Review Article

Determination of the Shear Strength of Rockfill from Small-Scale Laboratory Shear Tests: A Critical Review

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Determining the shear strength of rockfill is a key task for the design and stability analysis of rockfill structures. When direct shear tests are performed, the well-established ASTM standard requires that specimen width and thickness must be at least 10 and 6 times the maximum particle size (d_{max}), respectively. When the value of d_{max} is very large, performing such tests in laboratory with field rockfill becomes difficult or impossible. Four scaling-down techniques were proposed in the past to obtain a modeled sample excluding oversize particles: scalping, parallel, replacement, and quadratic. It remains unclear which of the four scaling-down techniques yields reliable shear strength of field rockfill. In this paper, an extensive review is presented on existing experimental results to analyze the capacity of each scaling-down technique to determine the field rockfill shear strength. The analyses show that previous researches followed an inappropriate methodology to validate or invalidate a scaling-down technique through a direct comparison between the shear strengths of modeled and field samples. None of the four scaling-down techniques was shown to be able or unable to predict the field rockfill shear strength by extrapolation. The analyses further show that the minimum ratios of specimen size to d_{max} dictated by well-established standards are largely used but are too small to eliminate the specimen size effect. In most cases, this practice results in shear strength overestimation. The validity or invalidity of scaling-down techniques based on experimental results obtained by using the minimum ratios is uncertain. Recommendations are given for future studies.

1. Introduction

Rockfill is usually considered as a good construction material for infrastructures. It is used to build dams for impounding water and reservoirs for hydroelectricity generation and prevent flooding [1–3]. Rockfill is also commonly used as ballast bed in the construction of railways to hold railway sleepers and provide high bearing capacity of foundations [4–6]. For steep slope terrains, rockfill permits slope protection from movement or scouring [7, 8]. In mining industry, large amounts of waste rocks are produced every year [9]. In most cases, this material is deposited on surface as rock piles and considered as a waste material. Over recent years, it is increasingly used as a construction material both in and out of the mine sites [10]. For instance, waste rocks have been more and more used to construct tailings dams [11–13] or waste rock inclusions in tailings storage facilities [14–20]. They are also used as rockfill to fill underground mine stopes [21, 22] or to construct barricades to retain backfill slurry in mine stopes [23–25]. All these structures made of waste rocks must be properly designed and constructed to ensure their long-term stability. Failure of such structures may result in serious consequences such as ecological devastations, damage to equipment and infrastructures, personal injury, and even loss of lives [7, 26–30]. Good knowledge of rockfill shear strength is fundamental in performing design and stability analyses of these structures.

Rockfill can be of natural origin (riverbed) or produced through rock blasting in a quarry or mine. Physical

properties of rockfill can vary significantly in terms of particle size distribution and particle shape [31]. In general, the particle size of rockfill can vary from material as fine as clay and silt to material as coarse as gravel and boulders [8, 32] while the particle shape can be qualitatively described as very angular, angular, subangular, subrounded, rounded, and well-rounded [33]. For natural rockfill, particles are often rounded with maximum particle sizes (d_{max}) typically varying from 4.75 to 80 mm [34]. For rockfills made of blasted rock from quarries or mines, particles are typically angular with d_{max} varying from 4.75 mm to sometimes over 1000 mm [31]. The content in fine particles may also differ from one rockfill to another [35–37]. All these factors are well known to influence the shear strength of rockfill.

Previous studies showed that the shear strength of granular materials depends on several influencing factors, including, for example, grain-grain contact friction, grain-grain interlock, compressive strength of solid grains, and possibility of dilation [38–40]. The grain-grain contact friction depends on the base or residual friction and asperity of the grain surfaces [41, 42]. The grain-grain interlock and dilation depend on the particle angularity, particle gradation (coefficient of uniformity, curvature, and d_{max}), degree of compaction, and confining pressure [40, 43, 44]. The mechanisms controlling the shear strength of granular material are important for understanding the role of each influencing factor. However, detailed discussion on this aspect is beyond the scope of the paper because the main purpose of this study is to see if it is possible to determine shear strength of rockfill from small-scale laboratory shear tests. Focus will be given on the influence of d_{max} on the shear strength of rockfill, especially friction angle.

Direct shear tests and triaxial compression tests are commonly used to measure the shear strength of geo-materials. For triaxial compression tests, ASTM D4767 [45] requires that the specimen diameter must be at least 6 times the maximum particle size, d_{max}. For direct shear tests, the minimum ratios of specimen width and thickness to d_{max}, as required by the commonly used standards, are presented in Table 1. For most soils such as clays, silts, and sands having d_{max} smaller than 2 mm, satisfying the standard requirements is not a problem because the ratio of specimen size to d_{max} can easily exceed 25 even with a small shear box of 50 mm. For rockfill and gravel materials with a d_{max} exceeding 75 mm, it is technically very difficult [3] and economically impracticable [50] to design testing equipment that can accommodate large size specimens that respect the requirements of testing standards.

To avoid such problems, one may try to perform in situ tests to directly obtain the field rockfill shear strength [51–59]. Goodrich [60] conducted in situ direct shear tests using a 300 mm × 300 mm shear box on a construction site to determine the friction angles of clay, sand, and gravel materials [61]. Tests were carried out by filling the box with the material and adding weights to the scale-pan. The upper half of the box was pulled until sliding. Tests were repeated by adding more weights to increase normal stress. The applied normal stress could not be very large. In addition, the box size is not suitable to test full-scale field materials. By

TABLE 1: Standards of direct shear tests regarding the maximum allowed particle size (d_{max}), specimen width (W), thickness (T), and diameter (D).

Standard	W	T	W/T	Allowed d_{max}
ASTM D3080 [46]	≥50 mm	≥13 mm	≥2	Min{$T/6$, $W/10$, $D/10$}
AS 1289.6.2.2 [47]	Not specified	≥12.5 mm	Not specified	$T/6$
BS 1377-7 [48]	60 mm	20 mm	3	2 mm
	100 mm	25 mm	4	2.5 mm
	305 mm	150 mm	≈2	15 mm–20 mm
Eurocode 7 [49]	Not specified	Not specified	—	$T/10$

performing such tests, Goodrich [60] showed that the friction angles of studied materials depend on particle size and degree of saturation. Similar results have been shown by Yu et al. [40] through laboratory direct shear tests.

The in situ testing approach of Goodrich [60] was followed by many other researchers [55, 57, 62]. This resulted in the modern direct shear test apparatus [61].

As direct shear tests impose a sliding (shear) plane, the measured friction angle usually includes a dilation angle. The dilation degree decreases as normal stress increases, and one usually observes a decrease in friction angle with an increase in normal stress [63]. Subsequently, one generally tends to obtain a high-friction angle when large normal stresses cannot be applied in in situ direct shear tests. The experimental results are not representative of those of large and high rockfill infrastructures [56, 64–66].

Barton and Kjaernsli [51] performed in situ tilt tests to measure the shear strength of rockfill with a rectangular open box composed of three parts. The instrumentation and test procedure are shown in Figure 1. The box was first placed on level rockfill and then filled and compacted. After having removed surrounding rockfill and the middle frame part of the box, one end of the filled box was lifted. The tilt angle at which the upper part of the filled box began to slide was taken as the maximum tilt angle (α), which corresponded to the friction angle ϕ at the applied normal stress σ_n.

Compared to other in situ direct shear tests, the method of Barton and Kjaernsli [51] is simple. The test box can be as large as necessary, depending on the largest particles of the rockfill. However, the applied normal stress is limited by the upper box thickness and cannot be very large. The instrumentation is heavy, and the tests are expensive. Furthermore, when the box is lifted at one end, particles can fall (due to the removal of the confinement initially provided by the middle part) before observing sliding of the upper part. The influence of particle fall on the measurement of shear strength has not yet been investigated.

Apart from the limitations specifically associated with each in situ shear test, other disadvantages associated with in situ shear tests include the difficulty in supplying equipment and transportation facilities, time-consuming, high costs, and generally intensive labor [7]. Finding a suitable and safe

Step 1: place tilt box on level rockfill

Rough interface

Step 2: fill and compact with next lift

Step 3: isolate from surrounding fill

Step 4: remove shear plane stiffeners and begin tilting

Step 5: measure tilt angle at which failure occurs
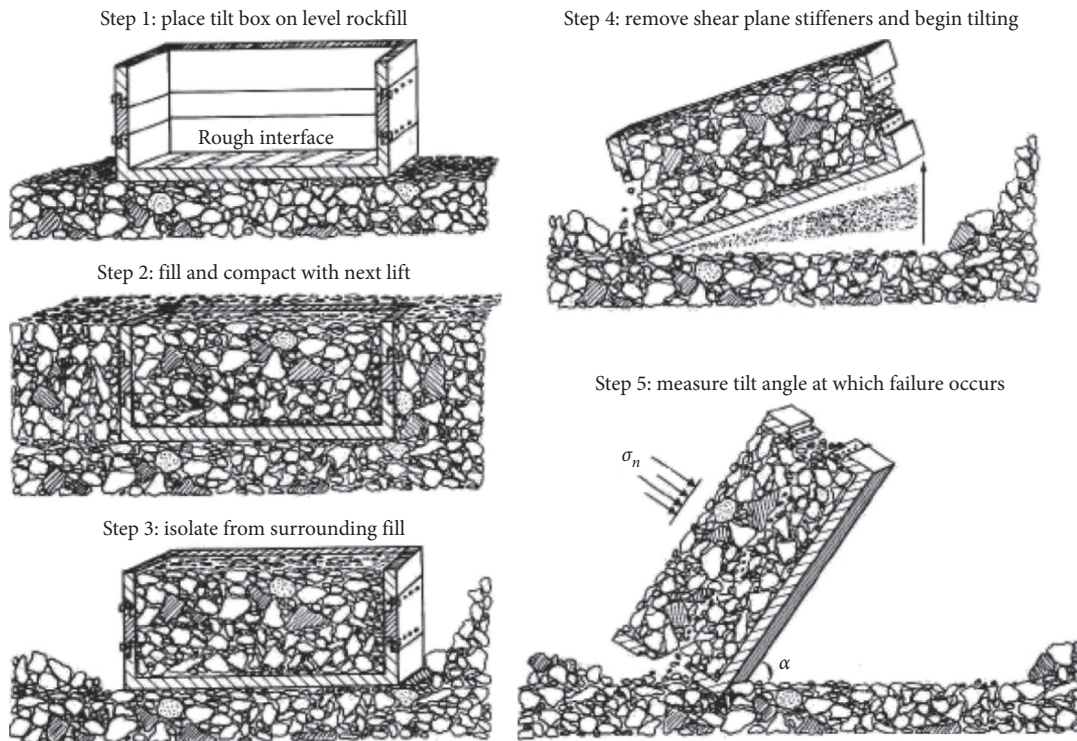
σ_n

α

FIGURE 1: Instrumentation and test procedure of in situ tilt tests on rockfill (reproduced from [51] with permission from ASCE).

location is another non-negligible challenge for in situ shear tests.

A simple and cost-effective alternative for obtaining the shear strength of field rockfill is to perform a series of laboratory shear tests on samples that are made from field rockfill with different d_{max} values [8, 67–70]. A relationship between shear strength and d_{max} can then be established and used to predict the shear strength of field rockfill by extrapolation technique, which can be realized with the graphical or regression-based method [31, 34, 71–78].

Sample preparation by eliminating the oversize particles to fit the capacity of laboratory equipment is known as scaling-down (gradation) method. Several scaling-down methods were proposed over the past years and used by researchers. It is unclear which scaling-down technique can be used to obtain reliable field rock shear strength by extrapolation. This is the main reason that motivates this review analysis. The initial and main objective of this paper is to identify a reliable scaling-down technique that can be used to predict the shear strength of field rockfill from small-scale laboratory tests.

To reach this objective, extensive review and comprehensive analyses on available experimental data are first presented, followed by an examination of the minimum ratios of specimen size to d_{max} suggested by well-established standards. Conclusions and recommendations are given at the end of the paper.

2. Laboratory Shear Tests

2.1. Large-Scale Laboratory Tests.
With a large project having the allowed budget, it is desirable to perform laboratory shear tests with large-scale apparatus to obtain the shear strength of in situ materials with less uncertainty. Large-scale laboratory tests can be direct shear tests or triaxial compression tests. The earliest research on large-scale tests was conducted by the South Pacific Division Laboratory (SPDL) of U.S. Army Corps of Engineers (USACE) [79, 80] and the US Bureau of Reclamation (USBR [68]).

Hall and Gordon [81] were among the first researchers having performed large-scale tests to estimate the static and kinetic internal friction angles of a rockfill containing natural alluvial deposits and coarse dredged tailings. Their test results showed that the friction angle decreases as the confining pressure increases because the particles can be crushed and dilation is diminished at high confining pressures. The same phenomenon was observed by other researchers through large-scale direct shear tests on rockfill [64, 82–84].

2.2. Scaling-Down Techniques.
Although large-scale shear tests may provide interesting results as the allowed maximum particles can be quite large, it is impossible for all projects to perform large-scale shear tests due to the requirement of special equipment, time-consumption, and high costs. With available testing equipment in laboratory, the maximum allowable specimen size is limited. This in turn limits the d_{max} value to meet the minimum required ratios of specimen size to d_{max} stipulated by several standards such as the ASTM D3080 [46] for direct shear tests and the ASTM D4767 [45] for triaxial compression tests.

As rockfill can contain boulders up to 1200 mm [85], large-scale shear tests are impossible for all cases. Alternatively, one can perform small-scale shear tests by excluding

the particles larger than the chosen d_{max} of rockfill. This is once again known as scaling-down or gradation technique [3, 31, 67, 69, 84, 86]. The variation of shear strength as a function of d_{max} can then be used to determine the shear strength of field rockfill through extrapolation.

The earliest scaling-down technique, called the scalping or truncated method, was proposed by Hennes [87]. In this technique, particles larger than the targeted d_{max} are simply removed, resulting in an increase of the percentages of all the particles smaller than the targeted d_{max} compared to those of the field material. To obtain a gradation curve similar to that of field material, Lowe [88] proposed a scaling-down technique, called parallel technique, in which the scalped sample is further modified in a way that the particle size distribution curve of modeled sample is parallel to that of the field material. Almost in the same time, another scaling-down technique called replacement technique was introduced by USACE [89] to keep the percentages of fine particles unchanged compared to those of the field material. In 1969, Fumagalli proposed a scaling-down technique to obtain a specific gradation curve. Details as well as the advantages and limitations of these scaling-down techniques are presented in the following sections.

2.2.1. Scalping Technique. During field sampling or sample preparation in laboratory, oversize particles (i.e., larger than the desired d_{max}) are simply excluded and removed. This method, called scalping or truncating, is the simplest and earliest scaling-down method. First introduced by Hennes [87], it is commonly used in sample preparation for laboratory tests [50, 67, 68, 80, 90–93].

Figure 2 shows the grain size distributions of a field rockfill and a scalped sample reported by Williams and Walker [50]. The field rockfill has a d_{max} of 200 mm while the targeted d_{max} of the scalped sample is 19 mm. To obtain the scalped sample, all the particle sizes larger than 19 mm were removed. After sieving analysis, the grain size distribution curve of the scalped sample is obtained. As seen in the figure, removal of the oversize particles results in different degrees of increase in the percentages of different size particles compared to the field material.

Zeller and Wullimann [93] performed triaxial compression tests to determine the shear strength of a rockfill. The samples were prepared by following scalping down technique. Figure 3(a) shows the particle size distribution curves of scalped samples at four d_{max} values (1, 10, 30, and 100 mm) and field material having a d_{max} of 600 mm. The diameters of all tested specimens prepared for the triaxial compression tests were at least 5 times the d_{max} of the scalped sample. Figure 3(b) presents the shear strengths of the scalped samples under a confining pressure of 88 kPa in function of their d_{max} value for porosities of 30% and 38%, respectively. For a given porosity, shear strength significantly decreases as the d_{max} value increases. By extrapolating the experimental data of the scaled down specimens, the shear strength of field rockfill with a d_{max} of 600 mm can then be predicted. However, no conclusion can be drawn to evaluate whether the field rockfill shear strength can be correctly predicted by using the scalping method
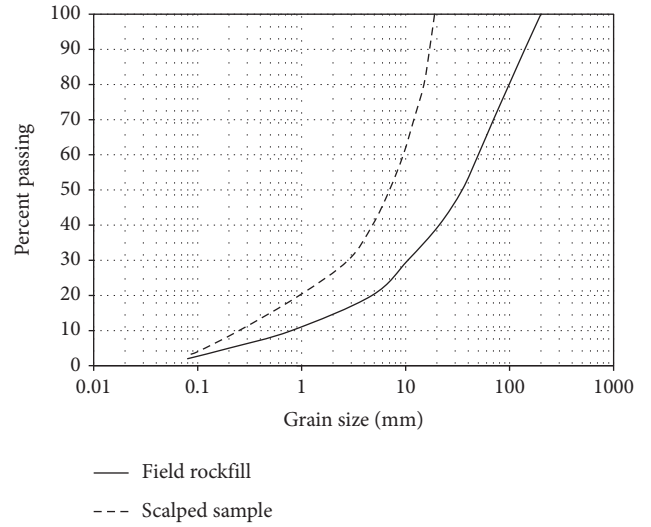

FIGURE 2: Grain size distribution curves of the field rockfill and scalped sample (data taken from [50]).

because the shear strength of field rockfill with d_{max} of 600 mm was not measured.

Through the previous analysis, one sees that sample preparation of scalping technique is very simple. However, application of the scalping procedure results in a significant change in the gradation curve. The percentages of all particles of the scalped sample increase and become higher than those of the field material.

2.2.2. Parallel Scaling-Down Technique. Similar to the scalping method, parallel scaling-down method also consists of excluding particles larger than the targeted d_{max}. However, the scalped sample is further modified to yield a particle size distribution curve that is parallel to that of the field material [88, 94, 95]. The obtained modeled sample thus has a gradation curve looking like a horizontal translation of the field material gradation curve towards the fine particles size side. If N is the ratio of the maximum particle size of field material to that of a modeled sample, the shift distance will be equal to $\log(N)$ along the logarithm axis of particle size [88].

For example, to produce a modeled sample of parallel scaling-down technique having a d_{max} value of $d_{max \cdot m}$ from a field material having a d_{max} value of $d_{max \cdot f}$, the ratio N is calculated as follows:

$$N = \frac{d_{max \cdot f}}{d_{max \cdot m}}. \qquad (1)$$

For a given percentage passing p, the grain size of modeled sample is calculated as follows:

$$d_{p \cdot m} = \frac{d_{p \cdot f}}{N}, \qquad (2)$$

where $d_{p \cdot m}$ and $d_{p \cdot f}$ are the particle sizes of modeled sample and field material having a percentage passing p, respectively.
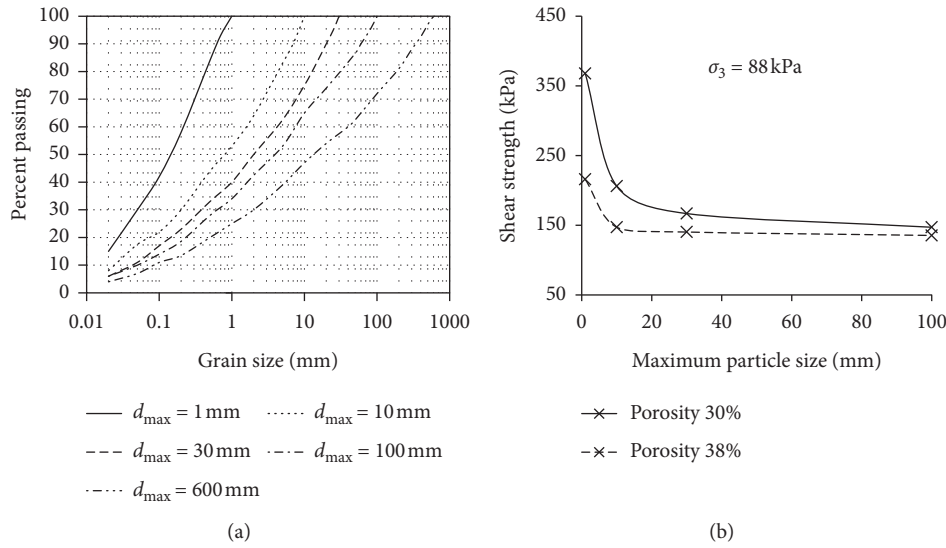
(a)

(b)

FIGURE 3: (a) Grain size distribution curves of the field material (rockfill) and the scalped samples and (b) variation of shear strength with the d_{max} for different porosities (data taken from [93]).

Once the target parallel gradation curve is determined, the required mass for each range of particle sizes can be obtained by considering the required portion and the total mass of the modeled sample. It is very possible that some particle size values obtained by equation (2) are missing in the available sizes of standard sieves. In this case, the sieves having the closest sizes to those calculated by equation (2) should be taken as an approximation. In addition, the production of parallel curves requires addition of particles finer than the minimum particle size of field material. Obviously, it is impossible without a grinding operation on the field material or without addition of needed fine particles from another material. In both cases, the origin of the modeled sample is different from that of the field material. In practice, the particle size distribution curves of parallel gradation samples can be nonparallel to that of field material near the fine particle part.

Figure 4 shows a particle size distribution curve of modeled sample by applying the parallel gradation method along with that of field material. The d_{max} value of the field material is 305 mm while the target d_{max} of the parallel gradation sample is 38 mm. The ratio between the d_{max} value of the field material and modeled sample is 8. The shift distance between the gradation curves of the field material and modeled sample is log(8) along the logarithm axis of particle size.

The parallel scaling-down method was proposed due to the necessity of determining the shear characteristics of several types of gravelly soils for the SPDL of USACE. Leslie [80] compared the friction angles obtained by triaxial compression tests on specimens prepared by parallel and scalping methods without any conclusive results. No recommendation could be made on the reliability of the two scaling-down methods.

Marachi et al. [86] applied the parallel scaling-down method to investigate the influence of d_{max} on the friction angle of three samples (Pyramid dam materials, crushed
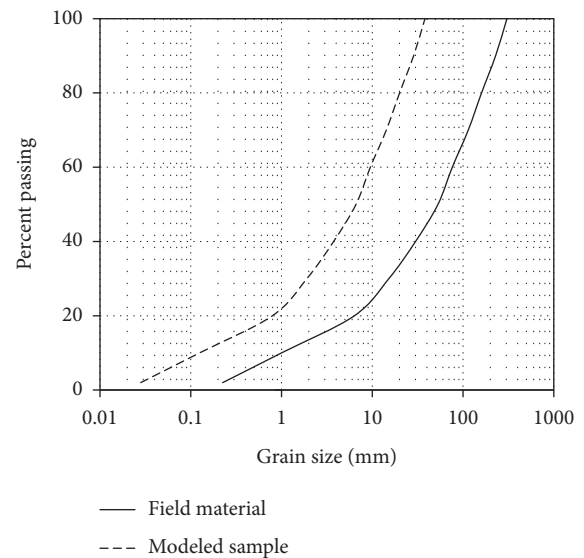

FIGURE 4: Grain size distribution curves of the field material and parallel sample (data taken from [88]).

basalt, and Oroville dam materials) through triaxial compression tests. Two samples were made of well-graded and angular particles. The third sample was prepared by a mixture of subangular and rounded particles. The content of subangular and rounded particles was not specified, and the angularity or roundness degree of the mixture was quantitatively unknown. Samples were prepared with diameters of 71, 305, and 914 mm for d_{max} value, respectively, of 12, 50, and 152 mm. The minimum required ratio of 6 [45] of specimen size to d_{max} was thus respected in all the tests. Triaxial compression tests were conducted under confining pressures of 207, 965, 2896, and 4482 kPa, respectively. The experimental results show a decreasing friction angle as d_{max} value increases (Figure 5). However, this study does not
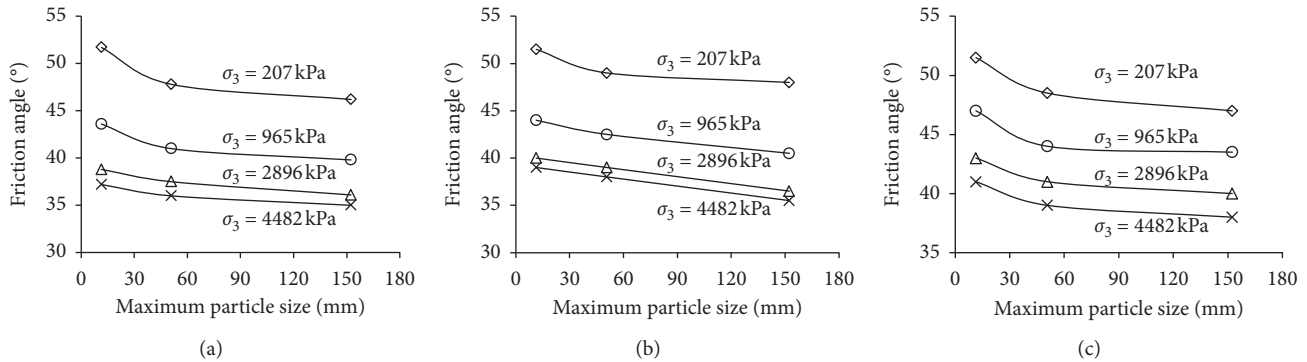
FIGURE 5: Variations of friction angle with the maximum particle size for (a) Pyramid dam materials, (b) crushed basalt, and (c) Oroville dam materials with different confining pressures (data taken from [86]).

confirm if the predicted friction angles through extrapolation correspond to the friction angle of the field materials since no tests were performed on the latter.

Charles [94] studied the friction angle of a rounded rockfill with a d_{max} of 900 mm by triaxial compression tests. The parallel method was used to scale down the field rockfill to three samples with d_{max} of 40 mm, 100 mm, and 300 mm, respectively. Samples were prepared at the same porosity as the field sample. A ratio of sample diameter to d_{max} of 5 was used for all the tests. Figure 6 shows the variations of friction angle with d_{max} for different confining pressures. The test results show that an increase in the confining pressure leads to a reduction in the friction angle for a given d_{max}. Same results were found by previous researchers (e.g., [44, 86]). This tends to indicate that confining pressure should be adequately chosen to mimic the stress condition of field material during the application of the scaling-down technique. Figure 6 also shows that the friction angle slightly increases as d_{max} increases. This trend is opposite to that found by Marachi et al. [86]. This may be attributed to the rounded particles of the tested rockfill while those of Marachi et al. [86] were angular or subangular.

Figure 7 further shows a collection of experimental results on the variations of friction angle with d_{max} for (alluvial) rounded (Figure 7(a)) and (quarried) angular (Figure 7(b)) materials obtained by applying the parallel scaling-down technique. The minimum ratio of specimen size to d_{max} of 10 as required by ASTM D3080 [46] was taken in all the tests. One sees that the friction angle of rounded material increases as d_{max} increases while the friction angle of angular material decreases with increasing d_{max} values.

The parallel scaling-down technique was proposed in order to reproduce the shape of the gradation of field materials. In practice, particle shape can change during sample preparation [31, 34, 99–103]. This can in turn result in a change in the friction angle [84, 104]. Moreover, the reproduction of modeled samples having gradation curves strictly parallel to that of field material requires addition of fine particles smaller than the minimum particle size of the field sample. This in turn requires grinding of field material or the addition of finer particle material of a different source. The modeled samples thus contain a portion of material
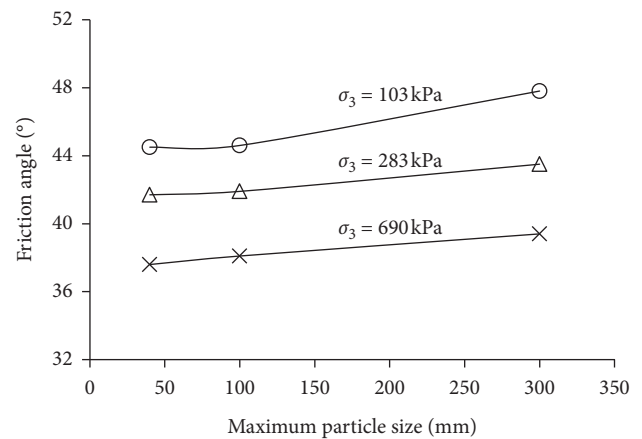

FIGURE 6: Variation of friction angle with maximum particle size of parallel modeled samples with different confining pressures (data taken from [94]).

which has a source different from that of the field material. In practice, the particle size distribution curves of parallel gradation samples can be nonparallel to that of field material, either due to the lack of required (nonstandard) sieve sizes or due to the lack of required fine particles smaller than the minimum particle size of the field material. All these indicate that the parallel scaling-down technique also has some drawbacks despite it is widely used in practice. At present, it is still far from being able to conclude whether the friction angle of field rockfill can be predicted by extrapolating test results obtained with the parallel scaling-down technique. More investigations are needed on this aspect.

2.2.3. *Replacement Technique*. When the portion of removed oversize particles exceeds 10% during the application of the scalping technique, USACE [89] suggested replacing the removed weight by particles finer than the targeted d_{max} but greater than No. 4 sieve (4.75 mm). The percentage of particles finer than the No. 4 sieve thus remains unchanged [89, 90, 105].

The replacement technique was first used by Frost [105] to conduct compaction tests of boulder-gravel fill for a large
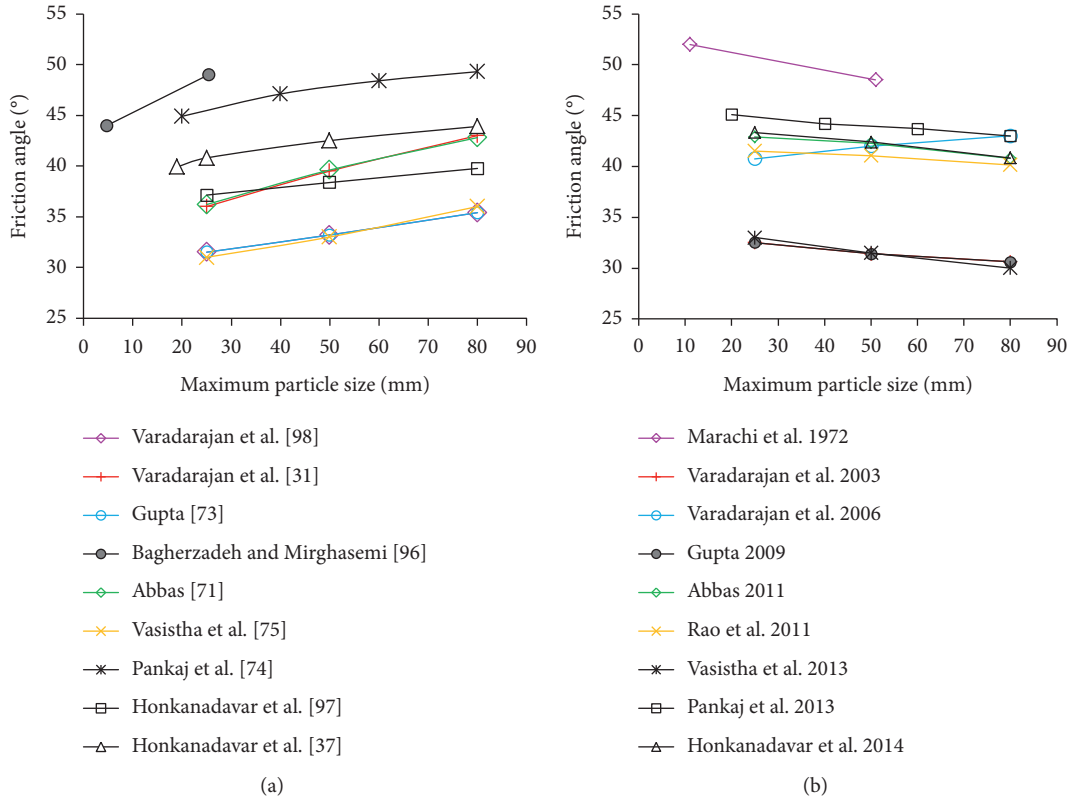
FIGURE 7: Variations of friction angle with maximum particle size for (a) rounded materials and (b) angular materials (data taken from [31, 32, 34, 71, 73–75, 86, 96–98]).

dam. Later, a few studies were conducted to verify the reliability of the replacement technique [90, 106, 107].

Figure 8 shows the grain size distribution curve of a field material having a d_{max} of 80 mm and that of modeled sample with a d_{max} of 20 mm [108]. The first one was obtained by measuring the masses of particles retained on different sieves and the total mass of the field material. The particles larger than the targeted d_{max} (i.e., 20 mm) were weighed and excluded. The same mass of particles having sizes between 4.75 mm (i.e., sieve No. 4) and d_{max} was added in the sample by applying the following equation to obtain the gradation curve of the modeled sample [108]:

$$P_{ij \cdot a} = \frac{P_{ij \cdot f}}{P_{d_{max}} - P_{No.4}} \times P_o, \qquad (3)$$

where $P_{ij \cdot a}$ is the percentage by mass of added particles passing sieve having size j ($\leq d_{max}$) and retained on the neighbor sieve having size i (≥ 4.75 mm); $P_{ij \cdot f}$ is the percentage by mass of field material particles passing sieve size j ($\leq d_{max}$) and retained on the sieve size i (≥ 4.75 mm); $P_{d_{max}}$ is the percentage of field material particles passing the targeted d_{max}; $P_{No.4}$ is the percentage of field material particles passing the sieve of 4.75 mm (i.e., sieve No. 4); and P_o is the percentage by mass of field material particles retained on the sieve having a size of the targeted d_{max}. The physical meaning of each symbol is shown in Figure 8 to ease their understanding.

The application of this procedure results in a change in the gradation curve shape of the modeled sample compared
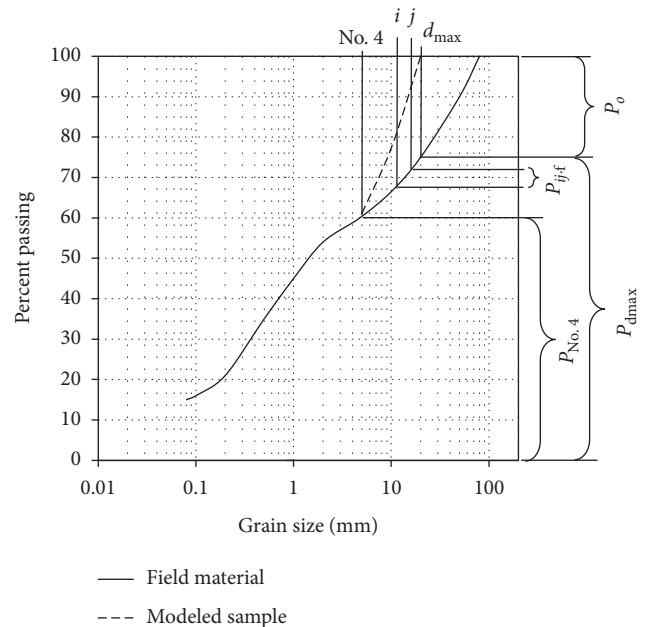

FIGURE 8: Grain size distribution curves of field material and a modeled sample (data taken from [108]).

to that of the field material for the particles greater than 4.75 mm. The replacement technique is thus considered as a scaling-down method that modifies the gradation of field material [108–110]. Only a few researchers have used this

technique [109, 111]. The validity or invalidity of this method for determining shear strength of field materials has not yet been demonstrated.

2.2.4. Quadratic Grain-Size Technique.

Quadratic grain-size technique was proposed by Fumagalli [112]. In this method, particle size distribution is defined by the following equation [112]:

$$P_Q = \sqrt{\frac{d}{d_{max}}} \times 100\%, \tag{4}$$

where d is a particle size of the modeled sample, smaller than the target d_{max} and P_Q is the percentage by mass of the particles smaller than d of the modeled sample.

Fumagalli [112] applied this technique on a rockfill with a d_{max} of 260 mm to obtain samples having d_{max} values of 10, 20, 30, 60, and 100 mm, respectively. Confined compression tests were performed. A chamber 100 mm in diameter and 200 mm high was used for the specimens with d_{max} of 10, 20, and 30 mm, respectively, and another chamber 500 mm in diameter and 1000 mm high on the specimens with d_{max} of 10, 60, and 100 mm, respectively, was used. The minimum ratios of chamber diameter to maximum particle size were 3.3 and 5, respectively, for the small and large chamber tests. The confined compression tests were conducted by filling the chosen chamber with a tested specimen. The filled chamber was then submitted to an axial pressure. The axial and hoop strains were monitored. The reliability of the tests is unknown because the obtained friction angles were in the range of 23° to 25°, which are abnormally small for granular materials.

According to Fumagalli [112], quadratic scaling-down technique could be applied to well-graded materials. By applying this method, one can note that a unique particle size distribution curve will be obtained, independently of the field material once the target value of d_{max} is chosen. Table 2 shows percentages by mass of different particle sizes normalized by the target d_{max} by applying quadratic scaling-down technique.

Figure 9 shows the grain size distribution curve of a modeled sample by applying quadratic scaling down technique with a d_{max} of 20 mm. At $P_Q = 20\%$, the target particle size of the modeled sample is 0.8 mm ($=0.04 \times 20$ mm).

Note that the particle size distribution curve of the scaled down sample spreads on a wide range of sizes. This may partly explain why Fumagalli [112] suggested that quadratic scaling-down technique is applicable to well-graded materials. However, as the scaled down sample does not depend on the field material once the allowed d_{max} is chosen, two situations can occur. First, if the particle size distribution curve of the scalped down sample (i.e., after removal of oversize particles) intercepts the target particle size distribution curve on the fine particle side, one or several of the following operations are needed:

(1) To add fine particles smaller than the minimum particle size of the field material

TABLE 2: Percentage by mass of different particle sizes by applying quadratic scaling-down technique (equation (4)).

d/d_{max}	P_Q (%)
1	100
0.81	90
0.64	80
0.49	70
0.36	60
0.25	50
0.16	40
0.09	30
0.04	20
0.01	10
0.0025	5
0	0

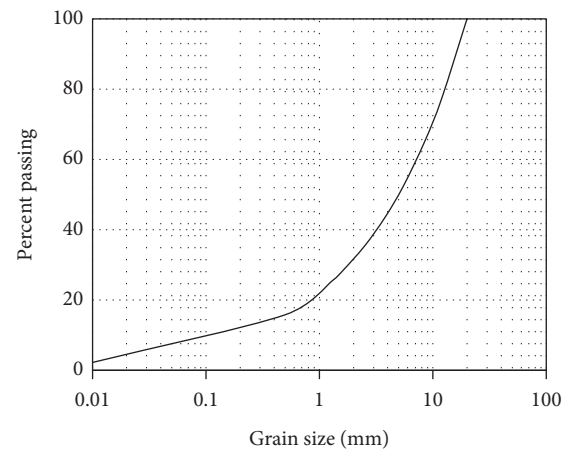

FIGURE 9: Grain size distribution curve of a modeled sample produced by using quadratic technique.

(2) To remove a portion of particles larger than the minimum particle size of the field material

(3) To add coarse particles smaller than the allowed d_{max} but larger than the particle size at which the target and scalped particle size distribution curves intercept.

Second, if the particle size distribution curve of a scalped down sample does not intercept the target particle size distribution curve, except at the chosen d_{max}, one has to add a wide range of particles from particles smaller than the minimum particle size of the field material to coarse particles smaller than the allowed d_{max}. In both cases, the resulting gradations highly differ from those of the original field sample. This may be why this quadratic scaling-down method has not been used by other researchers since 1969.

3. Validation of Scaling-Down Techniques

The four presented scaling-down techniques are not used at the same frequency. Parallel scaling-down technique is frequently used in practice. Use of the scalping technique is less frequent than that of the parallel scaling-down method but more than the replacement and quadratic scaling-down

techniques. The replacement method was used in a few researches while the quadratic scaling-down technique is seldom used due to its complex preparation and the non-representativeness of the modeled sample, as shown in Section 2.2.4. None of the four scaling-down techniques was shown to be able or unable to predict the shear strength of field rockfill by extrapolation.

To evaluate the validity of scalping and replacement techniques, Donaghe and Torrey [90] performed triaxial compression tests on a mixture of subrounded sand and subangular gravel having a d_{max} of 76 mm. The degree of roundness or angularity of the mixture is unknown. The scalping and replacement techniques were used to obtain samples with d_{max} of 4.75 and 19 mm, respectively. All the tested specimens were prepared according to the minimum requirement of the ASTM D4767 [45] in terms of the ratio between specimen size and d_{max}. Instead of performing more tests with specimens having different d_{max} and applying the extrapolation technique to predict the shear strength of field rockfill, Donaghe and Torrey [90] directly compared the test results of scalped and replaced samples with those of field sample, as shown in Figure 10. They concluded that the scalping and replacement methods are invalid because they found that the shear strengths of the scaled down samples differed from those of the field rockfill.

The methodology of Donaghe and Torrey [90] could be adequate in the case where the shear strength of the scalped and replaced materials is insensitive to the variation of the maximum particle size. This is only possible when the angularity or roundness of the sample particles reaches a critical degree. In general, the friction angle of rounded or subrounded particle samples increases as the d_{max} increases (Figure 7(a)) while the friction angle of angular or subangular particle samples decreases as the d_{max} increases (Figure 7(b)). The methodology taken by Donaghe and Torrey [90] to invalidate the scalping and replacement methods is therefore inappropriate.

The same methodology of Donaghe and Torrey [90] was followed by several other researchers [67, 113]. Linero et al. [113] measured the shear strength of a coarse granular material through triaxial compression tests. The field material with a d_{max} of 400 mm was scaled down by applying the parallel and scalping techniques, respectively. The tested specimens had a D/d_{max} ratio of 5, which was smaller than the minimum required ratio of ASTM D2850 [114]. Again, the methodology is incorrect to evaluate the reliability of the tested scaling-down techniques.

Hamidi et al. [67] performed a series of direct shear tests according to ASTM D3080 [46] to investigate the validity of the scalping and parallel techniques. A rounded sand and gravel mixture with a d_{max} of 25.4 mm was scaled down to samples having a d_{max} of 12.5 mm by applying the parallel and scalping techniques. Three normal stresses of 100, 200, and 300 kPa were used in the tests. Figure 11 shows the variations of the shear strength in terms of maximum shear stress under a normal stress of 100 kPa (Figure 11(a)) and friction angle (Figure 11(b)) as a function of the maximum particle size of the modeled and field samples for different relative densities. One first notes that the friction angle
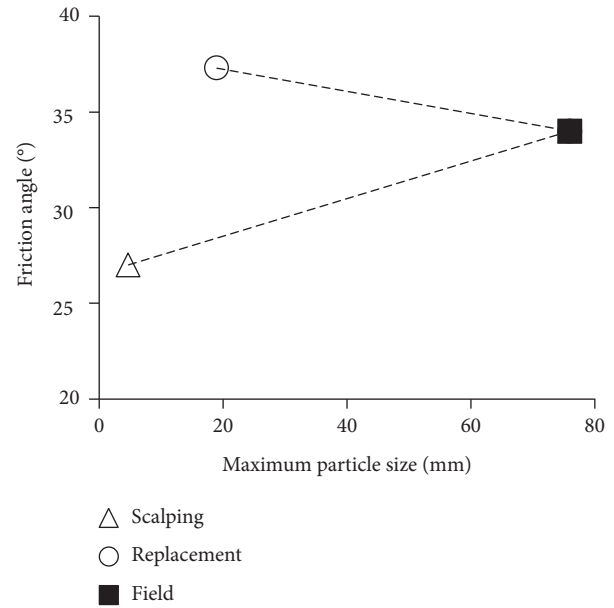

Figure 10: Comparison of friction angles of scalped and replaced samples with that of the field sample, all having a gravel content of 60% (data taken from [90]).

increases as the d_{max} increases from 12.5 to 25.4 mm for the rounded alluvium sand-gravel mixtures. This trend agrees with that of rounded materials (Figure 7(a)). Results further show that when the mixture is loose ($D_r = 35\%$), the maximum shear stress (Figure 11(a)) and friction angle (Figure 11(b)) remain constant when the d_{max} of the scalped specimens increases while there is an increasing trend for the intermediate ($D_r = 60\%$) or large ($D_r = 85\%$) relative densities of both scalped and parallel samples. Once again, direct comparison of the shear strengths of modeled and field samples is not a good way to validate or invalidate the tested scaling-down techniques. Consequently, one cannot conclude whether the scalping and parallel techniques are reliable to be used in an extrapolation to obtain the shear strength of field materials.

To correctly evaluate the capacity of a scaling-down technique, different shear tests on scaled down specimens with different d_{max} values should be done. The shear strength of field materials can then be obtained by extrapolating the shear strengths of specimens with different d_{max} values. Bagherzadeh and Mirghasemi [96] followed this approach and conducted a series of direct shear tests on coarse-grained material using 60 mm × 60 mm and 300 mm × 300 mm shear boxes to investigate the influence of scalping and parallel techniques on ellipsoidal gravel particles. The tested specimens were prepared to obtain a ratio value of 12 between specimen size and d_{max}. As shown in Figure 12(a), a field sample with a d_{max} of 50 mm was scaled down to samples 1 and 2 by applying the parallel scaling-down technique and to samples 3 and 4 by following the scalping technique. The small shear box was used for samples 2 and 4, and the larger shear box was used for samples 1 and 3. The maximum particle sizes were 25.4 mm for samples 1 and 3
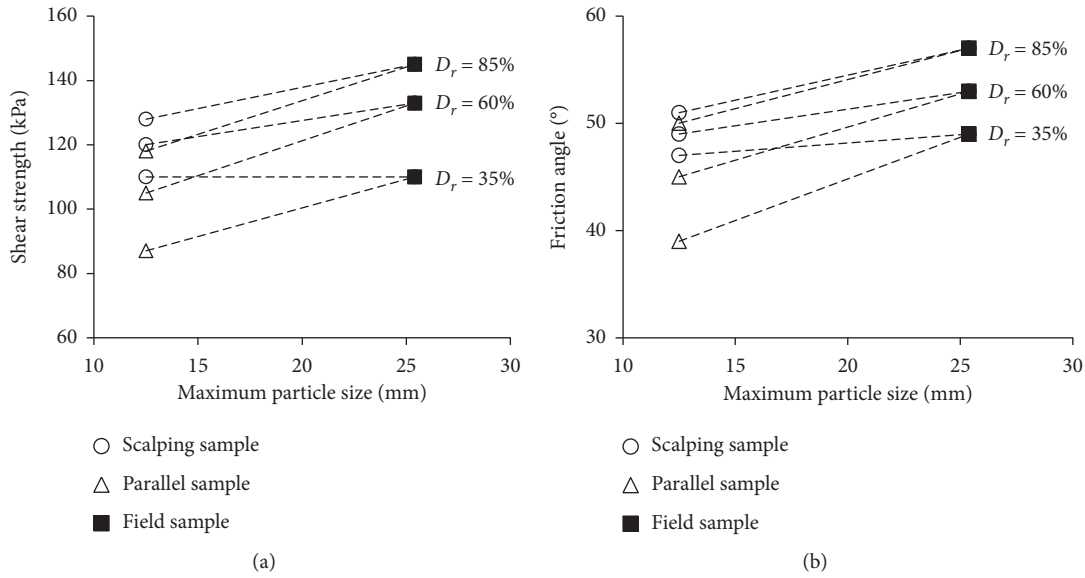
FIGURE 11: Variations of (a) shear strength (normal stress = 100 kPa) and (b) friction angle of field, parallel, and scalping samples as a function of maximum particle size (data taken from [67]).
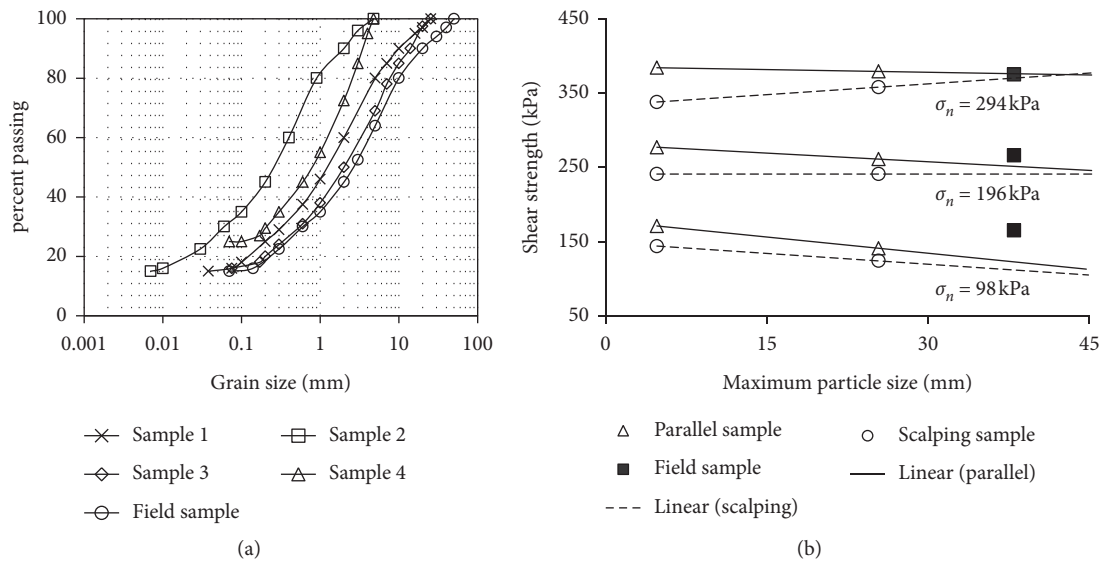

FIGURE 12: (a) Distribution curves of four samples and a field sample; (b) variation of shear strength as a function of d_{max} of parallel, scalping, and field samples under different normal stresses (data taken from [96]).

and 4.76 mm for samples 2 and 4, respectively. Figure 12(b) shows the shear stresses at failure versus the d_{max} value under different normal stresses. The results show that the shear strength of the field specimens having a d_{max} of 38 mm (number given in [96] but 50 mm according to Figure 12) can be predicted by extrapolating the test results of the parallel and scalped samples when the normal stress is high (294 kPa). When the normal stress is low (98 kPa) or intermediate (196 kPa), none of the two scaling-down techniques can be used to predict the shear strength of the field material.

Direct comparison between the shear strengths of modeled and field samples is not an appropriate methodology to validate or invalidate a scaling-down technique. The invalidity of replacement technique is not correctly shown. The previous analyses seem to show that both scalping and parallel techniques can be used to predict the shear strength of field rockfill through extrapolation on laboratory shear test results when normal stress is high. Both the techniques fail when the normal stress is intermediate or low. However, it is noted that most of the previous experimental shear tests have been

done by using the minimum (sometimes even smaller) required ratio of specimen size over d_{max} specified in ASTM D3080 [46] for direct shear tests or ASTM D4767 [45] for triaxial compression tests.

4. Specimen Size Effect

The variation of shear strength of granular materials with specimen size is known as a phenomenon of specimen size effect [115–118]. For the convenience of laboratory tests, one tends to use specimens as small as possible. When the specimen size is too small, the measured shear strength cannot represent that of the tested material in field conditions where the volume of the tested material can be very large. Therefore, the tested specimen should be large enough to avoid any specimen size effect, also known as a problem of representative volume element size [119–121]. That is why the diverse standards specify minimum required ratios between specimen dimensions and d_{max}.

To determine the shear strength of granular materials by direct shear tests, ASTM D3080 [46] requires specimens to be at least 50 mm wide and 13 mm thick (Table 1). In addition, the width and thickness should, respectively, be at least 10 and 6 times the maximum particle size (d_{max}). The standards AS 1289.6.2.2 [47] and Eurocode 7 [49] require, respectively, a thickness of at least 6 and 10 times the d_{max} value (Table 1). For fine particle soils such as clay, silt, and fine sand, the d_{max} values are smaller than 2 mm. The minimum required specimen sizes that are 50 mm in width and 13 mm in thickness give a ratio of 25 between specimen width and d_{max} and a ratio of 6.5 between specimen thickness and d_{max}; these satisfy the minimum required ratios. However, these requirements are not yet undoubtedly validated by experimental results.

Rathee [110] studied the influence of specimen size on the friction angle of mixtures of sand and gravel in four proportions (10, 30, 50, and 100%) by using two shear boxes of 60 mm × 60 mm and 300 mm × 300 mm. The tested specimens with d_{max} values of 50, 37.5, 25, 19, 12.5, and 6.3 mm were obtained by the parallel scaling-down technique on a field material with a d_{max} of 450 mm. The test results, not presented here, involved simultaneously the effects of d_{max} and specimen size. The methodology followed in this study is inappropriate to investigate the specimen size effect.

Palmeira and Milligan [122] performed direct shear tests on a sand with a d_{max} of 1.2 mm by using small (60 mm × 60 mm × 32 mm), medium (252 mm × 152 mm × 152 mm), and large (1000 mm × 1000 mm × 1000 mm) size shear boxes. The W/d_{max} ratios corresponding to the three shear boxes were 50, 126.7, and 833, respectively, while the T/d_{max} ratios were 26.7, 126.7, and 833, respectively. The results showed that the friction angles remained almost constant when the W/d_{max} ratio increased from 50 to 833. However, these ratios are much larger than the minimum values required in ASTM D3080 [46]. There were no shear test results on specimens prepared with the W/d_{max} ratios between 10 and 50. Thus, these results cannot be considered as a validity of the

minimum required specimen size ratio of ASTM D3080 [46].

Cerato and Lutenegger [123] studied the influence of specimen size on the friction angles of five sands with different d_{max} values considering compactness states of loose, medium, and dense sands. Three shear boxes were used. Table 3 shows the testing program and specimen sizes. All the ratios of specimen width and thickness to d_{max} met the requirements of AS 1289.6.2.2 [47], ASTM D3080 [46], and Eurocode 7 [49] except for the GP3 and winter sands with a d_{max} of 5 mm when using the smallest shear box. The ratio of 5 between specimen thickness and maximum particle size is slightly smaller than the minimum required value (Table 1).

Figure 13 shows the friction angle versus the ratios of specimen width and thickness over d_{max} for materials with different densities. For the Ottawa sand with a d_{max} of 0.9 mm (Figure 13(a)), the friction angle remains almost constant when the W/d_{max} ratio increases from 67 to 339 and the T/d_{max} ratio increases from 29 to 198. There was no specimen size effect for these ratios. However, there were no test results on specimens with the W/d_{max} ratio from 10 to 67. It is impossible to know whether a specimen size effect is removed for specimens with a W/d_{max} ratio of 10. Thus, the minimum ratio of specimen size to d_{max} as required in ASTM D3080 [46] is not validated.

For the material with a d_{max} of 1.7 mm (Figure 13(b)), the friction angle increases by more than 3 degrees when the W/d_{max} ratio increases from 36 to 179 and the T/d_{max} ratio from 16 to 105. For the material with a d_{max} of 2 mm (Figure 13(c)), the friction angle decreases by more than 2 degrees when the W/d_{max} ratio increases from 30 to 152 and the T/d_{max} ratio from 13 to 89. These results indicate that the specimen size effect on the friction angle of these materials is not eliminated in these ratio ranges, which invalidated the minimum requirements of ASTM D3080 [46] for these ratios.

For coarse grain materials with a d_{max} of 5 mm (Figures 13(d) and 13(e)), the friction angle decreases by more than 5 degrees as the W/d_{max} ratio increases from 12 to 20 and the T/d_{max} ratio from 5 to 8. The friction angle further decreases of 2 degrees when the W/d_{max} ratio increases from 20 to 61 and the T/d_{max} ratio from 8 to 36. These results further illustrate that the specimen size effect on the friction angle of the coarse grain materials is not eliminated for a W/d_{max} ratio between 12 and 61, which again invalidates the minimum requirements of ASTM D3080 [46].

Ziaie Moayed et al. [124] also performed direct shear tests by following ASTM D3080 [46] on a sand with a d_{max} of 0.8 mm mixed with different silt contents (0, 10%, 20%, and 30%). Three shear boxes 60 mm × 60 mm × 24.5 mm, 100 mm × 100 mm × 35 mm, and 300 mm × 300 mm × 154 mm were used. The W/d_{max} ratios were 75, 125, and 375, respectively, and the T/d_{max} ratios were 31, 44, and 192, respectively. For sand mixed with 30% silt, the friction angle only decreased by 1.3 degrees when the W/d_{max} ratio increased from 75 to 125 and remained almost constant when the W/d_{max} ratio further increased from 125 to 375. For

TABLE 3: Materials and specimen sizes used in direct shear tests by Cerato and Lutenegger [123].

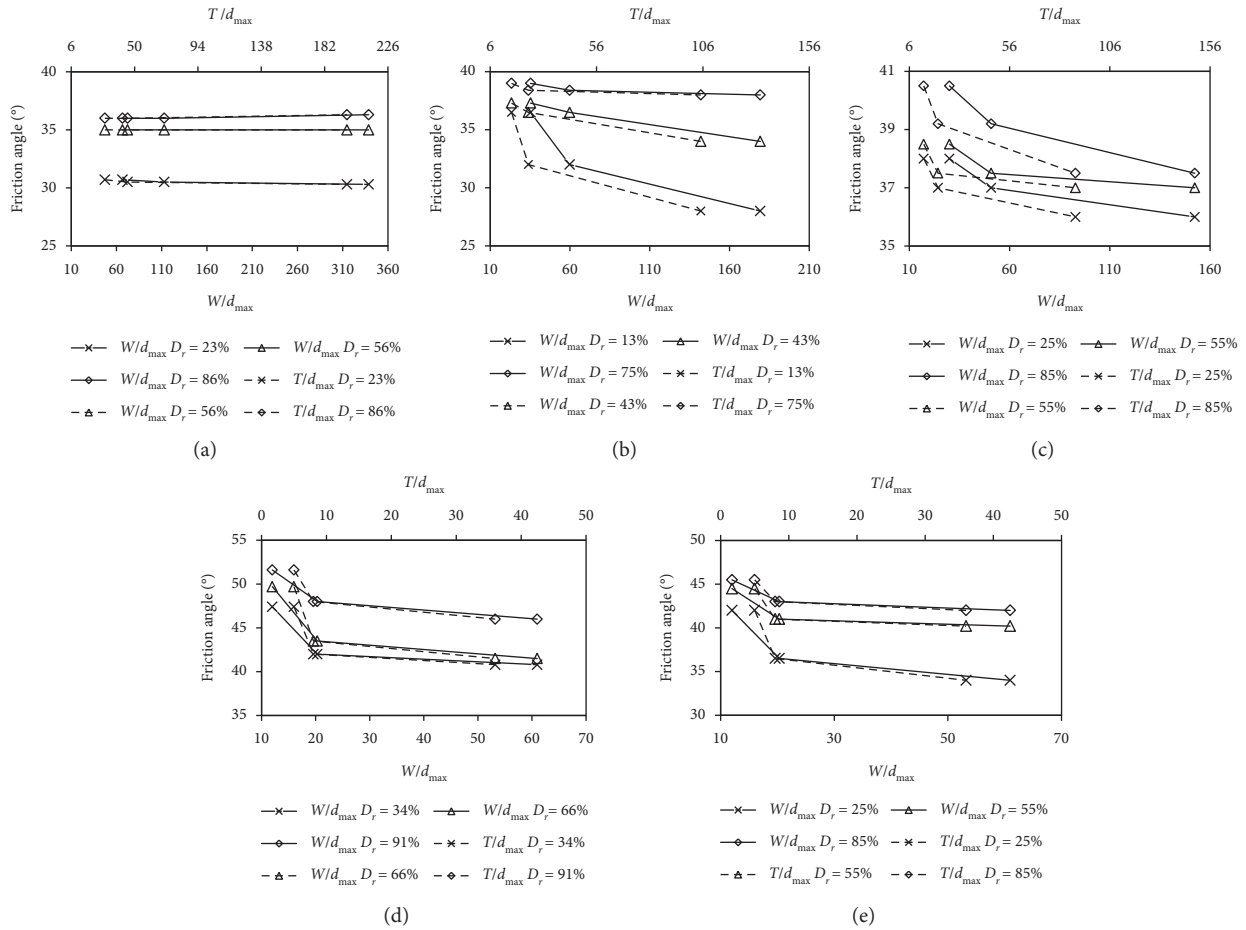| Materials	d_{max} (mm)	60 mm × 60 mm × 26.4 mm		101.6 mm × 101.6 mm × 40.64 mm		304.8 mm × 304.8 mm × 177.8 mm	
		T/d_{max}	W/d_{max}	T/d_{max}	W/d_{max}	T/d_{max}	W/d_{max}
Ottawa	0.9	29	67	45	113	198	339
FHWA	1.7	16	36	24	60	105	179
Morie	2.0	13	30	20	51	89	152
GP3	5.0	5	12	8	20	36	61
Winter	5.0	5	12	8	20	36	61

FIGURE 13: Variation of the friction angle in terms of specimen width and thickness to d_{max} ratios for specimens with different relative densities (data taken from [123]): (a) Ottawa sand, (b) FHWA (brown mortar), (c) Morie, (d) winter, and (e) gravel pack #3.

these tests, one cannot validate the minimum requirement of ASTM D3080 [46] because no specimen was tested with a W/d_{max} ratio between 10 and 75. For the pure sand specimens, the friction angle decreased by more than 3 degrees when the W/d_{max} ratio increased from 75 to 125 and then by 2 degrees when the W/d_{max} ratio further increased from 125 to 375. These results tend to indicate that a W/d_{max} ratio of 75 is not large enough to remove the specimen size effect. This invalidates the minimum requirements of ASTM D3080 [46].

Table 4 summarizes the previous studies regarding the specimen size effect on the friction angle of granular

materials. All specimen sizes met the minimum requirement ratios of the studied standards [46, 47, 49] except those highlighted by an asterisk. For the fine particle materials with d_{max} less than approximately 1.2 mm, the minimum specimen size ratio required by the studied standards is either invalidated or not validated. For the materials with d_{max} equal to or larger than 1.7 mm, the minimum required ratios of specimen sizes (width and/or thickness) to maximum particle sizes dictated by the studied standards (ASTM, AS, and Eurocode) are invalidated. More experimental works are needed to find the minimum required ratios that remove specimen size effect on friction angle of granular materials. The minimum required ratios in the diverse

TABLE 4: Summary of the previous studies on the specimen size effects of friction angle of granular materials.

Material type	d_{max} (mm)	W/d_{max}	T/d_{max}	Respect the standards		Validity of standards	Reference
				$W/d_{max} > 6$	$T/d_{max} > 10$		
Silty sand	0.8	75, 125, 375	30.6, 43.7, 192.5	Yes	Yes	Not validated	Ziaie Moayed et al. [124]
	0.8	75, 125, 375	30.6, 43.7, 192.5	Yes	Yes	Invalidated	
Sand	0.9	72, 119, 352	31, 45.2, 209	Yes	Yes	Not validated	Cerato and Lutenegger [123]
	1.2	50, 126.7, 833	26.7, 126.7, 833	Yes	Yes		Palmeira and Milligan [122]
	1.7	36, 60, 179	16, 24, 105	Yes	Yes		
	2	30, 51, 152	13, 20.5, 89	Yes	Yes	Invalidated	Cerato and Lutenegger [123]
Gravel	5	12, 20, 61	5*, 8*, 36	Yes	Yes for the largest box		

*Specimens not meeting the minimum requirement ratio of ASTM D3080 [46]. Standard BS 1377-7 [48] defines specific d_{max} for specified specimen sizes. The BS is not thus examined in this paper.

norms between specimen size and maximum particle size need to be revised upward.

5. Discussions

In this paper, the influence of d_{max} on shear strength of granular materials has been presented by considering scaling-down techniques and specimen size. Once the shear strengths of modeled samples with different d_{max} are obtained, graphical or equation relationship can be established between the shear strengths and d_{max}. The shear strength of field rockfill can then be obtained by extrapolation on the shear strength and d_{max} curve or equation.

However, one keeps in mind that the shear strength of granular materials can also be influenced by other influencing factors such as particle shape, fine particle content, gravel content, initial gradation (coefficient of uniformity and curvature), compactness (relative density), confining stress, strength of solid grain, and breakage of particles. In order to see the influence of d_{max} on shear strengths of granular materials, one has to keep other influencing parameters constant. More works are necessary to analyze the influence of d_{max} on the shear strength of granular material by considering different values of other influencing parameters. More works are also necessary to see the influences of other influencing parameters. Obviously, full and comprehensive analyses of the shear strength of granular materials still require considerable heavy work both in experimental and analyzing work. It is interesting and promising to see the application of machine learning models and approaches such as artificial neural network (ANN) model, random and cubist forest models, and genetic algorithm on this aspect [125–127]. The influences of different influencing parameters can be simultaneously considered. Of course, the application of these powerful models and approaches requires the input of big and reliable experimental data.

This is again closely related to the specimen size effect in the shear strength tests.

Finally, it is noted that a number of empirical equations have been proposed to relate the shear strength of granular materials and d_{max} based on experimental results obtained by applying parallel scaling-down technique [31, 34, 71, 74, 97, 98]. The review analyses presented in this paper indicate that it should be careful to use these models to predict shear strength of in situ field materials because the reliability of the parallel scaling-down technique has not yet been shown.

6. Conclusions

The review and analysis on experimental data of shear strengths obtained by performing direct shear tests and triaxial compression tests on samples prepared by applying different scaling-down techniques lead to the following conclusions:

(i) Applying any one of the four scaling-down techniques results in a modified gradation compared to that of the original field material. Unlike a common belief, the application of parallel scaling-down technique also results in a modification of the physical composition. In terms of complexity to obtain a target gradation curve, the scalping technique is the simplest to achieve, followed by the replacement method and the parallel scaling-down technique. The quadric scaling-down technique is the most complicated to achieve, and the target gradation curve is only a function of the targeted d_{max}, with no consideration to other gradation characteristics of the field material. Its physical justification and applicability are unclear.

(ii) Parallel scaling down is the most used technique in practice, followed in decreasing order by scalping and replacement methods. Quadratic scaling-down technique has never been used since its publication.

(iii) In previous studies, the validity or invalidity of a scaling-down technique was conducted by directly comparing shear strengths of modeled and field samples. This methodology is inappropriate and unreliable. The invalidity of the replacement technique by this methodology is uncertain.

(iv) The minimum ratios of specimen size to d_{max}, as suggested by well-established standards for direct shear tests, are too small to eliminate specimen size effect. The minimum size ratios given in the studied norms are thus not reliable. More specifically,

   (a) For fine particle materials, the minimum size ratios of well-established standards (ASTM, AS, and Eurocode) for direct shear tests are either invalidated or not validated.

   (b) For coarse granular materials, the minimum size ratios of well-established standards (ASTM, AS, and Eurocode) are invalidated.

(v) Almost all available shear test results on granular materials were obtained by following minimum required ratios specified in well-established norms. Conclusions based on these experimental results are thus uncertain. Shear strengths of field granular material were overestimated. Structure design based on such results is thus on the nonconservative side.

(vi) The primary analyses seem to show that both scalping and parallel techniques can be used to predict shear strength of field rockfill through extrapolation of laboratory shear test results when normal stress is high, but both techniques failed for low to intermediate normal stress. These conclusions are however uncertain since the experimental results were obtained by using the minimum ratio of specimen size to d_{max} specified by ASTM D3080 [46]

7. Recommendations

The review and analyses presented in this paper indicate that none of the four scaling-down techniques can be used in a reliable way to obtain the shear strength of field rockfill despite that the parallel scaling-down technique is the most used one. In addition, the minimum ratios of specimen size to d_{max} suggested by well-established standards for direct shear tests are too small to eliminate the specimen size effect. The shear strengths obtained by following these minimum ratios values are unreliable. More works are necessary, as indicated by the following recommendations:

(i) More direct shear tests using different shear box sizes are needed to determine the minimum required ratios of specimen size to d_{max} by which specimen size effect can be entirely eliminated or considered as negligible. The minimum specimen size to d_{max} ratios required in the well-established standards (ASTM, AS, and Eurocode) can thus be updated.

(ii) More experimental work is necessary to identify a reliable scaling-down technique that can be used to predict the shear strength of field materials by extrapolating the laboratory shear test results. Of course, this work can only be performed after the previous task to make sure that all shear tests are realized by using specimens large enough to eliminate any specimen size effect. The shear strength measurement of field rockfill using large enough specimens to avoid any specimen size effect is necessary to verify if the shear strength of field rockfill can be correctly predicted through extrapolation on the variation of shear strength as a function of d_{max}.

(iii) More experimental works are necessary to analyze the influences of other influencing parameters such as particle shape, fine particle content, gravel content, initial gradation (coefficient of uniformity, curvature, and d_{max}), compact (relative density), confining stress, strength of solid grain, and breakage of particles. Once again, this work can only be done when the minimum required ratios of specimen size to d_{max} are known to avoid any specimen size effect.

Notations

d: Particle size of modeled sample
$d_{p \cdot m}$:   Particle size of modeled sample having a percentage passing p
$d_{p \cdot f}$:   Particle size of field material having a percentage passing p
d_{max}:   Maximum particle size
$d_{max \cdot f}$:   Maximum particle size of field material
$d_{max \cdot m}$:   Maximum particle size of modeled (parallel) sample
D: Diameter of cylinder sample for direct shear tests or triaxial compression tests
D_r: Relative density
N: Ratio of $d_{max \cdot f}$ to $d_{max \cdot m}$
P_{dmax}:   Percentage of field material particles passing the targeted d_{max}
$P_{ij \cdot f}$:   Percentage by mass of field material particles passing sieve size j ($\leq d_{max}$) and retained on the sieve size i (≥ 4.75 mm)
$P_{No.4}$:   Percentage of field material particles passing the sieve of 4.75 mm
P_o: Percentage by mass of field material particles retained on the sieve having a size of the targeted d_{max}
P_Q: Percentage by mass of the modeled particle size
$P_{ij \cdot a}$:   Percentage by mass of added particles passing sieve having size j ($\leq d_{max}$) and retained on the neighbor sieve having size i (≥ 4.75 mm)
T: Thickness of specimen for direct shear tests
W: Width of specimen for direct shear tests
α: Tilt angle
σ_3:   Confining pressure for triaxial compression tests
σ_n: Normal stress for direct shear tests

ϕ: Friction angle.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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