Research Article

The Evaluation Method of Rock Mass Stability Based on Natural Frequency

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The limit equilibrium method's analysis index cannot be measured by on-site monitoring equipment and cannot be used for monitoring and early warning of rock instability. The existing rock stability evaluation methods based on vibration information cannot evaluate the stability of rocks quantitatively. In this paper, the slope's constraints on the rock were simplified to springs and a three-dimensional analysis model of rock vibration was established. The equation for calculating the natural frequency of rock that includes the spring stiffness as an indicator was derived. The rock stability calculation function containing the index of natural frequency was brought into the traditional rock stability coefficient calculation equation, and a new rock stability analysis method based on natural frequency was established. The experiment proved the measurability of the index of the natural frequency of rock and the method's effectiveness for the stability analysis of the rock based on natural frequency.

1. Introduction

Rock collapse is a common natural disaster [1], and the accurate analysis of rock stability is an essential and necessary work to warn the damage of dangerous rock and reduce the loss of collapse disaster [2, 3]. The limit equilibrium method is a standard method used in analyzing the stability of rock which uses the principle of static equilibrium to calculate the stability coefficients of rock by analyzing the force states of the rock under various damage modes [4, 5]. Strength parameters such as cohesion intercept (c) and angle of shearing resistance (φ) of the rock or its structural surfaces are important parameters for the analysis of the limit equilibrium method, but these parameters cannot be directly monitored by in situ equipment [6, 7].

The universal monitoring indicators for slope monitoring and early warning are two types of surface deformation and vibration information. However, unlike the deformation rate of landslide hazards with a slowly increasing phase, rock collapse has a strong suddenness. When monitoring the stability of rocks using external deformation indicators, it is possible to determine whether the rocks are

falling, but it is impossible to give an early warning of rock failure [8]. Because of the invalid external deformation indicators of rock collapse warning, some scholars have started to study the relationship between rock vibration information and its stability.

Spillmann et al. [9] analyzed the stability of rock slope in the Swiss Alps and used the microseismic technique. Xu et al. [10] found the damaged rocks by analyzing the spatial and temporal distribution pattern of microseismic activity in the slope. Salvoni and Dight [11] and Feng et al. [12] monitored rocks' failure on unstable slopes by microseismic networks. Mercerat [13, 14] et al. compared the vibration response of unstable rocks and slopes under ambient vibration and found a significant amplification effect of a rock's vibration on the verge of failure. These studies show that rocks' vibration information can reflect the changes in their stability, but the method for quantitative calculation of rock stability has not been given.

To solve this problem, Ma et al. [15, 16] and Xie et al. [17, 18] proposed some simple mechanical models of rocks and investigated the quantitative relationship between the natural frequency of rocks and their stability. In their study,

the rock was simplified as a rigid sphere, and the connection between the rock and the slope was simplified as a spring. As the connection area between the slope and the unstable rock decreases, the spring's stiffness gradually decreases and the rock's stability also decreases. The quantitative relationship between the rock's stability and its natural frequency was established by analyzing the spring stiffness [19]. Although the quantitative relationship between the natural frequency of the rock and its stability was given, the above study also has the following problems: (1) treating the rock as a rigid ball ignored the influence of the shape of the rock on its natural frequency; (2) the model can only be used to analyze the natural frequency of the rock in one direction; and (3) the calculation method of the spring stiffness of the connection between the rock and the slope has a large error [20, 21].

This paper proposes a new model for rock vibration analysis to remedy the above calculation model's deficiency. The model considers the form of rock vibration in three-dimensional space, and the constraints of the slope on the rock were simplified to multiple springs. By establishing the equilibrium equations of motion for the longitudinal, torsional, and transverse swinging vibrations of the rock and connecting them with the rock's boundary conditions, i.e., the springs' stiffness, the equations for solving the formation functions and vibration frequencies of the rock were established. The rock stability coefficients can be calculated using the simultaneous rock natural frequency calculation equation and the rock mechanical equilibrium equation. To analyze the validity of the theoretical calculation results, indoor experiments on the measurability of the rock's natural frequency were conducted. The experimental results proved the validity of the theory.

2. Proposal of the Method

2.1. Limit Equilibrium Analysis of Rock Stability. In the analysis of rocks using the limit equilibrium method, a rock's stability is expressed as the ratio of its resistance to the driving force. The rock's driving force is the action of external load such as gravity, and the resistance of the rock is the restraining force of the slope to the rock. For a rock formed by the cutting of a single joint plane, it can be considered that the restraining force is directly related to the area of the joint plane. The cause of the rock's instability and movement can be considered as the decrease in the joint plane area or the weakening of the joint plane's bond strength, leading to the deficiency of the slope's restraining force [22, 23].

A typical slope is shown in Figure 1, and the joint plane cuts a rock of weight W out of the slope of unit thickness. The length of the connection between this rock and the slope is l_1 , and the length of separation is l_2 . For this rock, the stability factor F_s is shown as follows:

$$F_s = \frac{\tau_1 l_1 + \tau_2 l_2}{W \sin \alpha}, \quad (1)$$

where τ_1 is the maximum shear stress of the rock connection region, τ_2 is the maximum shear stress of the rock separation

region, and α is the inclination angle of the joint plane. The value of τ_2 is equal to 0 when there is no filling material in the fracture.

Equation (1) indicates that the stability coefficient F_s of the rock will decrease as the connection length decreases. Usually, we consider that the rock will become unstable if $F_s \leq 1$ [24].

Because the rock material's stiffness is large, the rock's deformation in the decline of the connection length is minimal. It is difficult for the monitoring equipment to monitor the rock's small deformation, so the rock's stability cannot be judged by monitoring the deformation indexes such as displacement. With the decline of the rock and slope's connection length, the slope's restraint on the rock is weakened. As the constraint decreases, the dynamic indexes such as the rock's natural frequency will be changed. If we can give the mathematical expression of the rock's natural frequency and the connection length l_1 of the slope, we can derive the formula between the nature frequency f of the rock and its stability coefficient F_s by eliminating the parameter l_1 based on equation (1).

2.2. Calculation of Rock Stability Based on the Natural Frequency. The model proposed by Ma et al. [15] and Xie et al. [25, 26] for calculating the natural frequency of the rock is shown in Figure 2(a). This model simplifies the rock as a single-degree-of-freedom block and the rock-slope connection as a tension-compression spring K_0 . The calculation formula for the natural frequency w_0 of the rock is as follows:

$$w_0 = \sqrt{\frac{k_0}{M}}, \quad (2)$$

where k_0 is the stiffness of the tension-compression spring K_0 .

Equation (2) gives the formula for calculating the natural frequency of the rock when it is free to vibrate in the vertical joint plane's direction. However, there is not only one way of vibration of the rock. Along with the vertical vibration, the rock also performs swaying or rotational vibration. To analyze the rock's natural frequencies for multiple modes of vibration, it is necessary to create multiple springs to denote the slope restraints on the rock's translation and rotation in three-dimensional space.

As shown in Figure 2(b), the longitudinal vibration of rock along the Z -axis, torsional vibration around the Z -axis, swinging vibration in the XZ -plane, and swinging vibration in the YZ -plane were analyzed at the same time. It is necessary to establish springs K_1 , K_2 , K_3 , and K_4 to represent the boundary conditions of the above four vibrations of the rock, and the stiffnesses of these four springs can be expressed as k_1 , k_2 , k_3 , and k_4 . Combining the rock's motion equations and its boundary conditions, the rock vibration function and natural frequency expressions can be calculated.

For the rock shown in Figure 2(b) with length L , width B , and height H , the coordinate system is established with the center of the lower bottom surface of the rock as the origin, and the vibration function of the rock vibrating freely along

FIGURE 1: Calculation model of jointed rock.

FIGURE 2: Simplified model for rock vibration.

the Z-axis in the longitudinal direction can be written as follows [27]:

$$u(z) = B_1 \cos\left(\frac{w_1 z}{c_1}\right) + B_2 \sin\left(\frac{w_1 z}{c_1}\right), \quad (3)$$

where u is the displacement of the rock in the z -direction, B_1 and B_2 are unknown quantities representing the vibration function of the rock, w_1 is the frequency of the longitudinal vibration of the rock, and c_1 is the propagation velocity of the elastic wave along the Z -axis. The c_1 can be calculated by the following equation:

$$c_1 = \sqrt{\frac{E}{\rho}}, \quad (4)$$

where E is Young's modulus of the rock and ρ is the density of the rock.

For the rock shown in Figure 2(b), which is free at the top and connected to the slope foundation at the bottom, the boundary conditions can be written as follows:

$$\begin{cases} z = 0, & \text{EBL} \frac{du}{dz} = k_1 u, \\ z = H, & \frac{du}{dz} = 0. \end{cases} \quad (5)$$

Combining equation (5) with equation (3), the formula for calculating the intrinsic frequency of longitudinal vibration of rock can be derived as follows:

$$k_1 \cos\left(\frac{w_1 H}{c_1}\right) - \text{EBL} \left(\frac{w_1}{c_1}\right) \sin\left(\frac{w_1 H}{c_1}\right) = 0. \quad (6)$$

The longitudinal natural frequency w_1 of rock can be obtained by introducing the parameters such as size, density, Young's modulus, and Poisson's ratio of rock into equation (6). The satisfying condition (6) has multiple values of w_1 , which represent the frequencies of longitudinal vibration of the rock at different orders.

The calculation methods of the rock's frequencies for torsional vibration around the Z -axis swinging vibration in the XZ -plane and swinging vibration in the YZ -plane are similar to those of rock torsional vibration around the X -

axis. Because the calculation process of rock vibration is involved, it may not derive the analytical solution of rock's natural frequency, but the numerical solution of rock's natural frequency can be calculated by taking the parameters such as size and mass of rock.

For the rock shown in Figure 2(b), the constraint stiffness k_1 of Z-direction vibration of slope rock is as follows [28, 29]:

$$k_1 = \frac{4G}{1-\nu} 0.55 \sqrt{Bl_1}, \quad (7)$$

where G is the shear modulus of the rock and ν is Poisson's ratio of the rock.

The formula between the rock connection length l_1 and its intrinsic frequency w_1 can be derived from equations (7) and (6). And the stability of the rock can be calculated by bringing l_1 into equation (1) and assuming that the calculation formula of rock's connection length based on natural frequency is $l_1 = f(w)$. For the rock in Figure 1, the maximum shear stress τ_1 in the rock's connection area can be calculated by the following equation:

$$\tau_1 = c_1 + \frac{W \cos \alpha}{l_1} \tan \varphi_1, \quad (8)$$

where c_1 is the cohesion of the rock and φ_1 is the angle of shearing resistance of the rock.

A new expression of rock stability coefficient can be obtained by incorporating equation (8) into equation (1):

$$F_s = \frac{c_1 f(w)}{W \sin \alpha} + \cot \alpha \tan \varphi_1. \quad (9)$$

Equation (9) is the formula for calculating the stability coefficient of rock based on the indicator of natural frequency, which replaces the connection length l_1 of rock with the function of rock's natural frequency $f(w)$. Therefore, the stability of rock can be determined by the indicator of rock's natural frequency after the strength parameters and mass of rock are determined. The rock's strength parameters and mass usually do not change, and the damage caused by the external environment to the rock is mainly in its connection zone with the slope. Using on-site monitoring equipment to measure the change of rock's natural frequency in real-time, we can judge the state of the rock's connection area and realize the dynamic monitoring of rock's stability.

However, the expression $f(w)$ as a function of the rock's natural frequency in equation (9) cannot be given directly. When calculating the rock's connection length l_1 from its natural frequency, it is necessary to give the numerical solution of l_1 after bringing in the physical and mechanical parameters of the rock. The material of the rock is defined in Figure 2(b) as one kind of sandstone [30]. When $L = 3$ m, $B = 2$ m, and $H = 5$ m, the variation process line of natural frequency w of the four vibrations of the rock with the decrease in the length l_1 of the connection is shown in Figure 3. In the figure, FLV is used to represent the longitudinal vibration of rock along the Z-axis, FTov is used to represent the torsional vibration of rock around the Z-axis, FTrV1 is used to represent the swinging vibration rate of

rock in the XZ-plane, and FTrV2 is used to represent the swinging vibration of rock in the YZ-plane.

It can be seen from Figure 4 that the natural frequency of the rock decreases with the decline of the connection length. In other words, the natural frequency of the rock decreases as the stability of the rock declines. The results of this theoretical calculation remain consistent with the previous analysis.

3. Experiments

In order to analyze the validity of the theoretical model, a simple indoor experiment was carried out. The rock is granite with the length, width, and height of 25 cm, and a rubber pad of thickness $D = 3$ cm is laid under the granite with the same area as the cross-section of the rock. To simulate the reducing of rock's connection area, the rubber pad was cut with a knife to reduce its connection length l_1 . As shown in Figure 5, two triaxial accelerometer sensors were glued to the rock's top and side.

The rubber pad was cut 1 cm at a time, starting from the right side of the model, and then the rock was vibrated by striking the ground or the rock with a small hammer. These two acceleration sensors measured the vibration acceleration of the rock. During each experiment, the vibration acceleration of the rock was recorded for more than 10 seconds. After the vibration acceleration's time-domain curve was recorded, it was converted into a frequency-domain signal using the fast Fourier transform. The time-domain and frequency-domain curves of rock vibration recorded by the sensors when the rubber pad's area was the same as the rock's cross-sectional area are shown in Figure 6.

The rubber pad's length was decreased from 25 cm to 18 cm in the experiment, and the data of rock vibration were collected 8 times. The measurement results of the rock's vibration frequencies under various vibration modes during the length drop of the rubber pad are shown in Table 1.

4. Discussion

Based on the data in Table 1, the vibration frequencies of rock under different vibration modes are plotted against the rubber pads' length, as shown in Figure 7. From Figure 7, it can be seen that the torsional vibration frequencies of rock bodies located on rubber pads are significantly more than the frequencies of longitudinal vibration and side-swing vibration, and this difference can be maintained in the process of decreasing the length of rubber pads. The change of the frequency of side-swinging vibration of the rock in XZ- and YZ-plane has its apparent characteristics. When the length of the rubber pad $l = 25$ cm, the swing natural frequency of rock in the XZ-plane is equal to that in the YZ-plane. However, as the length of the rubber pad l decreases, both of them show different degrees of decrease, and the decrease in the swinging natural frequency of the rock in the YZ plane is more pronounced.

It is also necessary to determine the relationship equation between the rock's natural frequency and its connection length to analyze its stability by vibration data. In

FIGURE 3: Variation of the natural frequency of the rocks.

FIGURE 4: Calculation of natural frequency thresholds for rock instability.

FIGURE 5: Schematic diagram of the experimental model.

FIGURE 6: Monitoring data of measurement point one: (a) time-domain signals of vibration; (b) frequency-domain signal of vibration.

TABLE 1: Rock natural frequency and measurement conditions.

Number	FLV (Hz)	FToV (Hz)	FTrV1 (Hz)	FTrV2 (Hz)	l (cm)
1	34.5	55.5	12.0	12.0	25
2	32.0	54.5	11.5	10.5	24
3	31.0	53.5	11.0	10.0	23
4	30.0	50.0	10.5	8.5	22
5	30.0	48.5	10.5	7.5	21
6	29.5	46.5	10.0	6.5	20
7	29.0	45.5	10.0	6.0	19
8	28.0	42.0	9.5	4.5	18

FIGURE 7: Natural frequency of rock under different vibration methods.

the experiments, the rubber mat and the rock's physical and mechanical parameters are unknown quantities, and the experimental data are analyzed using function fitting to determine the equation between the natural frequency ω and

the connection length l . The power function curve's results fitting between the rock's natural frequency and its connection length for the four vibration modes are shown in Figure 8.

FIGURE 8: Rock natural frequency curve fitting.

The four calculation equations for the rock connection length l are as follows:

$$\begin{cases} w_{FLV} = -2.053l - 0.029l^2, \\ w_{FToV} = 2.693l - 0.018l^2, \\ w_{FTrV1} = 0.653l - 0.007l^2, \\ w_{FTrV2} = -0.280l + 0.030l^2, \end{cases} \quad (10)$$

where w_{FLV} , w_{FToV} , w_{FTrV1} , and w_{FTrV2} represent the rock's vibration frequencies under the four vibration modes of FLV, FToV, FTrV1, and FTrV2. They did not have adhesive force between the rock and the rubber pad in this experiment. Therefore, we can assume that the rock will be unstable when the center of gravity is losing support. The condition of the instability of the rock is $l = 12.5$ cm. The threshold value of the rock's natural frequency before its instability can be calculated by bringing the criterion of the instability of the rock into equation (2), as shown in Figure 4. The natural frequency thresholds of the rock instability in the experiment are $w_{FLV} = 21.10$ Hz, $w_{FToV} = 30.85$ Hz, $w_{FTrV1} = 7.02$ Hz, and $w_{FTrV2} = 1.25$ Hz.

The experiment shows that the natural frequency of rock can reflect the change of rock stability, and the sensitivity of the natural frequency of rock to the change of connection length is not the same in different vibration modes. Figures 3 and 7 both indicate that the frequency of torsional vibration of rock around the Z-axis is the largest when the connection area of rock and slope is complete. However, as the connection length decreases, the frequency of torsional vibration of the rock around the Z-axis is the most variable in Figure 3, and the rock swaying vibration in the YZ plane is the most variable in Figure 7. The parameters of the sandstone chosen for the calculation of the rock in Figure 3 and the parameters of the experimental material in Figure 7 are different, so we can make a conjecture that the sensitivity of the frequency variation of the different vibration modes of

the rock is related to the physical and mechanical parameters of the rock. The proof of this conjecture needs to be further investigated in the subsequent work.

5. Conclusion

The limit equilibrium method is a common method for rock stability analysis, but the calculated parameters in this method, such as bond and friction angle, cannot be measured by on-site monitoring equipment. It is necessary to determine the stability analysis methods with monitorable indicators to warn the rocks' failure. The method of rock stability analysis based on vibration information has been proposed, but it cannot accomplish the quantitative analysis of rock stability.

The stability of the rock is controlled by its joint plane. In this paper, the rock's restraint from the slope was simplified as springs, and the springs' stiffness was related to the rock and slope's connection length. Through the rock's vibration function, the method of calculating the natural frequency of the rock containing the variable of spring stiffness is determined. Finally, the calculating formula of the connection length based on the rock's natural frequency index is confirmed. The formula was brought into the rock's limit equilibrium equation, and a model for judging the stability of the rock based on the natural frequency was established.

The article analyzed the change process of rock's natural frequency with its connection length through theoretical calculation and experiment. The result proved the feasibility of rock stability evaluation based on the natural frequency index. However, there are different vibration forms of rocks, and when using the natural frequency index to analyze the stability of rocks, selecting the most sensitive one from the vibration frequencies of multiple vibration modes of rocks is still undetermined, and further research on this is needed.

Data Availability

The data used to support the findings of this study are included in the supplementary information files.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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Supplementary Materials

Frequency Curve Data of Rock Vibration in the Experiment. The vibration frequency data of the rock block are measured and calculated by the vibration sensors installed on the rock block. The first row of data 1 represents sensor 1, 2

represents sensor 2, and x , y , and z indicate the direction of rock vibration. A total of eight experiments were performed on the same block, and a total of eight sets of frequency data were generated. The rubber mat was not cut in the first set of experiments (VibrationMeterMonitoringData-1), and the cut length of the rubber mat was 7 cm in the eighth set of experiments (VibrationMeterMonitoringData-8). If you have questions about the data, you can contact LIU: grealwn@126.com. (Supplementary Materials)

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