Research Article

Stability Analysis of Surrounding Rock in Circular Tunnels Based on Critical Support Pressure

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1. Introduction

Tunnels are widely used in civil engineering, traffic engineering, and mining engineering [1, 2]. The excavation of the tunnels leads to the redistribution of the in situ stress. Once the redistributed stress is greater than the peak strength of the rock mass, the surrounding rock will be deformed and destroyed [3]. Generally, the surrounding rock around the tunnel suffers from the most severe damage, and this part is called the damage zone. The deep rock mass is still in an elastic state, which is called the elastic zone. And the transition from the failure zone to the elastic zone is called the plastic zone. Therefore, the range of the plastic zone and damage zone is very important to the design of the tunnel support system [4–6].

Strength criterion is the key to solving the radius of the plastic zone of the tunnel. In early work, linear Mohr–Coulomb strength criterion [7–10], nonlinear Hoek–Brown strength criterion [11–13], and generalized Hoek–Brown strength criterion [14–16] were used to analyzing the stresses distribution and deformations. However, the effect of intermediate principal stress on the surrounding rock was ignored by these criteria. A large number of test results show that the strength of rock mass is not only related to its own mechanical properties but also controlled by its stress state [17–22]. In practice, the underground rock mass is still in a triaxial stress state due to the existence of the supporting structure. Therefore, it is expected that the elastic-plastic analysis of tunnels should take the intermediate principal stress into consideration [23]. The unified strength criterion reasonably considers...
the influence of intermediate principal stress and is well suitable for many materials [24–27]. Thus, the unified strength criterion was chosen to analyze the tunnels in this study.

After the excavation of the tunnel, if enough support pressure can be applied to replace the excavated rock mass, the roadway will remain in a balanced state. However, if the support pressure is relatively small, the tunnel will suffer from different degrees of deformation [28]. Therefore, finding out the critical support pressure is of great significance for maintaining the stability of the tunnels surrounding rock. In the present study, a mechanical model of the circular tunnel was firstly established. Then, the critical support pressure when the plastic zone and damage zone begin to form is determined based on the strain-softening model and unified strength criterion. Finally, the sensitivity of the geomechanical parameters on the critical support pressure is discussed.

2. Problem Description

2.1. Mechanical Model of Circular Tunnels. An infinitely circular tunnel was excavated in uniform rock mass subjected to an initial field stress \((p_0)\) at infinity boundary and a support pressure \((p_i)\) at the tunnel surface (Figure 1). After excavation, there appeared the elastic zone, plastic zone, and damage zone around the tunnel. And the radii of the plastic and damage zones are denoted by \(R_p\) and \(R_d\), respectively.

2.2. Strain-Softening Model of the Rock Mass. As shown in Figure 2, the rock mass has gone through an elastic stage (OA), a strain-softening stage (AB), and a residual strength stage (BC) during the whole failure process. The elastic modulus in the elastic stage and softening modulus in the strain-softening stage are denoted by \(E\) and \(\alpha E\), respectively, where \(\alpha\) is the softening coefficient. The elastic stage, strain-softening stage, and residual strength stage of the full stress-strain curve correspond to the elastic zone, plastic zone, and damage zone of the surrounding rock in circular tunnels, respectively.

According to damage theory, the relationship between stress \(\sigma\) and strain \(\varepsilon\) of rock mass when considering damage can be written as

\[
\sigma = (1 - D)E\varepsilon, \quad (1)
\]

where \(D\) is the damage variable.

In one-dimensional case, the damage variable can be expressed as

\[
D = 0, \quad \varepsilon < \varepsilon_c,
\]

\[
D = \alpha \left( \frac{\varepsilon}{\varepsilon_c} - 1 \right), \quad \varepsilon_c < \varepsilon \leq \varepsilon^*, \quad (2)
\]

\[
D = D_{\text{max}}, \quad \varepsilon \geq \varepsilon^*,
\]

where \(\varepsilon_c\) is the maximum elastic strain. \(\varepsilon^*\) is the strain when the rock mass enters the residual strength stage. \(D_{\text{max}}\) is the maximum damage variable.

In the three-dimensional case, the equivalent strain \((\varepsilon_i)\) can be expressed as

\[
\varepsilon_i = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}. \quad (3)
\]

By using the equivalent strain instead of uniaxial strain, the damage variable in the plastic zone can be obtained as

\[
D = \frac{\lambda}{E} \left( \frac{\varepsilon_i}{\varepsilon_c} - 1 \right). \quad (4)
\]

2.3. Unified Strength Criterion. Assuming that the compressive stress is positive and the tensile stress is negative, the unified strength criterion can be expressed as
where \( c \) and \( \varphi \) are the cohesion and internal friction angle of the rock mass, respectively. \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the maximum, intermediate, and minimum principal stresses, respectively. \( b \) is the intermediate principal stress coefficient. When \( b = 0 \), the unified strength criterion degenerates to the Mogi–Coulomb strength criterion; when \( b = 1 \), the unified strength criterion degenerates to the general twin shear strength criterion; when \( 0 < b < 1 \), the unified strength criterion degenerates to a series of other new strength criteria.

Under plane strain conditions, the following can be easily obtained:

\[
\sigma_z = \mu (\sigma_r + \sigma_\theta),
\]

where \( \mu \) is Poisson’s ratio.

The radial stress \( (\sigma_r) \), axial stress \( (\sigma_z) \), and tangential stress \( (\sigma_\theta) \) in the surrounding rock of a tunnel can be regarded as \( \sigma_1, \sigma_2, \) and \( \sigma_3 \), respectively. Therefore, the unified strength criterion can be rewritten as

\[
\sigma_\theta - A\sigma_r - B = 0,
\]

where

\[
A = \frac{1 + b - b\mu + \sin \phi (-1b + b\mu)}{(1 - \sin \phi) (b\mu + 1)},
\]

\[
B = \frac{2c \cos \phi (1 + b)}{(1 - \sin \phi) (b\mu + 1)}.
\]

### 3. Elastic-Plastic Analysis of the Surrounding Rock

The excavation of the tunnel results in the redistribution of the stresses in the surrounding rock mass. When the redistributed stress is less than the peak strength of the rock mass, the surrounding rock only undergoes elastic deformation. When the redistributed stress is greater than the peak strength of the rock mass, the surrounding rock undergoes plastic damage and the bearing capacity decreases. The damage degree of the rock mass increases with the increase of stress until it enters the residual strength stage. At this time, a damage zone appears around the surrounding rock.

#### 3.1. Plastic Zone Begins to Form

The equilibrium differential equation in the plastic zone can be written as

\[
\frac{d\varepsilon_r}{dr} + \frac{\varepsilon_r - \varepsilon_\theta}{r} = 0.
\]  

The geometric equation can be expressed as

\[
\varepsilon_r = \frac{du}{dr},
\]

\[
\varepsilon_\theta = \frac{u}{r}
\]

According to the effective stress theory [29], the effective stress in the plastic zone meets the unified strength criterion. Therefore, equation (7) can be rewritten as

\[
\sigma_\theta' - A\sigma_r' - B = 0,
\]

where \( \sigma_\theta' \) and \( \sigma_r' \) are the effective radial stress and tangential stress, respectively.

Assuming that the damage of the surrounding rock is isotropic, the effective stresses can be expressed as

\[
\sigma_\theta' = \frac{\sigma_\theta}{1 - D}
\]

\[
\sigma_r' = \frac{\sigma_r}{1 - D}
\]

Substituting (12) in (11), the following can be obtained:

\[
\sigma_\theta - A\sigma_r - B = 0.
\]

In the plane strain state, assuming that the volume of the rock mass in the plastic zone is constant, therefore, the relationship between the radial and tangential strains can be given as

\[
\varepsilon_r + \varepsilon_\theta = 0.
\]

Combined with the boundary condition on the interface between the elastic and plastic zones, the equivalent strain can be deduced by superimposing (10) and (14):

\[
\varepsilon_i = \frac{R_p^2}{r^2} \varepsilon_r.
\]

Substituting (15) into (8), the damage variable can be rewritten as

\[
D = a \left( \frac{R_p^2}{r^2} - 1 \right).
\]
\[
\begin{align*}
\sigma_r &= B \left( \frac{p_i}{B} + \frac{1 + \alpha}{A - 1} - \frac{\alpha}{A + 1} \right) \left( \frac{r}{R_0} \right)^{A-1} - B \left( \frac{1 + \alpha}{A - 1} - \frac{\alpha}{A + 1} \right) \left( \frac{R_p}{r} \right) \frac{R_p}{R_0} \\
\sigma_\theta &= A \sigma_r + B \left( 1 + \alpha - \frac{R_p^2}{r^2} \right).
\end{align*}
\]

(17)

According to the elasticity theory [30], an assumed stress function is given as
\[
\Phi = M \ln r + Nr^2,
\]
where \(M\) and \(N\) are the constants. The radial and tangential stresses are then given by
\[
\sigma_r = \frac{1}{r} \frac{d\Phi}{dr} = \frac{M}{r} + 2N,
\]
\[
\sigma_\theta = \frac{d^2\Phi}{dr^2} = -\frac{M}{r^2} + 2N.
\]

(19)

Substituting the boundary conditions \(\sigma_r = \sigma_{r,p} = 0\) at \(r \to \infty\) and \(\sigma_r = \sigma_{r,\text{rep}}\) at \(r = R_p\), into (19), where \(\sigma_{r,\text{rep}}\) is the radial stress on the interface between the elastic and plastic zones, (19) can be rewritten as
\[
\sigma_r = \sigma_{r,\text{rep}} + \frac{R_p^2}{r} \left( 1 - \frac{R_p^2}{r} \right),
\]
\[
\sigma_\theta = -\sigma_{r,\text{rep}} + \frac{R_p^2}{r} \left( 1 + \frac{R_p^2}{r} \right).
\]

(20)

Combining with (7) and (20), the radial stress on the interface between the elastic and plastic zones can be deduced as
\[
\sigma_{r,\text{rep}} = \frac{2p_0 - B}{A + 1}.
\]

(21)

Substituting (21) into (17), the following equation can be obtained:
\[
\frac{2p_0 - B}{A + 1} = B \left( \frac{p_i}{B} + \frac{1 + \alpha}{A - 1} - \frac{\alpha}{A + 1} \right) \left( \frac{R_p}{R_0} \right)^{A-1} - B \left( \frac{1 + \alpha}{A - 1} - \frac{\alpha}{A + 1} \right).
\]

(22)

3.2. Damage Zone Begins to Form. In the damage zone, equation (7) can be rewritten as
\[
\sigma_\theta = A \sigma_r + B (1 - D_{\text{max}}).
\]

(23)

Combined with the boundary condition of \(\sigma_r = \sigma_i\) at \(r = R_0\), the radial and tangential stresses in the damage zone can be derived by solving (22). Then, the stresses distribution in the elastic and plastic zones can be obtained by introducing the value of \(R_p\) into (17) and (20).

\[
\begin{align*}
\sigma_r &= \frac{B(1 - D_{\text{max}})}{1 - A} \left( p_i - \frac{B(1 - D_{\text{max}})}{1 - A} \right) \left( \frac{r}{R_0} \right)^{A-1} \\
\sigma_\theta &= \frac{B(1 - D_{\text{max}})}{1 - A} + A \left( p_i - \frac{B(1 - D_{\text{max}})}{1 - A} \right) \left( \frac{r}{R_0} \right)^{A-1}.
\end{align*}
\]

(24)

The radial stress on the interface between the plastic and damage zones \(\sigma_{r,p,d}\) can be derived by combining (23) and (24):
\[
\sigma_{r,p,d} = \frac{B(1 - D_{\text{max}})}{1 - A} + \left( p_i - \frac{B(1 - D_{\text{max}})}{1 - A} \right) \left( \frac{R_d}{R_0} \right)^{A-1}.
\]

(25)

According to (17), the stresses in the plastic zone can be obtained as
Combined with (2), (16), and (23), the relationship between \( R_p \) and \( R_d \) can be derived as
\[
R_p = R_d \sqrt{1 + \frac{D_{\text{max}}}{\alpha}} \tag{27}
\]

Considering the continuity of radial stress in the elastic and plastic zones, the following equation can be obtained:
\[
R_d = R_0 \left( \left( \frac{(2p_0/B - 1 - \alpha)/(A + 1) + ((1 + \alpha)/(A - 1))}{(p_0/B) + ((1 - D_{\text{max}})/(A - 1))} \right)^{((A - 1)/2)} + \left( \frac{(\alpha + D_{\text{max}})/(A + 1) - ((\alpha + D_{\text{max}})/(A - 1))}{(p_0/B) + ((1 - D_{\text{max}})/(A - 1))} \right)^{((A - 1)/2)} \right)^{(1/(A - 1))}.
\]
\[
R_p = R_0 \left( \left( \frac{(2p_0/B - 1 - \alpha)/(A + 1) + ((1 + \alpha)/(A - 1))}{(p_0/B) + ((1 - D_{\text{max}})/(A - 1))} + \left( \frac{(\alpha + D_{\text{max}})/(A + 1) - ((\alpha + D_{\text{max}})/(A - 1))}{(p_0/B) + ((1 - D_{\text{max}})/(A - 1))} \right)^{((A - 1)/2)} \right)^{1/(1 + D_{\text{max}})} \right)^{(1/(A - 1))} \tag{29}
\]

The value of \( R_p \) and \( R_d \) can be expressed by combining (25), (27), and (28).

### 4. Example Study

#### 4.1. Case I: Stress Distribution around the Tunnel

Taking a circular tunnel with the radius \( R_0 = 2.5 \text{ m} \) as an example, the initial field stress is 15 MPa. The elastic modulus of the rock mass is 1350 MPa, softening coefficient is 2, cohesion is 2.5 MPa, internal friction angle is 30°, Poisson’s ratio is 0.3, and maximum damage variable is 70%.

The critical support pressure under different intermediate principal stress coefficients is shown in Table 1. Take \( b = 0.5 \) as an example, the stress distribution under different support pressures is shown in Figure 3. It can be seen that when \( p_i \geq 4.783 \text{ MPa} \), the tunnel surrounding rock only consists of the elastic zone. When \( 2.632 \leq p_i < 4.783 \text{ MPa} \), the plastic zone begins to form, and then, the tunnel surrounding rock is composed of the elastic and plastic zones. Once \( p_i < 2.632 \text{ MPa} \), the damage zone begins to develop and the tunnel surrounding rock finally displays three zones: elastic zone, plastic zone, and damage zone.

Figure 4 shows the stress distribution under different intermediate principal stress coefficients when the support pressure is 1 MPa. It can be seen that as \( b \) increases, the whole radial stress, the tangential stress in the damage and plastic zones, and the peak tangential stress all show an increase, while the tangential stress in the elastic zone and the radii of the plastic and damage zones show a decrease. For example, as \( b \) transforms from 0 to 1, the peak tangential stress increases by 1 MPa, and the \( R_p \) and \( R_d \) values decrease by 0.639 m and 0.550 m, respectively. Therefore, the intermediate principal stress should be properly considered in engineering applications.

#### 4.2. Case II: Effect of Parameters on the Critical Support Pressure

##### 4.2.1. Effect of the Initial Field Stress

The influence of initial field stress on the critical support pressure under different intermediate principal stress coefficients is shown in Figure 5. It can be seen that the initial field stress has a significant influence on the critical support pressure. The \( p_{i1} \)
Table 1: Critical support pressure under different intermediate principal stress coefficients.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$p_{\alpha}$</th>
<th>$p_{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.338</td>
<td>3.226</td>
</tr>
<tr>
<td>0.25</td>
<td>5.044</td>
<td>2.912</td>
</tr>
<tr>
<td>0.5</td>
<td>4.783</td>
<td>2.632</td>
</tr>
<tr>
<td>0.75</td>
<td>4.549</td>
<td>2.380</td>
</tr>
<tr>
<td>1</td>
<td>4.339</td>
<td>2.152</td>
</tr>
</tbody>
</table>

Figure 3: Stresses distribution under different support pressures. (a) $p_\alpha = 5$ MPa, (b) $p_\alpha = 3$ MPa, and (c) $p_\alpha = 1$ MPa.

Figure 4: Stresses distribution under different intermediate principal stress coefficients. (a) $\sigma_r$ and (b) $\sigma_\theta$. 

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Figure 5: Influence of initial field stress on the critical support pressure under different intermediate principal stress coefficients. (a) $p_{i1}$ and (b) $p_{i2}$.

Figure 6: Influence of the softening coefficient on the critical support pressure under different intermediate principal stress coefficients. (a) $p_{i1}$ and (b) $p_{i2}$.

Figure 7: Influence of the maximum damage variable on the critical support pressure under different intermediate principal stress coefficients. (a) $p_{i1}$ and (b) $p_{i2}$.
and $p_{i2}$ values both increase with increasing $p_0$. Take $b = 0.5$ as an example, as $p_0$ increases from 10 MPa to 20 MPa, $p_{i1}$ and $p_{i2}$ values increase by 5.113 MPa and 3.839 MPa, with an increment of 229.69% and 539.19%, respectively. Therefore, as the depth of the tunnel increases, greater support force needs to be applied to maintain the stability of the tunnel surrounding rock.

4.2.2. Effect of the Softening Coefficient. The influence of the softening coefficient on the critical support pressure under different intermediate principal stress coefficients is shown in Figure 6. It can be seen that the softening coefficient has a significant effect on the critical support pressure ($p_{i2}$) when the damage zone begins to form. Take $b = 0.5$ as an example, as $a$ increases from 1.0 to 3.0, the value of $p_{i2}$ transforms from 1.349 MPa to 3.216 MPa, with an increment of 138.40%. However, but the softening coefficient has no effect on the critical support pressure ($p_{i1}$) when the plastic zone begins to form.

4.2.3. Effect of the Maximum Damage Variable. Figure 7 shows the sensitivity of the maximum damage variable to the critical support pressure under different intermediate principal stress coefficients. It can be seen that the maximum damage variable has no influence on the value of $p_{i1}$. However, the value of $p_{i2}$ shows an increase with the continuous increase of the maximum damage variable. Take $b = 0.5$ as an example; as $D_{\text{max}}$ increases from 0 to 100%, the value of $p_{i2}$ decreases by 2.249 MPa, with a reduction of 55.52%. The results show that the lighter the damage degree of the rock mass, the more difficult it is for the tunnel surrounding rock to appear damage zone. Therefore, some measures, such as grouting, can be used to reduce the damage degree of the rock mass and ensure tunnel stability.

5. Conclusions

A mechanical model of the circular tunnel was firstly established. Considering the strain-softening characteristics of rock mass, the critical support pressure when the plastic zone and damage zone begin to form was deduced based on the unified strength criterion. The stress distribution under different support pressures and different intermediate principal stress coefficients were analyzed. The effects of initial field stress, softening coefficient, and maximum damage variable on the critical support pressure were discussed. The conclusions can be summarized as follows:

1. The critical support pressure and radii of plastic and damage zones all decrease with the increase of critical support pressure and the stress coefficient. Therefore, the support design should take the intermediate principal stress into consideration.

2. The critical support pressure increase with increasing initial field stress. Therefore, as the depth of the tunnel increases, greater support force needs to be applied to maintain the stability of the tunnel surrounding rock.

3. The softening coefficient and maximum damage variable of rock mass has no influence on the critical support pressure when the plastic zone begins to form, but has a significant effect on the critical support pressure when the damage zone begins to form. With the softening coefficient increasing, the $p_{i2}$ value increases. However, the $p_{i2}$ value decreases as maximum damage variable increases. Therefore, tunnel stability can be effectively controlled by reducing the damage degree of the rock mass.

Data Availability

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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