Research Article

Shear Strength of Flat Joint considering Influencing Area of Bolts

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1. Introduction

Joints widely exist in the natural rock mass, and the majority of failures in geotechnical engineering are caused by joint instability [1–9]. Bolting technology is the predominant means in geotechnical engineering reinforcement, which is adopted in mines, tunnels, slope, and dam foundation and other major projects. To date, scholars and experts have executed multitudinous theoretical and experimental studies on the bolted rock mass [10–15], which undoubtedly deepen the understanding of the bolting mechanism of jointed rock mass and provide certain foundation and conditions for further researches. However, those researches mainly focus on the tests without considering the group bolting effect, while bolts are usually systematically used in practical engineering. The arrangement of bolts significantly affects the bolting efficiency and engineering input, in which case it is inevitable to study the influencing area of the bolt to reach the optimal bolting efficacy. With respect to this point, there exist various opinions. For example, Chen et al. [16] believed that the influencing area of the bolt was the entire joint surface and established the calculating formula of the shear strength of bolted joint considering dilantancy characteristics. Liu et al. [17] considered the influencing area of bolt rectangular, which can be obtained by the product of bolt diameter and joint width or bolting interval. According to Teng et al. [18], however, it is less possible for the bolt to produce a rectangular influencing area in the actual shearing process, while the crack around the bolt will continuously extend in all directions until failure occurs under the interaction of shear force and axial force, ultimately forming a circular or elliptical section. In addition, the group bolting effect should also be taken into consideration when the number of bolts increases.

Based on the above considerations, the direct shear test on bolted joints was implemented, the number and inclination of the bolt were changed, respectively, and the test results were analyzed. Besides, a theoretical calculation model of bolted joint shear strength considering the influencing area was proposed, which was verified by the comparison between the calculated value and test results.
2. Laboratory Tests

2.1. Sample Preparation. A similar material was selected to simulate the jointed rock, the direct shear test was performed, and the shear stress-shear displacement curve was recorded under constant normal load [19–21]. The specific model contains two parts, and the joint was simulated by the interface of two rock blocks, forming a joint rock with the size of $150 \times 150 \times 120$ mm (see Figure 1). The bolt was represented by using the same material in Lin et al. [19] work. The installation angle of the bolt was set at 45° and 90°, respectively, and the number of bolts ranged from 1 to 3. The bolts were arranged as follows:

(1) When one bolt was used, the bolt was placed at the joint center.

(2) When two bolts were used, the bolts were arranged on the central axis perpendicular to the shearing direction, dividing the central axis into 3 parts on average at the interval of 50 mm.

(3) When three bolts were used, the bolts were arranged on the central axis perpendicular to the shearing direction, dividing the central axis into 3 parts on average at the interval of 37.5 mm.

During the process of producing jointed rock, the lower parts were poured first, and the bolts were inserted into the sample according to the predetermined inclination angle and number. 24 hours later, the mold was removed, and the upper parts were poured, as shown in Figure 1. Then, the sample was placed in an incubator for 28 days. To minimize the influence of the concrete strength difference of each sample on the test results, all the simulated rock blocks in this test were poured with the same proportion and the same batch of cement mortar.

RYL-600 microcomputer-controlled rock shear testing machine was adopted to conduct direct shear tests [22–25]. The normal load was set: 10, 20, 30, 40, and 50 kN, respectively, corresponding to normal stress of 0.45, 0.89, 1.33, 1.78, and 2.22 MPa. The loading speed was 1 mm/min. In order to obtain the basic parameters of the material under this ratio, three standard cylindrical samples with the diameter of 50 mm and the height of 100 mm, made of cement mortar with the same proportion, were produced to execute the uniaxial compression tests. And the mean value of the uniaxial compression tests was considered as the calculated uniaxial compressive strength (UCS = 21.86 MPa).

The direct shear tests of unbolted joints with different normal stresses were carried out to obtain the cohesion $c_j$ and the basic internal friction angle $\varphi_b$ of samples. The shear stress-shear displacement curves of unbolted joints were fitted by the Mohr-Coulomb stress-shear displacement curve [26–28], from which the cohesion of 0.306 MPa and the internal friction angle of 45.23° can be back-calculated. Nevertheless, for bolted joint, the bolt has not yielded when the sample reaches the peak strength, and only small deformation emerges. Therefore, it is believed that it is more reasonable to adopt the basic internal friction angle ($\varphi_b = 33.94°$) corresponding to the residual strength, instead of the internal friction angle corresponding to the peak strength, to calculate the shear strength of bolted joints. The mechanical parameters related to the sample and the bolt were shown in Table 1.

2.2. Test Results. A total of 30 bolted joint samples were prepared in the tests, divided into 6 groups, that is, one-bolted joints (inclination of 45° and 90°), two-bolted joint (inclination of 45° and 90°), three-bolted joint (inclination of 45° and 90°). Each group was subjected to direct shear tests under five different normal stresses, the shear stress-shear displacement curves were obtained as shown in Figure 3, and the corresponding shear strength was shown in Table 2.

3. Shear Strength Model of Bolted Joint

3.1. Bolt Stress and Deformation. Generally, attributed to the dilatancy effect, both the shear displacement and the normal displacement will emerge for the bolted joints under the action of shear load [28–30]. One common assumption was made that $U_0$ represented the shear displacement of the joint, $U_d$ was the normal displacement due to shear dilatancy, and $N_0$ was the axial force of bolt, while $Q_0$ was the shear force of bolt, as shown in Figure 4. When shearing, both axial force and shear force will be observed in the bolt under the compressive action of surrounding rock blocks, which subsequently produce axial deformation and shear deformation, ultimately resulting in approximate S-shaped deformation near the joint (see Figure 4). In the figure, $u_0$ is the axial displacement component of the bolt, $v_0$ is the tangential displacement component of the bolt, $P_u$ represents the ultimate reaction force of bolt subjected to surrounding rock material per unit length, and $\alpha$ is the inclination of the bolt. Point $O$ is the intersection between the joint and the bolt where the bending moment is zero and only axial and shear forces act in the bolt. The maximum curvature can be observed at both point $A$ and point $B$, with the maximum bending moment and no shear force.

Comprehensive analysis was conducted based on Figure 4. According to the geometric relation, it is easy to deduce the following relationship between the displacement of the bolt and joint [19, 22, 31]:

$$u_0 = 0.5(U_0 \cos \alpha + U_d \sin \alpha),$$

$$v_0 = 0.5(U_0 \sin \alpha - U_d \cos \alpha),$$

where $u_0$ is the axial displacement of point $O$; $v_0$ is the tangential displacement of point $O$; $U_d = U_0 \tan \psi$; and $\psi$ is the shear dilatancy angle of the joint.

Considering the bolt as a semi-infinite beam, Hamermesh [32] detailed the deformation of the bolt and illustrated that the following relationship (equations (3) and (4)) exists between the axial displacement $u(x)$,
tangential displacement $v(x)$ inside the bolt, and $u_0$ and $v_0$ when shearing:

$$u(x) = -u_0 \left( 1 - \frac{2x}{3l_0} \right),$$

$$v(x) = v_0 e^{-x/l_0} \cos \left( \frac{x}{l_0} \right),$$

where $l_0 = (\pi/4) \cdot I_A$, $I_A$, distance between point $O$ and point $A$.

Based on the theory of elastic foundation beam, Pellet and Egger [33] established the elastic foundation beam model of the bolt under shear load and systematically studied the relationship between internal force and deformation during bolt shearing. The expression of the total complementary energy was obtained from the calculation of the internal strain energy and the work of the external forces. When the total complementary energy is minimized with respect to the displacements, $u_0$ and $v_0$, the relations between forces and displacements are expressed as
3.2. Yield State of the Bolt. When the bolted joint is subjected to shear load and the shear displacement $U_0$ increases to a certain extent, there will be two yielding modes of the bolt: tensile shear yielding and bending yielding. When the bolt yields by tensile shear force, it obeys Von-Mises criterion and there exists a relationship between the yield stress $\sigma_t$ and the axial force as well as the tangential force in the bolt section, which can be expressed as follows:

$$\sigma_t = \sqrt{\left(\frac{N_0}{A_b}\right)^2 + 3\left(\frac{Q_0}{A_b}\right)^2},$$

where $N_0$ and $Q_0$ are the axial force and shear force acting at point $O$ at the yield stress of the bolt material, respectively, and $A_b$ is sectional area of bolt.

When bending to yield, the bolt yield point $A$ (or $B$) is subject to the combined action of axial force and bending moment. According to the beam theory of elastic foundation, the internal force of the bolt meets the following yield conditions:

$$1.7\sigma_t = \frac{M_A}{W} + \frac{N_{oy}}{A_b},$$

where $M_A = Q_{oy}^2/2P_u$, $W = \pi D_b^3/32$, $MA$ is the bending moment at point $A$ (or $B$) of the plastic hinge, and $W$ is the static moment at the interface of the bolt.

3.3. Relationship between Normal Stress and Dilatancy Angle. During joint shearing, the dilatancy effect will emerge, due to the roughness of the joint. And the dilatancy angle $\psi$ is generally used to indicate the magnitude of the dilatancy effect. Jing [35] used a parabolic equation to describe the relationship between peak dilatancy angle and normal stress (equation (10)):

$$\psi_p = \psi_{p0}\left(1 - \frac{\sigma_n}{\sigma_c}\right)^k,$$

where $\psi_{p0}$ is the initial dilatancy angle, $\sigma_n$ is the normal stress, and $k$ is the empirical parameter, depending on the material and the roughness of the joint, which was suggested to range from 0.2 to 5.0.

Additionally, Schneider [36] proposed a negative exponential formula to describe the relationship between peak dilatancy and normal stress (equation (11)):

$$\psi_p = \psi_{p0}e^{-k\sigma_n},$$

where $\psi_{p0}$, $\sigma_n$, and $k$ share the same meaning as equation (10), while $k$ ranges from 0.29 to 1.89.

Through comparison, Tang et al. [37] found that the result of equation (10) showed great consistency with that of equation (11). Therefore, both equations (10) and (11) were adopted to calculate the dilatancy angle in this paper, for guaranteeing the calculation accuracy of the dilatancy angle.

3.4. Analytical Solution for Shear Strength. Among the existing shear strength models of bolted joint, the ratio of the axial force and the shear force to the bolt sectional area $A_b$ is often taken as the reference to the averaging stress effect when considering the contribution of bolt to joint shear strength, while this approach that the stress in the bolt section area is simply added to the joint strength formula will lead to a large deviation to actual bolt strength contribution due to the size difference between the bolt section and the joint. Exactly, the influencing area of bolt may far exceed the total area of bolt section. Based on such an assumption, Liu et al. [17] proposed the concept of “equivalent shear area” and considered the influencing area of bolt rectangular. The equivalent shear area is defined as the product of bolt diameter ($D_b$) and joint width (rock block length) or bolt interval ($L$):

$$A_e = D_b \cdot L.$$

While the circular or elliptical shearing failure area of bolted joint generally emerges in practical tests, instead of rectangular, a more reasonable calculating model of shear strength of bolted joint by adopting circle as the influencing area of bolt was established in this paper (see equation (13)).
Figure 3: Shear stress-shear displacement curves of bolted joints under different conditions. (a) Joint-1-45°. (b) Joint-2-45°. (c) Joint-3-45°. (d) Joint-1-90°. (e) Joint-2-90°. (f) Joint-3-90°.
The bolt influence coefficient \( (m) \) was introduced, which was defined as the diameter ratio of the equivalent shear area to the bolt (see Figure 6). With the increase of bolt number, their intervals become smaller and the group bolting effect (the coincidence of the influencing area of bolt) should be taken into consideration. In this case, the reduction to the bolt strength is unavoidable, and the reduction factor is beyond the scope of this study, remaining to be further studied in the subsequent researches and investigations.

\[
A_e = \frac{\pi}{4} (m \cdot D_b)^2 \cdot a, \quad (13)
\]

where \( a \) is the number of the bolt.

According to the mechanical analysis of bolt, the shear strength of the bolted joint can be divided into three parts: (1) the joint shear strength; (2) the shear strength provided by the tangential force of bolt; (3) the shear strength provided by the axial force of bolt. Thus, the expression of the bolted joint shear strength \( \tau \) can be termed as

\[
\tau = \tau_j + \tau_{ba} + \tau_{bt}, \quad (14)
\]

where \( \tau_j = \sigma_j \tan(\phi_b + \psi) + c_j \), \( \tau_{ba} = \frac{N_0 y}{A_e} \{ \sin \alpha \tan(\phi_b + \psi) + \cos \alpha \} \), \( \tau_{bt} = \frac{Q_0 y}{A_e} \{ \sin \alpha - \cos \alpha \tan(\phi_b + \psi) \} \), \( \tau_j \) is the shear strength of joint, \( \sigma_j \) is the normal stress of joint, \( \tau_{ba} \) is the shear strength provided by the tangential force of bolt, \( \tau_{bt} \) is the shear strength provided by the axial force of bolt, \( \alpha \) is the inclination angle of bolt, and \( A_e \) is the equivalent shear area of bolt.

In this study, EXCEL programming was used to calculate the shear strength of bolted joints. Based on an initial shear displacement \( U_0 \), according to equations (1), (2), (5), and (6), the bolt axial force \( N_0 \) and the shear force \( Q_0 \) could be, respectively, obtained, which were adopted to measure whether equations (8) or (9) were satisfied. If not, iteratively calculation of the above steps was conducted until the satisfaction of equations (8) or (9). Then, it was possible to obtain the relevant computational parameters of \( U_0, U_t, N_0, Q_0, U_0, \) and \( v_0 \) of bolt when it yields by tension or bending. The bolt shear displacement when yielding determines the yielding mode. Finally, when the bolt was subjected to tensile shear yielding or bending yielding, the axial force \( N_0 \) and the shear force \( Q_0 \) were substituted into equation (14), and the shear strength of bolted joint under any normal stress could be obtained.

### Table 2: Shear strength of joints under different conditions.

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<tr>
<th>Normal stress (MPa)</th>
<th>Joint-1-45°</th>
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*Figure 4: Schematic of joint and bolt deformation [17].*

*Figure 5: n values corresponding to different rock mass strengths [34].*
Table 3: Comparison of test results with theoretical calculation results.

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<th>Normal stress (MPa)</th>
<th>Shear strength (MPa)</th>
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Note: \( m \) with \( \ast \) indicates the equivalent shear area covering the whole joint; \( m \) with \( \# \) indicates the equivalent shear area covering the inscribed circle area of joint.
4. Verification

As described in Figure 5, when the compressive strength is 21.86 MPa, the ultimate reaction coefficient \( n \) is 3.5. According to the calculation procedure of the bolted joint shear strength proposed in Section 3.4, through changing the value of the bolt influence coefficient \( m \) and using the EXCEL programming, the shear strength of the bolted joint under any normal stress could be obtained. Compared to the test results in Section 2.2, the relative error and its average value were selected to evaluate the accuracy of the calculation (see Table 3, wherein \( \tau_{\text{test}} \) is the test result, \( \tau_{\text{cal}} \) is the calculated result, \( E \) is the relative error, and \( E_{\text{ave}} \) is the average relative error).

The average relative error [21] is calculated as

\[
E_{\text{ave}} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\tau_{\text{test}} - \tau_{\text{cal}}}{\tau_{\text{test}}} \right| \times 100\%.
\] (15)

The comprehensive comparison of theoretical results and test results was carried out according to Table 3, which illustrates that the smallest difference between calculating values and test results, corresponding to the relative error of 0.5%, emerges for Joint-3-90° under the normal stress of 0.89 MPa (\( m = 32.57 \)), while the greatest difference, with the relative error of 40.9%, was observed for Joint-1-90° under the normal stress of 0.89 MPa (\( m = 56.42 \)). Besides, the maximum of average relative error is 22.1% and the minimum is 8.4%. Except for the individual abnormal test values, the relative errors are all basically less than 20%, indicating...

Figure 7: Relationship between \( E_{\text{ave}} \) and \( m \). (a) Joint with one bolt. (b) Joint with two bolts. (c) Joint with three bolts.
that the theoretical results are in great agreement with the

test results. Therefore, the value of the bolt influence co-
nefficient \( m \) with the smallest average relative error was

selected to calculate the equivalent shear area. If the average
relative errors are equal, then the influence coefficient \( m \)
with the smallest variance was taken.

As nonlinear curves fitting results shown in Figure 7,
there exists a negative correlation between the value of \( E_{\text{ave}} \)
and \( m \). And it can be observed that \( E_{\text{ave}} \) tends to stabilize
with slight fluctuations of the curve when \( m \) is greater than
30 (see Figure 7(a))). For two-bolted joints, \( E_{\text{ave}} \) stays steady
when \( m \) is greater than 25 (see Figure 7(b))), while for three-
bolted joint, the theoretical model agrees well with the test
results when \( m \) is greater than 20 (see Figure 7(c))). The
comparison between the optimal calculated results (the
underlined \( E_{\text{ave}} \) values in Table 3) and the test results in this
study is shown in Figure 8. It can be concluded that the
theoretical results are generally greater than the test results,
which can be attributed to the idealization of the stress state
of bolts during the calculation, as well as the ignorance of the
group bolting effect.

After the direct shear tests were completed, the bolts
were removed and the bolt deformations are shown in
Figure 9. It is found that the connection between the bolt and
the sample is intact, while an obvious crushing area between
the bolt and the joint can be observed. The bolts were nearly
S-shaped, and the shear deformation length is about 3 to 5
times the bolt diameter, which indicates that the tangential
deformation of the bolt is dominant during the joint
shearing process. According to the test observation, the bolt
basically showed bending failure, which is consistent with
the theoretical calculation results in Section 3.4. In the case
where the bolt was 45° obliquely intersected with the joint,
both the axial drawing effect and the tangential antishearing
effect were exerted, producing small tangential shear de-
formation of the bolt. And when 90° bolting was used to
reinforce the joint, the bolt mainly contributed to pin effect
during shearing, leading to great tangential shear
deformation.

To verify the validity of the circular influencing area of
bolt, a comprehensive comparison of calculating results was
made with the rectangular influencing area. According to the
experimental results of shear strengths in Section 2.2, the calculating results of shear strengths considering circular influencing area and rectangular influencing area were obtained, respectively. And the differences between the test results and calculating results for both influencing areas were solved according to equation (15), which are shown in Table 4. Obviously, considerable deviations can be observed from the values of $E$ for the rectangular influencing area, up to 210.5%, while the values of $E$ for the other approximate only to 12.8%, which makes it evident that the calculating model of shear strength considering the circular influencing area of the bolt is superior, with smaller $E$ and accurate calculating results.

### 5. Conclusion

(1) Considering the actual shear failure mode of bolted joint, based on the assumption of the circular influencing area of bolt, a theoretical calculation model for the shear strength of bolted joint was proposed.

(2) Changing the number and the inclination of bolt, the results obtained by direct shear tests on bolted joint present great consistency with the calculating results, which verified the validity of the proposed model.

(3) The influence coefficient $m$ significantly affects the shear strength of bolted joint. And when $m$ reaches a certain value, the theoretical relative error between the calculating results and test results tends to be stable and sufficiently small. In such case, the area, corresponding to $m$, can be considered as the actual influencing area of the bolt.

### Data Availability

Some or all data, models, or codes generated or used during the study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare no conflicts of interest.

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