Research Article

Analysis of Dynamic Response Mechanism of Roadway Bolt

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This work elucidates dynamic control equations of the anchoring system and the derivation of displacement equations and corresponding vibration modes. Furthermore, the anchoring system is found to be composed of three different vibration modes: (1) when $\omega < (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the anchoring section is an exponential function; (2) when $\omega = (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the anchoring section is a parabolic function; (3) when $\omega > (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the anchoring section is a trigonometric function, while all the free sections are trigonometric functions. With an increase of frequency, the amplitude of the bolt exhibits multipeak distribution characteristics and an intermittent amplification phenomenon. When the frequency reaches a certain value, the bolt of the free section exhibits only the amplified state. Under dynamic load, the amplitude of the bolt increases from end of bolt to the maximum in the root. On the other hand, when the frequency is low, the peak position of the roof bolt is stable, and the excitation wave component is the main influencing factor of the peak value of axial force at the root of the bolt, independent of frequency. When the frequency is relatively high, the peak value of the axial force is stable at the interface, and the higher the frequency, the greater the peak value of axial force. Axial force of the bolt has responded strongly to the frequency at the interface, and the farther away from the interface, the weaker the response.

1. Introduction

Anchor bolts' anchoring and failure mechanism have become the focal point of current anchoring technology because the standard static support system does not always face extreme dynamic load effects, and the roadway has a certain level of seismic resistance [1, 2]. Therefore, the static support design can often achieve the stability control of the surrounding rock under dynamic load. However, practice shows that the bearing capacity experimental results of the mentioned above factors may be inaccurate or even dangerous in applying roadway support design [3, 4].

Coal mining activities and rock dynamic phenomena (manifested as roof instability, gas outburst, coal burst, fault slip, and others) are always accompanied by dynamic load, and then, energy is released in the form of stress wave in the

coal and rock masses [3, 4]. Since the mechanical effects of acceleration, velocity, frequency, and time on the bolt members are rarely considered in support design, the resulting dynamic response of the bolted surrounding rock structure is often the main inducing factor for roadway failure and instability [4, 5]. Under the action of dynamic load, the force fluctuation of the bolt is large, and the failure of the anchoring structure is severe, which is mainly manifested by the release of the adhesive slip of the bolt body, the breaking of the bolt rod, the constant extension of the tray, and the tearing of the metal mesh, as shown in Figure 1.

Correspond to the stress environment under dynamic load in coal mining activities to obtain better control of the surrounding rock, for example, when the support tends to increase the length of anchor rod as the main means [6].

FIGURE 1: Failure of anchorage structure: support failure mode is caused by rockburst in Yi Ma mine, with obvious surface peeling and serious rock bolt debonding.

However, studies have shown that the neutral point position tends to stabilize after the anchor length exceeds a certain value, meaning that the bolt support effect cannot be achieved by simply extending the bolt length [7]. In addition, the multiple neutral points that may occur in the bolts under dynamic loading further suggest that the use of unilateral lengthened bolts to improve the support performance of the anchorage system may be unreliable [8].

Undoubtedly, a good anchorage system should be capable of challenging both the static and dynamic loads [9]. In the early 1980s, a joint research project was undertaken at the University of Aberdeen to study the mechanical response of bolted anchoring systems under transient impact loading [10], involving measurements at the active construction site and reinforced by laboratory and computer models. It is a preliminary research of the dynamic response characteristics of the bolts. The dynamic stability and mechanism of the dynamic response of anchored structures are broad topics involving rock mass mechanics, geology, dynamic materials, tribology, and others [11]. However, due to the late start, the diversity of anchorage forms, and the randomness and complexity of dynamic loads, the research on the dynamic response mechanism of anchored structures under dynamic loads far behind the engineering practice.

In recent years, the dynamic response mechanism of the bolt has aroused the research interest of many scholars and even become one of the focal points in the research field. In addition, recent findings on the dynamic response of bolts have provided new ideas for this research. For example, based

on considering the nonlinear characteristics of composite materials and loosening boundary conditions, Li H. studied the nonlinear vibration characteristics of cylindrical shells from both theoretical and experimental aspects, including the natural frequency, modal damping ratio, resonance response amplitude of bolts, and others [12]. Qin et al., based on the three-dimensional nonlinear finite element model, deduced the motion equation of bolt-loosening rotors when local stiffness changes after calculating the time-varying stiffness [13] and to establish the analytical model of the flexural stiffness of the bolted disk-drum joints, also performed the corresponding dynamic response analysis [14]. Furthermore, because of the sudden and random nature of dynamic loading in mining activities, there are some limitations in timely and extensively monitoring the dynamic response of anchorage characteristics, and creating external excitation conditions as dynamic loading seems to be gaining importance [3, 15, 16]. Therefore, research on the dynamic response characteristics of the anchoring system has mainly focused on three aspects:

- (1) An elastic or elastoplastic model with concentrated anchoring system parameters is established, and the dynamic characteristics of the anchoring system under external load excitation are discussed, including the effects of natural frequencies and anchoring system parameters. For example, Ivanovic et al. centralized the numerical calculation models based on the effective difference method and analyzed the effect of prestressing on the dynamic response of the anchor under the impact loading [17].

- (2) According to the geological conditions and dynamic load characteristics, similar material simulation experiments are carried out, or semicontrol blasting experiments are conducted on the engineering site to study the time-history variation rules of anchor shaft force [3, 9]. For example, through explosive loading, Tannant et al. [18] conducted an experimental research on the dynamic response of bolt and measured the response speed of bolt under two different explosive effects. In addition, [19] analyzed the mechanical characteristics of the full-length bolt and end-anchored bolt through the homogeneous material and concluded that the end-anchored bolt had a higher strengthening in the beginning performance.
- (3) A numerical simulation model was developed to investigate the dynamic response mechanism of the axial force of the bolt and the support effect of the roadway surrounding rock under dynamic loading [20]. Ivanovic et al. [17] established a model of the dynamic response of rock bolts under impact forces, performed simulation experiments and numerical analysis, and analyzed the effect of displacement and stress response of rock bolts under impact loads and frequencies. Through numerical simulation, Yang et al. [21] found by numerical simulation that the axial strain response varies with different bolt installation angles under dynamic blast loads. Xue et al. [22] studied the influence of ground vibration load on the axial force of supporting bolt by numerical simulation and assumed that the bolt installation angle had a greater impact on the dynamic load response mechanism of axial force, and the dynamic load failure of end-anchor support roadway was smaller than that of full-length anchorage roadway.

Generally, the main factors affecting the dynamic response characteristics of the bolts can be summarized into three levels [23–27]:

- (1) The physical and mechanical properties of the surrounding rock of the roadway are shear modulus, elastic modulus, Poisson's ratio, and rock mass density
- (2) Bolt morphology and physical and mechanical characteristics are rock bolt length, anchorage length, free length, installation angle, diameter, modulus of elasticity, and density
- (3) The influences of excitation wave are incident angle, propagation velocity, amplitude, frequency, vibration mode function, and the time and composition of excitation wave

After a long period of effective research, many scholars have made breakthroughs in the bolt stress characteristics, both under the static load or under the dynamic static load coupling, which have greatly enriched the basic theory of the anchoring system and guided roadway support design [28, 29]. For example, in terms of static load support, Lin

et al. established and revised the shear strength correction model by introducing and determining the influence coefficient [30]. At the same time, direct shear tests of bolts with different grouting states, quantities, and angles were carried out to study the mechanical behavior of bolts under load [31]. In terms of dynamic and static load coupling, the high-frequency excitation contributes little to the overall dynamic response of the anchoring system, and it is generally agreed that the seismic design of the anchoring system should pay attention to the mechanical response of low-frequency excitation [32, 33]. However, taking a comprehensive look at the existing research, it is undeniable that there are still some limitations in the stress state. Axial force distribution law of bolt under dynamic load, the influencing factors of axial force growth in terms of the dynamic response mechanism and time history, and the failure forms of an anchoring system have a certain theoretical value and the necessity of research [34, 35].

In this study, the axial dynamic model of the anchorage system is established by assuming conditions based on neglecting secondary factors, and the displacement equations of the anchoring system under steady-state conditions are derived by the method of separating variables. It is considered that the anchoring system can be regarded as a composite member with three different vibration modes under different frequency conditions. The feasibility of the theoretical model is verified through the theoretical calculation comparison of FLAC^{3D} and MATLAB programming. Next, the vibration characteristics and amplitude response of the bolts under different excitation conditions are analyzed. Based on this, the dynamic response mechanism of the axial force of the bolt under low-frequency excitation is analyzed. Moreover, by defining the axial force amplification coefficient, the dynamic response characteristics of the axial force of the bolt under multifactor coupling conditions are discussed.

2. Dynamic Model of the Anchoring System

2.1. Vibration Control Equation. In establishing the axial dynamic model of the anchoring system, as shown in Figure 2, the following assumptions are proposed for the model based on the neglecting of secondary factors [36–40]:

- (1) The anchor solid is a continuous elastic homogeneous rod with axial deformation satisfying the plane assumption
- (2) The anchor and the surrounding rock are isotropic homogeneous elastomers
- (3) The interaction between the anchor and the surrounding rock is achieved by a linearly elastic spring
- (4) The impact of other dynamic factors, such as additional stress generated by the deformation of the surrounding rock, is not considered
- (5) Neglecting the cumulative effect of the amount of change in the cross-section of the anchor in axial tension or compression

FIGURE 2: Axial dynamic model of the anchorage system.

- (6) In practical engineering, the factors such as the weak structural surface are not considered
- (7) The effect of waveform distortion is not considered, and the rock body can fully absorb the energy.

Let the mode function is u_g , and the angle between incident wave direction and bolt axial direction is α . Then, the motion equation of the microelement body is $u_i(x, t)$; length is dx , the mass is dm , the equivalent elastic modulus is E_i , the equivalent density is ρ_i , the cross-sectional area is A_i , the lateral spring stiffness is k_i , the diameter is D_i , the shear modulus of surrounding rock is G_s , and the equivalent mass of nut and tray is m_1 .

Therefore, the inertia force between nut and pallet bolt is $m_1 \cos \alpha \partial^2 u_g / \partial t^2$, the mass of microelement is $dm = \rho_i A_i dx$, the inertia force is $dm \cdot \partial^2 u_i / \partial t^2$, the axial forces on the left and right cross-sections are $E_i A_i \sigma_i$ and $E_i A_i (\sigma_i + \partial \sigma_i / \partial x \cdot dx)$, and the shear force is $k_i u_i(x, t) dx$. According to the dynamic equilibrium equation [41–43],

$$A_i \frac{\partial \sigma_i}{\partial x} dx - \frac{\partial^2 u_i(x, t)}{\partial t^2} \rho_i A_i dx = k_i u_i(x, t) dx - m_1 \frac{\partial^2 u_g \cos \alpha}{\partial t^2} dx. \quad (1)$$

According to the physical equation $\sigma_i = E_i \partial u_i / \partial x$, we can get

$$E_i A_i \frac{\partial^2 u_i(x, t)}{\partial x^2} - \frac{\partial^2 u_i(x, t)}{\partial t^2} \rho_i A_i = k_i u_i(x, t) - m_1 \frac{\partial^2 u_g \cos \alpha}{\partial t^2}. \quad (2)$$

Dividing both sides by $E_i A_i$, we can get

$$\frac{\partial^2 u_i(x, t)}{\partial x^2} - \frac{\rho_i}{E_i} \frac{\partial^2 u_i(x, t)}{\partial t^2} = \frac{k_i}{E_i A_i} u_i(x, t) - \frac{m_1}{E_i A_i} \frac{\partial^2 u_g \cos \alpha}{\partial t^2}. \quad (3)$$

In the formula, i is 1 or 2, when $i = 1$, the microelement is taken from the anchoring section, and $k_i = k_1 = 2.75 G_s / \pi D_1$, $u_1(x, t)$, E_1 , ρ_1 , A_1 , and D_1 are the motion equation, equivalent elastic modulus, equivalent density, cross-sectional area, and diameter of the anchoring body, respectively. When $i = 2$, the microelement is taken from the free

section, and $k_i = k_2 = 0$, $u_2(x, t)$, E_2 , ρ_2 , A_2 , and D_2 are the motion equation, elastic modulus, density, cross-sectional area, and diameter of the bolt, respectively.

According to the Fourier analysis results of complex source time history, it is shown that any aperiodic function can be expressed as a simple combination of sine and cosine functions. So, this study only considers the case of simple harmonic vibration. It is assumed that the movement equation of roadway bedrock is $u_g = U_g \cdot T(t)$, where U_g is the bedrock amplitude, and it is usually constant. The vibration time function $T(t) = a \cos \omega t + b \sin \omega t$ can be obtained by substituting it into equation (3); then, we can get

$$\frac{\partial^2 u_i(x, t)}{\partial x^2} - \frac{\rho_i}{E_i} \frac{\partial^2 u_i(x, t)}{\partial t^2} - \frac{k_i}{E_i A_i} u_i(x, t) = \frac{m_1}{E_i A_i} \omega^2 U_g T(t) \cos \alpha. \quad (4)$$

2.2. Solution of Bolt Displacement Equation in Anchorage Section. According to the method of separation variables $u_1(x, t) = U_1(x) \cdot T(t)$, then

$$\begin{cases} \frac{\partial^2 u_1(x, t)}{\partial x^2} = U_1''(x) T(t), \\ \frac{\partial^2 u(x, t)}{\partial t^2} = -\omega^2 U_1(x) T(t). \end{cases} \quad (5)$$

Substituting into equation (4), we can obtain equation (6):

$$\left[U_1''(x) + \left(\frac{\rho_1 \omega^2}{E_1} - \frac{k_1}{E_1 A_1} \right) U_1(x) \right] T(t) = \frac{m_1}{E_i A_i} \omega^2 U_g \cos \alpha T(t). \quad (6)$$

According to the basic principles of mathematical and physical equations, all solutions are nontrivial, and dividing both sides by $T(t)$, we can get

$$U_1''(x) + \left(\frac{\rho_1 \omega^2}{E_1} - \frac{k_1}{E_1 A_1} \right) U_1(x) = \frac{m_1}{E_i A_i} \omega^2 U_g \cos \alpha. \quad (7)$$

When $\rho_1/E_1\omega^2 - k_1/E_1A_1 = B$, $m_1/E_1A_1\omega^2 U_g \cos \alpha = D$; then, the solution of equation (7) is as follows.

If $B > 0$, then $\omega > (k_1/\rho_1A_1)^{1/2}$, characteristic equations have complex conjugate roots, and the solution is

$$U_1(x) = C_1 \cos(\sqrt{B}x) + C_2 \sin(\sqrt{B}x) + \frac{D}{B}. \quad (8)$$

If $B = 0$, $\omega = (k_1/\rho_1A_1)^{1/2}$; then, the solution of the equation is

$$U_1(x) = \frac{1}{2}Dx^2 + C_1x + C_2. \quad (9)$$

If $B < 0$, then $\omega < (k_1/\rho_1A_1)^{1/2}$, characteristic equations have different real roots, and the solution of it is

$$U_1(x) = C_1 e^{(\sqrt{-B}x)} + C_2 e^{(-\sqrt{-B}x)} + \frac{D}{B}. \quad (10)$$

2.3. Solution of Bolt Displacement Equation in a Free Segment. According to the principle of separation of variables, can set $u_2(x, t) = U_2(x) \cdot T(t)$; then,

$$U_2''(x) + \omega^2 \frac{\rho_2}{E_2} U_2(x) = \omega^2 \frac{\rho_2}{E_2} U_g \cos \alpha. \quad (11)$$

Because of $G = \rho_2\omega^2/E_2 > 0$, characteristic equations have complex conjugate roots, and the solution of it can be

$$U_2(x) = C_3 \cos(\sqrt{G}x) + C_4 \sin(\sqrt{G}x) + U_g \cos \alpha. \quad (12)$$

2.4. Initial Conditions and Undetermined Constants. By observing equations (8), (9), (10), and (12), the solution process of constant terms C_1 , C_2 , C_3 , and C_4 must be completed to solve the bolt displacement equation entirely. Let the bolt root axial force be F_0 at static load equilibrium, and according to the axial dynamic model of the anchorage system, the corresponding boundary condition is

$$\begin{cases} E_1 A_1 \frac{\partial u_1(x, t)}{\partial x} \Big|_{x=0} = k_0 A_1 u_1(0, t), \\ E_2 A_2 \frac{\partial u_2(x, t)}{\partial x} \Big|_{x=L_2, t=0} = F_0, \end{cases} \quad (13)$$

where k_0 is the bottom spring stiffness, $k_0 = 8G_s/\pi D_1(1 - \mu_s)$, and μ_s is Poisson's ratio of surrounding rock. At the same time, the displacement and strain continuity conditions should also be satisfied between the anchor segments as

$$\begin{cases} u_1(L_1, t) = u_2(L_2, t), \\ \frac{\partial u_1(x, t)}{\partial x} \Big|_{x=L_1} = \frac{\partial u_2(x, t)}{\partial x} \Big|_{x=L_1}. \end{cases} \quad (14)$$

Combining the initial and continuous conditions of the dynamic model, see Appendix for the results of solving for the constant terms C_1 , C_2 , C_3 , and C_4 .

Combined with the above analytical results, set the anchor bolt axial force of the anchorage section and the free section as $F_{N1}(x, t)$ and $F_{N2}(x, t)$. According to the equation $\sigma_i = E_i \partial u_i / \partial x$,

$$\begin{cases} F_{N1}(x, t) = E_1 A_2 \sqrt{-B} (C_1 e^{(\sqrt{-B}x)} - C_2 e^{(-\sqrt{-B}x)}) T(t), & 0 \leq x \leq L_1, \\ F_{N2}(x, t) = E_2 A_2 \sqrt{G} (C_4 \cos(\sqrt{G}x) - C_3 \sin(\sqrt{G}x)) T(t), & L_1 \leq x \leq L_2. \end{cases} \quad (15)$$

Defining the amplification factor of the anchor shaft force as $\beta(x, t)$, combined with the above, we can see that

$$\begin{cases} \beta_1(x, t) = \frac{F_{N1}(x, t)}{F_N(x, 0)}, & 0 \leq x \leq L_1, \\ \beta_2(x, t) = \frac{F_{N2}(x, t)}{F_N(x, 0)}, & L_1 \leq x \leq L_2. \end{cases} \quad (16)$$

3. Case Verification and Theoretical Analysis

3.1. Case Verification. The calculated model is 42 m long, 45 m high, 4.0 m roadway width, 3.0 m high, and with the embedded depth of 200 m. The research of Kuhlemeyer and Lysmer indicated that the frequency component would affect the numerical accuracy of stress wave propagation [44]. To obtain accurate simulation results, the model network

$\Delta l < 0.1 - 0.125 \lambda$, where λ is the wavelength corresponding to the highest frequency of the excitation wave, combined with geological engineering conditions, $\Delta l = 0.50$ m, and the bulk density of the overlying strata is applied equivalently by 25 MPa/km. Vertical stress above the model is 4.0 MPa. The lateral stress coefficients are all 0.75. Bolt spacing is 0.8×0.8 m, $L_2 = 12$ m, $L_1 = 6$ m, bolt diameter is 22 mm, modulus of elasticity is 200 GPa, and the bolt density is 7800 kg/m³. Physical and mechanical parameters of coal measure rock strata are listed in Table 1. The roadway, simulation model, and dynamic load are shown in Figure 3. The displacement of different sections of roof bolt is monitored during model calculation.

The dynamic load is applied to the roadway roof, and the oscillation time function is $T(t) = \cos \omega t + \sin \omega t$ with an amplitude of 0.20 m. Through the monitoring results of FLAC^{3D}, $F_0 = 80$ kN; the above parameters are put into the theoretical calculation formula, and through the monitor of

TABLE 1: Physical and mechanical parameters of coal measure rock strata.

Rock	Density ($\text{kg}\cdot\text{m}^{-3}$)	Elasticity modulus (GPa)	Shear elasticity (GPa)	Tensile strength (GPa)	Poisson's ratio	Cohesion (GPa)	Internal friction angle ($^{\circ}$)
Sandstone	2550	8.17	3.8	2.93	0.23	1.63	42
Mudstone	2600	5.00	0.7	2.10	0.30	1.35	36
Coal	1475	5.12	0.9	0.69	0.25	1.08	28
Packsand	2650	13.36	7.6	3.28	0.24	1.76	46

FIGURE 3: Roadway, simulation model, and dynamic load location. (a) Roadway and simulation model. (b) Roadway support and dynamic load position.

FLAC^{3D}, the displacement time-history curves of different section locations are shown in Figure 4. For example, let the displacement of FLAC^{3D} monitored is $u_{\text{simulation}}(x, t, \omega)$ and the displacement calculated theoretically is $u_{\text{theory}}(x, t, \omega)$; then, Δu is the following equation.

$$\Delta u = u_{\text{theory}}(x, t, \omega) - u_{\text{simulation}}(x, t, \omega). \quad (17)$$

- (1) The vibration laws of the theoretical calculation and numerical simulation are similar, and they are in good agreement. The difference is that the theoretical calculation amplitude is larger than the monitoring results of the FLAC^{3D} simulation. According to the analysis, the main reason may be that the theoretical model ignores the influence of the rock mass damping effect. Overall, the variance is controlled below 10%, which indicates that it is feasible to analyze the dynamic response of the anchoring system with the above theoretical model, and the results of the theoretical derivation and case calculations are reliable.
- (2) Further analysis reveals that after entering the steady-state vibration, the longer the time, $|\Delta u|_{\text{max}}$ gradually decreases, roughly decaying as a logarithmic function. The smallest difference in the amplitude of the bolt is free section due to the cumulative effect of the change in the length and cross-sectional area of the bolt after dynamic loading.

3.2. Theoretical Analysis of Mode Function. The shear modulus of the roadway surrounding rock is 0.7 GPa, modulus of elasticity is 5.0 GPa, Poisson's ratio is 0.3, and rock density is 2600 kg/m^3 . A screw bolt with a diameter of 22 mm is selected, bolt spacing is $0.8 \times 0.8 \text{ m}$, modulus of elasticity is 200 GPa, and the bolt density is 7800 kg/m^3 . The amplitude of bedrock in the roadway is 0.2 m. Let $L_2 = 12.0 \text{ m}$, $L_1 = 6.0 \text{ m}$, and $F_0 = 80 \text{ kN}$; then, the oscillation time function is $T(t) = a \cos \omega t + b \sin \omega t$, and the rock bolt vibration modes at different frequencies are shown in Figure 5. Combining the figure with the solution of the above displacement equation, we can know the following:

- (1) As an important parameter of the vibration time function, the influence of frequency on the vibration mode of the bolt in the anchoring section is understandable and corresponding to different frequencies, and the vibration mode of the bolt in the anchorage section is different. Usually, when $\omega < (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the bolt in the anchoring section is an exponential function, when $\omega = (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the bolt in the anchoring section is parabolic, and when $\omega > (k_1/\rho_1 A_1)^{1/2}$, the vibration mode of the bolt in the anchored section is a trigonometric function.
- (2) The frequency has no effect on the vibration mode of the anchor within the free section, which is always in the form of the triangular function, which is consistent with Dr. Duan study on the free vibration of the slope anchor [41, 45]. Therefore, no matter forced

(a)

(b)

(c)

(d)

(e)

(f)

FIGURE 4: Time-history curves of displacements of different sections. (a) Comparison results of vibration modes (300 rad/s). (b) Amplitude difference between theory and simulation (300 rad/s). (c) Comparison results of vibration modes (765 rad/s). (d) Amplitude difference between theory and simulation (765 rad/s). (e) Comparison results of vibration modes (1000 rad/s). (f) Amplitude difference between theory and simulation (1000 rad/s).

FIGURE 5: Modes of rock bolt vibration at different frequencies. (a) Mode of vibration ($\omega > (k_1/\rho_1 A_1)^{1/2}$). (b) Mode of vibration ($\omega = (k_1/\rho_1 A_1)^{1/2}$). (c) Mode of vibration ($\omega < (k_1/\rho_1 A_1)^{1/2}$).

vibration or free vibration, the vibration mode is the same in the free section, independent of frequency.

3.3. Theoretical Analysis of Equation of Motion. For the roadway bolt anchoring system, there is an empty surface at the root of a bolt, and the amplitude response of the bolt has an important influence on the stability control of the roadway surrounding rock. Therefore, it is essential to analyze the law of the amplitude response of any bolt section at different frequencies, as shown in Figure 6.

- (1) The response law of amplitude with frequency is complex and presents multippeak distribution characteristics. When the frequency is low, the law of the random bolt section with frequency response is consistent. However, as the frequency increases, the bolt in the anchoring section and free section may have different vibration directions. For example, when the frequency $\omega = 1900 \text{ rad/s}$, there is a point in the bolt with zero amplitude in this frequency time, i.e., the point is always at rest during the steady-state vibration.
- (2) Generally, the amplitude increases gradually from the end of bolt to maximum at the root of a bolt. Therefore, it may explain the failure, empty the surface at the root of the bolt, under dynamic loading, and the root of the bolt has the largest amplitude that inevitably leads to the typical failure phenomenon of obvious rock mass spall on the roadway surface and serious debonding failure of the bolt under the condition of failure.
- (3) When $\omega = 400 \text{ rad/s}$, the vibration amplitude of bolt is in the range of 3.10–4.20 m that is much larger than the amplitude of 0.2 m of roadway bedrock and much larger than the amplitude of bolt at other frequencies. As the dynamic load acting on the roadway is generally dominated by low-frequency excitation, the dynamic response of the bolt has an optimal frequency corresponding to the vibration amplitude of the bolt under certain conditions,

which is the best frequency between 300 and 500 rad/s in this example.

- (4) If we set $\xi = |U(x)|_{\max}/U_g$ as the amplitude of amplification coefficient, when the frequency is between 300 and 900 rad/s or 1900–2000 rad/s, the amplification factor of the anchored section is greater than 1.0. Similarly, when the frequency is between 0 and 200 rad/s, the amplitude of the amplification factor of the bolt in the free section is smaller than 1.0. While, when the frequency is larger than 300 rad/s, the amplitude of amplification coefficient of the bolt in the free section is larger than 1.0. Thus, it can be seen that the amplitude of the anchor segment bolts may be amplified intermittently as the frequency increases, although the amplitude of the free segment bolts is amplified when the frequency is greater than a certain value.

4. Dynamic Response Mechanism of the Bolt Axial Force

According to [46], monitoring on the occurrence of small roadway vibration, the natural vibration frequency of underground roadway engineering is generally low, and the response degree to high-frequency excitation is weak [45]. This conclusion has also been further confirmed by similar material simulation experiments conducted by [47]; the vibration frequency caused by roof collapse is 0–25 Hz. To illustrate the implication of problems, the dynamic response of the bolt axial force is when $\omega \leq 300 \text{ rad/s}$, and the corresponding parameters are taken from the above case. Because of $\omega < \sqrt{k_1/\rho_1 A_1}$, the vibration modes of anchor bolt in the anchor section are free section exponential and trigonometric functions, respectively.

By substituting corresponding parameters into the above theoretical equation, we can get that the law of the bolt axial force response with frequency is shown in Figures 7(a) and 7(b); the law of peak dynamic response of bolt at different sections is shown in Figures 7(c) and 7(d).

FIGURE 6: Amplitude response at different frequencies. (a) Whole section amplitude of bolt (20–180 rad/s). (b) Whole section amplitude of bolt (20–180 rad/s).

FIGURE 7: Continued.

FIGURE 7: Law of the bolt axial force response with frequency. (a) Axial force at different frequencies (20–160 rad/s). (b) Axial force at different frequencies (160–300 rad/s) (c) Peak value of axial force in different sections (20–160 rad/s). (d) Peak value of axial force in different sections (160–300 rad/s).

- (1) The bolt axial force presents a triangular single peak distribution pattern. When the frequency is low, the peak of axial force appears in the root of the bolt. As the frequency increases, in addition to the root of the bolt, a reduction in axial force of varying degrees occurs at any cross-sectional location of the bolt, consequently, the first appearance of a neutral point in the anchored section. Therefore, it may be one-sided, even invalid, and futile to attempt to obtain a better roadway control effect by increasing the length of bolt or anchoring section without considering the influence of the main frequency effect of dynamic load. In other words, the bolt length and anchoring length should be reduced while meeting the support strength.
- (2) At higher frequencies, the peak value of axial force appears and stabilizes at the interface. This phenomenon has been reflected in the analysis of axial stress distribution along the bolt body carried out by [48]. With the increasing frequency, the peak value of the axial force on the interface becomes more and more obvious, and the neutral point is closer to the interface.
- (3) The two sides of the zero-axis force section exhibit opposite dynamic response characteristics in tension and compression. It is worth noticing that frequency does not affect the peak axial force of the bolt root. Theoretically, with an increase of frequency, the peak

value of axial force at the root of the bolt proved as follows:

The axial force of the bolt during steady vibration is

$$F_{N2}(x, t) = E_2 A_2 \sqrt{G} (C_4 \cos(\sqrt{G} L_2) - C_3 \sin(\sqrt{G} L_2)) T(t). \quad (18)$$

According to the above constant C_3 and C_4 solving process, it can be known that

$$C_4 = C_3 \tan(\sqrt{G} L_2) + \frac{F_0}{E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)}. \quad (19)$$

That is,

$$E_2 A_2 \sqrt{G} (C_4 \cos(\sqrt{G} L_2) - C_3 \sin(\sqrt{G} L_2)) = \frac{F_0}{a}. \quad (20)$$

Submitting equation (19) in equation (18), we can get

$$F_{N2}(L_2, t) = F_0 \sqrt{\frac{(a^2 + b^2)}{a^2}} \sin(\omega t + \theta). \quad (21)$$

In the equation, $\theta = \arctan(a/b)$; then, we have

$$|F_{N2}(L_2, t)|_{\max} = F_0 \sqrt{\frac{(a^2 + b^2)}{a^2}}. \quad (22)$$

According to equation (21), the peak value of the bolt root axial force is certain after steady-state vibration, which is independent of the frequency but related to the mode function, i.e., the excitation wave component is the main influencing factor of the peak value of the bolt root axial force.

- (4) In the anchorage section, the axial force of the bolt gradually increases to the frequency response from the end of the bolt to the interface and is the strongest in the interface. Conversely, in the free section, the axial force of the bolt gradually decreases to the frequency response from the interface of the bolt to the root.

5. Conclusion

- (1) Frequency has a significant effect on the vibration mode of the anchoring section but does not affect the vibration mode of the bolt in the free section. The vibration mode of the bolt in the free section forms a trigonometric function, regardless of whether the vibration is forced or free, independent of frequency. The anchoring system can be regarded as a combined component with three different vibration modes, such as follows. When $\omega < \sqrt{k_1/\rho_1 A_1}$, the vibration mode of the anchoring section is an exponential function, and the vibration mode in the free section is a trigonometric function. When $\omega = \sqrt{k_1/\rho_1 A_1}$, the vibration mode of the anchored section is a parabolic function, and the vibration mode of the free section is a trigonometric function. When $\omega > \sqrt{k_1/\rho_1 A_1}$, the vibration mode of the anchored section is a trigonometric function.
- (2) Under the influence of frequency, the vibration amplitude of the bolt presents multipeak distribution characteristics. When the frequency is low, the vibration amplitude is consistent with the response law of frequency, and the vibration direction is the same at all times. The amplitude increases gradually from the end of the bolt to the maximum at the root of the bolt. At the same cross-section, the amplitude of anchoring is larger than that of anchoring. On the contrary, when the frequency increases, the amplitude of the bolt in the anchoring section is intermittently amplified, and the bolt in the free section is amplified when the frequency reaches a certain value, and when the frequency is higher, there may be a point with zero amplitude in the bolt.
- (3) The axial force of the bolt presents a triangular single peak distribution pattern. The response degree of bolt axial force to frequency gradually increases from the end of the bolt to the interface; while, from the interface to the root of the bolt, the response degree of bolt axial force to frequency gradually decreases.

Likewise, when the frequency is low, the peak position of the axial force is stable at the root of the bolt; when the frequency is large, the peak value of axial force is stable at the interface and becomes more obvious as the frequency increases. Therefore, the theoretical calculation shows that the peak of axial force at the bolt root is independent of frequency and is mainly influenced by the excitation wave, F_0 com

- (4) position, and linearly related to F_0 .

Abbreviations

ρ_1 :	Equivalent density of anchor solid
ξ :	Amplitude amplification factor
ω :	Circular frequency
β :	Bolt axial force amplification factor
ρ_2 :	Bolt density
μ_s :	Poisson's ratio of surrounding rock
α :	The angle between the direction of the incident wave and the axial direction of the bolt
λ :	Wavelength corresponding to the highest frequency of excitation wave
A_1 :	Cross-sectional area of the anchor solid
G_s :	Shear modulus of the surrounding rock
A_2 :	Bolt cross-sectional area
k_0 :	Stiffness of impedance spring at the bolt end
C_1 :	Constant term
k_1 :	Lateral spring stiffness of anchor solid
C_2 :	Constant term
L_1 :	Length of anchorage segment
C_3 :	Constant term
L_2 :	Anchor length
C_4 :	Constant term
$T(t)$:	Oscillation time function
dm :	Elementary mass
$u_i(x, t)$:	Equation of axial movement along the bolt
D_1 :	Anchor solids diameter
$u_1(x, t)$:	Equations of movement of anchor solids
E_1 :	Equivalent elastic modulus of anchor solid
$u_2(x, t)$:	Equation of bolt motion in the free section
F_0 :	Bolt root axial force
u_g :	Movement equation of roadway bedrock
$F_N(x, t)$:	Bolt axial force at t moment at any section
$u_{\text{simulation}}$:	Displacement of FLAC ^{3D} monitored
$F_{N1}(x, t)$:	Bolt axial forces in the anchorage section
u_{theory} :	Displacement calculated theoretically
$F_{N2}(x, t)$:	Bolt axial forces in the free section
U_g :	Bedrock amplitude.

Appendix

When $\omega > (k_1/\rho_1 A_1)^{1/2}$, the solution results of constant terms C_1 , C_2 , C_3 , and C_4 in Section 2.4 are as follows:

$$\left\{ \begin{aligned} C_1 &= \beta_4 C_3 + \beta_1, \\ C_2 &= \frac{k_0}{E_1 \sqrt{B}} CL_1 + \frac{k_0 D}{E_1 B \sqrt{B}}, \\ C_3 &= \frac{\beta_2 - \beta \beta}{\beta_3 \beta_4 - \beta_5}, \\ C_4 &= \frac{C_3 E_2 A_2 a \sqrt{G} \sin(\sqrt{G} L_2) + F_0}{E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)}, \end{aligned} \right. \quad (A.1)$$

$$\left\{ \begin{aligned} \beta_1 &= \frac{U_g \cos \alpha - (D/B) + (F_0 \sin(\sqrt{G} L_1)/E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)) - (k_0 D \sin(\sqrt{B} L)/E_1 B \sqrt{B})}{\cos(\sqrt{B} L_2) + (k_0 \sin(\sqrt{B} L)/E_1 \sqrt{B})}, \\ \beta_2 &= \frac{F_0 \sqrt{G} \cos(\sqrt{G} L_1)}{E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)} - \frac{k_0 (D/B)}{E_1} \cos(\sqrt{B} L), \\ \beta_3 &= \frac{\cos(\sqrt{G} L_1) + \tan(\sqrt{G} L_1) \sin(\sqrt{G} L_1)}{\cos(\sqrt{B} L) + (k_0 \sin(\sqrt{B} L)/E_1 \sqrt{B})}, \\ \beta_5 &= \sqrt{G} [\tan(\sqrt{G} L_2) \cos(\sqrt{G} L_1) - \sin(\sqrt{G} L_1)]. \end{aligned} \right.$$

When $\omega > (k_1/\rho_1 A_1)^{1/2}$, the solution results of constant terms C_1 , C_2 , C_3 , and C_4 in Section 2.4 are as follows:

$$\left\{ \begin{aligned} C_1 &= \frac{k_0}{E_1} C_2, \\ C_2 &= \beta_6 C_3 + \beta_7, \\ C_3 &= \frac{\beta_7 - \beta_8}{\beta_6 + \beta_9}, \\ C_4 &= \frac{C_3 E_2 A_2 a \sqrt{G} \sin(\sqrt{G} L_2) + F_0}{E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)}, \end{aligned} \right. \quad (A.2)$$

$$\left\{ \begin{aligned} \beta_6 &= \frac{[\cos(\sqrt{G} L_1) + \tan(\sqrt{G} L_2) \sin(\sqrt{G} L_1)]}{(1 + (k_0/E_1) L_1)}, \\ \beta_7 &= \frac{(E_1/k_0) F_0 \cos(\sqrt{G} L_1)}{E_2 A_2 a \cos(\sqrt{G} L_2)} - 2 \frac{E_1}{k_0} D L_1, \\ \beta_8 &= \frac{(F_0 \sin(\sqrt{G} L_1)/E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)) + U_g \cos \alpha = (1/2) D L_1^2}{(1 + (k_0/E_1) L_1)}, \\ \beta_9 &= \frac{E_1}{k_0} \sqrt{G} \sin(\sqrt{G} L_1) - \frac{E_1}{k_0} \sqrt{G} \tan(\sqrt{G} L_2) \sin(\sqrt{G} L_1). \end{aligned} \right.$$

When $\omega > (k_1/\rho_1 A_1)^{1/2}$, the solution results of constant terms C_1 , C_2 , C_3 , and C_4 in Section 2.4 are as follows:

$$\left\{ \begin{array}{l} C_1 = \frac{(E_1 \sqrt{-B} + k_0)}{(E_1 \sqrt{-B} - k_0)} C_2 + \frac{k_0 (D/B)}{(E_1 \sqrt{-B} - k_0)}, \\ C_2 = \frac{C_4 \sqrt{G} \sin(\sqrt{G} L_1) - C_3 \sqrt{G} \sin(\sqrt{G} L_1)}{((E_1 \sqrt{-B} + k_0)/(E_1 \sqrt{-B} - k_0)) \sqrt{-B} e^{\sqrt{-B} L_1} - \sqrt{-B} e^{\sqrt{-B} L_1}} + \beta_{14}, \\ C_3 = \frac{\beta_{10} - \beta_{11}}{\beta_{12} + \beta_{13}}, \\ C_4 = \frac{C_3 E_2 A_2 a \sqrt{G} \sin(\sqrt{G} L_2) + F_0}{E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2)}, \\ \beta_{10} = \frac{(D/B) + (k_0 (D/B) e^{(\sqrt{-B} L_1)}/E_1 \sqrt{-B} - k_0) - U_g \cos \alpha - (F_0 \sin(\sqrt{G} L_1)/E_2 A_2 a \sqrt{G} \cos(\sqrt{G} L_2))}{(E_1 \sqrt{-B} + k_0/E_1 \sqrt{-B} - k_0) e^{(\sqrt{-B} L_1)} + e^{(\sqrt{-B} L_1)}}, \\ \beta_{11} = \frac{(F_0 \cos(\sqrt{G} L_1)/E_2 A_2 a \cos(\sqrt{G} L_2)) + (k_0 D e^{(\sqrt{-B} L_1)}/\sqrt{-B} L_1 (E_1 \sqrt{-B} - k_0))}{(E_1 \sqrt{-B} + k_0/E_1 \sqrt{-B} - k_0) \sqrt{-B} e^{(\sqrt{-B} L_1)} - \sqrt{-B} e^{(\sqrt{-B} L_1)}}, \\ \beta_{12} = \frac{\cos(\sqrt{G} L_1) + \sin \sqrt{G} L_1 \tan \sqrt{G} L_2}{(E_1 \sqrt{-B} + k_0/E_1 \sqrt{-B} - k_0) e^{(\sqrt{-B} L_1)} + e^{(\sqrt{-B} L_1)}}, \\ \beta_{13} = \frac{\sqrt{G} \sin(\sqrt{G} L_1) \sqrt{G} \cos(\sqrt{G} L_1) \tan \sqrt{G} L_2}{(E_1 \sqrt{-B} + k_0/E_1 \sqrt{-B} - k_0) \sqrt{-B} e^{(\sqrt{-B} L_1)} - \sqrt{-B} e^{(\sqrt{-B} L_1)}}, \\ \beta_{14} = \frac{(k_0 D e^{(\sqrt{-B} L_1)}/\sqrt{-B} E_1 \sqrt{-B} + k_0)}{(E_1 \sqrt{-B} + k_0/E_1 \sqrt{-B} - k_0) \sqrt{-B} e^{(\sqrt{-B} L_1)} - \sqrt{-B} e^{(\sqrt{-B} L_1)}}. \end{array} \right. \quad (A.3)$$

Data Availability

The data, models, or code generated or used during the study are available from the corresponding author upon request, including the data used in Figures 3–7.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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