Research Article

Pile-Soil Stress Ratio and Settlement of Composite Foundation Bidirectionally Reinforced by Piles and Geosynthetics under Embankment Load

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The settlement calculation of composite foundation bidirectionally reinforced by piles and geosynthetics is always a difficult problem. The key to its accuracy lies in the determination of pile-soil stress ratio. Based on the theory of double parameters of the elastic foundation plate, the horizontal geosynthetics of composite foundation are regarded as the elastic thin plate, and the vertical piles and surrounding soil are regarded as a series of springs with different stiffness. The deflection equation of horizontal geosynthetics considering its bending and pulling action is obtained according to the static equilibrium conditions. The equation is solved by using Bessel function of complex variable, and the corresponding deflection function of horizontal geosynthetics is deduced. Then, the calculation formula of pile-soil stress ratio and settlement of composite foundation is derived by considering the deformation coordination of pile and soil. The results of engineering case analysis show that the theoretical calculation results are in good agreement with the measured values, which indicates that the proposed method is feasible and the calculation accuracy is good. Finally, the influence of composite modulus of horizontal geosynthetics, tensile force of geosynthetics, and pile-soil stiffness ratio on pile-soil stress ratio and settlement is further analyzed. The results show that the pile-soil stress ratio increases with the increase of the composite modulus of the horizontal geosynthetics, the tensile force of geosynthetics, and the pile-soil stiffness ratio, and the settlement decreases with the increase of the composite modulus of the horizontal geosynthetics, the tensile force of geosynthetics, and the pile-soil stiffness ratio. When the flexural stiffness of the horizontal geosynthetics is small, the influence of the tensile action of the geosynthetics on the pile-soil stress ratio and settlement of the composite foundation cannot be ignored.

1. Introduction

In recent years, with the rise of highway and railway construction in China, the problems of soft soil foundation have become increasingly prominent, the bearing capacity of the foundation is insufficient, the settlement is too large, and uneven settlement is particularly serious. The composite foundation reinforced by piles and geosynthetics, a new type of soft foundation treatment consisting of vertical piles and horizontal geosynthetics, has a good effect on the treatment of the abovementioned foundation problems, and the foundation treatment method has simple construction and rapid progress. In recent years, it has been widely used in railway soft foundation treatment [1].

Research on the working mechanism of composite foundation has achieved many results so far [2–4]. Among them, the pile-soil stress ratio and settlement are important parameters reflecting the working behavior of composite foundation reinforced by piles and geosynthetics, and the research on its calculation method has also been a hot spot in
this field. Among them, Yang et al. [5] uses the helix to simulate the deformation shape of the cushion, taking into account the net effect of the cushion; Chen et al. [6] considered the tensile effect of the geosynthetics, assumed that the shape of the cushion after deformation is a rotating paraboloid, and deduced the calculation formula of pile-soil stress ratio based on Winkler foundation; Abusharar et al. [7] assumed that the stress of soil between piles is uniformly distributed, and a simple method for calculating the stress ratio of pile to soil is put forward by using the arc assumption of cushion deformation. For a considerable part of the project, single-layer grille cannot meet the needs of the project, geogrid, or multilayer grille should be installed in the cushion. At the same time, reference [2] points out that when this kind of structure is different from the single-layer grille, the geosynthetics and the contained bulk fillers can form a "flexible raft" with strong bending resistance. Obviously, the above method based on the thin film theory is not suitable for geogrid or multilayer grille because it cannot take into account the bending effect. Therefore, Rao and Zhao [8] assume that the geosynthetics is a thin plate, and the shape of the thin plate is a parabola in order to consider the "flexible raft effect.” The formula for calculating the stress ratio of pile to soil is derived; Zheng et al. [9] think that this kind of cushion not only has a “flexible raft effect” but also has “pulling membrane effect,” so it is assumed that the deformation of geosynthetics is in the form of heavy trigonometric function, and the geometric nonlinearity in the deformation process is considered. As a result, the pile-soil stress ratio and settlement calculation method considering both “flexible raft effect” and “pulling membrane effect” of cushion are obtained. Ma et al. [10, 11] have done some work in the calculation of soft soil settlement. He et al. [12] made some attempts to chemically improve special soils. In addition, it can be seen from the above literature that the deformation shape of geosynthetics is the premise and key to calculating the pile-soil stress ratio and settlement, and the pile-soil stress ratio and settlement obtained by different deformation shapes are also different. Therefore, the closer it is to the actual deformation of the geosynthetics, the more accurate the pile-soil stress ratio and settlement will be.

However, the above literature all assume the deformation of the geosynthetics in advance, which cannot really reflect the stress state of the geosynthetics, which can easily lead to a large error between the calculated results and the measured results. For this reason, Tan et al. [13] derived the flexure function of horizontal stiffened body in the three-dimensional case based on Winkler elastic foundation rectangular plate theory, but due to the complexity of rectangular plate boundary conditions, it is difficult to consider the influence of pile settlement on geosynthetics. After that, Zhao et al. [14] took the composite foundation within the influence area of a single pile as the research object, discretized the pile and soil into a series of springs with different stiffness, and with the aid of the circular plate theory of elastic foundation and the deformation coordination of cushion at the pile-soil interface, the deflection function of geosynthetics considering the common deformation of pile and soil is derived, and on this basis, the calculation method of pile-soil stress ratio is put forward. However, the above two analysis methods still only consider the “flexible raft effect” of geosynthetics, but not the “pulling membrane effect” of the reinforcement.

From the above analysis, we can know how to comprehensively consider the flexible raft effect, net effect, and pile-soil joint deformation of cushion without assuming the deformation of geosynthetics is the key to accurately solve the pile-soil stress ratio and settlement of composite foundation reinforced by piles and geosynthetics. Therefore, on the basis of the above research work, based on the Filonenko–Borodich two-parameter foundation model [15], this paper comprehensively considers the “flexible raft effect” and net effect of the geosynthetics, considers the pile-soil deformation at the same time, deduces the analytical function of the flexural deformation of the geosynthetics according to the Bessel complex function, and then obtains the calculation expression of pile-soil stress ratio and settlement of composite foundation. In order to further improve the calculation method of pile-soil stress ratio and settlement of composite foundation.

2. Calculation Model

Take the typical unit within the influence range of the single pile as shown in Figure 1 for analysis. The diameter of the pile is \( d \), the center distance is \( s_h \), the diameter of single pile reinforcement range is \( d_r \), square pile \( d_r = 1.13s_h \), and piles are arranged as the shape of plum blossoms: \( d_r = 1.65s_g \).

In order to further simplify the calculation, the following assumptions are made:

(1) As shown in Figure 2, the geosynthetics within the range of single pile reinforcement is regarded as an elastic circular thin plate placed on the Filonenko–Borodich two-parameter foundation model [16], and among them, the tension \( T \) of the reinforcement can be obtained by the following formula:

\[ T = E_g \cdot \varepsilon_g, \]

where \( \varepsilon_g \) is the average strain of the reinforcement and \( E_g \) is the tensile stiffness of the reinforcement.

(2) It is assumed that the vertical supporting force \( P_p \) of the pile to the thin plate is uniformly distributed, and according to references [6, 13, 14], the deformation of pile and soil between piles accord with Winkler foundation model.

(3) The distribution of embankment load caused by differential settlement is not considered, that is, the embankment load is uniformly distributed [7, 8, 14].

3. Analysis of Geosynthetics

Let \( u_1(r) \) represent the flexure function of the pile top reinforcement. According to the Filonenko–Borodich two-parameter foundation model, under the joint action of the embankment load \( q \) and the pile top reaction force \( P_p \), the
control differential equation of the horizontal geosynthetics is as follows:

\[ DV^4 w_1 (r) - T \cdot V^2 w_1 (r) = q - p_p, \]  

(2)

where \( D \) is the bending stiffness of the thin plate, \( D = (E\delta^3/12(1-v^2)) \), where \( E \) is the elastic modulus of the plate, \( v \) is Poisson's ratio of the ratio of the plate, and \( \delta \) is the thickness of the plate: \( V^2 = (d^2/dr^2) + ((1/r)(d/dr)) \).

Let \( a_1 = \sqrt{r^2/D_0} \). Then, the homogeneous equation corresponding to equation (2) can be transformed into

\[ V^2 w_1 (r)(V^2 w_1 (r) + a_1^2 w_1 (r)) = 0. \]  

(3)

The general solution of the homogeneous equation corresponding to equation (2) is

\[ w_1 (r) = C_1 J_0 (a_1 r) + C_2 Y_0 (a_1 r) + C_3 \ln r + C_4, \]  

(4)

where \( J_N \) and \( Y_N \) are the first and second kinds of \( N \)-order Bessel functions and \( C_1, C_2, C_3, \) and \( C_4 \) are undetermined constants.

Under the action of uniformly distributed load \( q \) and supporting force \( p_p \), the special solution of equation (2) is

\[ w_1^* = -((q - p_p)r^2/4T). \]

Combining formula (4), the flexural deformation function of the thin plate at the top of the pile is

\[ w_1 (r) = C_1 J_0 (a_1 r) + C_2 Y_0 (a_1 r) + C_3 \ln r + C_4 - \frac{(q - p_p)r^2}{4T}. \]  

(5)

4. Deformation Analysis of Thin Plate

Let \( w_2 (r) \) denote the flexural function of the thin plate at the top of the pile, under the action of uniformly distributed load \( q \) and foundation soil support, the governing differential equation of thin plate at the top of interpile soil is as follows:

\[ DV^4 w_2 (r) - T \cdot V^2 w_2 (r) + k_s w_2 = q, \]  

(6)

where \( k_s \) is the spring coefficient of soil foundation between piles.

To simplify the calculation,

\[ a_{21} = \frac{1}{2} \left[ \frac{T}{D} + \sqrt{\left(\frac{T}{D}\right)^2 - \frac{4 k_s}{D}} \right], \]

\[ a_{22} = \frac{1}{2} \left[ \frac{T}{D} - \sqrt{\left(\frac{T}{D}\right)^2 - \frac{4 k_s}{D}} \right]. \]

(7)

The homogeneous equation corresponding to equation (6) can be transformed into the following form:

\[ \left( V^2 w_2 (r) + a_{21} w_1 (r) \right)\left( V^2 w_2 (r) + a_{22} w_1 (r) \right) = 0. \]

(8)

It can be seen from formula (7) that the values of \( a_{21} \) and \( a_{22} \) are divided into real and imaginary numbers according to the positive and negative of \( T^2 - 4 D k_s \). The solution of equation (8) is different in the two cases, which are discussed as follows.

1. When \( T^2 \geq 4 D k_s \), order:

\[ l_{21} = \sqrt{a_{21}} \cdot i, \]

\[ l_{22} = \sqrt{a_{22}} \cdot i. \]

2. The general solution of equation (8) is

\[ w_2 (r) = C_5 \phi_0 (l_{21} r) + C_6 N_0 (l_{21} r) + C_7 \phi_0 (l_{22} r) + C_8 N_0 (l_{22} r), \]

(9)

where \( N_0 \) is the second kind of \( N \)-order virtual variable Bessel function and \( C_5, C_6, C_7, \) and \( C_8 \) are undetermined constants.

Under the action of uniformly distributed load \( q \), the special solution of equation (6) is \( w_2^* = q/k_s \), and the combination formula (11) can be used to obtain the flexural deformation function of thin plate at the top of soil between piles at \( T^2 \geq 4 D k_s \):

\[ w_2 (r) = C_5 \phi_0 (l_{21} r) + C_6 N_0 (l_{21} r) + C_7 \phi_0 (l_{22} r) + C_8 N_0 (l_{22} r) + \frac{q}{k_s}. \]

(11)

When \( T^2 < 4 D k_s \), the general solution of equation (6) can be constructed as follows:

\[ w_2 (r) = C_5 \phi_0 (r) + C_6 \phi_0 (r) + C_7 \phi_0 (r) + C_8 \phi_0 (r) + \frac{q}{k_s}, \]

(12)
where \( u_n \) and \( v_n \) are the real parts of the first kind of Bessel function of order \( n \) and the real parts of Hankel function and \( f_n \) and \( g_n \) are the imaginary part of Bessel function of order \( n \) and the imaginary part of Hankel function, respectively. \( C_5, \rho_0, C_7, \) and \( C_9 \) are undetermined constants.

\[
\begin{align*}
&u_0(r) = \text{Re}J_0(\sqrt{\alpha_1}r) = \frac{1}{2}\left[J_0(\sqrt{\alpha_1}r) + J_0(\sqrt{\alpha_2}r)\right], \\
n_0(r) = \text{Im}J_0(\sqrt{\alpha_1}r) = \frac{1}{2i}\left[J_0(\sqrt{\alpha_1}r) - J_0(\sqrt{\alpha_2}r)\right], \\
f_0(r) = \text{Re}H_0^{(1)}(\sqrt{\alpha_1}r) = \frac{1}{2}\left[H_0^{(1)}(\sqrt{\alpha_1}r) + H_0^{(2)}(\sqrt{\alpha_2}r)\right], \\
g_0(r) = \text{Im}H_0^{(2)}(\sqrt{\alpha_2}r) = \frac{1}{2i}\left[H_0^{(1)}(\sqrt{\alpha_1}r) - H_0^{(2)}(\sqrt{\alpha_2}r)\right].
\end{align*}
\]

(13)

5. Solution of Undetermined Coefficient

In order to determine the bending function of the thin plate, it is also necessary to compare the unknown parameters \( C_1, C_2, \ldots, C_9 \) and \( p_p \) in equations (5), (11), and (12), which are solved by boundary and continuous conditions.

As the turning angle of the thin plate at the center of the circular plate is \( 0, C_3 = C_5 = 0 \) can be obtained. At the pile-soil junction, the continuity conditions of thin plate deflection, rotation angle, radial bending moment, and shear force are as follows:

\[
\begin{align*}
\omega_1\left(\frac{d}{2}\right) &= \omega_2\left(\frac{d}{2}\right), \\
\theta_1\left(\frac{d}{2}\right) &= \theta_2\left(\frac{d}{2}\right), \\
M_{r1}\left(\frac{d}{2}\right) &= M_{r2}\left(\frac{d}{2}\right), \\
Q_{r1}\left(\frac{d}{2}\right) &= Q_{r2}\left(\frac{d}{2}\right).
\end{align*}
\]

(14)

In addition, according to hypothesis (2), the deflection \( \omega_1\left(\frac{d}{2}\right) \) of the circular plate at the pile-soil interface is

\[
\omega_1\left(\frac{d}{2}\right) = \frac{\pi d^2 p_p}{4K_p}.
\]

(15)

The value of pile top reaction \( p_p \) is the sum of the upper embankment load and the load transferred from the cushion to the pile, that is,

\[
p_p = \frac{4Q_{r1}(r=d/2)}{d} + q.
\]

(16)

According to reference [2], the corner and shear boundary conditions at \( r = d/2 \) of the element are as follows:

\[
\begin{align*}
\theta_1\left(\frac{d}{2}\right) &= 0, \\
Q_{r1}\left(\frac{d}{2}\right) &= 0.
\end{align*}
\]

(17)

At the same time, according to the bending functions (5), (11), and (12) of the thin plate, the expressions of the rotation angle, radial bending moment, and shear force of the thin plate can be obtained.

Within the range of the top of the pile, that is, \( 0 \leq r \leq (d/2) \),

\[
\begin{align*}
\theta_1 &= -C_1a_1J_1(a_1r) \cdot \frac{(q-p_p)r}{2T}, \\
M_{r1} &= D_1a_1\left[C_1\frac{1-v}{a_1}J_1(a_1r) - J_0(a_1r) + \frac{(1+v)(q-p_p)}{2T}a_1\right], \\
Q_{r1} &= C_1D_1a_1\frac{J_1(a_1r)}{r} + \frac{2-r^2}{r}J_1(a_1r) - \frac{J_0(a_1r)}{r}.
\end{align*}
\]

(18)

Within the range of the top of the pile, that is, \( (d/2) \leq r \leq (d/2) \),

(1) When \( T^2 \geq 4Dk_3 \),

\[
\begin{align*}
\theta_2 &= I_{21}\left[C_5I_1(I_{22}r) - C_6N_1(I_{22}r)\right] + I_{21}\left[C_7I_1(I_{22}r) - C_8N_1(I_{22}r)\right], \\
M_{r2} &= D_{21}\left[C_5\frac{1-v}{I_{22}r}I_1(I_{22}r) - J_0(I_{22}r) - C_6\frac{1+v}{I_{22}r}N_1(I_{22}r) + N_0(I_{22}r)\right], \\
Q_{r2} &= C_5D_{21}\frac{I_{22}I_1(I_{22}r)}{r} + \frac{2-r^2}{r}I_1(I_{22}r) - \frac{I_0(I_{22}r)}{r} + C_6D_{21}\frac{N_2(I_{22}r)}{r} + \frac{2+r^2}{r}N_1(I_{22}r) + \frac{N_0(I_{22}r)}{r}.
\end{align*}
\]

(19)
(2) When \( T^2 \geq 4D_k \), in order to simplify the expression, order \( l_2 = \sqrt{\frac{k_0 D}{T}} \cos 2 \varphi = -(T/2 \sqrt{k_0 D}) \), according to Euler’s formula:

\[
\begin{align*}
    a_{21} &= l_2^2 e^{2w}, \\
    a_{22} &= l_2^2 e^{-2w}.
\end{align*}
\]

Combining formula (20),

\[
\begin{align*}
\theta_{r_2} &= \sum_{M=1}^{4} I_2 C_{M+4} \left\{ Z_{M1}(r) \left[ \cos \varphi - (-1)^M \sin \varphi \right] \right\}, \\
M_{r_2} &= Dl_2^3 \sum_{M=1}^{4} C_{M+4} \left\{ Z_{M0}(r) \left[ \cos 2 \varphi - (-1)^M \sin 2 \varphi \right] - \frac{(1 - v)}{l_2^2} Z_{M1}(r) \left[ \cos \varphi - (-1)^M \sin \varphi \right] \right\}, \\
Q_{r_2} &= -Dl_2^3 \sum_{M=1}^{4} C_{M+4} \left\{ Z_{M1}(r) \left[ \cos 3 \varphi - (-1)^M \sin 3 \varphi \right] \right\},
\end{align*}
\]

where \( Z_{MN}(r) \) (among them \( M = 1, 2, 3, 4 \)) denotes \( u_n(r), v_n(r), f_n(r), \) and \( g_n(r) \), respectively.

According to the above analysis, the simultaneous equations \((14)\)–\((17)\) can be used to calculate \( C_1, C_4, \ldots, C_8 \) and \( p_p \). Thus, the deflection expression of the plate is obtained.

### 6. Pile-Soil Stress Ratio and Settlement

From the above analysis, if the strain of the steel bar is known before the calculation, the tension of the steel bar can be calculated directly by using formula (1), and then the undetermined coefficient can be solved according to the steps in Section 5. However, in most projects, the deformation of reinforcement is not measured beforehand. In view of this situation \([4]\) through field test and study, it is found that the relationship between average strain and settlement of reinforcement under embankment load is approximately as follows:

\[
\varepsilon_{g} = \frac{\alpha}{50} \varphi^g,
\]

where \( \alpha \) and \( \beta \) are the relevant fitting parameters, respectively, which can be selected according to the settlement calculation position. According to the table listed in reference \([4]\), 50 kN/m is the tensile stiffness of the grid used in the test.

According to the above research results, the maximum deflection \( w_{M}(d_e/2) \) of geosynthetics is selected as the settlement \( S \) in formula (22). Combined with the method in Section 5, when the tension of the reinforcement is unknown, the steps of solving the undetermined coefficient are as follows:

(1) First of all, assuming an initial tension value \( T_0 \) (\( T_0 \) can be a very small value), it is substituted into equations (2) and (6) to solve the flexural function of geosynthetics, and the values of \( w_1(d/2) \) and \( w_2(d_e/2) \) are obtained.

(2) Then, take the value of \( w_2(d_e/2) \) obtained before as \( S \), replace (22), combine the formula (1) to obtain the tension \( T \), then replace \( T \) into equations (2) and (6), solve the flexural function of geosynthetics, and get the values of \( w_1(d/2) \) and \( w_2(d_e/2) \).

(3) Select an error value and compare the obtained \( w_1(d/2) \) and \( w_2(d_e/2) \) with the previous one. If the comparison results are less than the error value, stop the calculation; otherwise, continue the iterative calculation according to the steps of second and third.

The flowchart of the above steps is shown in Figure 3 as follows.

After obtaining the flexural function and \( p_p \) of the geosynthetics, it can be seen from the calculation model shown in Figure 2 that, according to the stress balance in the z direction, the average vertical stress \( p_s \) at the top of the soil between piles can be obtained as follows:

\[
p_s = \frac{\left( qn \pi d_e^2 / 4 \right) - \left( p_p \pi d_e^2 / 4 \right)}{\pi \left( d_e^2 - d_i^2 \right) / 4}.
\]

Then, the pile-soil stress ratio \( n \) can be calculated according to the following formula \([16]\):

\[
n = \frac{p_p}{p_s}.
\]
Reference [8] thinks that, in the case of large area overloading, the settlement at the top of the pile is relatively small, and the settlement of soil between piles can be regarded as the settlement of the foundation.

\[ S = w_2 \left( \frac{de}{2} \right). \]  

(25)

7. Determination of Parameters

7.1. Stiffness Coefficient of Pile \( k_p \). In order to comprehensively consider the nonlinearity in the process of pile deformation, the deformation stiffness of pile is taken as the secant slope of the load test \( q-s \) curve of pile foundation.

7.2. Spring Coefficient of Surrounding Soil \( k_s \). In order to comprehensively consider the nonlinearity in the process of soil deformation between piles, the coefficient of soil foundation between piles \( k_s \), takes the secant slope on the load test \( q-s \) curve. If there is no measured data, it can also be taken in accordance with regional experience or determined according to the following formula [13]:

\[ k_s = \frac{E_s}{H_s}, \]  

(26)

where \( E_s \) is the deformation modulus of soil, the weighted average of multilayer soil is depth, and \( H_s \) is the thickness of soil layer.

7.3. Calculated Thickness of Cushion \( \delta \). From the point of view of the joint action of the geosynthetics and the bulk material pile, the thickness of the composite structure formed by the geosynthetics and its wrapped filler is taken as the thin plate thickness if it is a multilayer grille, that is, the distance between the top layer and the bottom grille. If it is a geotechnical cell, the thickness of the geotechnical cell can be taken directly.

7.4. Composite Elastic Modulus of Cushion \( E \). For the grid cushion, Zheng et al. [9] gave a method to determine its composite elastic modulus, that is, the weighted average value of the grid elastic modulus and the deformation modulus of the cushion filler. In the case of geotechnical cell, Yang et al. [17] gave the average elastic modulus of different types of geotechnical cell combined with various common fillers through stacked beam test.

8. Example and Parameter Analysis

8.1. Example 1. Technical treatment of soft soil foundation according to foundation reinforced by piles and geosynthetics in DK10 + 320 and DK10 + 336 test section of Sùníng-Chóngqíng railway [4], and the pile is powder injection pile, the diameter of the pile is 0.50 m, the center of the pile is 1.0 m, piles are arranged as the shape of plum blossoms and \( de = 1.05 \) m, and powder injection pile is laid with a double-layer geogrid with 50 kN/m tensile stiffness and a distance of 30 cm. The foundation is quaternary aluvial soft soil, the bedrock is mudstone, the upper filling load is 20 kN/m³, the filling height of roadbed is 10 m, and the crushed stone cushion is 25 kN/m³. According to the static load test, the stiffness coefficient of pile is \( k_p = 2000 \) kN/m. The thickness of the thin plate is 0.3 m, the composite elastic modulus \( E \) is 52 MPa, and Poisson’s ratio \( \nu \) is 0.3. Other calculation parameters are shown in Table 1.

The comparison between the calculated and measured values of the central settlement of the embankment and the pile-soil stress ratio under the geogrid geosynthetics is shown in Table 2.

The development trend of foundation settlement \( S \) of DK10 + 336 test section with the increase of embankment filling height \( H \) is shown in Figure 4.

As can be seen from Tables 2 and 3, the pile-soil stress ratio and settlement of double-layer geogrid-reinforced composite foundation obtained by this method are close to the measured values.

8.2. Example 2. The test section of an expressway lying on soft soil foundation in Hunan is treated with geocell + mixing piles. The pile is arranged in the form of plum blossoms, and its diameter is 0.50 m with its spacing 1.2 m. The pile top is filled with a thick 30 cm sand cushion, and the center is equipped with a geotechnical cell with the thickness of 10 cm. The foundation is muddy clay, the load of the upper fill is 20 kN/m³, and the filling height of the roadbed in the test section is 4 m. The measured settlement value \( S \) is 5.3 cm, and the pile-soil stress ratio is \( n \). The stiffness...
coefficient of pile $k_p$ is 2355 kN/m. After treatment, the coefficient of soil foundation between piles $k_s$ is 1024 kN/m$^3$, the thickness of thin plate $\delta$ is 0.10 m, the composite elastic modulus $E$ is 40 MPa, and the composite Poisson’s ratio $\nu$ is 0.3.

The pile-soil stress ratio and settlement are calculated as shown in Table 3.

It can be seen from Table 3 that the pile-soil stress calculated in this paper is closer to the measured value than the method in reference [14]. In addition, the settlement is also close to the measured value. This is due to the fact that compared with the method in reference [14], the tensile effect of the cell body is considered in this paper.

8.3. Parameter Analysis. In order to further discuss and analyze the effects of geosynthetics composite elastic modulus, grille tension and pile-soil stiffness ratio on the settlement and pile-soil stress ratio of composite foundation reinforced by piles and geosynthetics, based on the parameters in example 2, the corresponding parameters are analyzed according to the abovementioned factors.

8.3.1. Influence of Pile-Soil Stiffness Ratio on Settlement and Pile-Soil Stress Ratio. Without considering the influence of tension of geosynthetics, $k_p = 4k_p/(\pi d^2)$ is introduced to characterize the comprehensive influence of pile

### Table 1: The parameters for calculation.

<table>
<thead>
<tr>
<th>Station</th>
<th>Type of soils</th>
<th>Depth, $H_s$ (m)</th>
<th>Deformation modulus, $E_s$ (MPa)</th>
<th>Grid tension, $T$ (kN/m)</th>
<th>$T^2 - 4D_{k_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DK10 + 320</td>
<td>Soft soil</td>
<td>7.0</td>
<td>3.1</td>
<td>60.3</td>
<td>&lt;0</td>
</tr>
<tr>
<td>DK10 + 336</td>
<td>Soft soil</td>
<td>10.6</td>
<td>3.1</td>
<td>82.7</td>
<td>&lt;0</td>
</tr>
</tbody>
</table>

### Table 2: Results of settlement and pile-soil stress ratio.

<table>
<thead>
<tr>
<th>Station</th>
<th>Settlement, $S$ (cm)</th>
<th>Pile-soil stress ratio, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated value</td>
<td>Measured value</td>
</tr>
<tr>
<td>DK10 + 320</td>
<td>28.8</td>
<td>26.6</td>
</tr>
<tr>
<td>DK10 + 336</td>
<td>33.9</td>
<td>32.0</td>
</tr>
</tbody>
</table>

### Table 3: Results of settlement and pile-soil stress ratio.

<table>
<thead>
<tr>
<th>Items</th>
<th>$T^2 - 4D_{k_s}$</th>
<th>Pile-soil stress ratio, $n$</th>
<th>Settlement, $S$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured value</td>
<td>—</td>
<td>6.5</td>
<td>5.3</td>
</tr>
<tr>
<td>Method of literature [14]</td>
<td>—</td>
<td>5.8</td>
<td>—</td>
</tr>
<tr>
<td>The proposed method</td>
<td>&lt;0</td>
<td>6.1</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Figure 4: Comparison between calculated and measured results of embankment settlement.
Figure 5: Relation between settlement and \( k_p/k_s \).

Figure 6: Relation between pile-soil stress and \( k_p/k_s \).

Figure 7: Relation between settlement and tension.
deformation stiffness and replacement ratio, and other parameters remain unchanged.

As shown in Figure 5, the influence of pile-soil stiffness ratio on settlement increases with the increase of elastic modulus $E$. At the same elastic modulus, the settlement of composite foundation decreases with the increase of pile-soil stiffness ratio. However, when the pile-soil stiffness ratio exceeds a certain value, continuing to increase the pile-soil stiffness ratio, it has little effect on reducing settlement.

As can be seen from Figure 6, the relationship between pile-soil stiffness ratio and pile-soil stress ratio is nonlinear under different elastic modulus, and the smaller the elastic modulus is, the more obvious the nonlinear phenomenon is. In addition, the larger the elastic modulus is, the more obvious the influence of pile-soil stiffness ratio on pile-soil stress ratio is.

8.3.2. Effect of Reinforcement Tension on Settlement and Pile-Soil Stress Ratio. As can be seen from Figures 7 and 8, for the same elastic modulus $E$, the settlement decreases with the increase of reinforcement tension $T$, while the pile-soil stress ratio $n$ increases with the increase of tension $T$.

In addition, the effect of $T$ on both pile-soil stress ratio and settlement decreases with the increase of $E$. It can be seen that when the elastic modulus of geosynthetics-reinforced cushion is small, using graded material and properly increasing the interface friction can effectively increase the pile-soil stress ratio and reduce the settlement.

9. Conclusions

Based on theory of the Filonenko–Borodich two-parameter elastic foundation model, the horizontal geosynthetics of composite foundation are regarded as the elastic thin plate, and the vertical piles and surrounding soil are regarded as a series of springs with different stiffness. The deflection equation of horizontal geosynthetics considering its bending and pulling action is obtained according to the static equilibrium conditions. The equation is solved by using Bessel function of complex variable, and the corresponding deflection function of horizontal geosynthetics is deduced. The calculation method for pile-soil stress ratio and settlement of composite foundation is derived by considering the deformation coordination of pile and soil.

1. The settlement of composite foundation decreases with the increase of pile-soil stiffness ratio, geosynthetics tension, and composite elastic modulus of geosynthetics. The pile-soil stress ratio of composite foundation increases with the increase of pile-soil stiffness ratio, elastic modulus of geosynthetics, and its tension. When the bending stiffness of geosynthetics is small, the influence of tensile action of reinforcement on pile-soil stress ratio and settlement cannot be ignored.

2. The key to calculating with this method is to accurately measure the composite elastic modulus of geosynthetics-reinforced cushion, the strain of geosynthetics, and the stiffness of pile and soil.

3. Although this presented method does not rely on preassumed deformation when calculating the pile-soil stress ratio and settlement of composite foundation, the net effect of geosynthetics-reinforced cushion can be considered only when the strain of geosynthetics is known, which brings defects to the convenience of calculation. The proposed method still cannot consider the coupling relationship between deformation of geosynthetics-reinforced cushion and tension of geosynthetics, which can be carried out in the next work.

Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>The tension of geosynthetics</td>
</tr>
<tr>
<td>$\varepsilon_g$</td>
<td>The average strain of geosynthetics</td>
</tr>
<tr>
<td>$E_g$</td>
<td>The tensile stiffness of geosynthetics</td>
</tr>
<tr>
<td>$D$</td>
<td>The bending stiffness of the thin plate</td>
</tr>
</tbody>
</table>
$E$: The elastic modulus of the thin plate
$
u$: Poisson’s ratio of the thin plate
$\delta$: The thickness of the thin plate
$I_N$: The first kind of $N$-order Bessel functions
$Y_N$: The second kind of $N$-order Bessel functions
$q$: The distributed load
$P_F$: The supporting force
$k_s$: The spring coefficient of surrounding soil
$N_N$: The second kind of $N$-order virtual variable
$C_{11}$: The real part of the first kind of Hankel function of order $N$
$C_{12}$: The real part of the first kind of Bessel function of order $N$
$f_n$: The value of pile top reaction
$\alpha, \beta$: The relevant fitting parameters
$T_0$: The initial tension value
$s$: The settlement
$p_s$: The average vertical stress at the top of the soil between piles
$k_p$: The stiffness coefficient of pile
$E_s$: The deformation modulus of soil
$H_s$: The thickness of soil layer
$\delta$: The calculating thickness of cushion
$E$: The composite thickness of cushion
$C_{11}, C_{22}, C_{33}$: The undetermined constants
$C_{44}$: $C_{55}, C_{66}, C_{77}$: The undetermined constants.

**Data Availability**

The [Data Type] data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


