Research Article

A Nonlinear Statistical Damage Constitutive Model for Porous Rocks

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In the current paper, the deformation behaviours of rocks during compression are studied by testing 10 groups of sandstone samples with different porosity characteristics. According to the energy theory, the rock material was divided into two parts: solid skeleton and voids. A statistical damage-based approach was adopted to establish a nonlinear statistical damage constitutive model. The validity of the statistical damage constitutive model is verified by the test data. The statistical damage constitutive model performs well in each stage of rock compression before failure. For different types of rocks, different confining pressures, and different water contents, the statistical damage constitutive model fits well. This model can be applied to most types of rocks and in most engineering environments.

1. Introduction

Previous research [1–7] has shown that compression of rock materials typically involves four stages: (a) initial compression stage, (b) linear elastic deformation stage, (c) yield stage, and (d) failure stage. The stress-strain behaviour of rock is significantly influenced by the size and number of voids [8–10]. During the compression process, the volume changes continuously, leading to variation in the deformation modulus [11, 12]. Therefore, the stress-strain relationship of rock is nonlinear during compression. A thorough understanding of the deformation behaviour of rock during compression is essential for establishing an accurate numerical model for rock engineering.

As an imperfect material, rock includes pores and cracks. The porosity of rock has a significant influence on its mechanical properties and permeability [13–15]. In essence, rock damage is caused by the expansion of cracks and the propagation of voids in the rock leading to a decrease in its strength. At present, the methods for researching rock damage mainly include (a) using meso-damage theory and computed tomography scanning and (b) using continuum damage mechanics from the macroscopic point of view [16–18].

Continuum damage mechanics has been increasingly applied to model the constitutive behaviours of brittle rock [16, 18–23]. Great progress has been made in the incorporation of statistical theory in the establishment of damage constitutive models [16, 18–23]. Many recent statistical damage constitutive models have assumed that the strength of microcells follows a distribution law [16, 17, 24–26]. The most commonly applied distributions describing the microunit intensity distribution of rock are log-normal distributions, normal distributions, and Weibull distributions [17]. Chen et al. [17] explored a statistical damage constitutive model of rock and verified and compared the model with others following different kinds of distributions. The results showed that the Weibull distribution is more reasonable. However, these models assume that the stress-strain relationship of each mesoscopic element obeys Hooke's law before the load reaches the peak. Thus, these models can simulate only elastoplastic materials, which show a quasilinear elastic stage and a yield stage during the compression of rock and cannot effectively simulate the plastic deformation process in the initial compression stage, which is when the initial voids in the rock are compacted. At the initial compression stage, the initial voids in the rock are diminishing during loading, and plastic deformation occurs.

Thus, the stress-strain curve is an upper concave curve that is nonlinear in shape.

Commercial software (such as ANSYS and ABAQUS) for the numerical simulation of geotechnical engineering applications can find nonlinear solutions, as they usually have nonlinear material models. However, as the materials in these models are commonly regarded as a set of units, springs, or beams bonded together, these models can simulate only elastoplastic materials. Fortunately, such software usually offers an interface where user-defined material models can be used, allowing us to construct a constitutive model more suitable for porous rock materials. Such a model would be significant for the numerical simulation of rock engineering applications. Several scholars [27, 28] have tried to import custom functions into numerical simulations.

In this paper, we aimed to propose a statistical damage constitutive model that can effectively simulate the plastic deformation process during the initial compression stage. To study the deformation behaviour of porous rock during compaction, 10 groups of sandstone samples with different porosity characteristics were prepared for uniaxial compression tests. According to the energy principle, the rock material was divided into two parts: solid skeleton and void. Then, it is assumed that the strength of microcells of the solid skeleton follows the Weibull distribution. In this way, we proposed a nonlinear statistical damage constitutive model of rock that considers the initial compression stage.

2. Experimental Test

2.1. Test Materials and Methods. To study the deformation behaviour of rock during compression, we selected 10 groups of sandstone samples with different porosity characteristics for uniaxial compression tests. These sandstones were strongly weathered with high porosity; thus, the initial compression stage was very obvious. The sandstone samples were collected from Longchang County, Sichuan Province, China, and consisted of quartz, chlorite, feldspar, mica, and other auxiliary minerals. The clastic particles were sub-angular, with particle sizes between 200 and 500 microns.

The porosity of this kind of sandstone varies widely, ranging from 12 to 28%. The sandstone material was processed into standard samples with a size of 50×100 mm. According to porosity, the samples were divided into 10 groups, with 12 samples in each group, as shown in Table 1. Through uniaxial compression testing, the stress-strain relationships of the samples were obtained. The test instrument is shown in Figure 1. Axial stress is applied through the hydropress, and the deformation of the rock sample is monitored with strain gauges. The loading rate is 0.5 MPa/s. As the water content is also an important factor affecting the mechanical properties of rocks [29], the samples for the uniaxial compression test were divided into two states: dry and saturated. The test results are shown in Table 1.

2.2. Analyses of Experimental Results. The experimental results show that the porosity of sandstone is closely related to its mechanical properties. Figure 2 shows that there is a

linear relationship between porosity and uniaxial compressive strength, whether the samples are dry or saturated.

The failure processes and stress-strain curves of the sandstone samples with different porosities are very similar. The principle of energy can help us better understand the deformation and failure process of rock. Energy dissipation in the initiation and propagation of voids and cracks in rock during the compression process causes damage in rock [30–35]. According to previous research [33, 35], the total work done by the external force on the rock during compression is

$$U = \int_0^{\varepsilon_1} \sigma_1 d\varepsilon_1 + \int_0^{\varepsilon_2} \sigma_2 d\varepsilon_2 + \int_0^{\varepsilon_3} \sigma_3 d\varepsilon_3, \quad (1)$$

where U is the total work done by the external force; σ_1 , σ_2 , and σ_3 are the principal stresses; and ε_1 , ε_2 , and ε_3 are the principal strains.

During compression, some of the energy is stored in the rock and converted into elastic strain energy, while some energy dissipates due to plastic deformation. The elastic strain energy accumulated in rocks can be calculated by equation (2). The dissipated energy can be calculated by

$$U_e = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)], \quad (2)$$

$$U_d = U - U_e, \quad (3)$$

where U_e is the elastic strain energy, U_d is the dissipated energy, E is Young's modulus, and ν is Poisson's ratio.

The stress-strain curve is an upper concave curve, as shown in Figures 3(a) and 3(b), which shows the energy change curve during the deformation process. At the initial compression stage (OA), a certain number of initial voids are diminished. The rock accumulates little elastic strain energy, and most of the input energy is dissipated. At the elastic stage (AB), the energy absorbed by the rock is stored as the elastic strain energy and dissipates very little. At the yield stage (BC), elastic energy increases at a diminishing rate. Dissipated energy begins to increase as the rock begins to undergo plastic deformation and microcracks begin to form and expand. During this stage, the proportion of elastic strain energy to total energy decreases gradually. Overall, the deformation of the solid skeleton under the load is elastic deformation before reaching the yield stage. Thus, the rock sample can be considered to be composed of voids and the solid skeleton at the macroscopic scale, as shown in Figure 3. The initial length of the sample before loading is defined as l_0 , the length of the void is l_0^v , and the length of the solid skeleton is l_0^s . The deformation under the axial stress σ is Δl . Thus, the deformation of the solid skeleton is Δl^s , and the deformation of the voids is Δl^v . Then, the strain of the porous rock can be expressed as (Figure 4).

$$\varepsilon_1 = \frac{\Delta l}{l_0} = \frac{(\Delta l^v + \Delta l^s)}{l_0}. \quad (4)$$

The axial strain of the solid skeleton ε_1^s and the axial strain of the voids ε_1^v can be expressed as (5) and (6), respectively. The relationship between ε_1^s and ε_1^v is defined in

TABLE 1: Test data of sandstone samples.

Group	Porosity (%)	Density (kg/m ³)		Compressive strength (MPa)		Young's modulus (GPa)		Poisson's ratio	
		Dry	Saturated	Dry	Saturated	Dry	Saturated	Dry	Saturated
1	12.39–12.96	2174–2283	2312–2410	21.3–28.1	5.4–9.9	3.45–4.00	0.86–2.22	0.19–0.23	0.26–0.29
2	13.23–13.98	2146–2285	2287–2418	18.9–29.7	7.1–11.4	2.95–3.82	1.32–2.53	0.18–0.23	0.25–0.28
3	14.01–14.98	2128–2314	2251–2457	15.2–28.0	5.2–14.8	2.59–3.73	0.92–2.37	0.19–0.24	0.25–0.29
4	15.01–15.99	2140–2288	2311–2454	15.4–27.2	4.8–12.8	2.03–3.58	0.71–2.94	0.18–0.24	0.26–0.30
5	16.00–16.90	2134–2340	2291–2516	13.0–26.7	4.7–10.0	1.63–2.94	0.49–1.61	0.19–0.24	0.27–0.31
6	17.37–17.78	2155–2341	2358–2479	17.4–21.5	4.4–5.1	1.45–2.37	0.03–0.49	0.19–0.22	0.30–0.31
7	18.11–18.98	2173–2235	2365–2447	13.3–21.3	2.5–4.6	1.07–2.09	0.20–0.43	0.22–0.27	0.30–0.31
8	19.08–19.96	2129–2265	2326–2475	11.6–19.7	2.1–2.3	0.98–0.14	0.16–0.18	0.20–0.28	0.30–0.31
9	20.02–23.28	2173–2281	2372–2482	11.5–14.1	0.8–2.2	0.67–0.91	0.06–0.13	0.23–0.28	0.30–0.32
10	26.21–27.89	2161–2233	2425–2529	4.7–10.5	0.4–0.7	0.28–0.43	0.02–0.07	0.25–0.28	0.31–0.32

FIGURE 1: A view of the test instrument.

FIGURE 2: The relationship between porosity and strength.

(7). The deformation characteristics of the rock sample, solid skeleton, and voids are shown in Figure 5. When the load reaches the critical point A (in Figure 3(a)), compaction of the voids is basically completed. At this time, the axial strain of the voids reaches a maximum of ε_{1A}^v , and then the axial

strain will remain unchanged until the rock completely fails, as shown in Figure 5.

$$\varepsilon_1^s = \frac{\Delta l^s}{l_0}, \quad (5)$$

$$\varepsilon_1^v = \frac{\Delta l^v}{l_0}, \quad (6)$$

$$\varepsilon_1 = \varepsilon_1^s + \varepsilon_1^v. \quad (7)$$

The proportional coefficient is defined as $\gamma = \varepsilon_1^v / \varepsilon_1$, and the coefficient represents the proportion of the void axial strain to the total axial strain. According to our test results, the value of γ decreases with the increase in axial strain, and the relationship between the axial strain and γ is roughly linear. Our test materials have different porosities and water contents. From the results, we can see that the variation law of the proportionality coefficient is consistent during compression, as shown in Figure 6, and is not affected by the properties of the rock and the water it contains.

3. The Establishment of the Nonlinear Statistical Damage Constitutive Model

3.1. The Principle of the Statistical Damage Constitutive Model. Previous studies [16, 26] have shown that using the Weibull distribution to describe the strength of the microscopic unit of rock is reasonable; thus, the statistical damage constitutive model we propose will follow the Weibull distribution. If the strength of mesoscopic elements is assumed to follow the Weibull distribution, then the probability density function $P(F)$ can be written as [16, 26]

$$P(F) = \frac{m}{F_0} \left(\frac{F}{F_0} \right)^{m-1} \exp \left[- \left(\frac{F}{F_0} \right)^m \right], \quad (8)$$

where m is a shape parameter and F_0 is a scale parameter.

Let N_f denote the number of elements that have failed and N denote the total number of elements. Then, the damage variable can be defined as follows [26]:

$$D = \frac{N_f}{N}. \quad (9)$$

(a)

(b)

FIGURE 3: The process of deformation: (a) complete stress-strain curve; (b) energy evolution curve.

FIGURE 4: Diagram of the compression process of rock voids.

The number of failed elements increases by $NP(F)dF$ when the stress level F increases to $F + dF$. If the external load increases from zero to F , then the total number of failed elements can be written as [26]

$$N_f(F) = \int_0^F NP(y)dy = N \left\{ 1 - \exp \left[- \left(\frac{F}{F_0} \right)^m \right] \right\}. \quad (10)$$

Substituting (10) into (9), the damage variable can be written as

$$D = 1 - \exp \left[- \left(\frac{F}{F_0} \right)^m \right]. \quad (11)$$

Based on the continuum damage mechanics, the statistical damage constitutive model of rock can be expressed as [26]

$$\sigma_1 = E_t \varepsilon_1 (1 - D) + \nu(\sigma_2 + \sigma_3), \quad (12)$$

FIGURE 5: Schematic diagram of the deformation of each part.

where E_t is Young's modulus of the undamaged rock material; ν is Poisson's ratio; D is the damage variable, which takes a value ranging from 0 for the intact or undamaged state ($D=0$) and 1 for the fully damaged state.

Assuming that (a) the strength of each mesoscopic element of the rock follows the Weibull distribution during the loading process and (b) the stress-strain relationship of each mesoscopic element obeys Hooke's law before the load reaches the peak, the statistical damage constitutive model of rock can be expressed as

$$\sigma_1 = E_t \varepsilon_1 \exp \left[- \left(\frac{F}{F_0} \right)^m \right] + \nu(\sigma_2 + \sigma_3). \quad (13)$$

FIGURE 6: The relationship between and strain: (a) the porosity is 13.90% when the saturation is 0. (b) The porosity is 15.27% when the saturation is 1. (c) The porosity is 14.27% when the saturation is 0. (d) The porosity is 19.56% when the saturation is 1.

The complete stress-strain curve of the rock under compression can be simplified as shown in Figure 7. Assume that the confining pressure is 0. According to the geometric conditions shown in Figure 7 and by taking the derivative of (13), we can obtain

$$F_0 = \frac{\epsilon_{1c}}{(1/m)^{(1/m)}}, \quad (14)$$

$$m = \frac{1}{\ln(E_t \epsilon_{1c} / \sigma_{1c})}. \quad (15)$$

Then, the constitutive model of the rock under compression can be written as

$$\sigma_1 = E_t \epsilon \exp \left[-\frac{1}{m} \left(\frac{\epsilon_1}{\epsilon_{1c}} \right)^m \right] + \nu (\sigma_2 + \sigma_3). \quad (16)$$

Equation (16) is the constitutive model of the rock under compression, which has been widely adopted [17]. Previous studies [16, 17] have proved that this model can describe the stress-strain behaviour of rock during the process of compression. From Figure 8, we can see that this constitutive model fits well for hard rock with few voids. However, for rock with high porosity and low strength, this constitutive model does not fit the test data well. Some scholars [16, 17] have improved this model but have not considered the plastic strain of rock during the initial compression stage

FIGURE 7: The geometric conditions of the deformation modulus.

(OA). Thus, the coefficient of determination (R^2) of the fitting line at the OA stage is not high enough if the rock is not hard. Therefore, to effectively simulate the plastic deformation process during the initial compression stage, the statistical damage constitutive model should be improved.

3.2. The Statistical Damage Constitutive Model considering Plastic Deformation at the Initial Compression Stage. In Section 2, we suggest that the rock is composed of voids and a solid skeleton based on energy theory. The proportional coefficient γ , which represents the ratio of void strain to total strain, was defined. From Figure 6 we can see the value of γ decreases with the increase in axial strain, and the relationship between the axial strain and γ is roughly linear. Furthermore, γ is close to 1 when the axial strain is very low. Therefore, we define that

$$k = \frac{d\gamma}{d\epsilon_1}. \quad (17)$$

Coefficient k can be calculated as

$$k = \frac{\gamma_A - 1}{\epsilon_{1A} - 0}. \quad (18)$$

In the compression stage (section OA in Figure 3(a)), the relationship between γ and the axial strain ϵ_1 can be written as

$$\gamma = 1 + \frac{\gamma_A - 1}{\epsilon_{1A}} \epsilon_1 \quad (\epsilon_1 < \epsilon_{1A}). \quad (19)$$

Thus, ϵ_1^v can be calculated by

$$\epsilon_1^v = \gamma \epsilon_1 = \left(1 + \frac{\gamma_A - 1}{\epsilon_{1A}} \epsilon\right) \epsilon_1 \quad (\epsilon_1 < \epsilon_{1A}). \quad (20)$$

When reaching critical point A (in Figure 3(a)), the deformation of the voids reaches a maximum of ϵ_{1A}^v , and the deformation will remain unchanged. ϵ_1^v can be defined as

$$\epsilon_1^v = \epsilon_{1A}^v \quad (\epsilon_1 \geq \epsilon_{1A}). \quad (21)$$

According to the evolution law of elastic strain energy, before reaching the yield stage, the deformation of the solid skeleton under the load is elastic. The following is assumed:

$$\sigma_1 = E_t \epsilon_1^s + \nu(\sigma_2 + \sigma_3). \quad (22)$$

Based on the assumption above, the value of ϵ_{1A}^s can be calculated using (23), and γ_A can be calculated using

$$\epsilon_{1A}^s = \frac{\sigma_{1A}}{E_t}, \quad (23)$$

$$\gamma_A = 1 - \frac{\epsilon_{1A}^s}{\epsilon_{1A}} = 1 - \frac{\sigma_{1A}}{E_t \epsilon_{1A}}. \quad (24)$$

Combining (19)–(24), the expression of ϵ_1^s at any time can be written as (25) and (26).

$$\epsilon_1^s = \frac{\sigma_{1A}}{E_t \epsilon_{1A}^2} \epsilon_1^2, \quad \epsilon_1 \leq \epsilon_{1A}, \quad (25)$$

$$\epsilon_1^s = \epsilon_1 - \epsilon_{1A} + \frac{\sigma_{1A}}{E_t}, \quad \epsilon_1 > \epsilon_{1A}. \quad (26)$$

When the axial stress reaches the peak, the corresponding strain on the solid skeleton is

$$\epsilon_c^s = \epsilon_c - \epsilon_{1A} + \frac{\sigma_{1A}}{E_t}. \quad (27)$$

When the load reaches the critical point A (in Figure 3(a)), the deformation of the voids reaches a maximum of ϵ_{1A}^v , and the deformation remains unchanged. Thus, the stress-strain curve of the sample conforms to the previous statistical damage constitutive model, as defined in (16). It is assumed that (a) the strength of each mesoscopic element of the solid skeleton follows the Weibull distribution during the loading process, and (b) the stress-strain relationship of each mesoscopic element obeys Hooke's law before failure. By replacing ϵ_1 with ϵ_1^s in (16), the nonlinear statistical damage constitutive model that considers the initial compression stage can be obtained as

$$\sigma_1 = E_t \epsilon_1^s \exp \left[-\frac{1}{m} \left(\frac{\epsilon_1^s}{\epsilon_{1c}^s} \right)^m \right] + \nu(\sigma_2 + \sigma_3), \quad (28)$$

$$m = \frac{1}{\ln(E_t \epsilon_{1c}^s / \sigma_{1c})}. \quad (29)$$

The statistical damage constitutive model requires 6 parameters, all of which can be directly obtained by conventional mechanical tests. Therefore, there are no difficulties related to obtaining parameters.

4. Discussion

The statistical damage constitutive model (see (28)) we established considers the plastic deformation during the initial compression stage and can better simulate the process of rock compression than the previous constitutive model (see (16)). To verify the reasonability of this statistical damage constitutive model, we compared the calculated

FIGURE 8: Fitting degree analysis of the statistical damage constitutive model from [26] (see (16)). (a) Altered rock [36] $\sigma_c = 88.57$ MPa, $\sigma_3 = 12$ MPa. (b) Quartz [37] $\sigma_c = 385$ MPa, $\sigma_3 = 5$ MPa. (c) Coal [37] $\sigma_c = 65$ MPa, $\sigma_3 = 8$ MPa. (d) Sandstone ($n = 21.1\%$) $\sigma_c = 12.97$ MPa, $\sigma_3 = 0$ MPa.

values with the test data. Table 2 shows the error analysis of the statistical damage constitutive model (see (28)) at each stage during compression. It can be seen that, in each stage, the statistical damage constitutive model has a high fitting degree. The statistical damage constitutive model written as (16) is an elastoplastic constitutive model that is widely adopted at present. As it does not consider the plastic deformation during the initial compression stage, the degree of fit is low. If the voids of the rock are not developed or only slightly developed, then the initial compression stage may not be as notable or may not occur. Even if this stage is ignored, the initial compression stage has little impact on the results. However, if the rock develops microcracks or voids, then the stress-strain curve will be an upper concave curve in the initial compression stage. Thus, the statistical damage

constitutive model written as (16) cannot effectively simulate the plastic deformation process of porous rocks during the initial compression stage.

To verify the performance of the statistical damage constitutive model on the simulation of other types of rocks, we referenced some published test data [36–38] of different types of rocks, as shown in Figure 9. As shown in Figure 9, for sandstone samples with different porosities, the constitutive model fits well. Thus, the statistical damage constitutive model also performs well for saturated samples. Furthermore, it can be seen that the statistical damage constitutive model also performs well when used to simulate the compression processes of other types of rocks. From Figure 9, we can see that the degree of fit is good even if the confining pressure is considered.

TABLE 2: Error analysis of the statistical damage constitutive model.

No.	Rock type	Confining pressure (MPa)	Water content (%)	OA section		AB section Refer to Figure 3		CD section		Whole stage	
				Equation (16)	Equation (28)	Equation (16)	Equation (28)	Equation (16)	Equation (28)	Equation (16)	Equation (28)
1	Sandstone ($n = 13.9\%$)	0	0	—	0.987	—	0.999	—	0.999	—	0.996
2	Sandstone ($n = 21.1\%$)	0	0	—	0.999	—	0.999	—	0.999	—	0.998
3	Sandstone ($n = 15.2\%$)	0	9.88	—	0.956	—	0.999	—	0.991	—	0.990
4	Sandstone ($n = 21.3\%$)	0	6.95	—	0.983	—	0.997	—	0.903	—	0.998
5	Sandstone [38]	0	2.05	—	0.996	—	0.961	—	0.987	—	0.998
6	Sandstone [38]	0	3.41	—	0.984	—	0.995	—	0.989	—	0.999
7	Coal [37]	0	—	—	0.999	—	0.939	—	0.664	0.405	0.994
8	Coal [37]	3	—	—	0.997	—	0.938	—	0.947	0.639	0.998
9	Coal [37]	5	—	—	0.994	—	0.992	—	0.996	0.613	0.999
10	Coal [37]	8	—	—	0.990	—	0.992	—	0.981	0.660	0.998
11	Norite [37]	0	—	—	0.991	—	0.821	0.203	0.833	0.724	0.980
12	Norite [37]	3	—	0.176	0.995	—	0.879	0.627	0.916	0.840	0.988
13	Norite [37]	5	—	0.298	0.995	—	0.894	0.900	0.927	0.896	0.992
14	Norite [37]	8	—	0.371	0.997	—	0.941	0.880	0.970	0.914	0.994
15	Quartzite [37]	0	—	—	0.931	—	0.992	—	0.988	0.673	0.999
16	Quartzite [37]	3	—	—	0.995	—	0.995	0.258	0.939	0.769	0.997
17	Quartzite [37]	5	—	0.05	0.995	—	0.994	0.364	0.960	0.837	0.998
18	Quartzite [37]	8	—	—	0.982	0.127	0.999	0.745	0.980	0.875	0.998
19	Altered rock [36]	4	—	0.581	0.979	0.628	0.997	0.495	0.994	0.914	0.999
20	Altered rock [36]	12	—	0.780	0.781	0.910	0.993	0.940	0.993	0.978	0.997

FIGURE 9: Continued.

FIGURE 9: The comparison of the theoretical curve and the test data. *Equation (16) is the statistical damage constitutive model from [26], and (28) is the statistical damage constitutive model we proposed; the specimen numbers correspond to Table 2. (a) No. 1. (b) No. 2. (c) No. 3. (d) No. 4. (e) No. 9. (f) No. 10. (g) No. 13. (h) No. 17. (i) No. 19. (j) No. 20.

In conclusion, the nonlinear statistical damage constitutive model established in this study performs well in each stage of rock compression process before failure. For different types of rocks, different confining pressures, and different water contents, the statistical damage constitutive model fits well. Thus, this model can be applied to most types of rocks and most engineering environments. Since the stress-strain behaviour of rocks in the postpeak stage is too complex, it is not considered in this paper. Therefore, the model proposed in this paper still has room for improvement.

5. Conclusions

In this paper, particular attention is paid to the deformation behaviour of rock in the initial compression stage. A relatively reasonable nonlinear statistical damage constitutive model could be developed with the aid of the statistical damage theory. The following points summarize the present study:

- (1) Based on energy theory, the rock material was divided into two parts: solid skeleton and void. From

the test results, the variation law of strain in each part of the rock during compression was obtained, which has nothing to do with rock properties or test conditions.

- (2) Based on the theory of damage mechanics, a nonlinear statistical damage constitutive model following the Weibull distribution was developed. This model can simulate the deformation behaviour of rock in the initial compression stage, as the strain of the void and solid skeleton are calculated separately.
- (3) The validity of the nonlinear statistical damage constitutive model is verified by the existing test data. The statistical damage constitutive model established in this study performs well regardless of which stage of the rock compression process is simulated. For different types of rocks, different confining pressures, and different water contents, the statistical damage constitutive model fits well. Thus, this model can be applied to most types of rocks and in most engineering environments.
- (4) The statistical damage constitutive model established in this paper is for porous rocks. However, the results of contrastive analysis show that the statistical damage constitutive model also has a high fitting degree for rocks with high stiffness and dense structure, so the statistical damage constitutive model can also be suitable for such rocks.

Data Availability

The test data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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