

Research Article

Formulation of Anisotropic Strength Criterion for Geotechnical Materials

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A new nonlinear unified strength (NUS) criterion is obtained based on the spatially mobilized plane (SMP) criterion and Mises criterion. New criterion is a series of smooth curves between SMP curved triangle and Mises circle in the π plane and thereby unifies the strength criteria. The new criterion can reflect the effect of the intermediate principal stress and consider the strength nonlinearity of a material. Based on the fabric tensor, the anisotropic parameter A is defined, and the anisotropic equation is proposed and introduced into the NUS criterion to form a nonlinear unified anisotropic strength criterion. The new criterion can be used to predict the strength variation of granular materials and cohesive materials under three-dimensional stress and can present the strength anisotropy of the geomaterials. The validity of the new criterion was checked using rock and soil materials. It is shown that the prediction results for the criterion agree well with the test data.

1. Introduction

The research of strength criterion theory [1–11] is an important topic in geotechnical engineering. There are usually three types of methods for establishing strength criteria, namely, theoretical methods, empirical methods, and combined theoretical-empirical methods. Theoretical methods are usually based on a hypothesis, and there is a model to interpret the failure mechanism of geomaterials. All the model parameters in the strength criteria established by this method have clear physical meanings. This type of strength criterion uses the law of friction to explain the failure of geomaterials, including the Mohr–Coulomb strength theory, the D-P criterion [11], the spatially mobilized plane (SMP) criterion [2], and the twin shear unified strength theory [9, 12, 13]. Strength criteria established by empirical methods are usually based on the fitting of test

data and include the famous Willam–Warnke criterion [14] and the Hoek–Brown criterion [6, 15]. Compared to the criteria that are based on theoretical methods, the physical meanings of some of the parameters of the strength criteria established by empirical methods are often not very clear. Finally, combined theoretical-empirical methods are usually used to establish special strength criteria for specific material properties. This type of criterion applies the Mohr–Coulomb strength theory to jointed rock materials [16] by introducing additional parameters [17–19].

However, the above strength criteria are associated with a single shape in the π plane and are thus unable to reflect the factors of material strength that may vary with the change of internal factors. To address this problem, researchers [10, 11, 20–26] have proposed to unify the strength criteria. The common method is to introduce some parameters to change the size of the shape function.

The above criteria are all isotropic strength criteria and cannot depict the strength anisotropy of geomaterials. However, numerous geomaterials exhibit transverse isotropy due to deposition; that is, the strength of the material is the same in the deposition surface but different in the depositional surfaces of different directions. The directional shear test of most soils shows that the change in the direction of the principal stress also leads to strength anisotropy, reflecting the initial anisotropy of the material. Therefore, it is necessary to establish a uniform strength criterion that considers the depositional characteristics or initial anisotropy of geomaterials.

The above previous studies show that the strength criterion is researched through a simple single form to a unified strength theory system with a relatively wide scope of application. Consequently, it is important to explore a widely applicable strength theory system. In the present paper, a new anisotropic nonlinear unified strength criterion is established based on Mises criterion and SMP criterion. The new criterion is approximated as a series of smooth curves between the SMP curved triangle and the von Mises circle in the π plane to unify the strength criteria. Based on the fabric tensor, a nonlinear anisotropic strength criterion is proposed. The correctness of the criterion is verified using sand, clay, and rock materials.

2. Form of the NUS Criterion

2.1. Linear Unified Strength (LUS) Criterion. SMP criterion (Matsuoka and Nakai [2]) has been widely used to check the strength of geomaterials because of its simple form and clear physical meaning. In the present study, the SMP criterion is modified to make it more widely applicable. The SMP criterion is shown in the following equation:

$$\frac{I_1 I_2}{I_3} = C, \quad (1)$$

where I_1 , I_2 , and I_3 are stress invariants:

$$\left. \begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_1 \sigma_3 \\ I_3 &= \sigma_1 \sigma_2 \sigma_3 \end{aligned} \right\}. \quad (2)$$

C is a model parameter. Under conventional triaxial compression ($\sigma_2 = \sigma_3$), the sine of the friction angle φ_0 of a granular material is

$$\sin \varphi_0 = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}. \quad (3)$$

Substituting equations (2) and (3) into equation (1) gives

$$\frac{I_1^3}{I_3} = \frac{(3 - \sin \varphi_0)^3}{1 - \sin \varphi_0 - \sin^2 \varphi_0 + \sin^3 \varphi_0}. \quad (4)$$

Using the trigonometric identity, the expression of q is obtained as

$$3p^3 - L_2 q^2 p + L_3 \cos(3\theta) q^3 = 0, \quad (5)$$

where

$$L_2 = \frac{3 + \sin^2 \varphi_0}{12 \sin^2 \varphi_0}, \quad (6)$$

$$L_3 = \frac{9 - \sin^2 \varphi_0}{108 \sin^2 \varphi_0}, \quad (7)$$

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}, \quad (8)$$

$$q = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]}, \quad (9)$$

where p is the mean stress and q is generalized shear stress. Thus, the shape function of the SMP criterion is

$$g_0(\theta) = \frac{q}{q_0} = \frac{\cos[(1/3)\arccos(-3\sqrt{3}L_3/2L_2^{3/2})]}{\cos[(1/3)\arccos(-3\sqrt{3}L_3 \cos(3\theta)/2L_2^{3/2})]}, \quad (10)$$

where q_0 is the data for q when $\theta = 0$. Therefore, the forms of p and q of the SMP criterion are

$$q = M g_0(\theta) p, \quad (11)$$

where $g_0(\theta)$ is obtained by equation (10) and M is the coefficient of friction:

$$M = \frac{6 \sin \varphi_0}{3 - \sin \varphi_0}. \quad (12)$$

The shape function of the SMP criterion cannot reflect the influence of internal factors on material; therefore, the application of the criterion is limited. In this report, the shape function is rewritten as the following expression:

$$q = M g(\theta) (p + \sigma_0), \quad (13)$$

where M is obtained from equation (12), σ_0 which is the triaxial tensile strength of the material is given by

$$\sigma_0 = c \cot \varphi_0, \quad (14)$$

and p and q are the average stress and the generalized shear stress, as defined by equations (9) and (10), respectively.

Therefore,

$$g(\theta) = \frac{r_\theta}{r_0}, \quad (15)$$

where

$$\left. \begin{aligned} r_\theta &= \frac{\sqrt{3}(1-\alpha)}{2\sqrt{L_2} \cos \delta_\theta} + \frac{6\alpha \sin \varphi_0}{3 - \sin \varphi_0} \\ r_0 &= \frac{\sqrt{3}(1-\alpha)}{2\sqrt{L_2} \cos \delta_0} + \frac{6\alpha \sin \varphi_0}{3 - \sin \varphi_0} \end{aligned} \right\}, \quad (16)$$

and α is the model parameter determined by the strength ratio under the triaxial compression and extension conditions. The value of α ranges from 0 to 1. One has

$$\left. \begin{aligned} \delta_\theta &= \frac{1}{3} \arccos\left(-\frac{3\sqrt{3}L_3 \cos(3\theta)}{2L_2^{3/2}}\right) \\ \delta_0 &= \frac{1}{3} \arccos\left(-\frac{3\sqrt{3}L_3}{2L_2^{3/2}}\right) \end{aligned} \right\}, \quad (17)$$

where L_2 and L_3 are obtained from equations (6) and (7), respectively.

Figure 1 illustrates the influences of parameters φ_0 and α on linear criterion in the deviatoric plane. As Figure 1(b) shows, the shape of the criterion is shifted from the SMP curved triangle to the von Mises circle when α gradually increases from 0 to 1.

2.2. Nonlinear Isotropic Strength (NIS) Criterion. In the triaxial test of the rockfill material tests [27], the large confining pressure causes the granules to break [28–30], and the material exhibits remarkable strength nonlinearities. Therefore, to obtain the nonlinear of the rockfill material, the above new criterion equation (13) need be further modified as expressed in the following equation:

$$q = M_f g(\theta) p_r \left(\frac{p + \sigma_0}{p_r} \right)^n. \quad (18a)$$

That is,

$$\ln\left(\frac{q}{p_r}\right) = n \ln\left(\frac{p + \sigma_0}{p_r}\right) + \ln[M_f g(\theta)], \quad (18b)$$

where p_r is reference stress usually taken as $p_r = 101$ kPa; M_f is usually obtained from the intercept of equation (18b) and the shape function; and n is a curvature parameter.

In the space of $\ln(q/p_r) - \ln(p + \sigma_0/p_r)$, $\ln[M_f g(\theta)]$ is the intercept and n is the slope.

When $n = 1$,

$$M_f = M = \frac{6 \sin \varphi_0}{3 - \sin \varphi_0}, \quad (19)$$

where n is the curvature parameter, and the shape function in the deviatoric plane is

$$g(\theta) = \frac{(\sqrt{3}(1-\alpha)/2\sqrt{L_2} \cos \delta_\theta) + (6\alpha \sin \varphi_0/3 - \sin \varphi_0)}{(\sqrt{3}(1-\alpha)/2\sqrt{L_2} \cos \delta_0) + (6\alpha \sin \varphi_0/3 - \sin \varphi_0)}. \quad (20)$$

2.3. Nonlinear Unified Strength (NUS) Criterion. Numerous geomaterials exhibit transverse isotropy due to deposition, as shown in Figure 2; that is, the strength of the materials is the same within the depositional surface but different in the depositional surfaces of different directions, reflecting the initial anisotropy of the material. Therefore, it is necessary to establish a strength criterion that considers the depositional characteristics or initial anisotropy of the geomaterial. The premise of establishing the anisotropic strength criterion is to quantify the degree and direction of

the initial anisotropy of the geomaterial. The fabric tensor proposed by Brewer is a good choice. The anisotropic function in the present study refers to the form in [31–33]:

$$f(A) = \exp[\eta_1(A - A_0) + \eta_2(A - A_0)^2], \quad (21)$$

where A is the anisotropic parameter and A_0 is the value that A takes from $b=0$ and $\alpha=0$. η_1 and η_2 in the anisotropic function are selected to depict the anisotropy of the material. The anisotropic function and the nonlinear isotropic strength criterion are combined to obtain an anisotropic strength criterion, as shown in the following equation:

$$q = f(A) M_f g(\theta) p_r \left(\frac{p + \sigma_0}{p_r} \right)^n. \quad (22)$$

2.4. Effects of η_1 and η_2 on ANUS. The effect of isotropic parameters on the shape of the strength criterion in the deviatoric plane has been discussed above. This section focuses on the effect of anisotropic parameters on the shape of the strength criterion. According to the characteristics of the test data, the parameters η_1 and η_2 in the anisotropic function are selected to depict the anisotropy of the material. Different formulas can be selected for the anisotropic parameter A in equation (22) according to the true triaxial test or the hollow torsional shear test. In the present study, the anisotropic strength in the true triaxial test is mainly discussed.

In the true triaxial test, when the stress tensor and the fabric tensor are coaxial ($\beta=0$), as shown in Figure 2, the shape of the strength criterion in the π plane varies with the parameters η_1 and η_2 , as shown in Figures 3(a), 3(c), and 3(e). In this section, with η_2 fixed at -0.25 , -0.333 , and 0 , the effect of varying η_1 on the strength curve in the π plane is discussed. As shown in Figures 3(a) and 3(b), with $\eta_2 = -0.25$, the strength values of the anisotropic criterion and the isotropic criterion at $\theta = 180^\circ$ are equal with different η_1 values, and the criterion is symmetric about the σ_z axis. The criterion expands in the π plane when $\eta_1 > 0$ and shrinks in the π plane when $\eta_1 < 0$, and the degrees of expansion or shrinkage are related to the value of η_1 . The anisotropic function variation in the π plane is consistent with that of the strength criterion, as shown in Figure 3(b). With $\eta_2 = -0.333$, as shown in Figure 3(c), the strength values of the anisotropic criterion and the isotropic criterion at $\theta = 120^\circ$ and 240° , respectively, are equal with different η_1 values, and the criterion is symmetric about the σ_z axis. Relative to the isotropic criterion, the anisotropic criterion expands in regions I and II and shrinks in region III of the π plane when $\eta_1 > 0$, and it shrinks in regions I and II and expands in region III of the π plane when $\eta_1 < 0$, with the degrees of expansion or shrinkage being related to the value of η_1 . The anisotropic function variation in the π plane is consistent with that of the strength criterion, as shown in Figure 3(d). When $\eta_2 = 0$, as shown in Figure 3(e), the criterion at different η_1 values is symmetric about the σ_z axis; when $\eta_1 > 0$, the anisotropic criterion expands in the π plane relative to the isotropic criterion; and when $\eta_1 < 0$, the anisotropic

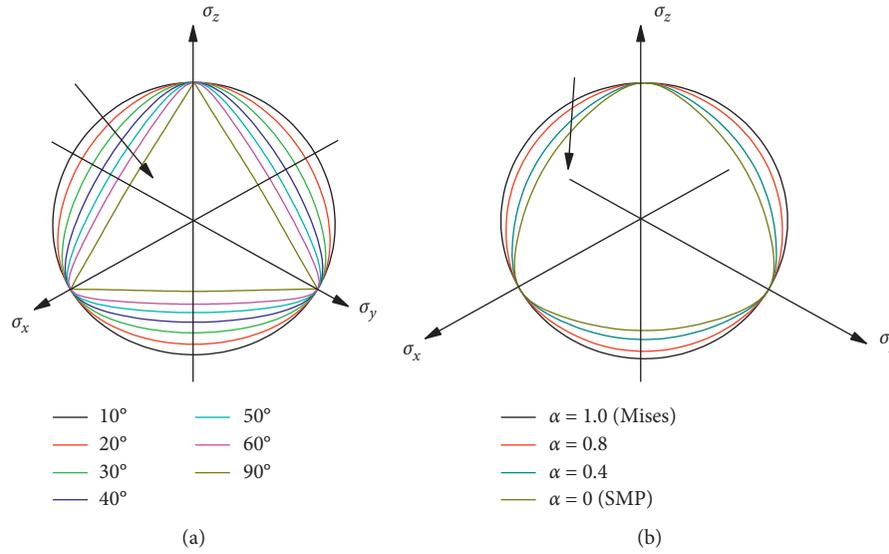


FIGURE 1: Variations of criterion with φ_0 and α on the octahedral plane.

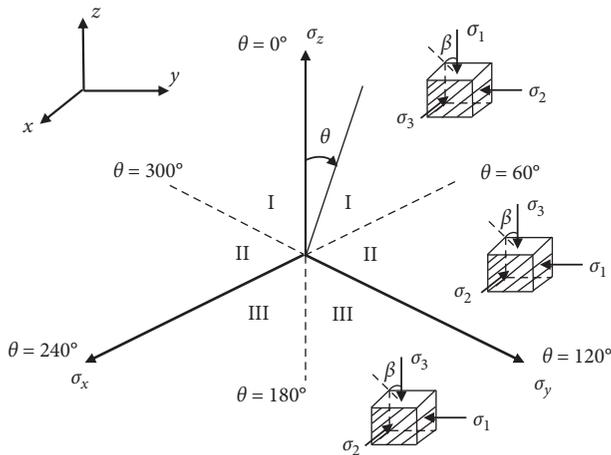


FIGURE 2: Application of true triaxial tests on cross-anisotropic soils in the octahedral plane.

criterion shrinks in the π plane relative to the isotropic criterion. The degrees of expansion or shrinkage are related to the value of η_1 . The anisotropic function variation in the π plane is consistent with that of the strength criterion, as shown in Figure 3(f).

When the stress tensor and the fabric tensor are not coaxial ($\beta \neq 0$), the changes in the shape of the strength criterion in the π plane at different included angles are shown in Figures 4(a) and 4(c). With fixed η_1 and η_2 values ($\eta_1 = -0.08$, $\eta_2 = 0$), the effect of the change in the included angle between the deposition surface and the vertical stress on the strength curve in the π plane is discussed. When $\beta = 0$, as shown in Figure 4, the strength curve is symmetric about the σ_z axis in the π plane. When the deposition surface is rotated in the $x - z$ plane, as shown in Figure 4(a), the strength curve is symmetric with respect to the σ_y axis in the π plane when $\beta = 90^\circ$. The changes in the strength curve when $\beta = 30^\circ$ and 60° are shown in Figure 4(a). As the figure

shows, when the deposition surface is rotated in the $x - z$ plane, the strength values of the anisotropic strength criterion are equal at $\theta = 60^\circ$ and 240° . The included angle has a large effect on the position and size of the strength curve. The method proposed in the present study can predict the trend and position of the strength curve at different included angles. When the deposition surface is rotated in the $y - z$ plane, as shown in Figure 4(c), the strength curve is symmetric about the σ_x axis in the π plane when $\beta = 90^\circ$. The changes in the strength curve when $\beta = 30^\circ$ and 60° are shown in Figure 4(c). As the figure indicates, when the deposition surface is rotated in the $y - z$ plane, the strength values of the anisotropic strength criterion are equal at $\theta = 120^\circ$ and 300° . The variations of the anisotropic function in the π plane in both cases are shown in Figure 4(b) and 4(d), where a similar situation with the change in the strength criterion is observed.

3. Test Verification

3.1. *Parameter Determination.* To facilitate the application of the criteria, the model parameters should be determined as much as possible using conventional triaxial compression or extension tests. In this section, the proposed ANUS criterion is applied to various geomaterials. The geomaterial model parameters need to be determined only by conventional triaxial compression or extension tests. In this section, the strength parameters and anisotropy parameters in the anisotropic criterion are mainly determined. Callisto et al. [34] performed a large number of conventional triaxial tests and true triaxial tests on Pietrafitta clay to determine all the parameters in the anisotropic criterion. In the present study, the parameters are determined by conventional triaxial test data, and other test data are used for model verification. The specific determination steps are as follows:

- (1) Determination of M_f , n , and σ_0 . The $p - q$ curve is drawn using the failure data of the conventional

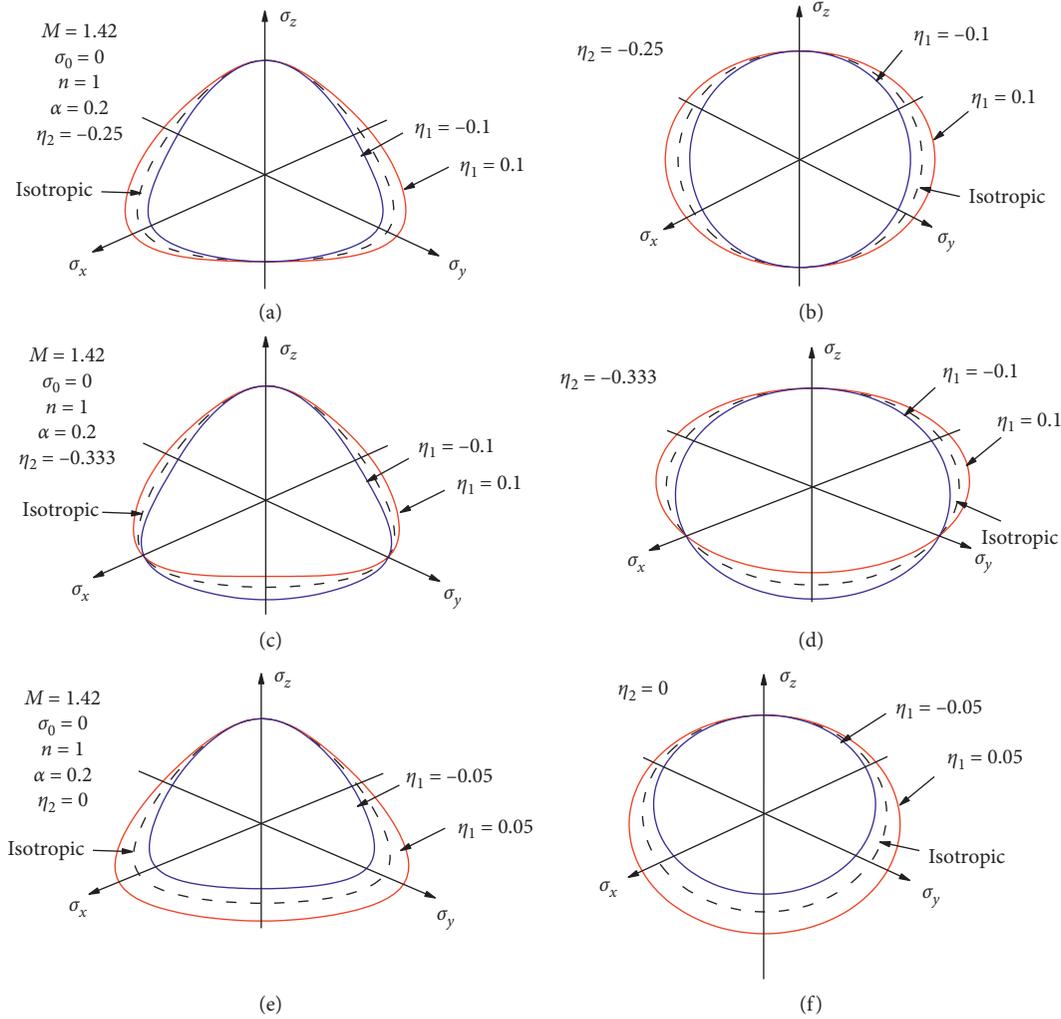


FIGURE 3: Effect of η_1 on the failure loci and $f(A)$ in the deviatoric plane with fixed η_2 in the octahedral plane: (a, c, e) the failure curves and (b, d, f) the $f(A)$.

triaxial test, as shown in Figure 5(a). As shown in the figure, $p - q$ exhibits a linear relationship. Therefore, $n = 1$, the slope of the straight line is $M(M = 6 \sin \varphi_0 / 3 - \sin \varphi_0)$, and the intercept is C ($C = 6c \cos \varphi_0 / 3 - \sin \varphi_0$), where c is the cohesion, and φ_0 is the friction angle. From the fitted linear relation in the figure, it can be seen that, for the Pietrafitta clay, $M = 0.5472$ and $C = 105.59$ kPa. Therefore, it can be determined that $c = 49.98$ kPa and $\varphi_0 = 14.478^\circ$, and $\sigma_0 = c \cot \varphi_0 = 193.58$ kPa can be obtained from the values of c and φ_0 , taking $\xi = 0$.

- (2) Determination of η_1 and η_2 , η_1 and η_2 can be determined by the q value of other stress paths with $b \neq 0$. Usually, the strength of triaxial extension is taken as the reference value for calibration. In most triaxial tests, only one of the terms in equation (21) is needed to satisfy the prediction accuracy requirement. For this reason, only the η_1 term is taken in this test. In the triaxial extension test, when $b = 1$ and $\theta = 180^\circ A - A_0 = 4$. Using $q_e = f(B)M_f g(\theta)(p + \sigma_0)$, $q_e = 254.41$ kPa ($\theta = 180^\circ p = 250$ kPa) can be obtained

using the same method. Therefore, $g(\theta) = 0.8462$, $f(A) = \exp(4\eta_1)$, and $\eta_1 = 0.0417$.

Using the above steps, all parameters of the anisotropic criterion for Pietrafitta clay are determined. The test parameter determination method is shown in Figure 5(a). The comparison of the prediction results by the isotropic criterion and the anisotropic criterion in Figure 5(b) shows that the anisotropic criterion proposed in the present study better predicts the test data.

3.2. Strength Nonlinearity. A series of large triaxial tests for rockfill materials were conducted to research strength nonlinearity of this material [27]. In the meridional plane ($p - q$), the test results show the strength nonlinearities. Therefore, it is necessary to use nonlinear strength criteria to depict the strength development pattern. The two strength criteria are compared by using them to predict the test data, as shown in Figure 6. As the figure shows, the nonlinear strength criterion proposed this paper can depict the strength nonlinearity of this material.

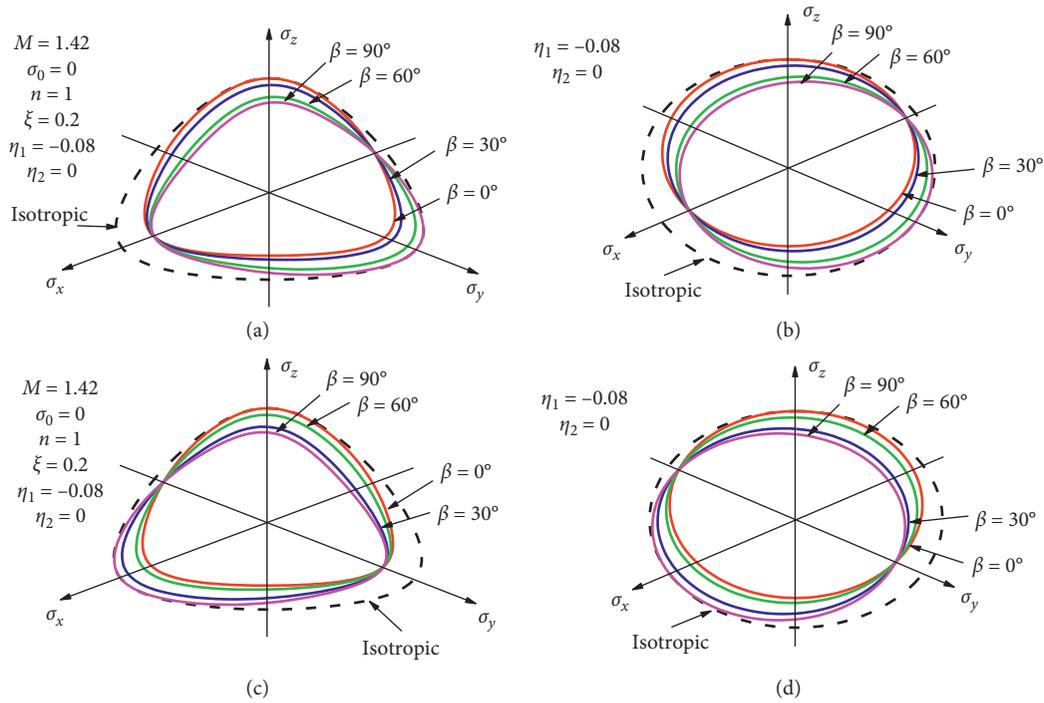


FIGURE 4: The variations of the failure surface and $f(A)$ in the octahedral plane: (a, b) the $y - z$ plane and (c, d) the $x - z$ plane.

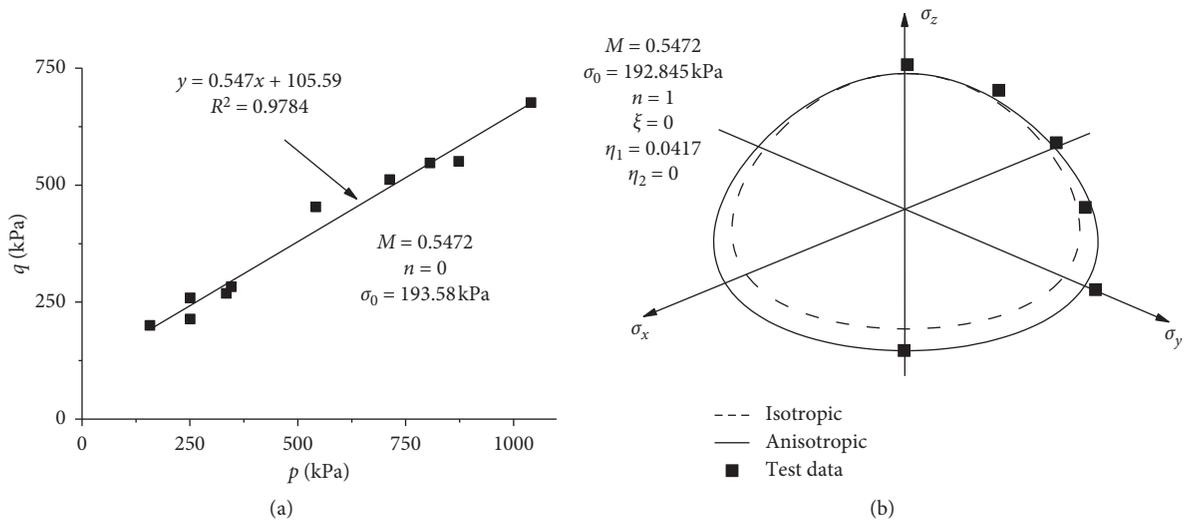


FIGURE 5: Illustration on calibrating the parameters M_f , c , σ_0 (b) Comparison of the two criteria for natural Pietrafitta clay (test data from [34]).

3.3. Anisotropy

3.3.1. *True Triaxial Test of Soil.* In addition to the above true triaxial test of Pietrafitta clay, fine glass-bead sand is then used to verify the anisotropic criterion. Haruyama [35] conducted a large number of conventional triaxial tests and true triaxial tests to study the initial anisotropy of fine glass-bead sand. The effective stress of the test is $p = 294$ kPa. The conventional triaxial compression test and the triaxial tensile test are used to determine the parameters. The parameters determined by the test data are as follows: $M = 1.142$, $\sigma_0 = 0$,

$n = 1$, $\eta_1 = -0.038$, and $\eta_2 = 0$. The prediction results and test data in the deviatoric plane and $b-\phi$ plane are shown in Figure 7. It can be seen from the figure that the anisotropic criterion of the present study can reasonably predict the strength of the fine glass sand in the π plane. However, the isotropic criterion fails to predict the strength of regions II and III in the deviatoric plane and overestimates the material strength. In addition, in the $b-\phi$ plane, the isotropic criterion can only predict one variation curve of the friction angle, whereas the anisotropic criterion results in a continuous prediction curve in each region, reflecting the anisotropy of

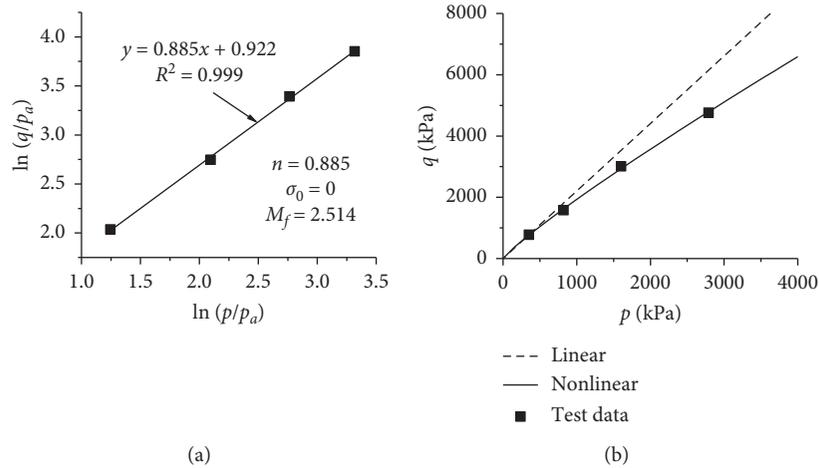


FIGURE 6: Verifications of the two criteria. (a) Parameter determination. (b) Test data in the $p - q$ diagram.

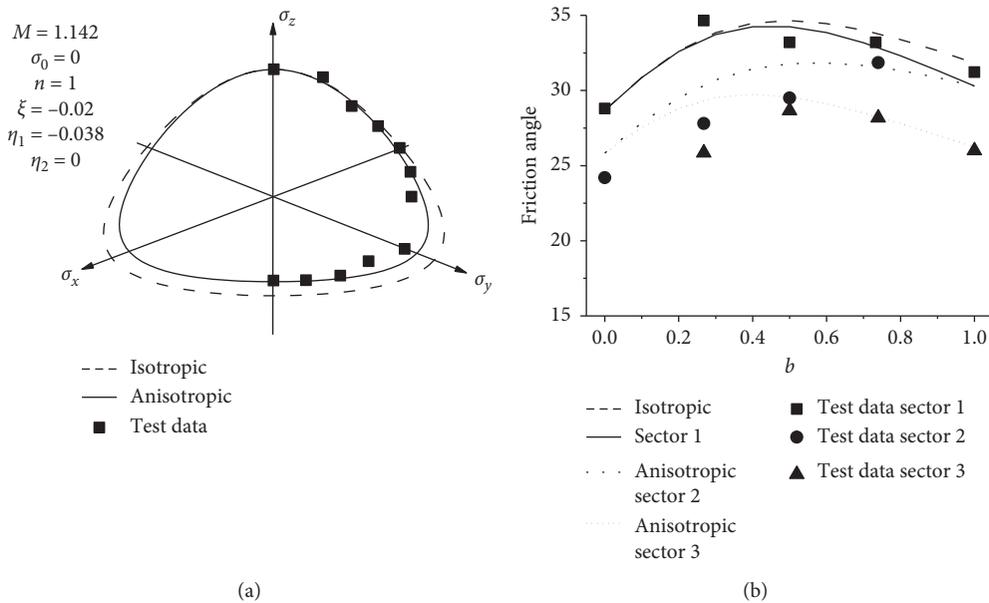


FIGURE 7: Verifications of the two criteria for glass beads in (a) the octahedral plane; (b) the $b - \phi$ diagram (test data from [35]; this figure is reproduced from [36]).

the material. Therefore, the anisotropic criterion can reasonably predict the peak strength of the fine glass-bead sand compared to the isotropic criterion.

3.3.2. *Torsional Shear Tests for Soils.* The failure friction angle measured from torsional shear tests on Leighton Buzzard sand of different densities and on spherical glass beads conducted by Yang et al. [37] was analyzed using the proposed ANUS criterion. The effective stress of the test is $p = 200$ kPa. The parameters determined by the test data are as follows: for glass ballotini, $M = 1.43$, $\sigma_0 = 0$, $n = 1$, $\eta_1 = -0.368$, and $\eta_2 = 0.1187$; for medium dense LB sand, $M = 1.805$, $\sigma_0 = 0$, $n = 1$, $\eta_1 = -0.2463$, and $\eta_2 = 0.0593$; for

dense LB sand, $M = 1.91$, $\sigma_0 = 0$, $n = 1$, $\eta_1 = -0.2668$, and $\eta_2 = 0.0655$. The prediction results (ANUS criterion and ALD criterion [38]) and test data in the $\alpha - \phi$ plane are shown in Figure 8. As shown in Figure 8, under torsional shear tests, the sand shows obvious anisotropy, the new anisotropic criterion can predict the failure friction angle well, and the prediction curve is smooth.

3.3.3. *Rock Triaxial Test considering Deposition Surface.* This test is mainly used to verify the peak strength of the rock considering the different angles between the rock deposition surface and the principal stress. The isotropic strength criterion and the anisotropic strength criterion were used for

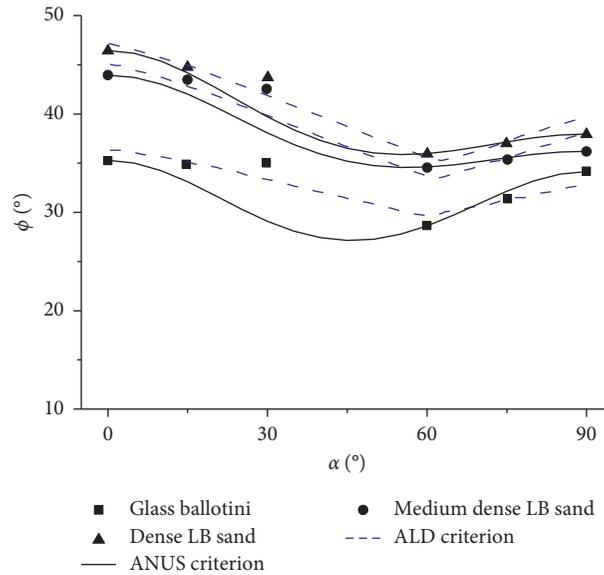


FIGURE 8: Verifications of ANUS criterion for different granular materials (data from [37]).

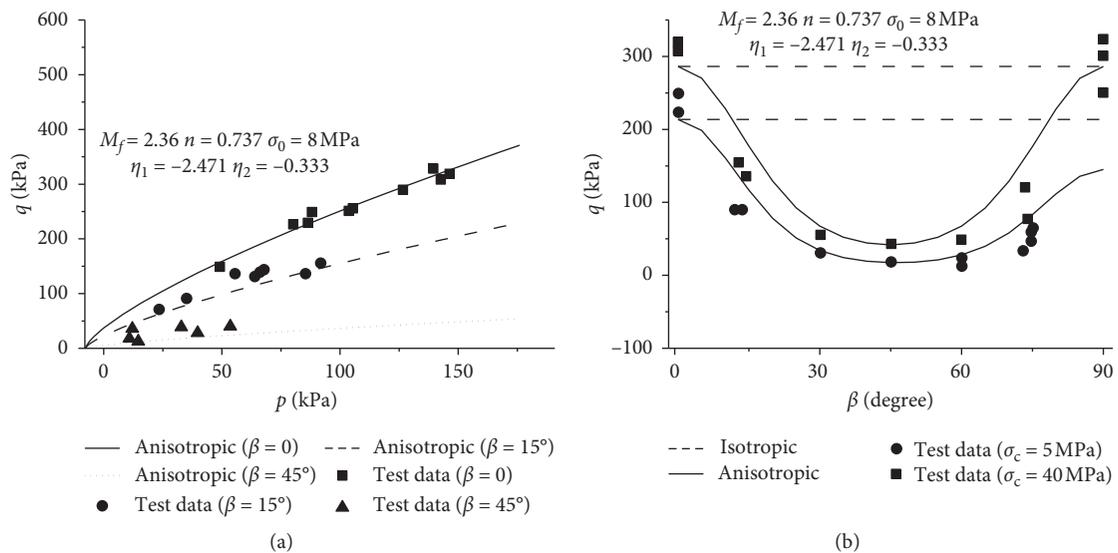


FIGURE 9: Comparison of the two criteria for Tournemire shale in (a) the $p - q$ diagram and (b) the $\beta - q$ diagram (test data from [39]; this figure is reproduced from [40]).

prediction, and a comparison of the prediction results with the test data shows the superiority of the anisotropic strength criterion. Here, based on the schist triaxial tests with different deposition surface angles in the literature, the parameters of the anisotropic strength criterion are $M = 2.36$, $\sigma_0 = 8 \text{ MPa}$, $n = 0.74$, $\eta_1 = -2.47$, and $\eta_2 = -0.33$. It can be seen from the test results that the two types of rock exhibit strong strength anisotropy in different deposition directions. In comparison, the use of the isotropic strength criterion to predict strength variations can lead to large errors, making it difficult to promote the isotropic criterion in engineering applications.

As shown in Figure 9, the anisotropic criterion can reasonably predict the strength failure line with different

deposition surface angles in the $p - q$ plane. In contrast, the isotropic strength criterion can only predict the failure line when $\beta = 0^\circ$, cannot reflect the strength anisotropy, and overestimates the strength values with different deposition surface angles, especially for $\beta = 45^\circ$. Meanwhile, the anisotropic criterion can reasonably predict the trend of strength variation under different confining pressures at different deposition surfaces, as shown in Figure 9(a). It can be seen from Figure 9(b) that the strengths of rocks with different deposition surface angles differ considerably. The peak strength of the rock is the largest when $\beta = 0^\circ$ or 90° and the smallest when the deposition surface angle is approximately 45° . In contrast, the isotropic criterion is a straight line in the $\beta - q$ plane

and, hence, cannot reflect the strength anisotropy caused by the direction of the deposition surface.

4. Conclusion

- (1) In this paper, a new strength criterion is proposed on the basis of the SMP criterion. In the π plane, the new criterion is a series of smooth curves and can unify the strength criteria.
- (2) The new criterion can depict the 3D strength variation of the material and reflect the effect of the intermediate principal stress and the nonlinear characteristics of the material strength.
- (3) The unified anisotropic strength criterion based on the pattern of the fabric evolution of granular materials can reflect the strength anisotropy caused by the deposition characteristics of the material. This criterion can be applied to the true triaxial test to consider the direction angle of the deposition surface. The correctness of the criterion was verified using sand, clay, and rock materials.

Data Availability

The underlying data used in the presented study were obtained from the literature.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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