2 \xi_2^2 - \xi_1^2 - \xi_2^2 + 1$	a_{29}	-0.01
13	$\xi_3^3 - 3\xi_3$	a_{12}	0	31	$\xi_1^2 \xi_3^2 - \xi_1^2 - \xi_3^2 + 1$	a_{30}	0.02
14	$\xi_1 \xi_2^2 - \xi_1$	a_{13}	-0.01	32	$\xi_2^2 \xi_3^2 - \xi_2^2 - \xi_3^2 + 1$	a_{31}	0.04
15	$\xi_1 \xi_3^2 - \xi_1$	a_{14}	-0.01	33	$\xi_1^2 \xi_2 \xi_3 - \xi_2 \xi_3$	a_{32}	0.07
16	$\xi_1^2 \xi_2 - \xi_2$	a_{15}	0.06	34	$\xi_1 \xi_2^2 \xi_3 - \xi_1 \xi_3$	a_{33}	-0.03
17	$\xi_2^2 \xi_3 - \xi_2$	a_{16}	0.03	35	$\xi_1 \xi_2 \xi_3^2 - \xi_1 \xi_2$	a_{34}	-0.03
18	$\xi_1^2 \xi_3 - \xi_3$	a_{17}	0.09

FIGURE 5: Continued.

FIGURE 5: Validation of the surrogate models underlying three representative excavation steps. (a) Comparison of u_1 . (b) Comparison of the PDF of u_1 . (c) Comparison of u_2 . (d) Comparison of the PDF of u_2 . (e) Comparison of u_5 . (f) Comparison of the PDF of u_5 .

illustration. Figure 10 presents the variation of the posterior probability of ground collapse with the excavation step. As seen from Figure 10, the posterior probability of ground collapse increases continuously as the tunnel starts to advance. Then it increases dramatically at the 2nd excavation step and exceeds the prior probability of ground collapse (5.152×10^{-5}) and increases furthermore at the 3rd excavation step until reaching 0.36% at the 5th excavation step. The variation trend of the posterior probability indicates a safety check and necessary support measures shall be timely taken at the 3rd excavation step to control the monotonous increase of ground settlement. Otherwise, the occurrence probability of ground collapse due to the shield tunneling will eventually be large and unacceptable. Moreover, the

variation trend of the posterior probability with the time is consistent with that of the time-series monitoring data as shown in Figure 3.

For the case of the 5th excavation step, the posterior probability of ground collapse ($P_{f,5}$) estimated from the proposed approach is 0.36%. To calculate such a probability, the proposed approach needs performing 5×70 runs of 3D deterministic finite difference analyses of the tunneling-induced ground settlements to construct five surrogate models and additional probabilistic back analysis and reliability updating. For the same problem, the direct MCS requires more than 27677 runs of 3D deterministic finite difference analyses for achieving a target $COV_{P_{f,5}}$ below 10%. This is because the least number of samples

FIGURE 6: Comparison of the posterior PDFs of Young's modulus of gravel layer for different excavation steps.

FIGURE 7: Comparison of the posterior PDFs of Young's modulus of silty clay layer for different excavation steps.

required for the MCS to estimate $P_{f,5}$ is calculated by $N_{\text{sim}} \geq (1 - P_{f,5}) / (P_{f,5} (\text{COV}_{P_{f,5}})^2)$ [22]. The computational time required for one run of 3D deterministic finite difference analysis is 800 seconds when the computations are

FIGURE 8: Comparison of the posterior PDFs of ground vehicle load for different excavation steps.

FIGURE 9: Variation of the coefficients of variation of input parameters with the excavation step.

performed on a desktop with 8 GB RAM and one Intel Core i7-4790 CPU clocked at 3.6 GHz. The computational time taken on the probabilistic back analysis and reliability updating with the constructed surrogate models equals 18 seconds, which is only 1/44 of that required for one run of 3D deterministic finite difference analysis. Based on these, about 6150 hours will be required for the direct MCS while $5 \times 800 + 18$ seconds (1.1 hours) are required for the proposed approach to calculate the posterior probability of

FIGURE 10: Comparison of prior and posterior probabilities of ground collapse induced by shield tunneling.

ground collapse at the 5th excavation step. This confirms that the proposed approach is much more efficient in the probabilistic back analysis of the uncertain geomechanical parameters and the reliability updating. Such high efficiency will greatly facilitate the applications of the proposed approach in geotechnical engineering.

4. Conclusions

A BUS-based sequential probabilistic back analysis is proposed to estimate the uncertain geomechanical parameters and update the reliability of tunneling-induced ground settlements. The shield tunnel project of No. 1 Nanchang Metro Line in China is investigated to assess the effectiveness of the proposed approach. Several conclusions can be drawn from this study:

- (1) The proposed approach can well infer the posterior distributions of uncertain geomechanical parameters by fully utilizing the time-series monitoring data. The reliability of tunneling-induced ground settlements is updated in a real-time manner. The computational efficiency has been improved through transforming the Bayesian back analysis problem into an equivalent structural reliability problem and constructing the surrogate models of the output responses of shield tunnels by the Hermite polynomial chaos expansion.
- (2) By employing the proposed approach, the variation trends of the means of uncertain geomechanical parameters and the posterior probability of ground collapse match well with those of time-series monitoring data and the post-event investigations. The probability distributions of geomechanical parameters gradually converge to the target distribution, and the uncertainties of geomechanical parameters are reduced successively after updating. These demonstrate the effectiveness of the proposed approach.

- (3) The sequential probabilistic back analysis and reliability updating results can provide an important reference for the reduction of the uncertainties of geomechanical parameters during shield tunnel excavation and, consequently, the mitigation of the potential risk of ground collapse. For the considered real example, the probability of ground collapse increases markedly from October 1, 2012, 7:00 to October 1, 2012, 15:00, which can provide valuable information for the practitioners to formulate early warning measures to prevent the occurrence of ground collapse accident.

Data Availability

Some or all data, models, or code generated or used during this study are available to the readers upon request. The items are listed as follows:

- (1) Time-series monitoring data of tunneling-induced ground settlement
- (2) Hermite polynomial chaos expansion code that is used for constructing the surrogate models of the output responses of shield tunnels
- (3) BUS code that is used for inferring the posterior distribution of geomechanical parameters and estimating the posterior probability of ground collapse

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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