Hindawi
Advances in Civil Engineering
Volume 2020, Article ID 8875029, 9 pages
https://doi.org/10.1155/2020/8875029

Research Article

Study of Damage Constitutive Model of Brittle Rocks considering Stress Dropping Characteristics

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Received 13 August 2020; Revised 22 September 2020; Accepted 5 October 2020; Published 28 October 2020

Academic Editor: Fengqiang Gong

Deep brittle rock exhibits characteristics of rapid stress dropping rate and large stress dropping degree after peak failure. To simulate the whole process of deformation and failure of the deep brittle rock under load, the Lemaitre strain equivalent theory is modified to make the damaged part of the rock has residual stress. Based on the damage constitutive model considering residual strength characteristics, a correction factor reflecting stress dropping rate is added, the Weibull distribution is used to describe the inhomogeneity of rock materials, and Drucker–Prager criterion is used to quantitatively describe the influence of stress on damage; a damage constitutive model of deep brittle rock considering stress dropping characteristics is established. According to the geometric features of the rock stress-strain curve, the theoretical expressions of model parameters are derived. To verify the rationality of the model, triaxial compression experiments of deep brittle rock under different confining pressures are conducted. And the influence of model parameters on rock mechanical behaviour is analysed. The results show that the model reflects the stress dropping characteristics of deep brittle rock and the theoretical curve is in good agreement with the experimental results, which indicates that the proposed constitutive model is scientific and feasible.

1. Introduction

With the reduction of mining resources in the shallow strata of the world, the mining depth of resources is gradually increased [1, 2]. In order to meet the needs of production and life, China has proposed the "deep earth" strategy during the 13th Five-Year period. The number and scale of deep rock engineering will increase significantly in the future [3, 4]. The deep rock is in the deep stratum, and its internal damage is continuously generated and developed under the action of external high stress, and it still has a certain bearing capacity after the peak failure [5]. Therefore, the whole process of deep rock failure under load is studied, and the stress dropping characteristics in the postpeak stage are analysed, which provides an important theoretical model for the construction of deep rock engineering.

The mechanical behaviour and constitutive relationship of the rock in the postpeak stage have always been a frontier research topic in the field of rock mechanics. Wawersik and Fairhurst [6] found that with the increases of confining pressure, the mechanical behaviour of marble transforms from brittleness to ductility in the postpeak stage. Yang et al. [7] carried out a series of postpeak cyclic loading and unloading experiments on sandstone to study the mechanical behaviour, deterioration parameters, energy indexes, and failure modes of rock samples in the postpeak stage. Tang et al. [8] carried out three-point bending experiments combined with digital image correlation and acoustic emission technology on notched granite beams, and obtained the crack development characteristics of rocks under postpeak cyclic loading. Yang et al. [9] studied the energy evolution trend of sandstone under cyclic loading and unloading, and obtained the energy consumption law and damage characteristics under different confining pressures. Lin et al. [10] established a new postpeak strain-softening model of rock, which can predict the postpeak mechanical behaviour of rock, and obtained the evolution law of the rock strength parameters. Based on rock damage model founded by the Lemaitre strain equivalent theory, Cao et al. [11–13] established a damage statistical constitutive model that can consider the effect of damage threshold and simulate the whole process of strain-softening deformation of rock. However, these models cannot show a phenomenon that rock has residual strength in the postpeak stage, because Lemaitre strain equivalent theory does not consider that the rock still has a certain bearing capacity after damage, so scholars have revised the damage model in recent years. Cao et al. [14] established a statistical damage constitutive model that can simulate the characteristics of the whole process of rock deformation, which can reflect the residual strength of the rock in the postpeak stage. Cao et al. [15] considered the residual strength of rock in the postpeak stage, modified the damage variable, and established a statistical damage constitutive model, which can describe the stress-strain relationship and residual strength of rock. Jiang et al. [16] added the damage threshold and residual strength of the rock to the existing statistical damage model, and established the statistical damage model on the rock damage and the strain softening.

The existing damage constitutive models seldom consider the stress dropping rate after peak failure. When the research object is brittle rock, the stress dropping rate after peak failure is faster [17]. The above damage models cannot reflect the characteristics of brittle rock. Therefore, in this paper, the brittleness index that reflects the stress dropping characteristics is added to modify the damage constitutive model, and a new damage constitutive model of deep rock considering the stress dropping characteristics is established. The model not only reflects the stress dropping rate but also reflects the stress dropping degree.

2. Damage Constitutive Model of Deep Brittle Rock considering Stress Dropping Characteristics

2.1. Modification of Strain Equivalent Theory. According to the strain equivalence theory [18], the axial load acting on the rock microelements is completely supported by the undamaged part. When the rock microelements are damaged, the strength decreases to zero, showing the characteristics of no residual strength. In fact, stress dropping phenomenon occurs in the postpeak stage, and the macrosections are formed inside the rock. The residual stress is supported by the friction between the sections and no longer changes with the deformation growth. To establish the constitutive model of residual strength, the following assumptions are made:

(1) The rock microelements under axial load in the axial direction are abstracted into two parts: the undamaged part and the damaged part, and the axial stress is supported by the two parts together.

(2) The strength of the rock microelements after the failure decreases to residual stress σ_r, and the undamaged part follows generalized Hooke's law, i.e.,

$$\sigma_1^* = E\varepsilon_1^* + \mu\left(\sigma_2^* + \sigma_3^*\right), \tag{1}$$

where σ_i^* is the effective stress of the undamaged part in the i direction; ε_i^* is the effective stress of the undamaged part in the i direction; E is the elastic modulus; and μ is the Poisson ratio.

(3) The undamaged part shares full lateral load.

2.2. Establishment of Damage Constitutive Model. The determination of the damage variable is the key to establishing a damage constitutive model. According to continuum damage mechanics, the damage variable D can be defined in a variety of ways, such as the axial strain of the rock specimen and the area of the microelements [19, 20]. The article uses the second method to define the damage variable. For the rock microelement analysis, the total area of the rock microelements in the axial direction is set as A, and its nominal axial stress and nominal axial strain are σ_1 and ε_1, respectively. The area of the rock microelements in the undamaged part is A_1, and its effective axial stress and effective axial strain are σ_i^* and ε_i^*, respectively. The area of the rock microelements in the damaged part is A_2, the axial stress is residual stress σ_r, and the axial strain is ε_r. From assumption (1), it is solved that

$$\sigma_1 A = \sigma_1^* A_1 + \sigma_r A_2, \tag{2}$$

$$A = A_1 + A_2. \tag{3}$$

The ratio of the damaged area to the total area is defined as the damage variable D:

$$D = \frac{A_2}{A} = 1 - \frac{A_1}{A}. \tag{4}$$

Substituting equations (3) and (4) into equation (1),

$$\sigma_1 = \sigma_1^* (1 - D) + \sigma_r D. \tag{5}$$

Because the undamaged part and the damaged part of the rock microelements are closely mixed with each other in the macroscale, which conforms to the deformation coordination principle, i.e.,

$$\varepsilon_1 = \varepsilon_1^* = \varepsilon_r. \tag{6}$$

From assumption (3), it is solved that

$$\sigma_2 = \sigma_2^*, \tag{7}$$

$$\sigma_3 = \sigma_3^*. \tag{8}$$

Substituting equations (6) to (8) into equation (1),

$$\sigma_1^* = E\varepsilon_1 + \mu\left(\sigma_2 + \sigma_3\right). \tag{9}$$

Substituting equation (9) into equation (5) and the damage constitutive model considering residual strength characteristics can be obtained as follows:

$$\sigma_1 = E\varepsilon_1(1-D) + ND + \mu(\sigma_2+\sigma_3), \tag{10}$$

$$N = \sigma_r - \mu(\sigma_2+\sigma_3). \tag{11}$$

When the research object is deep brittle rock, the damage constitutive model should reflect not only the stress dropping degree in the postpeak stage, but also the stress dropping rate in the postpeak stage. Therefore, a correction factor K is added to the model, which includes the brittleness index η reflecting the stress dropping rate and the fitting parameter n reflecting the rock homogeneity. It should be noted that deep brittle rock only undergoes stress dropping phenomenon in the postpeak stage, so the influence of the stress dropping rate on the deformation characteristics of the rock in the prepeak stage can be ignored:

$$K = \begin{cases} 1, & \varepsilon_1 \le \varepsilon_p, \\ K_0, & \varepsilon_1 > \varepsilon_p, \end{cases} \tag{12}$$

$$K_0 = (1-\eta)^n, \tag{13}$$

$$\eta = \frac{\lg l}{10}, \tag{14}$$

$$l = \left|\frac{\sigma_p - \sigma_r}{\varepsilon_p - \varepsilon_r}\right|, \tag{15}$$

where l is the stress dropping rate and n is the fitting parameter, which is related to the confining pressure.

Modifying equation (10), the following model can be obtained:

$$\sigma_1 = KE\varepsilon_1(1-D) + ND + \mu(\sigma_2+\sigma_3). \tag{16}$$

It is assumed that the rock microelement strength follows the Weibull random distribution, and the damage variable can be expressed as

$$D = \int_0^F P(F)\mathrm{d}F = 1 - \exp\left[-\left(\frac{F}{F_0}\right)^m\right], \tag{17}$$

where F is the rock microelement strength; $P(F)$ is the distribution function of the probability density of the rock microelement strength; and m and F_0 are Weibull distribution parameters.

Combining equations (16) and (17), a damage constitutive model considering the characteristics of stress dropping rate and residual strength is obtained:

$$\sigma_1 = KE\varepsilon_1 \exp\left[-\left(\frac{F}{F_0}\right)^m\right] + N\left\{1 - \exp\left[-\left(\frac{F}{F_0}\right)^m\right]\right\} + 2\mu\sigma_3. \tag{18}$$

To measure the strength of the rock microelements, the Drucker–Prager criterion is introduced:

$$F = \alpha I_1^* + \sqrt{J_2^*} = \frac{\sin\varphi}{\sqrt{9+3\sin^2\varphi}} I_1^* + \sqrt{J_2^*}, \tag{19}$$

where φ is the internal friction angle of rock and I^*1 and J^*2 are the first invariant of the stress tensor and the second invariant of the deviatoric stress tensor, respectively. I^*1 and J^*2 can be expressed:

$$I_1^* = \sigma_1^* + \sigma_2^* + \sigma_3^*, \tag{20}$$

$$J_2^* = \frac{1}{6}\left[(\sigma_1^*-\sigma_2^*)^2 + (\sigma_2^*-\sigma_3^*)^2 + (\sigma_1^*-\sigma_3^*)^2\right]. \tag{21}$$

Substituting equations (7) to (9) into equation (19),

$$F = PE\varepsilon_1 + Y\sigma_3, \tag{22}$$

$$P = \alpha + \frac{\sqrt{3}}{3}, \tag{23}$$

$$Y = \left(2\alpha + \frac{2\sqrt{3}}{3}\right)\mu + 2\alpha - \frac{\sqrt{3}}{3}. \tag{24}$$

2.3. Determination of Model Parameters. Based on the geometric features of rock stress-strain curve, the following boundary conditions are obtained:

$$\begin{cases} \varepsilon_1|_{\sigma_1=\sigma_p} = \varepsilon_p, \\ \dfrac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon_1}\Big|_{\sigma_1=\sigma_p} = 0. \end{cases} \tag{25}$$

Solving the partial differential of equation (16),

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon_1} = KE(1-D) + (N - KE\varepsilon_1)\frac{\partial D}{\partial \varepsilon_1}. \tag{26}$$

Combining equations (25) and (26),

$$\frac{\partial D}{\partial \varepsilon_1}\Big|_{\sigma_1=\sigma_p,\varepsilon_1=\varepsilon_p} = \frac{KE(1-D_p)}{KE\varepsilon_p - N} \tag{27}$$

D_p is the damage variable of rock at the peak stress, which is solved by equation (16):

$$D_p = \frac{\sigma_p - KE\varepsilon_p - 2\mu\sigma_3}{N - KE\varepsilon_p}. \tag{28}$$

Solving the partial differential of equation (17),

$$\frac{\partial D}{\partial \varepsilon_1} = \exp\left[-\left(\frac{F}{F_0}\right)^m\right]\left[m\left(\frac{F}{F_0}\right)^{m-1}\right]\frac{1}{F_0}\frac{\partial F}{\partial \varepsilon_1}. \tag{29}$$

Solving the partial differential of equation (22),

$$\frac{\partial F}{\partial \varepsilon_1} = PE. \tag{30}$$

Changing equation (17),

$$\left(\frac{F}{F_0}\right)^{m-1} = -\frac{F_0}{F}\ln(1-D) \tag{31}$$

Substituting equations (29) to (31) into equation (27), m can be solved, i.e.,

$$m = \frac{KF_p}{\left(N - KE\varepsilon_p\right)\ln\left(1 - D_p\right)P}, \quad (32)$$

where F_p is the rock microelement strength at the peak point, which can be solved by equation (22):

$$F_p = PE\varepsilon_p + Y\sigma_3. \quad (33)$$

Substituting D_p and F_p into equation (31), then

$$F_0 = F_p\left[-\ln\left(1 - D_p\right)\right]^{-(1/m)}. \quad (34)$$

3. Verification of Damage Constitutive Model

3.1. Triaxial Compression Experiment of Deep Brittle Rock. The rock samples were taken from the granite cores at the depth of 1200 m in the Shaling gold mine, Shandong province, China. The composition analysis result shows that the content of the broken matrix is more than 50%, and the granite is classified as granitic cataclasite. The porosity of the rock is determined to be 1.68% by the vacuum saturation test and nuclear magnetic resonance experiment. According to standards of the International Society for Rock Mechanics (ISRM) [21], the cores are processed into standard cylindrical samples with a diameter of 50 mm and a height of 100 mm. The samples with similar wave velocity are selected by ultrasonic testing to reduce the mechanical experiment errors.

Firstly, the rock sample is placed in the TAW-2000 electrohydraulic servocontrolled triaxial test machine, and the confining pressure is applied to the predetermined value at the loading rate of 100 N/s in the initial stage, and then the confining pressure is kept constant during the experiment. After the confining pressure is stable, the axial deviatoric stress is applied at the loading rate of 0.015 mm/min until the sample is damaged. The confining pressure values used in the experiment are 15 MPa, 25 MPa, and 35 MPa. Different confining pressures are used to simulate the stress environment of rock mass in different depths.

As shown in Figure 1, the deformation of granitic cataclasite under high confining pressure can be divided into five stages: elastic stage (OA section), yield stage (AB section), expansion stage (BC section), strain-softening stage (BD section), and residual stress stage (DE section). Granitic cataclasite has typical characteristics of deep brittle rock. In the stress-strain curve, it shows no obvious crack closure stage, longer yield stage, higher expansion point, and faster stress dropping rate in strain-softening stage. It should be noted that due to the low porosity of deep brittle rock and the high confining pressure, the crack closure stage of the rock is not obvious.

The complete stress-strain curves of rocks under different confining pressures are shown in Figure 2. It can be found that the peak stress of granitic cataclasite increases with the increase of confining pressure, while the residual strength shows an opposite trend. Because the confining

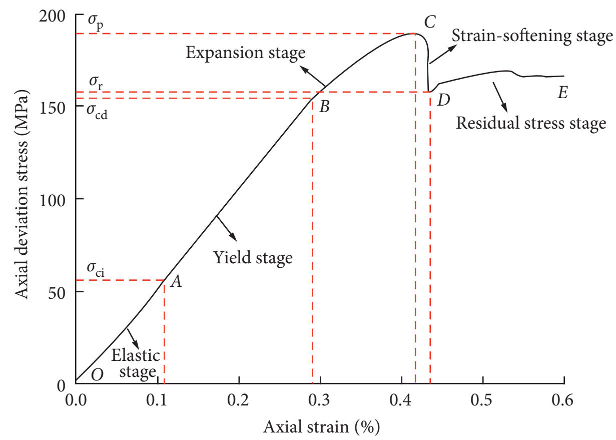

Figure 1: Stress threshold and stress stage division.

pressure increases below the critical confining pressure value for the brittle-ductile transition, the energy accumulation degree of the rock increases. When the peak failure occurs, the energy is released violently and impacts the overall structure of the rock, causing the rock to become more brittle. The elastic modulus and Poisson's ratio increase with the increase of confining pressure.

4. Evolution Law of Strength Parameters

Stress thresholds are very important in the investigating rock deformation and failure. In the 1960s, Hoek and Bieniawski [22] began to study the mechanical properties of brittle rocks. Brace et al. [23] found that when the stress exceeds the elastic limit of the rock sample, the rock sample exhibits nonlinear mechanical behaviour, and as the stress continues to increase, the volume of the rock sample changes from the compressed state to the expanded state. Cook [24] has proved that dilatancy is a volume property of brittle rocks through mechanical tests. In order to determine the crack initiation stress of the rock sample, Martin and Chandler [25] proposed the concept of crack volumetric strain and obtained the stress threshold of the rock sample through the transformation of crack volumetric strain trend. In recent years, many researchers have proposed new methods to obtain stress thresholds. Peng et al. [26] proposed the axial strain response method to determine the crack initiation stress. Gong and Wu [27, 28] obtained crack damage stress of rock more accurately by means of acoustic emission signal detection and load-unload response ratio.

As the confining pressure increases from 15 MPa to 35 MPa, the damage stress and the peak stress increase linearly (Figure 3), because the confining pressure can improve the compactness of rock, limit the development and expansion of cracks to a certain extent, increase the friction force between rock particles, and enhance the rock support capacity, so the volume expansion and peak failure of rock are delayed.

Both the damage stress and the peak stress follow the Mohr–Coulomb strength criterion. Based on the criterion, the strength parameters φ_{cd} and c_{cd} at the damage stress and

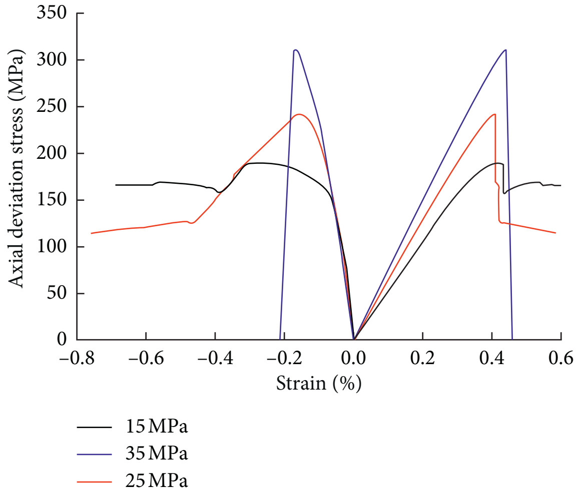

FIGURE 2: The complete stress-strain curve.

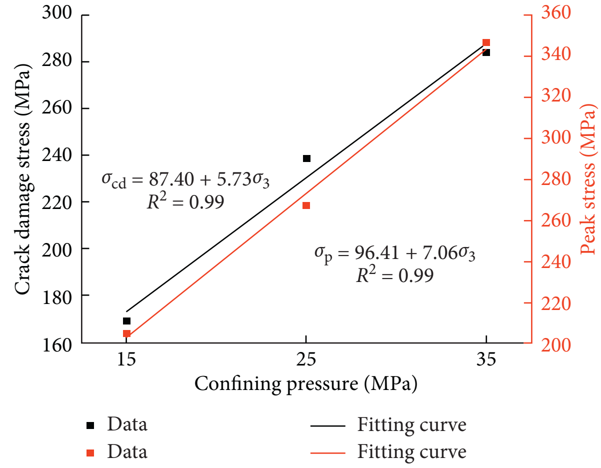

FIGURE 3: Relationship between damage stress, peak stress, and peri-pressure.

the strength parameters φ_p and c_p at the peak stress are obtained, respectively. The criterion is as follows:

$$\sigma_1 = A + B\sigma_3, \tag{35}$$

$$A = \frac{2c\cos\varphi}{1-\sin\varphi}, \tag{36}$$

$$B = \frac{1+\sin\varphi}{1-\sin\varphi}, \tag{37}$$

where c is the cohesion of rock.

The regression analysis method is used to calculate φ_{cd}, c_{cd}, φ_p, and c_p, which are 40.61°, 20.09 MPa, 48.75°, and 18.14 MPa, respectively.

Equation (38) can quantitatively describe the stress dropping degree in the strain-softening stage:

$$\omega = \frac{\sigma_p - \sigma_r}{\sigma_p}, \tag{38}$$

where ω is the stress dropping degree.

As shown in Figure 4, it can be found that the stress dropping degree of rock increases with the increase of confining pressure. This is because confining pressure can reduce the degree of inhomogeneity of rock, enhance the friction between particles, and increase the level of stored energy, which leads to the increase of energy released during rock failure and the impact on the overall structure. Therefore, the stress dropping degree increases.

4.1. Model Verification. Combining the experiment curves and the equations of parameters in damage constitutive model, the values of the parameters under different 1confining pressures are calculated, as shown in Table 1.

Based on the model parameters, the damage constitutive model considering the stress dropping characteristics is calculated from equation (18). To reflect the advanced and reasonable of the damage constitutive model proposed in

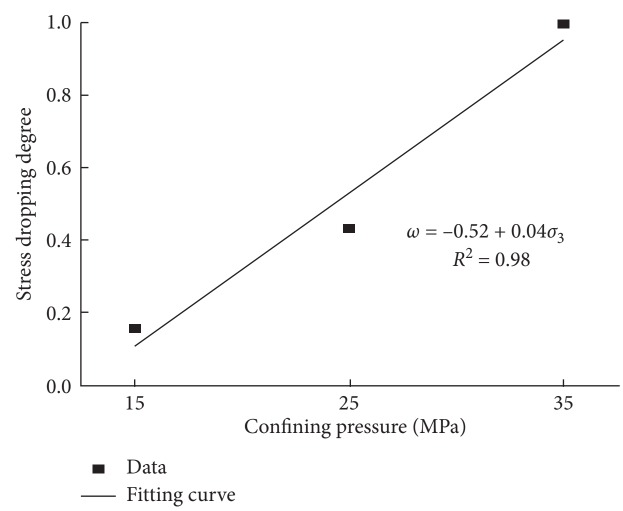

FIGURE 4: Relationship between peak strength and stress dropping degree.

this paper, the damage constitutive models established in [13, 14] are compared with the model in this paper, which is shown in Figure 5. The theoretical curve of the model established in this paper is in good agreement with the experiment curve, which can reflect the whole process of deep brittle rock deformation, especially in the strain-softening stage.

5. Discussion of Model Parameters

5.1. Influence of Weibull Parameters on Experimental Result. The microelement strength of rock follows Weibull distribution, and the damage constitutive model based on the principle includes Weibull parameters F_0 and m which affect the geometry of the stress-strain curve. Based on the rock constitutive parameters under confining pressure of 15 MPa, we keep other parameters constant and change the values of F_0 and m, respectively. The stress-strain curves are shown in Figures 6 and 7.

Table 1: The value of parameters in the constitutive model at different confining pressures.

Number	σ_3 (MPa)	E (GPa)	μ	σ_{cd} (MPa)	σ_p (MPa)	ε_p (%)	σ_r (MPa)	ε_r (%)	$K = 1.00$		$K = K_0$		
									m	F_0	m	F_0	K_0
1	15	55.58	0.20	169.44	205.17	0.44	172.77	0.45	3.79	203.90	81.31	202.00	0.83
2	25	61.62	0.27	238.64	267.19	0.43	150.66	0.44	12.38	260.61	139.75	232.26	0.93
3	35	74.94	0.34	284.09	346.37	0.46	0	0.47	19.31	331.98	179.87	292.90	0.95

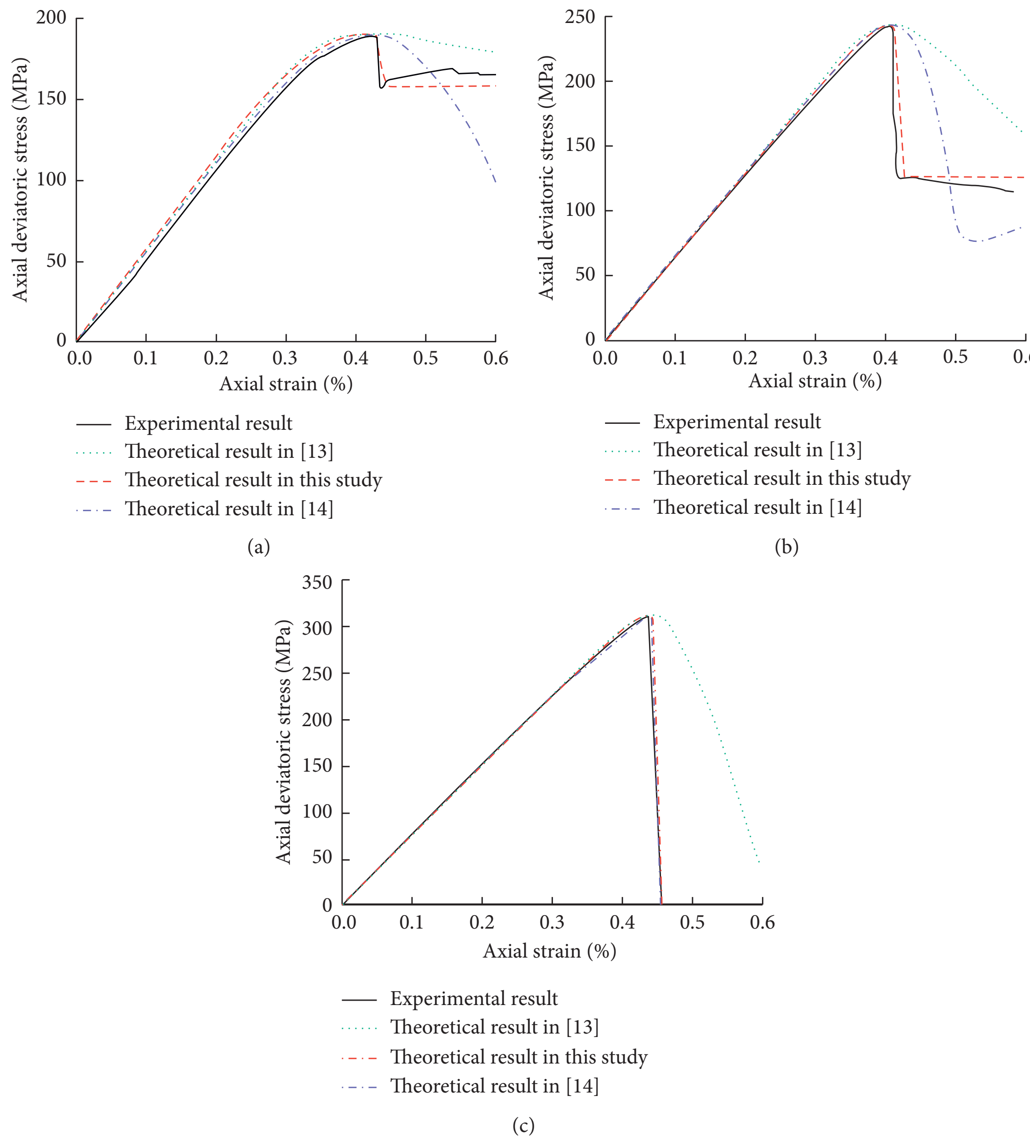

Figure 5: The complete stress-strain curve: (a) 15 MPa; (b) 25 MPa; (c) 35 MPa.

As shown in Figure 6, the change of F_0 and m does not affect the evolution trend of the curve in the prepeak stage, that is, the axial deviatoric stress increases with the increase of axial strain, and the stress growth rate decreases when it approaches the peak stress. The change of F_0 and m can slightly affect the stress-strain curve of rock before damage, but it has a significant effect on the curve geometry of the rock after damage. With the increase of F_0 and m, the peak stress increases, while the peak strain shows an opposite trend. The increase of F_0 and m enhances the ductility and brittleness of rocks, respectively.

It can be seen from Figure 7 that the change of F_0 and m does not affect the evolution trend of the curve in the postpeak stage, that is, with the increase of axial strain, the axial deviatoric stress decreases, and the stress does not change when entering the residual strength stage. The change of F_0 and m does not change the residual strength of rock. With the increase of F_0, both the peak stress and its

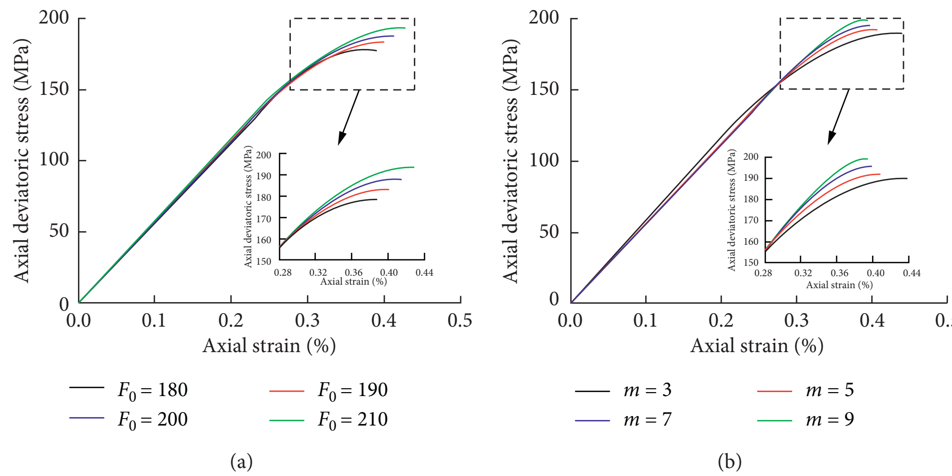

Figure 6: The influence of Weibull parameters on the stress-strain curve in the prepeak stage: (a) different F_0; (b) different m.

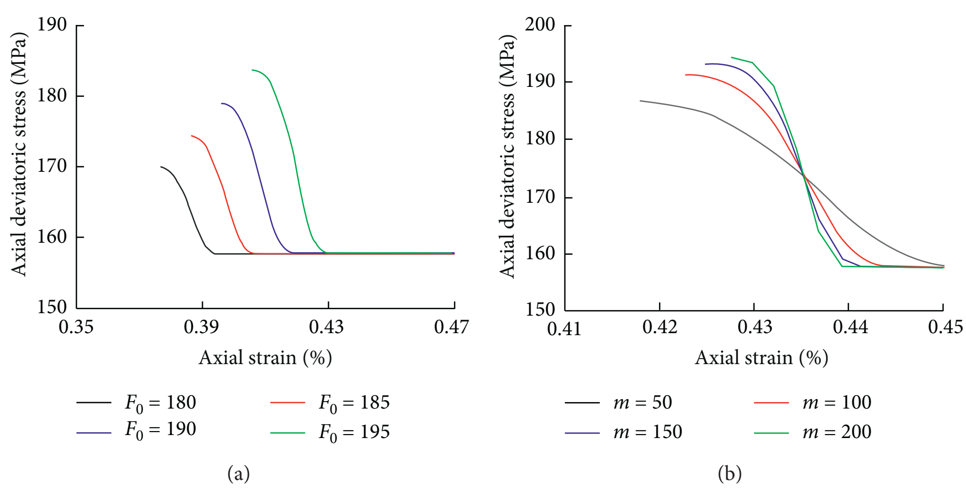

Figure 7: The influence of Weibull parameters on the stress-strain curve in the postpeak stage: (a) different F_0; (b) different m.

corresponding strain increase. And m has the same effect. The increase of F_0 and m increases the stress dropping rate.

5.2. Influence of Correction Factor on Experimental Result. The correction factor is closely related to the properties of deep brittle rocks and characterizes the brittleness of rocks. The correction factor K includes the brittleness index η considering stress dropping rate and fitting parameter n related to homogeneity. Both η and n are related to confining pressure. The values are shown in Table 2. η reflects the brittleness of rock, and η increases with the increase of confining pressure, which indicates that the brittleness of rock gradually develops. n reflects the inhomogeneity of rock, and n shows the opposite trend, which indicates that the rock is more compact. Both of them lead to the increase of stress dropping rate and stress dropping degree. To analyse the influence of the correction factor on the constitutive model, taking the complete stress-strain curve of rock under 15 MPa confining pressure as an example, the theoretical curves with different K values are selected for comparative analysis.

Table 2: Parameters related to correction factor.

Number	σ_3 (MPa)	η	n
1	15	0.33	0.46
2	25	0.39	0.14
3	35	0.43	0.09

As shown in Figure 8, the change of K value does not affect the evolution law of the theoretical curve, nor does it change the position of the peak stress and peak strain of the theoretical curve. With the decrease of K, the stress corresponding to the same strain decreases in the postpeak stage, which is because K reflects the brittleness of rock. The increase of confining pressure reduces the inhomogeneity of rock. The stress dropping phenomenon becomes more obvious, and the stress dropping rate and stress dropping degree increase. The larger the K value, the earlier the theoretical curve reaches the residual strength stage.

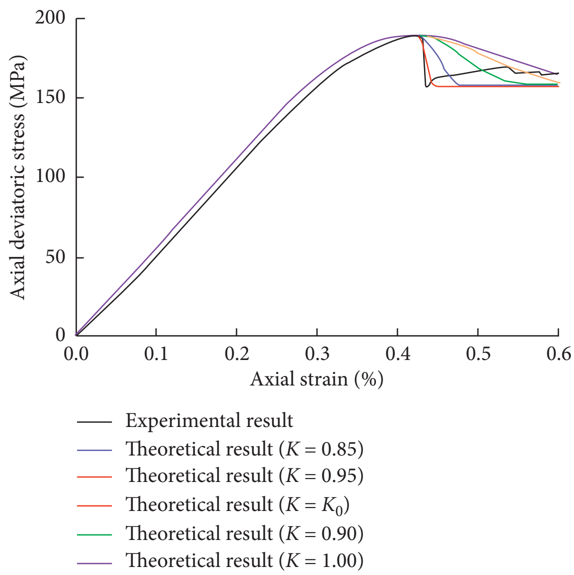

Figure 8: The influence of correction factor on the stress-strain curve.

6. Conclusions

The study of the constitutive model of deep brittle rock is of great theoretical significance for deep rock mass engineering. In this paper, the main conclusions are as follows:

(1) Based on the damage constitutive model considering residual strength, the correction factor reflecting the stress dropping rate is added, the damage constitutive model considering the stress dropping characteristics is established, and the theoretical expression of the model parameters is derived.

(2) Triaxial compression tests of granitic cataclasite under different confining pressures are carried out to verify the rationality of the model. The results show that with the increase of confining pressure, the stiffness and strength of rock increase, and its brittleness and stress dropping characteristics are enhanced. The model established in this paper can reflect the stress-strain relationship of deep brittle rock, especially the characteristics of rapid stress dropping rate and large stress dropping degree in the postpeak stage.

(3) The change of Weibull parameters does change the overall change trend of the stress-strain curve, hardly affects the curve of the undamaged stage of the rock, and also does not change the residual strength of rock, but it has a greater impact on the damage stage. The change of the correction factor K can neither affect the development trend of the theoretical curve of the constitutive model, nor change the position and size of the peak point. And with the decrease of K, the stress corresponding to the same strain decreases in the postpeak stage.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was financially supported by the National Key Research Development Program of China (2016YFC0600801), the National Natural Science Foundation of China (51774021), and the Major Scientific and Technological Innovation Project of Shandong Province (2019SDZY05).

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