

Research Article

A V_s -Based Logistic Regression Method for Liquefaction Evaluation

Xiaofei Yao ^{1,2}, Lu Liu,^{1,2} Zhihua Wang,^{1,2} Zhifu Shen,^{1,2} and Hongmei Gao ^{1,2}

¹Urban Underground Space Research Center, Nanjing Tech University, Nanjing 210009, China

²Institute of Geotechnical Engineering, Nanjing Tech University, Nanjing 210009, China

Correspondence should be addressed to Hongmei Gao; hongmei54@163.com

Received 6 February 2021; Revised 28 March 2021; Accepted 20 May 2021; Published 30 May 2021

Academic Editor: Shuang Yang Li

Copyright © 2021 Xiaofei Yao et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The current liquefaction evaluation methods mainly focus on the success rate for liquefied sites so that the evaluation result tends to be conservative at different seismic intensities. Therefore, a new formula about liquefaction evaluation by introducing logistic regression theory is proposed to solve the deficiencies of the current evaluation method, which is based on 225 sets of shear wave velocity data reported by Andrus. The reliability of the new formula is verified based on 336 sets of V_s data collected from the Kayen database. The performance of the new formula on liquefaction evaluation is compared with existing liquefaction evaluation methods including the Andrus method and the Chinese code method. Compared with the Andrus method and Chinese code method, the success rates of liquefaction evaluation given by the new formula under different seismic intensities are more balanced between liquefied site and nonliquefied site. The new formula at 50% probability of liquefaction is more adaptable for a wide range of seismic intensities, ground water table, and sand buried depth. In addition, the new formula at different probabilistic levels of liquefaction can be adopted based on the importance of the engineering site in risk analysis.

1. Introduction

Soil liquefaction induced by the earthquake may often occur on some sites. The damage induced by soil liquefaction was not fully recognized until the occurrence of the 1964 Niigata and the 1964 Alaska earthquake [1], which destroyed structures and infrastructures and caused loss of life. It is necessary to develop a proper liquefaction evaluation method in order to prevent or minimize such damage.

At present, numerous researchers have tried to evaluate the soil liquefaction by standard penetration test (SPT), cone penetration test (CPT), Becker hammer test (BHT), and shear wave velocity (V_s) test. Among these methods, the shear wave velocity (V_s) test has wider applicability for liquefaction evaluation than other methods [2]. The shear wave velocity test can also be conducted in gravelly soils where SPT and CPT are unreliable [3–6] since they cannot be conducted in soils containing gravels, cobbles, or boulders. In contrast to SPT and CPT, V_s -based method has the advantage that no corrections are required based on soil

characteristics [7–11]. In addition, shear wave velocity tests measure the shear modulus (stiffness) of the soil at low strains, which can also be obtained in the laboratory, so that comparison can be made between laboratory test and field test [5, 6]. Thanks to these advantages, V_s -based methods have great prospects for liquefaction evaluation [4, 12].

Dobry et al. [13] confirmed that the shear wave velocity is relevant to liquefaction resistance. In addition, both shear wave velocity values and blow counts are similarly influenced by many factors such as void ratio, state of stress, and stress history. Accordingly, a simple procedure incorporating a cyclic stress ratio (CSR)- V_s triggering resistance curve was proposed by Seed et al. [14]. Andrus and Stokoe [15] proposed guidelines for using the V_s -based liquefaction evaluation procedure, which is subsequently updated by Andrus et al. [16]. Yamazaki et al. [17] presented a new simplified liquefaction prediction and assessment method considering the influence of the waveforms and durations of earthquakes. El-Sekelly et al. pointed out that the existing V_s -based liquefaction charts could favorably evaluate the

liquefaction of uncompacted clean and silty sand recent fills but might be too conservative for heavily preshaken deposits in highly seismic areas. Chen et al. [18] confirmed that the cyclic resistance and the corrected shear wave velocity are uniquely related for a range of sandy soils and presented a new V_s -based approach to assess the liquefaction triggering of saturated sandy soils. Akbari-Paydar and Ahmadi [19] indicated that the commonly used CRR (cyclic resistance ratio)- V_s correlation curves might be conservative for silty and clayey sands; thus, it can only be used as an initial estimation of liquefaction resistance by performing a series of cyclic triaxial and bender element tests on reconstituted specimens of clean sand with different contents.

The deterministic methods discussed above only give the result of liquefaction or nonliquefaction of the site. However, it is unreasonable to give a deterministic evaluation result due to the uncertainty of seismic characteristics and the nonlinearity of soil. In addition, these methods commonly pursue the success rate of liquefaction, leading to a low success rate of nonliquefaction. Using these deterministic methods to treat the foundation or design buildings is likely to increase the construction cost. A probabilistic liquefaction evaluation method can potentially account for the uncertainties of soil properties, model parameters, and seismic input characteristics, which is also in line with the current development on earthquake engineering [1, 2, 5]. Thus, numerous scholars pay more attention to the research on probabilistic approaches to liquefaction analysis. Liao et al. [20] recommended regression models for evaluating the probability of liquefaction through corrected/normalized SPT $(N_1)_{60}$ values (the corrected SPT resistance normalized to a hammer energy ratio of 60%, as per Seed et al. [21]) and comparisons are made with other methods of liquefaction analysis. Similar studies have been conducted by logistic regression analysis based on field performance data [22–24]. Shen et al. [5] developed an updated log-log model for liquefaction potential evaluation based on shear wave velocity and concluded that the probabilistic models are quite satisfactory when they are checked against the case histories. Rahmanian and Rezaie [25] presented that the severity of liquefaction occurrence in the studied area by the V_s -based method was lower than that by SPT-based method through the comparison of the safety factor and liquefaction potential index. Johari et al. [2] developed a soil liquefaction model based on shear wave velocity using the jointly distributed random variables (JDRV) method. They pointed out that the probability distribution of the liquefaction safety factor obtained by the JDRV method is very close to that predicted by the Monte Carlo simulation. Andrus et al. [26, 27] concluded that the wave velocity-based method is more conservative than the penetration-based method for liquefaction evaluation based on compiled Holocene data including 43 Holocene-age sand layers in California, South Carolina, Canada, and Japan.

The existing probability methods usually use CSR and CRR [7, 27] to establish the liquefaction evaluation formula. However, this is not in line with the Code for Investigation of Geotechnical Engineering in China (GB 50021-2009) for liquefaction evaluation [28], which is referred to as Chinese

code hereafter. Therefore, a new liquefaction evaluation model combined with logistic regression theory, which is based on 225 sets of V_s data from the Andrus database [16], is proposed to solve the deficiencies of the current liquefaction evaluation method. The reliability of the new model is verified by 329 sets of data from the Kayen database and the performance of the new model on liquefaction evaluation is compared with the existing liquefaction evaluation methods.

2. Chinese Code Method and Andrus Method on Liquefaction Evaluation

2.1. Data Collection for Liquefaction Evaluation. The liquefaction evaluation method established in this study is based on the authoritative database published by Andrus et al. [16], which is summarized in Table 1. Many scholars use the Andrus database to develop a new model for liquefaction evaluation and the authority of the database is widely recognized [2, 29]. The database was involved in 225 groups of shear wave velocity data, which were collected from 26 earthquakes and more than 70 sites. The fundamental parameters such as M_w (moment magnitude), CSR (cyclic stress ratio), V_{s1} (overburden pressure-corrected shear wave velocity), d_s (sandy-layer buried depth), d_w (depth of ground water table), and a_{max} (peak horizontal ground surface acceleration) are provided in the database. According to the Chinese seismic intensity table [30], these data are divided into seismic intensities VI, VII, VIII, and IX, as shown in Table 2. It should be noted that all sites that are distributed in seismic intensity VI are nonliquefied. At present, the existing code for liquefaction evaluation in China does not consider the liquefaction in seismic intensity VI sites. On the one hand, few data about shear wave velocity are available in seismic intensity VI sites located in China. On the other hand, the site distributed in intensity VI is basically not liquefied. However, there are 34 sites provided in the Andrus database in seismic intensity VI and they will be considered in establishing a new model for liquefaction evaluation in this study in order to determine the lower boundary of the liquefaction critical curve more reasonably.

Figure 1 presents the data frequency distributions with the buried depth of sandy layer for case histories based on 225 groups of sites [16]. It is shown that there exists a slight difference in the distribution of sandy-layer buried depth between liquefied site and nonliquefied site. At liquefied sites and nonliquefied sites, most of the sandy-layer buried depth ranges from 2 m to 10 m. Figure 2 depicts the data frequency distributions with the depth of ground water table. The depth of ground water table ranges from 0 m to 8 m. At liquefied sites, the depth of water table mainly distributes from 1 m to 4 m, while at nonliquefied sites mainly from 0 to 3 m.

2.2. Chinese Code Method for Liquefaction Evaluation. In Chinese code [28], the critical shear wave velocity value is used as the index for the soil liquefaction evaluation. The liquefaction evaluation formula based on shear wave velocity has been widely used in China, which is written as follows:

TABLE 1: Authoritative database published by Andrus et al. [16].

| No. | Earthquake | M_w | No. of cases | CSR | d_s (m) | d_w (m) | V_{S1} (m/s) | a_{max} (g) |
|-----|----------------------------------|-------|--------------|-----------|-----------|-----------|----------------|---------------|
| 1 | 1906 San Francisco | 7.7 | 12 | 0.22~0.3 | 4.2~8 | 2.4~6.1 | 131~168 | 0.32~0.36 |
| 2 | 1957 Daly City | 5.3 | 5 | 0.08~0.09 | 3.5~7.9 | 2.7~5.9 | 105~220 | 0.11 |
| 3 | 1964 Nigita Japan | 7.5 | 4 | 0.12~0.15 | 3.2~6.2 | 1.2~5 | 112~162 | 0.16 |
| 4 | 1975 Haicheng China | 7.3 | 6 | 0.12~0.14 | 3~10.2 | 0.5~1.5 | 98~147 | 0.12 |
| 5 | 1979 imperial Valley, Calif | 6.5 | 11 | 0.09~0.42 | 3~4.7 | 1.5~2.7 | 90~173 | 0.12~0.51 |
| 6 | 1980 Mid-Chiba Japan | 5.9 | 2 | 0.08 | 6.1~14.8 | 1.3 | 155~195 | 0.08 |
| 7 | 1981 Westmorland, Calif | 5.9 | 11 | 0.02~0.29 | 3~4.7 | 1.5~2.4 | 90~173 | 0.02~0.36 |
| 8 | 1983 Borah Peak, Idaho | 6.9 | 18 | 0.16~0.41 | 1.9~3.7 | 0.8~3 | 94~274 | 0.23~0.46 |
| 9 | 1985 Chiba-Ibaragi-Kenkyo, Japan | 6.0 | 2 | 0.05 | 6.1~14.8 | 1.3 | 155~195 | 0.05 |
| 10 | 1986 Taiwan (event Lsst2) | 5.3 | 4 | 0.08 | 5.3~6.1 | 0.5 | 127~156 | 0.05 |
| 11 | 1986 Taiwan (event Lsst3) | 5.5 | 4 | 0.03 | 5.3~6.1 | 0.5 | 127~156 | 0.02 |
| 12 | 1986 Taiwan (event Lsst4) | 6.6 | 4 | 0.34 | 5.3~6.1 | 0.5 | 127~156 | 0.22 |
| 13 | 1986 Taiwan (event Lsst6) | 5.4 | 4 | 0.06 | 5.3~6.1 | 0.5 | 127~156 | 0.04 |
| 14 | 1986 Taiwan (event Lsst7) | 6.6 | 4 | 0.27~0.28 | 5.3~6.1 | 0.5 | 127~156 | 0.18 |
| 15 | 1986 Taiwan (event Lsst8) | 6.2 | 4 | 0.06 | 5.3~6.1 | 0.5 | 127~156 | 0.04 |
| 16 | 1986 Taiwan (event Lsst12) | 6.2 | 4 | 0.27~0.28 | 5.3~6.1 | 0.5 | 127~156 | 0.18 |
| 17 | 1986 Taiwan (event Lsst13) | 6.2 | 4 | 0.08 | 5.3~6.1 | 0.5 | 127~156 | 0.08 |
| 18 | 1986 Taiwan (event Lsst16) | 7.6 | 4 | 0.21~0.22 | 5.3~6.1 | 0.5 | 127~156 | 0.14 |
| 19 | 1987 Chiba-Toho-Okai, Japan | 6.5 | 1 | 0.06 | 9 | 6.2 | 150 | 0.1 |
| 20 | 1987 Elmore Ranch, Calif | 5.9 | 11 | 0.03~0.19 | 3.4~4.7 | 1.5~2.7 | 90~173 | 0.03~0.24 |
| 21 | 1987 Superstition Hills Calif | 6.5 | 11 | 0.15~0.20 | 3.0~4.7 | 1.5~2.7 | 90~173 | 0.18~0.21 |
| 22 | 1989 Loma Prieta, Calif | 7.0 | 67 | 0.13~0.42 | 2.3~9.9 | 0.6~6.1 | 91~209 | 0.1~0.45 |
| 23 | 1993 Kushiro-Okai, Japan | 8.3 | 2 | 0.35~0.46 | 4.2~4.5 | 0.9~1.9 | 135~152 | 0.41 |
| 24 | 1993 Hokkaido-Nansei-Okai, Japan | 8.3 | 4 | 0.16~0.18 | 2.0~7.0 | 1.0~1.4 | 74~143 | 0.15~0.19 |
| 25 | 1994, Northridge, Calif | 6.7 | 3 | 0.37~0.4 | 4.4~5.6 | 3.4 | 129~160 | 0.51 |
| 26 | 1995 Hyogo-Ken Nanbu, Japan | 6.9 | 19 | 0.1~0.6 | 3.3~11.5 | 1.5~7 | 110~214 | 0.12~0.65 |

TABLE 2: Data statistics based on the database by Andrus et al. [16].

| Seismic intensity | L | NL | Total |
|-------------------|-----|-----|-------|
| VI | 0 | 34 | 34 |
| VII | 23 | 46 | 69 |
| VIII | 35 | 35 | 70 |
| IX | 38 | 14 | 52 |
| Total | 96 | 129 | 225 |

Note. L represents liquefied sites; NL represents nonliquefied sites.

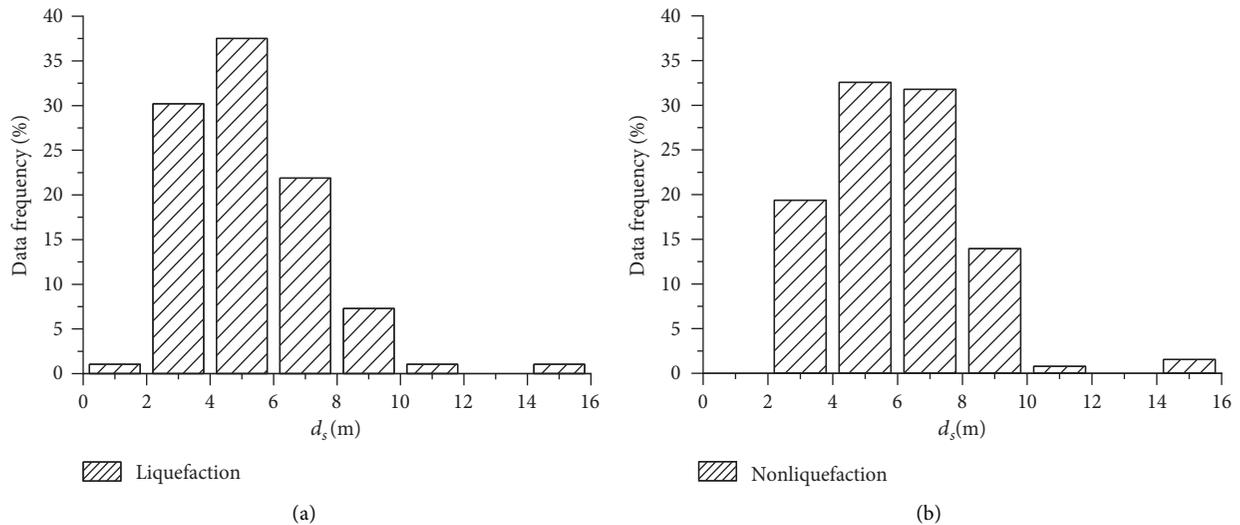


FIGURE 1: Histogram of sandy-layer buried depth versus data frequency: (a) liquefaction; (b) nonliquefaction.

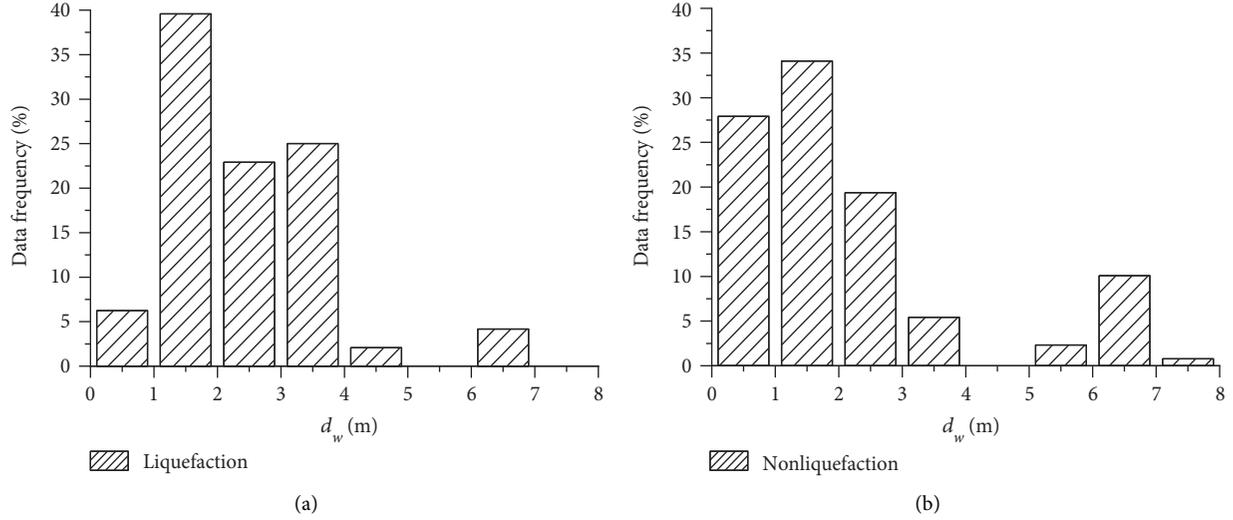


FIGURE 2: Histogram of ground water table versus data frequency: (a) liquefaction; (b) nonliquefaction.

$$V_{scri} = V_{s0} \sqrt{(d_s - 0.0133d_s^2)} \left[1 - 0.185 \frac{d_w}{d_s} \right], \quad (1)$$

where V_{scri} is the critical shear wave velocity and V_{s0} is a reference value for shear wave velocity, which are 65 m/s, 95 m/s, and 130 m/s corresponding to seismic intensity VII, VIII, and IX, respectively.

When the measured shear wave velocity of the soil layer is less than the corresponding V_{scri} , the site is judged as a liquefied site. Otherwise, the site is judged as a nonliquefied site.

2.3. Andrus Method for Liquefaction Evaluation. Based on 225 sets of data, Andrus et al. [16] proposed a liquefaction evaluation method which is widely used all over the world and referred to as the Andrus method hereafter. The Andrus method is expressed as follows:

$$CRR = \left(0.022 \left(\frac{V_{s1}}{100} \right)^2 + 2.8 \left(\frac{1}{V_{s1}^* - V_{s1}} - \frac{1}{V_{s1}^*} \right) \right) \cdot MSF, \quad (2)$$

$$V_{s1} = V_s \left(\frac{P_a}{\sigma_r'} \right)^{0.25}, \quad (3)$$

$$MSF = \frac{M_w}{7.5}^{-2.56}, \quad (4)$$

where CRR is soil liquefaction resistance; V_s is in situ measured shear wave velocity in m/s, which can be obtained by several seismic tests [6]; P_a is reference stress of 100 kPa; σ_r' is overburden effective stress in kPa; MSF is a magnitude scaling factor related to M_w ; and V_{s1}^* is limiting upper value of V_{s1} in m/s, which is dependent of fine content, F_c , in percent by mass.

V_{s1}^* can be estimated as follows:

$$V_{s1}^* = 215 \text{ m/s, for sands with } F_c \leq 5\%,$$

$$V_{s1}^* = 215 - 0.5(F_c - 5) \text{ m/s, for sands with } 5\% < F_c < 35\%,$$

$$V_{s1}^* = 200 \text{ m/s, for sands with } F_c \geq 35\%.$$

(5)

Liquefaction is predicted to occur when $CSR > CRR$, and liquefaction is predicted not to occur when $CSR < CRR$. CSR is cyclic shear stress ratio which can be calculated by the simplified formula proposed by Seed and Idriss [31] and written as follows:

$$CSR = \frac{\tau_{eq}}{\sigma_r'} = 0.65 \frac{a_{max}}{g} \frac{\sigma_r}{\sigma_r'} r_d, \quad (6)$$

where τ_{eq} is the average equivalent uniform cyclic shear stress suffered by an earthquake in kPa; σ_r is overburden total stress in kPa; g is the acceleration of gravity in m/s^2 ; and r_d is a shear stress reduction coefficient to adjust for the flexibility of the soil profile, which can be calculated as follows [32]:

$$\begin{aligned} \gamma_d &= 1 - 0.00765d_s, & \text{for } d_s \leq 9.15 \text{ m,} \\ \gamma_d &= 1.174 - 0.0267d_s, & \text{for } 9.15 \text{ m} < d_s \leq 23 \text{ m,} \\ \gamma_d &= 0.744 - 0.008d_s, & \text{for } 23 \text{ m} < d_s \leq 30 \text{ m.} \end{aligned} \quad (7)$$

2.4. Liquefaction Evaluation Comparison between Chinese Code and Andrus Method. Chinese code method evaluates liquefaction sites which locate in seismic intensity VII, VIII, and IX. Thus, the 34 sets of data in seismic intensity VI are removed when compared with the Andrus method. It should be noted that clay content is not provided in the Andrus database. Therefore, the value of clay content is taken as 3% when using the Chinese code method [29]. The fine content is assumed as $FC \leq 5\%$ when it is not provided in the database [29].

Table 3 shows the success rates of liquefaction evaluation based on the Andrus database by Chinese code method and Andrus method. It should be noted that the total success rate is defined as the ratio of site number evaluated successfully to total site number. Taking the total success rate in seismic intensity VII as an example, as displayed in Table 2, there are 23 liquefied sites and 46 nonliquefied sites in seismic intensity VII. Of these, 21 liquefied sites and 19 nonliquefied sites are successfully evaluated by the Andrus method. Thus, the total success rate, S_{total} , is obtained as follows:

$$S_{\text{total}} = \frac{21 + 19}{23 + 46} \times 100\% = 58\%. \quad (8)$$

In seismic intensity VII, the success rates for liquefied sites given by the Chinese code method and Andrus method are 56.5% and 91.3%, respectively. The success rates at nonliquefied sites reach 39.1% and 41.3%, respectively. That is to say, the Chinese code method tends to be dangerous in seismic intensity VII. In seismic intensity VIII and IX, for liquefied sites, the success rates of the two methods are both 100%; for nonliquefied sites, the success rates of both methods are lower than 35%. The success rate for liquefied sites is satisfactory, but these two methods stay conservative in seismic intensity VIII and IX. For liquefied sites, the total success rates using the Chinese code method and the Andrus method are 89.6% and 97.9%, respectively. However, the total success rates for nonliquefied sites are only 29.5% and 34.7%, respectively. These two methods with total success rates smaller than 70% tend to be conservative and reach an unacceptable level. Therefore, it is essential to improve the existing liquefaction evaluation methods.

3. A New Liquefaction Evaluation Formula

3.1. The Theoretical Basis. The existing liquefaction evaluation methods based on shear wave velocity have made great progress; however, they are not widely used in China. This mainly results from that CSR and CRR indexes are not used in existing Chinese codes. This study intends to introduce the parameters with higher liquefaction discrimination to establish a new liquefaction evaluation formula. The parameters affecting the liquefaction probability are analyzed herein.

According to Seed and Idriss [31], the minimum cyclic shear stress of soil to resist liquefaction, τ_d , can be expressed as follows:

$$\tau_d = C_r \frac{\sigma_{\text{ad}}}{2\sigma_3} \sigma'_r, \quad (9)$$

where σ_{ad} is the dynamic stress amplitude in kPa, σ_3 is the effective confining pressure in cyclic triaxial test in kPa, and C_r is a stress correction factor related to the relative density of soil. The values of C_r in equation (14) are summarized in Table 4 which are recommended by Seed and Idriss [31].

Furthermore, the cyclic stress ratio, $\sigma_{\text{ad}}/2\sigma_3$, is expressed as follows:

$$\frac{\sigma_{\text{ad}}}{2\sigma_3} = \frac{D_r}{50} \left[\frac{\sigma_{\text{ad}}}{2\sigma_3} \right]_{50}, \quad (10)$$

TABLE 3: Success rates of liquefaction evaluation by Chinese code method and Andrus method.

| Evaluation method | Seismic intensity | L (%) | NL (%) | Total (%) |
|---------------------|-------------------|-------|--------|-----------|
| Andrus method | VII intensity | 91.3 | 41.3 | 58 |
| | VIII intensity | 100 | 34.3 | 67.1 |
| | IX intensity | 100 | 14.3 | 76.9 |
| | Total | 97.9 | 34.7 | 66. |
| Chinese code method | VII intensity | 56.5 | 39.1 | 44.9 |
| | VIII intensity | 100 | 22.9 | 61.4 |
| | IX intensity | 100 | 14.3 | 76.9 |
| | Total | 89.6 | 29.5 | 59.7 |

TABLE 4: Stress correction factor.

| Relative density (%) | 30 | 40 | 50 | 60 | 70 | 80 | 85 |
|----------------------|------|------|------|------|------|------|------|
| C_r | 0.52 | 0.55 | 0.58 | 0.61 | 0.65 | 0.68 | 0.70 |

where D_r is the relative density of sand and $[\sigma_{\text{ad}}/2\sigma_3]_{50}$ is cyclic stress ratio at a relative density of 50%.

Liquefaction potential, $f_{M_w=7.5}$, can be expressed as follows [33]:

$$f_{M_w=7.5} = \frac{\tau_{\text{eq}}}{\tau_d}. \quad (11)$$

Umar et al. [34] pointed out that earthquake magnitude, M_w , plays an important role in the assessment of liquefaction. To consider the influence of M_w on $f_{M_w=7.5}$ and facilitate comparison $f_{M_w=7.5}$ of sites from different earthquakes, f is obtained as follows:

$$f = \frac{f_{M_w=7.5}}{\text{MSF}}. \quad (12)$$

Combining equations (9)–(12) yields the following:

$$f = \frac{0.65r_d}{C_r(D_r/50)[\sigma_{\text{ad}}/2\sigma_3]_{50}} \frac{a_{\text{max}}}{g} \frac{\sigma_r}{\sigma'_r}. \quad (13)$$

For the sake of brevity, a new parameter, k , was defined as follows:

$$k = \frac{\sigma_r}{\sigma'_r}. \quad (14)$$

Furthermore, the liquefaction potential is obtained as follows:

$$f = \frac{0.65r_d}{C_r(D_r/50)[\sigma_{\text{ad}}/2\sigma_3]_{50}} \frac{a_{\text{max}}}{g} k \frac{1}{\text{MSF}}, \quad (15)$$

where r_d is a coefficient related to d_s ; MSF is relevant to M_w ; and σ_r and σ'_r are dependent on d_s and d_w . The sandy-layer buried depth and depth of ground water table can reflect the effective overburden stress. It is difficult to directly obtain the value of D_r . The shear wave velocity or SPT counts can be alternative to reflect the relative density of the soil.

D_r can be calculated from

$$D_r = \frac{e - e_{\text{min}}}{e_{\text{max}} - e_{\text{min}}}, \quad (16)$$

where e , e_{max} , and e_{min} are the natural, maximum, and minimum void ratios, respectively.

Hardin and Drnevich [35] have shown that for sands, the shear modulus at low strain, G_{\max} , can be calculated from

$$G_{\max} = A \frac{(e_g - e)^2}{1 + e} (\sigma'_m)^n, \quad (17)$$

where n is a parameter related to soil type and is usually taken as 0.5 and σ'_m is the principal effective stress in kPa. The fitting parameters, A and e_g , are optimized in order to obtain the best fit of the experimental points. A is dependent on the particle shape and gradation of soil; e_g is dependent on void ratio. The optimization technique is a classical minimization approach [36].

G_{\max} can be calculated as [14, 37]

$$G_{\max} = \rho V_s^2. \quad (18)$$

Combining equations (16)–(18) yields

$$V_s = \left[\frac{A (\sigma'_m)^{0.5}}{\rho (1 + e_{\max} - D_r (e_{\max} - e_{\min}))} \right]^{0.5} \cdot [D_r (e_{\max} - e_{\min}) + e_g - e_{\max}]. \quad (19)$$

It should be noted that σ'_m is equal to σ'_r for the isotropic consolidation of the cyclic triaxial test. By combining equation (3) with equation (19), V_{s1} can be written as follows:

$$V_{s1} = \left[\frac{10A}{\rho (1 + e_{\max} - D_r (e_{\max} - e_{\min}))} \right]^{0.5} \cdot [D_r (e_{\max} - e_{\min}) + e_g - e_{\max}]. \quad (20)$$

It is worth noting that D_r can be obtained by substituting A , ρ , V_{s1} , e_{\max} , e_{\min} and e_g into equation (20). The value of C_r can be obtained from Table 4. It can be seen from equation (20) that V_{s1} largely depends on the relative density of the soil

so that overburden stress-corrected shear wave velocity (V_{s1}) is used to represent D_r since V_{s1} is easy to get. Therefore, liquefaction potential can be expressed by the five descriptive parameters including d_s , k , a_{\max} , V_{s1} , and M_w . These parameters can be regarded as independent variables influencing liquefaction occurrence. The result of liquefaction evaluation can be classified into liquefaction and non-liquefaction so that liquefaction evaluation is a two-category variable event. The logistic regression method can deal with complex and multivariable event, which is suitable for liquefaction evaluation by giving the results of probability.

3.2. Establishment of New Liquefaction Evaluation Formula.

Due to the uncertainty of earthquake occurrence and the fact that soil is a highly nonlinear material, there is no clear boundary between liquefaction and nonliquefaction, so the expression of the probability of liquefaction evaluation is more suitable for reflecting the liquefaction issue. Based on the derivation of the liquefaction potential, the five parameters including d_s , k , a_{\max} , V_{s1} , and M_w , are selected to establish the logistic regression-based probability model.

The logistic regression model proposed by Liao et al. [20] can be expressed as follows:

$$\ln \left[\frac{P_L(X)}{1 - P_L(X)} \right] = \theta_0 + \theta_1 a_{\max} + \theta_2 M_w + \theta_3 d_s + \theta_4 V_{s1} + \theta_5 k, \quad (21)$$

where X is a vector of explanatory variable, which includes d_s , k , a_{\max} , V_{s1} , and M_w ; $P_L(X)$ is the probability of liquefaction, which is a function of X ; and θ_0 , θ_1 , θ_2 , θ_3 , θ_4 , and θ_5 are regression coefficients of the logistic model.

The vector of coefficients $\theta = [\theta_0, \theta_1, \dots, \theta_5]$ is estimated through maximization of the likelihood of function $L(X; \theta)$, which is given by

$$\begin{aligned} L(X; \theta) &= \prod_{i=1}^m [P_L(X)]^{y_i} [1 - P_L(X)]^{1-y_i} \\ &= \prod_{i=1}^m \left\{ \frac{1}{1 + \exp[-\theta_0 + \theta_1 a_{\max} + \theta_2 M_w + \theta_3 d_s + \theta_4 V_{s1} + \theta_5 k]} \right\}^Y \\ &\quad \times \left\{ \frac{\exp[-\theta_0 + \theta_1 a_{\max} + \theta_2 M_w + \theta_3 d_s + \theta_4 V_{s1} + \theta_5 k]}{1 + \exp[-\theta_0 + \theta_1 a_{\max} + \theta_2 M_w + \theta_3 d_s + \theta_4 V_{s1} + \theta_5 k]} \right\}^{1-Y}, \end{aligned} \quad (22)$$

where m is the number of total sites and Y is equal to 1 or 0 corresponding to liquefied sites and nonliquefied sites, respectively.

According to the principle of maximization of the likelihood function, the obtained $\hat{\theta} = [\hat{\theta}_0, \hat{\theta}_1, \dots, \hat{\theta}_5]$ is the best estimate of $\theta = [\theta_0, \theta_1, \dots, \theta_5]$ when $L(X; \theta)$ takes the maximum value. By solving the partial derivative of $L(X; \theta)$ to θ , a series of likelihood function equations can be established as follows:

$$\frac{\partial \ln[L(X; \theta)]}{\partial \theta_i} = 0, \quad i = 0, 1, 2, 3, 4, 5. \quad (23)$$

Substituting 225 sets of V_s data from the Andrus database into equation (23) yields the vector of coefficient $\theta = [\theta_0, \theta_1, \dots, \theta_5]$ through the Newton–Raphson algorithm [20]. The probability of liquefaction based on V_s can be written as follows:

$$P_L = \frac{1}{1 + \exp(5.75 - 8.592a_{\max} - 2.096M_w + 0.287d_s + 0.051V_{s1} + 0.846k)} \quad (24)$$

The critical shear wave velocity, V_{scr} , at a given probabilistic level can be obtained by equation (24) as follows:

$$V_{scr} = \frac{-5.75 + 8.592a_{\max} + 2.096M_w - 0.287d_s - 0.846k - \ln(P_L^*/1 - P_L^*)}{0.051} \quad (25)$$

where P_L^* is the given probabilistic level.

When the overburden pressure-corrected shear wave velocity, V_{s1} , is less than V_{scr} obtained by equation (25), the site is judged as being liquefied; otherwise, the site is judged as not liquefied.

4. Validation of the New Formula

4.1. Liquefaction Evaluation at Different Probabilistic Levels.

In comparison with the Andrus method and the Chinese code method, the reliability of the new formula is verified based on the Andrus database. In order to evaluate the balance of the liquefaction evaluation between liquefied site and nonliquefied site, the index of DI is defined herein as the difference in success rate between liquefied site and nonliquefied site. Figure 3 lists the success rate of liquefaction evaluation given by the new formula at different probabilistic levels. As the probabilistic level increases from 10% to 90%, the total success rate for liquefaction evaluation first increases and then decreases; correspondingly, the success rate for liquefied sites decreases from 100% to 19.8%, and nonliquefied sites increase from 34.7% to 96.8%, respectively. It can be concluded from equation (25) that the critical shear wave velocity of the site becomes smaller with the increase of given probabilistic levels. Thus, as the probabilistic level increases, the success rate for liquefied sites calculated by the new formula decreases and the success rate for nonliquefied sites increases.

The probability of liquefaction for saturated ground can be divided into five categories, which is shown in Table 5. It indicates the greater the P_L of the site, the more likely the site will liquefy. Thus, a lower probabilistic level can be adopted in order to enhance the success rate for liquefied sites. Important projects related to national security and buildings which may have serious secondary disasters induced by earthquake require special fortification. It can be seen from Figure 3 that the new formula at 10% probabilistic level gives a satisfying success rate for liquefied sites. Thus, the formula with a probabilistic level equal to 0–10% can be proposed for important engineering facilities, such as nuclear power plants and the three Gorges Dam in China. When the liquefaction probability level is between 10% and 30%, the new formula can satisfactorily predict the liquefaction with a success rate greater than 90% and nonliquefaction with 35%, respectively. Lifeline buildings whose functions cannot be

interrupted during an earthquake and should be recovered as soon as possible, such as hospitals and schools, can adopt the new formula with the probabilistic level ranging from 10% to 30% for seismic design. The new formula with a probabilistic level ranging from 30% to 60% can be used for civil buildings that will not cause large losses after an earthquake. The probabilistic level can be adjusted according to the importance of the projects.

4.2. Reevaluation Based on Andrus Database. The proposed formula at a given 50% probabilistic level is compared with the Andrus method and Chinese code method. The 50% probabilistic level is chosen because the critical line of the liquefaction evaluation represents the same probability of liquefaction and nonliquefaction. When the probabilistic level is set to be 50%, the difference in the success rate between liquefied and nonliquefied sites given by the new formula is only 0.2%. The success rates of liquefied sites and nonliquefied sites both approximate 80%, which fulfill the requirement of the critical line of liquefaction evaluation. It is proved that the new formula at a probabilistic level of 50% is theoretically safe and reliable.

Table 6 presents the success rates of liquefaction evaluation by the new formula. In seismic intensity VII, the success rates for liquefied sites and nonliquefied sites are 78.3% and 89.1%, respectively. In seismic intensity VIII, the success rates for liquefied sites and nonliquefied sites are 65.7% and 80%, respectively. Liquefaction evaluation gives a satisfactory success rate of 94.7% at liquefied sites, while 50% at nonliquefied sites in seismic intensity IX. The results are slightly conservative due to the lack of actual data of nonliquefied sites in seismic intensity IX. In conclusion, the new formula overcomes the conservative disadvantage of the Chinese code method or the Andrus method for liquefaction evaluation.

As mentioned in Section 2.4, the Andrus method and Chinese code method tend to be conservative, which are not optimal. For the Andrus method and Chinese code method, the higher success rate of liquefaction evaluation comes at the cost of the low success rate of nonliquefied sites. Obviously, it is not advisable or beneficial to cost control in engineering design. Therefore, the balance of the success rate of the new formula needs to be achieved between the liquefied sites and nonliquefied sites. It is undoubted that the new formula at $P_L^* = 50\%$ is more competitive than the

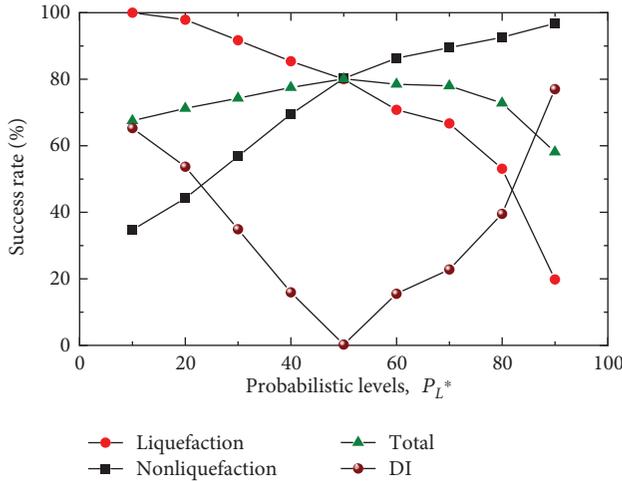


FIGURE 3: Success rate of liquefaction evaluation by new formula at different probabilistic levels.

TABLE 5: Probability of liquefaction for saturated ground and its describes.

| Probability | Class | Description (likelihood of liquefaction) |
|--------------------|-------|---|
| $0% < P_L < 10%$ | 1 | Almost certain that it will not liquefy |
| $10% < P_L < 30%$ | 2 | Liquefaction unlikely |
| $30% < P_L < 60%$ | 3 | Liquefaction and nonliquefaction equally likely |
| $60% < P_L < 80%$ | 4 | Liquefaction very likely |
| $80% < P_L < 100%$ | 5 | Almost certain that it will liquefy |

TABLE 6: Success rates of liquefaction evaluation by the new formula ($P_L^* = 50\%$).

| Seismic intensity | L (%) | NL (%) | Total (%) |
|-------------------|---------|--------|-----------|
| VII | 78.3 | 89.1 | 85.5 |
| VIII | 65.7 | 80 | 72.9 |
| IX | 94.7 | 50 | 82.7 |
| Total | 80.2 | 80 | 80.1 |

Andrus method and Chinese code method because it can fulfill the balance requirement.

5. Comparison of Liquefaction Evaluation with Different Methods

5.1. Liquefaction Data from Kayen Database. Andrus et al. [16] have compiled a significant number of liquefied case histories and recommended deterministic boundary curves with different fine contents for liquefaction evaluation. A total of 415 groups of liquefaction databases based on shear wave velocity published by Kayen et al. [9] provide an opportunity to compare the new formula proposed in this study with other methods. It should be noted that 75 groups of data are repetitive between the Andrus database and Kayen database. In addition, 4 sites in the Kayen database are around the boundary line between liquefaction and nonliquefaction, which are removed from the database. Thus, the remaining 336 groups of

TABLE 7: Data used in this study based on Kayen database [9].

| Seismic intensity | L | NL | Total |
|-------------------|------|------|-------|
| VI | None | 4 | 4 |
| VII | 23 | 17 | 40 |
| VIII | 80 | 29 | 109 |
| IX | 145 | 35 | 180 |
| X | 3 | None | 3 |
| Total | 251 | 85 | 336 |

data summarized in Table 7 are used to compare the performance of liquefaction evaluation between the new formula and other methods.

Figure 4 presents the data frequency distribution with the sandy-layer buried depth. Nearly 94% of sandy-layer buried depths at liquefied sites lie between 2 m and 10 m. Approximate 80% of sandy-layer buried depths range from 2 m to 8 m for nonliquefied sites. Figure 5 presents the data frequency distribution with ground water table. Statistically, most of the ground water table for both liquefied and nonliquefied sites lies in depths of 0 m to 4 m. Overall, the liquefied sites do not differ obviously from nonliquefied sites in the distribution of the sandy-layer buried depth and ground water table.

5.2. Comparison with Different Methods. In the Kayen database, there are 4 groups of nonliquefied sites in seismic intensity VI and 3 groups of liquefied sites in seismic intensity X. All 7 groups of samples were successfully predicted by using the new formula. Chinese code method based on shear wave velocity only covers the areas of seismic intensity VII, VIII, and IX. Thus, the samples in the seismic intensity VI and X are not discussed herein.

Table 8 compares the success rates of liquefaction evaluation based on the Kayen database in seismic intensity VII, VIII, and IX. In seismic intensity VII, the new formula gives a more dangerous evaluation result than the other two methods. The lower success rate at liquefied sites is mainly due to the lack of enough data in seismic intensity VII than other seismic intensities. However, in seismic intensity VIII and IX, the new formula can satisfactorily evaluate the liquefaction with success rates greater than 80% at liquefied sites. Moreover, in seismic intensity VIII or IX, the success rate obtained by the new formula at nonliquefied sites is far higher than that by the Chinese code method and Andrus method.

As displayed in Table 8, the total success rate at liquefied sites is 97.2% and 96.4%, corresponding to the Chinese code method and Andrus method, respectively. But the Chinese code method and Andrus method only give success rates of 23.5% and 15.4% for unliquefied sites, respectively. These two methods are obviously conservative. To be noted, the new formula proposed in this study gives the success rate of 79% and 67.9% for liquefied sites and nonliquefied sites, respectively. Therefore, considering the balance of the liquefaction evaluation between liquefied and nonliquefied sites, it is undoubtedly that the new formula proposed is more advantageous than the Chinese code method and Andrus method.

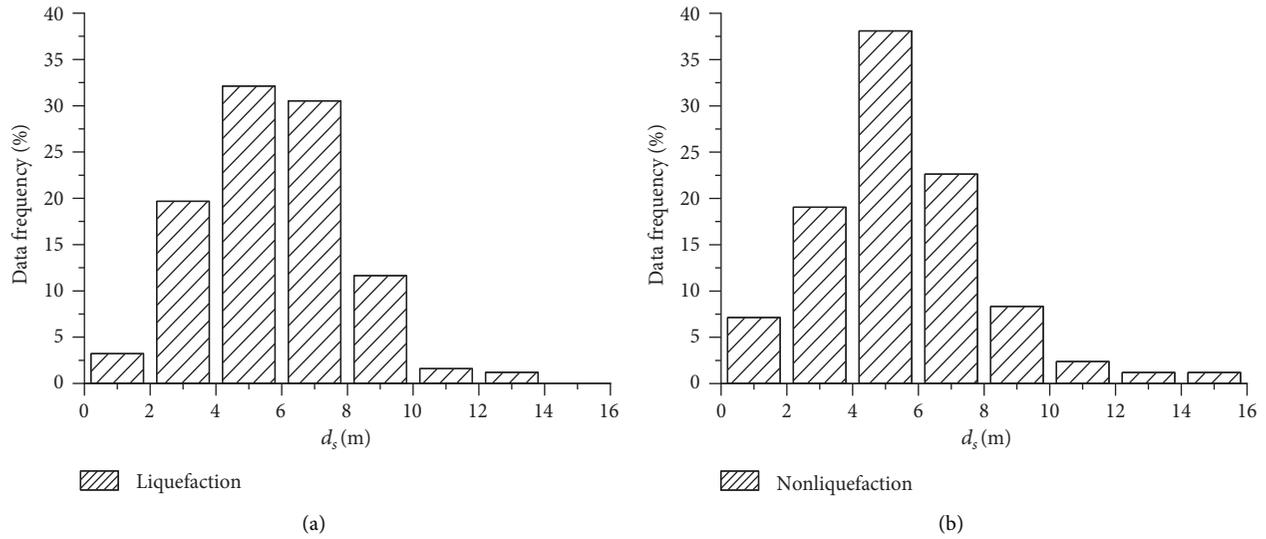


FIGURE 4: Histogram of d_s versus data frequency based on Kayen database: (a) liquefaction; (b) nonliquefaction.

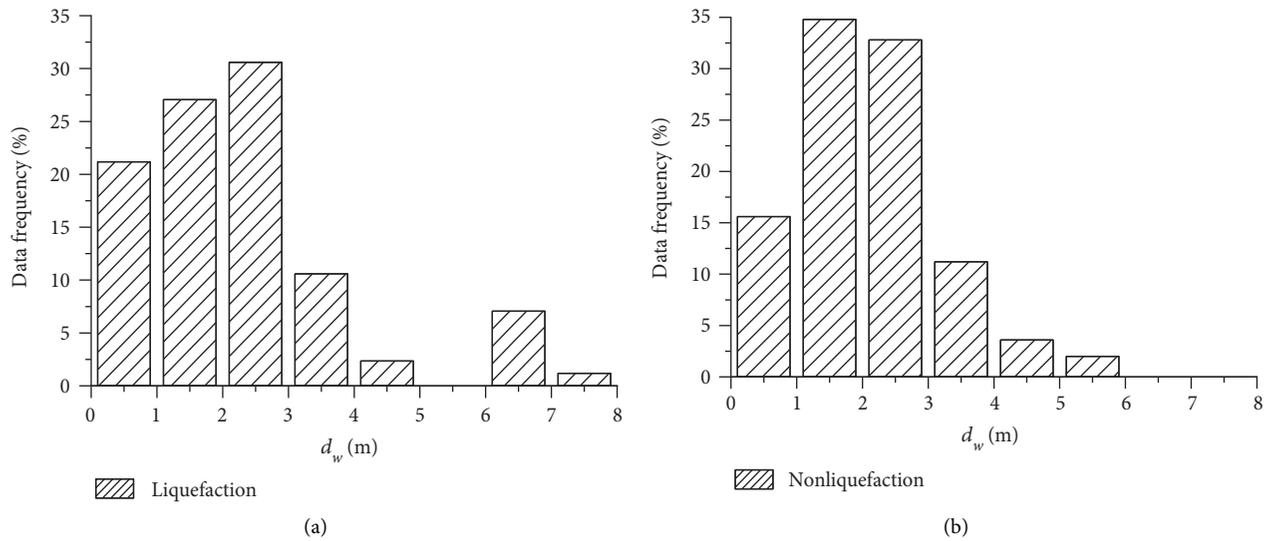


FIGURE 5: Histogram of d_w versus data frequency based on Kayen database: (a) liquefaction; (b) nonliquefaction.

TABLE 8: Success rates given by different methods based on Kayen database.

| Seismic intensity | L/NL | Chinese code method (%) | Andrus method (%) | The new formula ($P_L^* = 50\%$) (%) |
|-------------------|-------|-------------------------|-------------------|--|
| VII | L | 78.3 | 100 | 52.2 |
| | NL | 52.9 | 52.9 | 88.2 |
| | Total | 67.5 | 80 | 67.5 |
| VIII | L | 97.5 | 98.8 | 81.3 |
| | NL | 13.8 | 3.4 | 55.2 |
| | Total | 75.2 | 73.4 | 74.3 |
| IX | L | 100 | 94.5 | 82.1 |
| | NL | 17.1 | 11.4 | 68.6 |
| | Total | 83.9 | 78.3 | 79.4 |
| Total | L | 97.2 | 96.4 | 79 |
| | NL | 23.5 | 15.4 | 67.9 |
| | Total | 79 | 76.9 | 76.3 |

6. Conclusions

This study discussed the influence of several parameters on the liquefaction potential. Based on the logistic regression theory, a new formula was established to evaluate the site liquefaction. The new formula gave more balanced evaluation results between liquefied and nonliquefied sites by comparing with the Andrus method and Chinese code method. The main conclusions can be drawn as follows:

- (1) The liquefaction potential is explained by the five descriptive variables including d_s , k , V_{s1} , a_{max} , and M_w . These parameters are selected to build the new formula based on the logistic regression method.
- (2) The new formula can evaluate the liquefaction at different probabilistic levels. The result at the probabilistic level of 50% is relatively balanced than other probabilistic levels based on 225 sets of V_s data from the Andrus database. The success rates of liquefied sites and nonliquefied sites given by the new formula are both above 80% and the total success rate is 80.1%, which can be applied to a general project. For another type of project, the probabilistic level can be adjusted based on the importance.
- (3) The proposed formula at a $P_L^* = 50\%$ is verified by 336 case histories collected from the Kayen database. Three methods discussed in this study almost have the same success rate for liquefaction evaluation at all sites. However, the evaluating result obtained by the Chinese code method and the Andrus method is obviously conservative. The proposed formula predicts a success rate of 67.9% for nonliquefied sites, which is far higher than that of Chinese code method and Andrus method.

In summary, the evaluating results gained by the Chinese code method and Andrus method tend to be conservative in different seismic intensities. The new formula overcomes the disadvantage and gives a more balanced evaluation result.

Data Availability

The data of the Andrus database used to build the new formula are on pages 113–121 of the literature [16] (<https://nehrpsearch.nist.gov/static/files/NIST/PB99117897.pdf>). And the data of the Kayen database collected from Kayen et al. [9] are available online in the ASCE Library (<https://ascelibrary.org/doi/abs/10.1061/%28ASCE%29GT.1943-5606.0000743>).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

The authors would like to express their thanks to the financial support for this study from the Project of the National Natural Science Foundation of China (Grant nos.

51678300 and 51378257), Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant nos. 19KJB560015 and 18KJA560002), and Middle-Aged & Young Science Leaders of Qinglan Project of Universities in Jiangsu Province, China (Grant nos. QL20200203 and QL20210210).

References

- [1] H. Shahnazari, Y. Jafarian, M. A. Tutunchian, and R. Rezvani, "Probabilistic assessment of liquefaction occurrence in calcareous fill materials of kawaihae harbor, Hawaii," *International Journal of Geomechanics*, vol. 16, no. 6, Article ID 5016001, 2016.
- [2] A. Johari, A. R. Khodaparast, and A. A. Javadi, "An analytical approach to probabilistic modeling of liquefaction based on shear wave velocity," *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, vol. 43, no. 1, pp. S263–S275, 2019.
- [3] A. Johari and A. R. Khodaparast, "Analytical reliability assessment of liquefaction potential based on cone penetration test results," *Scientia Iranica*, vol. 21, no. 5, pp. 1549–1565, 2014.
- [4] W. J. Chang, "Evaluation of liquefaction resistance for gravelly sands using gravel content-corrected shear-wave velocity," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 142, no. 5, Article ID 04016002, 2016.
- [5] M. Shen, Q. Chen, J. Zhang, W. Gong, and C. Hsein Juang, "Predicting liquefaction probability based on shear wave velocity: an update," *Bulletin of Engineering Geology and the Environment*, vol. 75, no. 3, pp. 1199–1214, 2016.
- [6] S. Bhattacharya, R. P. Orense, and D. Lombardi, *Seismic Design of Foundations: Concepts and Applications*, London, UK, 2019, https://www.google.co.in/search?hl=en&q=inpublisher:%22ICE+Publishing%22&tbm=bks&sa=X&ved=2ahUKewi274_to9jwAhVOcCsKHe8ED6wQmxMoADALegQICBAC.
- [7] I. Idriss and R. Boulanger, "Soil liquefaction during earthquakes," in *Monograph MNO-12p*. 261, Earthquake Engineering Research Institute, Oakland, CA, USA, 2008.
- [8] W. EI-Sekelly, R. Dobry, T. Abdoun, and J. H. Steidl, "Two case histories demonstrating the effect of past earthquakes on liquefaction resistance of silty sand," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 143, no. 6, Article ID 04017009, 2017.
- [9] R. Kayen, R. E. S. Moss, E. M. Thompson et al., "Shear-wave velocity-based probabilistic and deterministic assessment of seismic soil liquefaction potential," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 140, no. 4, pp. 407–419, 2014.
- [10] P. K. Robertson, "Comparing CPT and vs liquefaction triggering methods," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 141, no. 9, Article ID 04015037, 2015.
- [11] K. S. Vipin and S. D. Anitha Kumari, "Probabilistic approach of liquefaction assessment for guwahati based on shear wave velocity values," *Lecture Notes in Civil Engineering*, vol. 86, pp. 299–308, 2021.
- [12] M. B. Asadi, M. S. Asadi, R. P. Orense, and M. J. Pender, "Shear wave velocity-based approach for estimating liquefaction resistance of natural pumiceous sands," *Geotechnique Letters*, vol. 8, no. 4, pp. 1–21, 2018.
- [13] R. Dobry, R. S. Ladd, F. Y. Yokel, R. M. Chung, and D. Powell, *Prediction of Pore Water Pressure Buildup and Liquefaction of Sands During Earthquakes by the Cyclic Strain Method*,

- Vol. 138, National Bureau of Standards, Gaithersburg, MD, USA, 1982.
- [14] H. B. Seed, I. M. Idriss, and I. Arango, "Evaluation of liquefaction potential using field performance data," *Journal of Geotechnical Engineering*, vol. 109, no. 3, pp. 458–482, 1983.
- [15] R. D. Andrus and K. H. I. Stokoe, "Liquefaction resistance based on shear wave velocity," in *NCEER Workshop on Evaluation of Liquefaction Resistance of Soils*, T. L. Youd and I. M. Idriss, Eds., pp. 89–128, National Center for Earthquake Engineering Research, 1997.
- [16] R. D. Andrus, K. H. Stokoe, R. M. Chung, and C. H. Juang, "Draft guidelines for evaluating liquefaction resistance using shear wave velocity measurements and simplified procedures," *NIST Interagency/Internal Report (NISTIR)–6277*, NCEER Publications, Taipei, Taiwan, 1999.
- [17] H. Yamazaki and S. Shinji, "Simplified liquefaction prediction and assessment method considering waveforms and durations of earthquakes," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 2, no. 143, 2017.
- [18] G. X. Chen, Q. Wu, K. Zhao, Z. F. Shen, and J. Yang, "A binary packing material-based procedure for evaluating soil liquefaction triggering during earthquakes," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 146, no. 6, Article ID 04020040, 2020.
- [19] N. Akbari-Paydar and M. M. Ahmadi, "Correlation of shear wave velocity with liquefaction resistance for silty sand based on laboratory study," *Japanese Geotechnical Society Special Publication*, vol. 2, no. 21, pp. 794–799, 2015.
- [20] S. S. C. Liao, D. Veneziano, and R. V. Whitman, "Regression models for evaluating liquefaction probability," *Journal of Geotechnical Engineering*, vol. 114, no. 4, pp. 389–411, 1988.
- [21] H. B. Seed, K. Tokimatsu, L. F. Harder, and R. M. Chung, "Influence of SPT procedures in soil liquefaction resistance evaluations," *Journal of Geotechnical Engineering*, vol. 111, no. 12, pp. 1425–1445, 1985.
- [22] T. L. Youd and S. K. Noble, "Liquefaction criteria based on statistical and probabilistic analyses," *National Center for Earthquake Engineering Research*, vol. 22, no. 97, pp. 201–216, 1997.
- [23] S. Toprak, T. L. Holzer, M. J. Bennett, and J. J. Tinsley, "CPT- and SPT-based probabilistic assessment of liquefaction potential," in *Proceedings of the 7th US-Japan Workshop on Earthquake Resistant Design of Lifeline Facilities and Countermeasures against Liquefaction*, Tokyo, Japan, 1999.
- [24] K. O. Cetin, A. D. Kiureghian, and R. B. Seed, "Probabilistic models for the initiation of seismic soil liquefaction," *Structural Safety*, vol. 24, no. 1, pp. 67–82, 2002.
- [25] S. Rahmanian and F. Rezaie, "Evaluation of liquefaction potential of soil using the shear wave velocity in Tehran, Iran," *Geosciences Journal*, vol. 21, no. 1, pp. 81–92, 2016.
- [26] R. D. Andrus, P. Piratheepan, B. S. Ellis, J. Zhang, and C. H. Juang, "Comparing liquefaction evaluation methods using penetration-Vs relationships," *Soil Dynamics and Earthquake Engineering*, vol. 24, no. 9–10, pp. 713–721, 2004.
- [27] R. D. Andrus, K. H. Stokoe, and C. Hsein Juang, "Guide for shear-wave-based liquefaction potential evaluation," *Earthquake Spectra*, vol. 20, no. 2, pp. 285–308, 2004.
- [28] Code for Investigation of Geotechnical Engineering, GB 50021–2009, China Architecture and Building Press, Beijing, 2009.
- [29] S. Rui and X. M. Yuan, "Depth-consistent Vs-based approach for soil liquefaction evaluation," *Chinese Journal of Geotechnical Engineering*, vol. 41, no. 3, pp. 439–447, 2018.
- [30] Seismic Ground Motion Parameters Zonation Map of China, GB 18306–2015, Beijing, 2015.
- [31] H. B. Seed and I. M. Idriss, "Simplified procedure for evaluating soil liquefaction potential," *Journal of the Soil Mechanics and Foundations Division*, vol. 97, no. 9, pp. 1249–1273, 1971.
- [32] S. S. C. Liao and R. V. Whitman, "Overburden correction factors for SPT in sand," *Journal of Geotechnical Engineering*, vol. 112, no. 3, pp. 373–377, 1986.
- [33] Z. Y. Li, X. M. Yuan, and R. Sun, "Variation tendencies and general rules for critical curve of liquefaction evaluation," *Chinese Journal of Rock and Soil Mechanics*, vol. 40, no. 9, pp. 3603–3609, 2019.
- [34] S. K. Umar, P. Samui, and S. Kumari, "Deterministic and probabilistic analysis of liquefaction for different regions in Bihar," *Geotechnical and Geological Engineering*, vol. 36, no. 5, pp. 3311–3321, 2018.
- [35] B. O. Hardin and V. P. Drnevich, "Shear modulus and damping in soils: design equations and curves," *Journal of Soil and Mechanics and Foundations Division*, vol. 98, no. 118, pp. 667–692, 1972.
- [36] T. Kokusho, "Cyclic triaxial test of dynamic soil properties for wide strain range," *Soils and Foundations*, vol. 20, no. 2, pp. 45–60, 1980.
- [37] K. Tokimatsu and A. Uchida, "Correlation between liquefaction resistance and shear wave velocity," *Soils and Foundations*, vol. 30, no. 2, pp. 33–42, 1990.