Research Article

A Strain Rate-Dependent Damage Evolution Model for Concrete Based on Experimental Results

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There are various definitions of damage variables from the existing damage models. The calculated damage value by the current methods still could not well correspond to the actual damage value. Therefore, it is necessary to establish a damage evolution model corresponding to the actual damage evolution. In this paper, a strain rate-sensitive isotropic damage model for plain concrete is proposed to describe its nonlinear behavior. Cyclic uniaxial compression tests were conducted on concrete samples at three strain rates of 10^{-3}s^{-1} , 10^{-4}s^{-1} , and 10^{-5}s^{-1} , respectively, and ultrasonic wave measurements were made at specified strain values during the loading progress. A damage variable was defined using the secant and initial moduli, and concrete damage evolution was then studied using the experimental results of the cyclic uniaxial compression tests conducted at the different strain rates. A viscoelastic stress-strain relationship, which considered the proposed damage evolution model, was presented according to the principles of irreversible thermodynamics. The model results agreed well with the experiment and indicated that the proposed damage evolution model can accurately characterize the development of macroscopic mechanical weakening of concrete. A damage-coupled viscoelastic constitutive relationship of concrete was recommended. It was concluded that the model could not only characterize the stress-strain response of materials under one-dimensional compressive load but also truly reflect the degradation law of the macromechanical properties of materials. The proposed damage model will advance the understanding of the failure process of concrete materials.

1. Introduction

Concrete is a heterogeneous and discontinuous multiphase composite material. Shrinkage and bleeding can occur in concrete during concrete hardening due to the nonuniformity of material constituents and lead to inevitable development of excessive microcracking and microdefects. The microcracks and microdefects dispersed in the material will gradually expand, evolve, converge, penetrate, and eventually form macrocracks, which will lead to failure under the action of external loads. Studying the damage evolution process will help deepen the understanding of the deterioration of macromechanical properties of concrete.

The internal microcracking and debonding between cement paste and aggregates are the main reasons for the

nonlinear behavior of concrete materials. The occurrence and development of damage are considered as the process of damage evolution and appropriate damage variables are regarded as the internal variables in the constitutive relationships that describe the physical and mechanical properties. The authors in [1–3] assumed the damage of material is isotropic and proposed an elastic constitutive equation of isotropic damage. However, [4] recommended adopting different order tensors to describe the anisotropic damage and proposed a model of anisotropic elastic damage. Brünig and Michalski [5] established a model of anisotropic damage for concrete according to irreversible thermodynamics. However, these elastic damage models could not characterize the irreversible behavior of materials.

Researchers have treated irreversible deformations as plastic and proposed elastoplastic damage constitutive equations. A constitutive model for nonlinear characteristics of concrete was recommended by [6]. Oñate et al. [7] proposed a local constitutive model to analyze the failure of solid materials. Zheng et al. [8] obtained a new plastic damage model with two damage variables for concrete crack failure. Jefferson et al. [9] suggested a new comprehensive 3D plastic damage contact model. Feng et al. [10] adopted the elastoplastic damage energy release rates as the driving force of damage. These plastic damage models are used to describe the irreversible behavior of concrete on the basis of yield surface theory and nonassociated flow rules to characterize the occurrence of inelastic deformations. However, there is no clear yield point in the stress-strain curve for concrete under low triaxial stress conditions. An isotropic model for creep damage in concrete under uniaxial compression was proposed by [11]. Pedersen et al. [12] presented a rate-independent viscoelastic and viscoplastic constitutive relationship for the dynamical behavior of concrete. All the models above are based on the phenomenological approaches of field theory methods. This macroapproach is based on continuum mechanics and continuum thermodynamics and determines phenomenologically the material constitutive equations and damage evolution using the thermodynamic theorems and internal variable theory. The method does not need to derive the theoretical relationships between macroscopic quantities directly from the microscopic mechanisms and requires only that the established model be consistent with the actual behavior.

Many researchers utilized the mesoscopic approaches to analyze the damage of concrete. These methods are based on the individual mechanical behavior and interaction of the microscopic material components [13, 14]. Kassner et al. [15] proposed that the mechanics of microcracks in material is based on a multiscale mechanical model and studied the brittle-ductile material behavior at a mesoscale. Contrafatto et al. [16] established a mesoscale model to analyze the nonlinear behavior of concrete with a randomly assigned distribution of phases. These mesoscopic models could describe the physical processes behind damage variables and damage evolution and clearly explain the mechanism of concrete crack initiation. However, the mechanics of heterogeneous microscopic materials requires many simplifying assumptions for transition to macroscopic homogeneous materials. The understanding of the microscopic composition is insufficient because microscopic damage mechanisms are very complicated, and, consequently, it is difficult to obtain their mechanical parameters.

There are various definitions of damage variables used in the existing damage models. On the other hand, the damage evolution models are always established from experience; some are based on a macroscopic perspective and some on a microscopic viewpoint. In fact, damage values calculated by the models based on these two approaches do not agree well with the actual damage values. Therefore, it is necessary to establish a damage evolution model corresponding to the actual damage evolution. In this paper, the damage variable of the material is defined as the ratio of the decrease in the elastic modulus of

concrete to the initial elastic modulus. The experimental damage evolution values were quantified and the corresponding damage evolution model was established based on experimental damage data. Finally, the rationality of the proposed viscoelastic damage constitutive model of concrete was verified by the uniaxial cyclic compression test results.

2. Experiments

2.1. Materials and Mix Proportion. The concrete specimens used were 150 mm cubes. The concrete mix proportion is shown in Table 1. A Grade 42.5 ordinary Portland cement, supplied by HaiLuo Cement Co. Ltd., Anhui Province, China, was used in the experiments as the binder. The fine aggregate was desalinated sea sand, the coarse aggregate was crushed stone, and the size of aggregates was kept below 17.5 mm. Ordinary tap water was used. Mixtures were cast in identical plastic molds. To remove entrapped air, the fresh concrete mix was vibrated on a vibrating table after having been poured into the molds. The cast specimen was kept under laboratory conditions for 24 h and then demolded. They were then cured in a laboratory room at 20°C and relative humidity of 95% for 28 days.

2.2. Test Setup. A cyclic uniaxial compression test was conducted in a microprocessor-controlled electrohydraulic servo universal testing machine WAW-2000 produced by Shanghai Bairuo Testing Instrument Co., Ltd. The vertical displacement was measured by two linear variable displacement transducers (1# and 2# LVDT), and the loading rate was set to control by displacement. Nine specimens were divided into three groups and loaded under three strain rates (10^{-5}s^{-1} , 10^{-4}s^{-1} , and 10^{-3}s^{-1}). The test setup was schematically shown in Figure 1.

Ultrasonic testing was used to detect cracking and deterioration in concrete during the cyclic compression loading experiments. As the ultrasonic wave velocity in concrete is much faster than in the air, the ultrasonic velocity of concrete contains small cracks that would be reduced compared with the initial velocity. Hence, the microcracks nucleation in concrete can be detected by the reduction in ultrasonic velocity [17]. The unloading started at preselected points and the ultrasonic measurements were taken when the compression displacement reached the preselected unloading points. Two ultrasonic probes were arranged vertically in the direction of loading and Vaseline was used as a coupling agent between the probes and concrete to reduce the loss of acoustic energy. Ultrasonic experimental equipment supplied by Beijing Zhibo was adopted.

2.3. Experimental Results. The stress-strain relationships of concrete under cyclic compression load with different strain rates are illustrated in Figure 2. The overall characteristics of the cyclic compression experimental results can be summarized as follows.

A single loading and unloading cycle generally contains two distinct paths: the unloading path and the reloading

TABLE 1: Mix proportion of concrete samples.

Cement type	Quantity of concrete materials per cubic meter (kg/m^3)				Water/cement ratio
	Cement	Sand	Crushed stone	Water	
PO42.5	451	558	1186	185	0.41

FIGURE 1: Schematic diagram of a cyclic uniaxial compression test.

FIGURE 2: Stress-strain curves of concrete under cyclic compression loading with different strain rates: (a) strain rate of 10^{-3} s^{-1} ; (b) strain rate of 10^{-4} s^{-1} ; (c) strain rate of 10^{-5} s^{-1} .

path. As presented in Figure 2, concrete stiffness gradually reduced due to the cumulative effect of internal damage of the material caused by the increased number of load cycles.

The stiffness of the unloading curve was significantly less than the stiffness of the reloading curve. This interesting result indicates that some of the internal microdefects of the material closed during the unloading process.

The loading curve can be divided into two stages during the reloading progress. The stress-strain curve was almost linear before reaching the unloading strain. When the value of strain exceeds the unloading point strain, the stress-strain curve exhibited a nonlinear shape. This result may be explained by the fact that the loading process did not cause new damage before the former unloading point. When the strain was greater than the strain at the unloading point, the stress-strain curve showed nonlinearity due to new damage nucleation.

The relative ultrasonic wave velocity results of concrete subjected to compression loading process with different strain rates are presented in Figure 3. From Figure 3, some of the main characteristics of concrete behavior under cyclic compression loading can be deduced as follows.

During the initial stage of experimental loading, the ultrasonic velocity had no obvious change or only a partial increase due to the internal microdefects closing. These findings corroborate the results of the cyclic compression experiments, which show that the unloading paths were almost overlapping with the loading paths.

As the load increased, the ultrasonic propagation velocity started to decrease slowly as the microcracks at the interface between aggregates and cement mortar began to expand gradually. When the load was close to the peak strength, the ultrasonic wave velocity dropped sharply, indicating that the internal cracks in concrete had spread from the interface cracks between aggregates and mortar to the interior of mortar. The material exhibited obvious nonlinear characteristics at this stage and hysteretic energy dissipation was found more and more evident in Figure 2.

When the load was further increased, the cracks became interconnected and obvious macroscopic cracks could be observed on the outer surface of the concrete (Figure 4). The ultrasonic propagation velocity was significantly reduced due to the instability of the crack propagation and it was difficult to detect any ultrasonic wave velocity change in the later stage.

Taken together, these results suggest that there is a relationship between the ultrasonic velocity and the hysteretic energy dissipation. More obvious hysteretic energy dissipation means that more energy was consumed to create more cracks, which in turn would lead to a longer time required for an ultrasonic frequency wave pulse to travel through a concrete sample. Hence, a reduction in ultrasonic velocity will be observed when concrete cracks.

3. Damage Evolution in Concrete under Cyclic Compression

It can be seen from the stress-strain curves obtained in the concrete cyclic loading experiments that the stiffness of the material degenerates due to the development of microvoids

FIGURE 3: Relative ultrasonic wave velocity of concrete samples subject to the cyclic compressive loading process.

FIGURE 4: Macroscopic cracks on the outer surface of the concrete.

and defects as the deformations increase. Therefore, a damage factor, D , was defined as the relative attenuation of the elastic modulus of concrete under cyclic compressive loading:

$$D = 1 - \frac{\tilde{E}}{E} \quad (1)$$

where $\tilde{E}$ is the elastic modulus of the damaged material and E is the initial elastic modulus.

The modulus of material can be calculated from the ultrasonic wave velocity if the material is homogeneous, isotropic, and elastic. Obviously, concrete does not satisfy these physical requirements [18, 19]. In this paper, the secant modulus was considered as the elastic modulus of damaged material. Equation (1) was used to quantify the experimental results of damage versus strain under three strain rates, which reflected the actual degradation of material stiffness during loading. The calculation results are shown in Figure 5.

The damage evolution curves shown in Figure 5 indicate that the entire damage evolution process in concrete can be divided into three stages. During the initial stage of loading,

FIGURE 5: Experimental and model results of damage evolution in concrete under different strain rates.

the initial microcracks between aggregates and cement mortar remained relatively stable and some microcracks closed under loading, which can lead to an increase in density. During this stage, the modulus of material hardly decreased or even increased compared to the initial modulus. The degree of damage can be considered as 0 at this stage; obviously, a damage threshold existed in the damage evolution process. With the increase in load, the internal damage began to increase and the modulus of material began to decrease slowly, mainly due to the gradual increase of cracks in the transition zones between mortar and aggregates. When the loading increased further, the rate of modulus decline began to accelerate and the damage of material began to increase rapidly. This happened mainly due to microcracks in the transition zone nucleated and expanded into the mortar. At the stage of residual strength, microcracks are formed due to the interconnection of

microcracks in the interfacial zone between aggregates and mortar. At this stage, the specimen cracked and then the attenuation of modulus of elasticity became gentle and damage stabilized.

It can be seen from Figure 5 that a damage threshold existed in the damage evolution process, which increased with the increase in strain ratio. When the strain ratio was small, the microcracks in the material had enough time to coalesce and propagate. On the other hand, the damage threshold increased with the increase in strain ratio due to the internal microcracks not having enough time to propagate and evolve. Therefore, damage evolution in concrete was related not only to the material rheology but also to the strain rates. This conclusion is consistent with literature [20]. Therefore, the strain-rate dependence should be considered when establishing a damage evolution model.

The damage evolution model for concrete assumed in

this study is as follows:

$$D = \begin{cases} 0, & \varepsilon \leq \varepsilon_{th}, \\ 1 - \exp\left(-\left(\frac{\varepsilon - \varepsilon_{th}}{\varepsilon_u}\right)\right), & \varepsilon > \varepsilon_{th}, \end{cases} \quad (2)$$

$$m = \alpha \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^\beta,$$

where ε_{th} is the damage threshold value, ε_u is the undetermined parameter, $\dot{\varepsilon}_0$ is the reference strain rate which was taken as 1 s^{-1} , and m is the shape factor.

By substituting the parameter values listed in Table 2 into equation (2), the damage evolution results can be obtained, as shown in Figure 5. The results of the damage evolution model agreed well with the experimental damage results of modulus attenuation.

4. Model Validation

The experimental results of cyclic the uniaxial compression test of concrete show that some irrecoverable deformation existed after unloading. Song et al. [21] regarded this kind of irrecoverable deformations as viscous conformation. Therefore, the total strain of concrete during compression can be divided into elastic strain and viscous strain as follows:

$$\varepsilon = \varepsilon_e + \varepsilon_v, \quad (3)$$

where ε_e is elastic strain and ε_v is the viscous strain.

Internal variables were introduced to characterize the development of viscous properties. The uniaxial compression experiment was a quasi-static compression process at room temperature, which can be considered as an isothermal. Based on the theory of irreversible thermodynamics, the Helmholtz free energy density function can be expressed as follows:

$$\Phi = \Phi(\varepsilon, D, \xi). \quad (4)$$

Considering the Helmholtz free energy density function of materials as consisting of elastic and irreversible viscous components, equation (4) can be written as follows:

$$\Phi = \Phi_e(\varepsilon_e, D) + \Phi_v(\varepsilon_v, \xi). \quad (5)$$

Based on Lemaitre's equivalent strain principle,

$$\varepsilon_e = \frac{\bar{\sigma}}{E} = \frac{\sigma}{(1-D)E}, \quad (6)$$

where $\bar{\sigma}$ is the effective stress, σ is the nominal stress of the material, and E is the elastic modulus of intact material. Therefore, the elastic part of the free energy density function can be expressed as follows:

$$\Phi_e = \frac{1}{2\rho} (1-D)E\varepsilon_e^2, \quad (7)$$

where ρ is the density of concrete.

TABLE 2: Parameters of damage evolution model.

Strain rate (s^{-1})	α	β	ε_u	ε_{th}
10^{-3}	2.268	0.05809	0.0009	0.0013
10^{-4}	2.268	0.05809	0.0013	0.0011
10^{-5}	2.268	0.05809	0.0024	0.0006

The stress-strain relationship can be obtained from the thermodynamic governing equation as follows:

$$\sigma = \rho \frac{\partial \Phi_e}{\partial \varepsilon_e} = (1-D)E\varepsilon_e = (1-D)E(\varepsilon - \varepsilon_v). \quad (8)$$

According to the nonlinear iterative solutions of internal variables evolution equations [22],

$$\varepsilon_e = \varepsilon - \varepsilon_v = E \int_{-\infty}^t \text{Exp}\left(-\frac{t-\tau}{\tau_m}\right) \dot{\varepsilon}(\tau) d\tau, \quad (9)$$

where τ_m is the material relaxation time.

The viscoelastic constitutive model with damage can be obtained by substituting equation (9) into equation (8) as follows:

$$\sigma = (1-D)E \int_{-\infty}^t \text{Exp}\left(-\frac{t-\tau}{\tau_m}\right) \dot{\varepsilon}(\tau) d\tau. \quad (10)$$

The parameters of the damage evolution model are shown in Table 2. Sima et al. [22] proposed that the envelope of the stress-strain curve of concrete under cyclic compression is essentially consistent with that under uniaxial compression. Therefore, to verify the applicability of the model, the envelope of cyclic compression load curves was fitted by the least-squares method according to the established model. The parameters of the viscoelastic constitutive model obtained by the fitting are shown in Table 3.

The curve fitting results demonstrate that the relaxation time was correlated with the loading rate and that it decreased with the increase in strain ratio on account of the multiplicity of molecular motion. The large-scale unit of motion will not respond to a high strain ratio, while the small-scale unit of motion can be activated by a high strain rate, which would lead to a small relaxation time. On the contrary, under a low strain ratio, the large-scale unit of motion and the small-scale unit of motion have sufficient time response; thus, the relaxation time is longer. The curve fitting results are shown in Figure 6.

Figure 6 shows that the viscoelastic constitutive model with damage was in good agreement with the experimental results. In the present study, the R-square of model calculation results and test results is 0.9018, 0.7573, and 0.9313, respectively. Therefore, the proposed model can well characterize the stress-strain response behavior of concrete under uniaxial compression. The previously proposed constitutive models with damage were only required to coincide with the actual stress-strain curves for the verification of their adaptability. However, damage models often do not capture the actual material stiffness degradation. The model proposed in this paper not only

TABLE 3: Concrete mechanical parameters under uniaxial loading obtained by curve fitting.

Strain rate (s^{-1})	E (GPa)	τ_m (s)
10^{-3}	36.00	1.048×10^{-5}
10^{-4}	31.47	1.907×10^{-6}
10^{-5}	32.55	2.797×10^{-6}

FIGURE 6: Experimental and model stress-strain curves of concrete under different strain rates.

reflects the rheological degradation of material stiffness but can also characterize the stress-strain response behavior of concrete. The proposed model can characterize the stress-strain response of concrete under quasi-static loading. The proposed model in the present study was not suitable for the strain response behavior of concrete under dynamic loading. Our findings suggested that the strength, stiffness, and damage evolution of concrete were related to the loading rate. Therefore, the dynamic mechanical response behavior of concrete needs to be further studied.

5. Conclusions

The quantitative experimental results of damage evolution in concrete subjected to cyclic uniaxial compression loads were obtained. The results showed that there existed a damage threshold related to the strain ratio. The damage evolution model was established, which was consistent with the quantitative damage experimental results. A viscoelastic constitutive relationship with damage was recommended for concrete. The analyses showed that the model can not only

characterize the stress-strain response of materials under one-dimensional compressive loads but also reflect accurately the degradation of the macromechanical properties.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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