

Research Article

A Damage Constitutive Model of Rock Subjected to Freeze-Thaw Cycles Based on Lognormal Distribution

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The constitutive model of rock is closely connected with the mechanical properties of rock. To achieve a more accurate quantitative analysis of the mechanical properties of rock after the action of freeze-thaw cycles, it is necessary to establish the constitutive models of rock subjected to freeze-thaw cycles from the view of rock damage. Based on the assumption of rock couple damage, this study established a statistical damage constitutive model of rock subjected to freeze-thaw cycles by combining the lognormal distribution, which is commonly used in engineering reliability analysis, and the strain strength theory. Then, the coordinates and derivative at the peak of the stress-strain curve of the rock after the action of freeze-thaw cycles were obtained through experiments to solve the statistical distribution parameters μ_ε and S of the model, whereafter, the theoretical curves by the established model were compared with the experimental curves to verify the validity of it, which shows a great agreement. Finally, the sensitivity analysis of the statistical distribution parameters was implemented. The results indicate that μ_ε reflects the strength of the rock, which shows a positive relation, and S stands for the brittleness of the rock, which shows a negative relation.

1. Introduction

Freeze and thaw effect is widely encountered in cold areas, which shows significant influences on the rock mass when constructing geotechnical engineering. In order to ensure the stability of the projects, in addition to considering the inherent mechanical properties of rock mass, it is also necessary to consider the deterioration of the rock mechanical properties under the low-temperature environment and the repeated freezing and thawing action, which has been a research focus in the field of rock mechanics. Extensive research studies have been carried out on the degradation of rock mechanical properties caused by freeze-thaw cycles. Typically, the changes in the compressive mechanical properties [1–5], shear mechanical properties [6–10], tensile mechanical properties [11–13], and dynamic compressive mechanical properties [14–16] of rocks were studied under the influence of freeze-thaw cycles. Despite these studies providing crucial conclusions about the degradation effect of freeze-thaw cycles on the mechanical properties of rocks, the majority of them still

remain in the qualitative stage, which may not be used for accurately predicting the mechanical behaviors of rocks subjected to freeze-thaw cycles. As a result, it is necessary to establish a constitutive model of rock subjected to freeze-thaw cycles to reflect the mechanical behaviors more precisely.

Since the concept of rock statistical damage constitutive was put forward by combining continuous damage theory and statistical strength theory, it has become an efficient method to studying the rock stress-strain relationships [17–19]. Current damage constitutive models mainly adopt the Weibull distribution to describe the statistical characteristics of rock microunits [20–23], while research evidence indicates that Weibull statistical theory may not apply to quasi-brittle materials such as rock. Comparing with the Weibull distribution, the lognormal distribution, as a classic distribution commonly used in engineering reliability analysis, has a wider applicability. Therefore, on the basis of the previous statistical models, this paper established a statistical damage constitutive model of rock subjected to freeze-thaw cycles based on lognormal distribution, which

was verified by the comparison between the theoretical curves and experimental curves.

2. Derivation of the Damage Constitutive Model of the Rock after Freezing and Thawing Action

2.1. Freeze-Thaw Damage Variable D_n . There exist multitudinous microdefects (cracks, pores, etc.) inside the rock. The water in these microdefects freezes into ice when the temperature around the rock decreases below 0°C, causing volume expansion, while it melts into water when the temperature rises above 0°C, in which progress part of the water will migrate [24, 25]. Under the repeated action of freeze and thaw, the water in the rock continuously goes through phase transitions and migrates, resulting in the expansion, connection, and destruction of these microdefects in the rock, which ultimately manifests as the deterioration of the macromechanical properties of rock [26, 27]. Therefore, the degradation of the rock macroscopic mechanical properties can be used to characterize the freeze-thaw damage variable. Because the elastic modulus of the rock after freeze-thaw cycles is easy to measure and analyze, so this study used the following formula to define the rock freeze-thaw damage variable:

$$D_n = 1 - \frac{E_n}{E_0}, \quad (1)$$

where E_n is the elastic modulus of the rock after different freeze-thaw cycles and E_0 is the initial elastic modulus of the rock.

2.2. Damage Variable under Loading D_p . For a quasi-brittle material such as rock, the strain caused by loading is often used to analyze the rock damage. Due to the rock discontinuity and the random distribution of load-bearing particles, it can be assumed that the strain limit of the rock microunits after the action of freeze-thaw cycles obeys a certain statistical distribution, and the failure occurs when the strain of the microunit exceeds the limit. In order to calculate the damage variable D_p of the rock at a certain strain level after the action of freeze-thaw cycle, the initial number of rock microunits after the action of freeze-thaw cycles was denoted as N_0 , and the failure number of rock microunits under a certain strain level is N_p . Definition:

$$D_p = \frac{N_p}{N_0}. \quad (2)$$

For the statistical distribution that the strain limit of rock microunits after the action of freeze-thaw cycles obeys, the previous research studies mainly employed the Weibull distribution. However, rock is a kind of quasi-brittle material; due to the obvious characteristic length, the problem of size effect in Weibull statistical theory does not apply to quasi-brittle material [28]. Therefore, this paper intended to select the normal distribution to describe the distribution of

the strain limit of the microunits. In general, the strain limit of the rock microunits is greater than 0. If the normal distribution is directly applied, the negative value is definitely unreasonable. In the structural reliability theory, the lognormal distribution is generally used as the structural resistance probability distribution, so this article finally adopted the lognormal distribution as the statistical distribution of the strain limit. Suppose $\ln \varepsilon \sim N(\mu_\varepsilon, S^2)$; then, its probability density function is

$$f(\varepsilon) = \frac{1}{\varepsilon S \sqrt{2\pi}} \exp\left[-\frac{(\ln \varepsilon - \mu_\varepsilon)^2}{2S^2}\right], \varepsilon \geq 0, \quad (3)$$

where μ_ε and S are the statistical parameters of the lognormal distribution.

In the process of the rock microunits, strain increases from 0 to ε_1 , and the number of the damaged rock microunits is

$$\begin{aligned} N_p &= N_0 \int_0^{\varepsilon_1} f(\varepsilon) d\varepsilon \\ &= N_0 \int_0^{\varepsilon_1} \frac{1}{\varepsilon S \sqrt{2\pi}} \exp\left[-\frac{(\ln \varepsilon - \mu_\varepsilon)^2}{2S^2}\right] d\varepsilon \\ &= N_0 \Phi\left(\frac{\ln \varepsilon_1 - \mu_\varepsilon}{S}\right), \end{aligned} \quad (4)$$

where Φ refers to the distribution function of the standard normal distribution.

From formulas (2) and (4), the following formulas can be obtained:

$$D_p = 1 - \Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_1}{S}\right). \quad (5)$$

2.3. Damage Constitutive Model of Rock Subjected to Freeze-Thaw Cycles Based on Lognormal Distribution. The damage variables of the rock subjected to the freeze-thaw cycles and the loading separately were deduced above. According to the generalized equivalent strain principle proposed in [29], taking the damage state of the rock after the action of freeze-thaw cycles as the reference state and the state of the rock after being loaded as the damaged state, the coupling damage can be gained:

$$\begin{aligned} D &= 1 - (1 - D_n)(1 - D_p) \\ &= D_n + D_p - D_n D_p, \end{aligned} \quad (6)$$

where D is the total damage variable of the rock under the coupling effect of freeze-thaw cycles and loading.

From formulas (1), (5), and (6), the following formula can be obtained:

$$D = 1 - \frac{E_n}{E_0} \Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_1}{S}\right). \quad (7)$$

When the rock is not subject to freeze-thaw cycles ($E_n = E_0$),

$$D = 1 - \Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_1}{S}\right) = D_p. \quad (8)$$

When the rock is unloaded ($\varepsilon_1 = 0$),

$$\Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_1}{S}\right) = 1, \quad (9)$$

$$D = 1 - \frac{E_n}{E_0} = D_n.$$

According to the Lemaitre equivalent strain principle [30] and the generalized Hooke's law, the constitutive equation of the rock under conventional triaxial compression can be obtained:

$$\sigma_1 = E_0 \varepsilon_1 (1 - D) + 2\nu_n \sigma_3. \quad (10)$$

From formulas (7) and (8), the constitutive equation of freeze-thaw damage of rock based on lognormal distribution is

$$\sigma_1 = E_n \varepsilon_1 \Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_1}{S}\right) + 2\nu_n \sigma_3, \quad (11)$$

where ν_n is Poisson's ratio of the rock after different freeze-thaw cycles.

2.4. Determination of Model Parameters μ_ε and S . As shown in Figure 1, the general compression stress-strain curve of rock after the action of freeze-thaw cycles also includes five stages (the compaction stage of the OP section, the elastic stage of the PA section, the stable expansion section of the AB section, the unstable and unstable expansion section of the BC section, and the postpeak stage of the CD section).

It can be seen from Figure 1 that the stress-strain curve passes the peak point and the derivative at the peak point is equal to zero: ① $\varepsilon_1 = \varepsilon_c, \sigma_1 = \sigma_c$; ② $d\sigma_1/d\varepsilon_1|_{\varepsilon_1=\varepsilon_c} = 0$.

Then, the following formulas can be obtained:

$$\sigma_c = E_n \varepsilon_c \Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_c}{S}\right) + 2\nu_n \sigma_3, \quad (12)$$

$$\frac{\Phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_c}{S}\right) - \phi\left(\frac{\mu_\varepsilon - \ln \varepsilon_c}{S}\right)}{S} = 0, \quad (13)$$

where ϕ is the probability density function of the standard normal distribution:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), x \in R. \quad (14)$$

Assuming $X = (\mu_\varepsilon - \ln \varepsilon_c)/S$, it can be obtained that $(\sigma_c - 2\nu_n \sigma_3)/E_n \varepsilon_c = \Phi(X)$ from formula (12). Substituting the peak point $(\varepsilon_c, \sigma_c)$ of the rock stress-strain curve after different freeze-thaw cycles and different confining pressures, the values of $\Phi(X)$ can be obtained; then, the values of X can be obtained by checking the standard normal distribution function table.

From formulas (12)–(14), the following formula can be obtained:

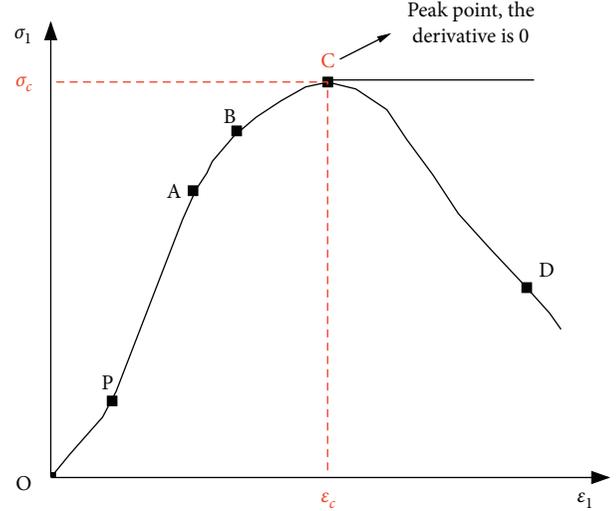


FIGURE 1: The general compression stress-strain curve of rock after the action of freeze-thaw cycles.

$$\sqrt{2\pi} \frac{\sigma_c - 2\nu_n \sigma_3}{E_n \varepsilon_c} S = \exp\left(-\frac{X^2}{2}\right). \quad (15)$$

Taking the logarithm of both sides of formula (1), we obtain

$$\ln\left[\sqrt{2\pi} \frac{\sigma_c - 2\nu_n \sigma_3}{E_n \varepsilon_c}\right] + \ln S = -\frac{X^2}{2}. \quad (16)$$

From formula (14), the following formulas can be obtained:

$$S = \exp\left\{-\frac{X^2}{2} - \ln\left[\sqrt{2\pi} \frac{\sigma_c - 2\nu_n \sigma_3}{E_n \varepsilon_c}\right]\right\}, \quad (17)$$

$$\mu_\varepsilon = XS + \ln \varepsilon_c. \quad (18)$$

3. Model Verification

The test results of red sandstone triaxial compression after different freeze-thaw cycles conducted in [31] were used to verify the rationality and applicability of the damage constitutive model of the rock subjected to freeze-thaw cycles based on lognormal distribution proposed in this paper.

According to the test results of [31], the model parameters established in this paper were shown in Table 1. The lognormal distribution parameters under the different freeze-thaw cycles and the different confining pressure were calculated by formulas (17) and (18).

For example, when the number of freeze-thaw cycles $n = 5$, $\sigma_3 = 2$ MPa. From Table 1, it can be obtained that

$$\begin{aligned} E_n &= 1.295 \text{ GPa}, \\ \nu_n &= 0.259, \\ \sigma_c &= 13.101 \text{ MPa}, \\ \varepsilon_c &= 10.6. \end{aligned} \quad (19)$$

TABLE 1: Parameter values of the rock damage constitutive model under different freeze-thaw cycles.

F-T cycles	σ_3 (MPa)	ϕ_n (°)	E_n (GPa)	ν_n	σ_c (MPa)	$\varepsilon_c/10^{-3}$	$S/10^{-3}$	$\mu_\varepsilon/10^{-3}$
0	2	38.4	1.387	0.258	14.572	11.0	0.2144	2.6588
	4		1.628	0.255	19.652	13.0	0.3018	2.8553
	6		1.649	0.254	24.866	16.0	0.3078	3.0644
5	2	35.8	1.295	0.259	13.101	10.6	0.2295	2.6289
	4		1.452	0.257	19.132	13.2	0.2099	2.8386
	6		1.565	0.255	24.347	17.0	0.3499	3.1278
10	2	33.6	1.156	0.262	12.701	11.3	0.2078	2.6821
	4		1.289	0.260	18.910	14.5	0.1950	2.9239
	6		1.325	0.259	23.519	19.2	0.3465	3.2494
20	2	30.9	0.890	0.268	11.356	12.1	0.0985	2.6606
	4		1.066	0.264	18.100	18.0	0.2996	3.1804
	6		1.240	0.260	22.903	21.1	0.4157	3.3369
40	2	29.2	0.710	0.277	10.570	15.2	0.2325	2.9907
	4		0.917	0.273	17.121	19.0	0.2627	3.2253
	6		0.932	0.269	21.274	24.9	0.3832	3.5076

Then, $\Phi(X) = (\sigma_c - 2\nu_n\sigma_3) / E_n\varepsilon_c = 13.101 - (2 * 0.259 * 2) / 1.295 * 10.6 = 0.8789$. By checking the normal distribution table, $X = 1.168$.

Substituting it into formula (17), we obtain

$$S = \exp \left\{ -\frac{1.168^2}{2} - \ln \left[\sqrt{2\pi} \frac{13.101 - (2 * 0.259 * 2)}{1.295 * 10.6} \right] \right\}$$

$$= 0.2295. \quad (20)$$

Finally, substituting $S = 0.2295$ into formula (18), we obtain

$$\mu_\varepsilon = XS + \ln \varepsilon_c = 1.168 * 0.2295 + \ln 10.6 = 2.6289. \quad (21)$$

Then, according to formula (11), the theoretical stress-strain curves of the rock under different freeze-thaw cycles and confining pressure were calculated, which were then compared with the experimental curves. The results were shown in Figure 2, wherein Figures 2(a)–2(e), respectively, represent the comparisons corresponding to the 0, 5, 10, 20, and 40 cycles of freeze-thaw.

It can be seen from Figure 2 that the theoretical values and the experimental values are relatively close, and the error is within a reasonable range, indicating that the statistical damage constitutive model is reasonable. The proposed model can accurately reflect the elasticity and yield stage of the rock. However, it must be pointed out that the theoretical curve cannot describe the compaction stage of the rock well. This is because the statistical constitutive model assumes that the loaded material are the rock microunits while the pores of the rock are not considered. Therefore, the effect of gas pressure in the pores of the rock during the compaction stage is ignored. In addition, the theoretical curve cannot describe the postpeak stage of the rock. This is because the statistical constitutive model believes that once the strain of rock microunits reaches the strain limit, it is completely

damaged and no longer bears any load. In fact, due to the friction between microunits, they still bear a certain residual stress after failure. Thus, this statistical constitutive model is not suitable for the postpeak stage of the rock.

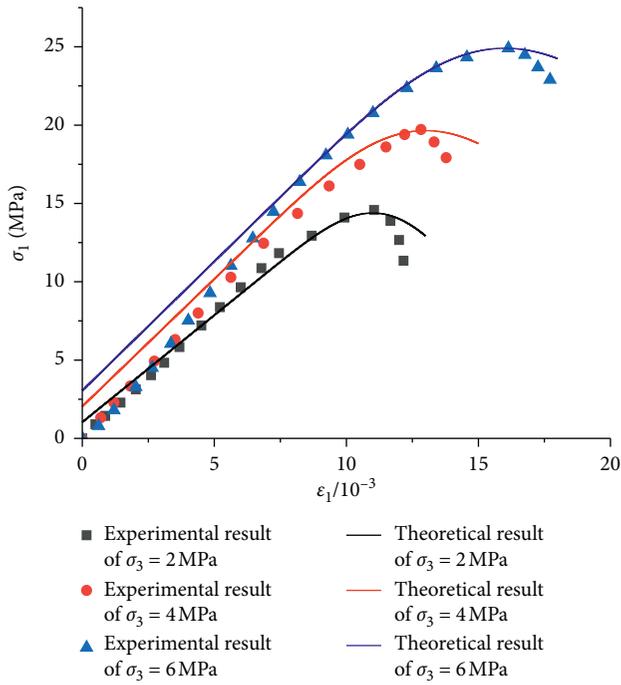
4. The Physical Meaning of Model Parameters μ_ε and S

Through carrying out sensitivity analysis of parameters μ_ε and S , respectively, the physical meanings they represent were discussed.

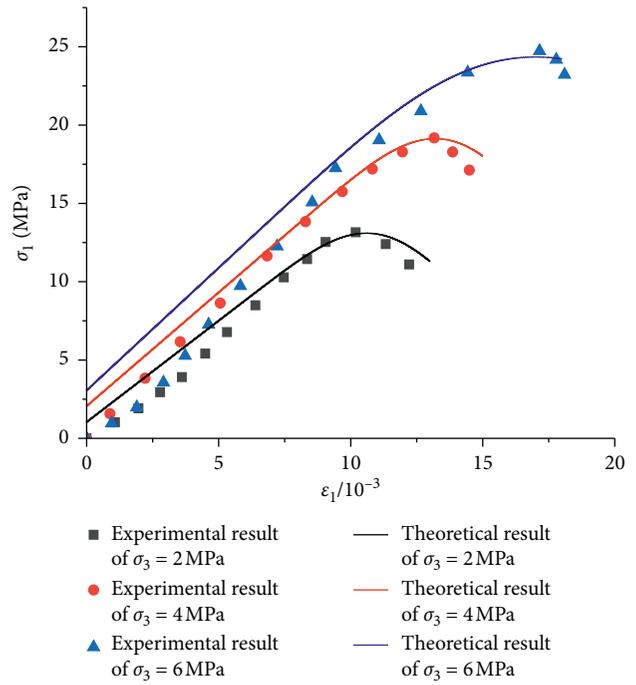
By fixing $E_n = 2$ GPa, $S = 1$, and $\sigma_3 = 0$ MPa and supposing $\mu_\varepsilon = 0.8, 0.9, 1.0, 1.1, \text{ and } 1.2$, respectively, the different stress-strain curves were shown in Figure 3. It can be seen from Figure 3 that when other parameters remain unchanged, as the parameter μ_ε increases, the elastic segments of the stress-strain curve basically overlap, the peak strength of the rock increases significantly, and the postpeak curves are roughly parallel. So, it can be considered that μ_ε reflects the strength of the rock, which is positively related with the strength of the rock.

Similarly, keeping $E_n = 2$ GPa, $\mu_\varepsilon = 1$, and $\sigma_3 = 0$ MPa and supposing $S = 0.8, 0.9, 1.0, 1.1, \text{ and } 1.2$, respectively, the different stress-strain curves were shown in Figure 4. It can be seen from Figure 4 that when other parameters are fixed, with the continuous increase of the parameter S , the elastic segments of the stress-strain curve basically coincide, the peak strength of the rock increases, and the curve after the peak becomes more and more gentle, indicating that the ductility of the rock is enhancing. So, it can be considered that the parameter S represents the brittleness of the rock. As S increases, the ductility of the rock increases while the brittleness of the rock decreases.

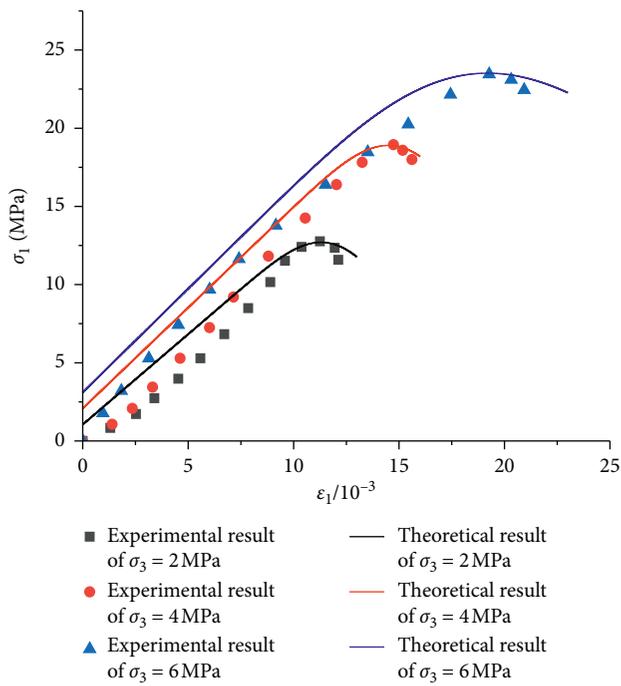
In the lognormal distribution, S represents the variance, which means the dispersion degree of the variable. When the value of S is small, the dispersion of the rock strain limit is also small, and the rock is more likely to undergo



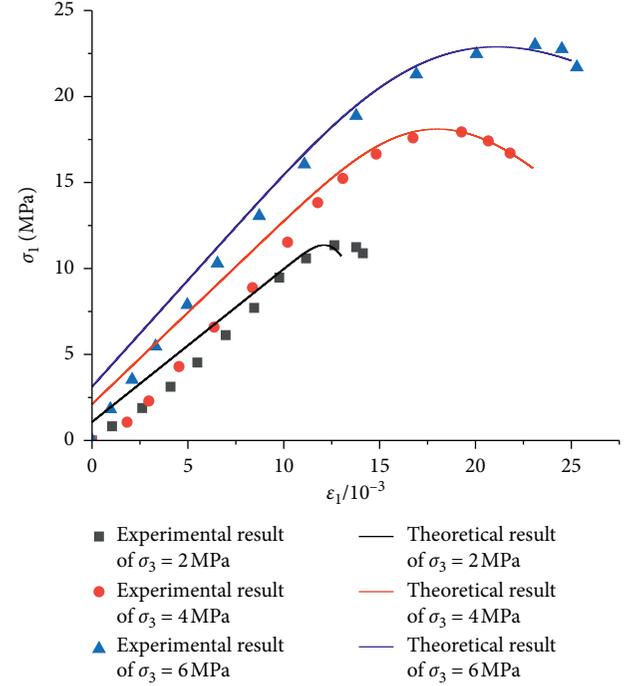
(a)



(b)



(c)



(d)

FIGURE 2: Continued.

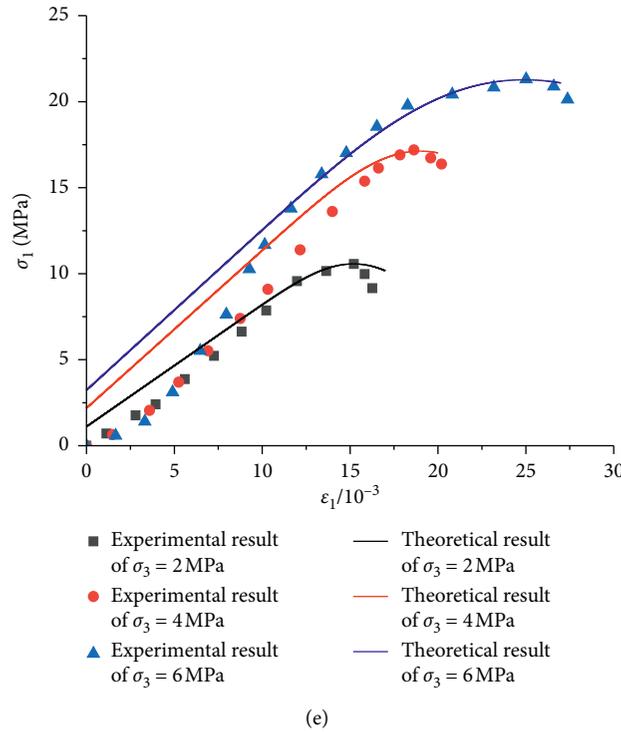


FIGURE 2: The stress-strain curves of rock subjected to different freeze-thaw cycles. (a) $n = 0$, (b) $n = 5$, (c) $n = 10$, (d) $n = 20$, and (e) $n = 40$.

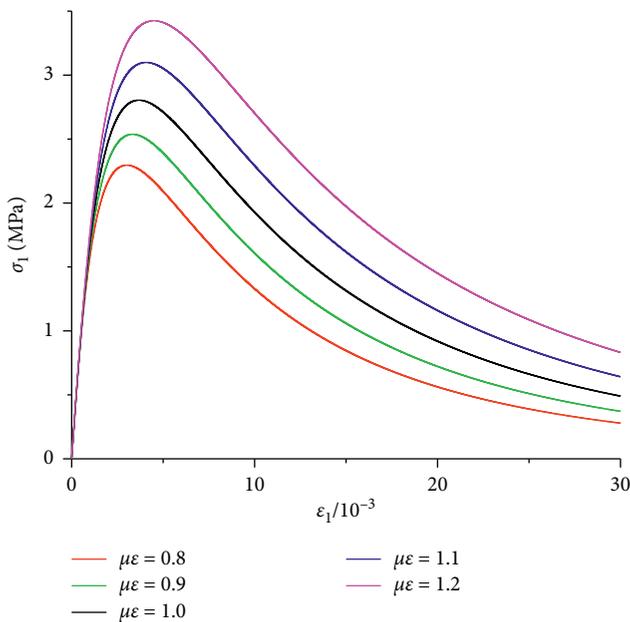


FIGURE 3: The sensitivity analysis of parameters μ_ϵ .

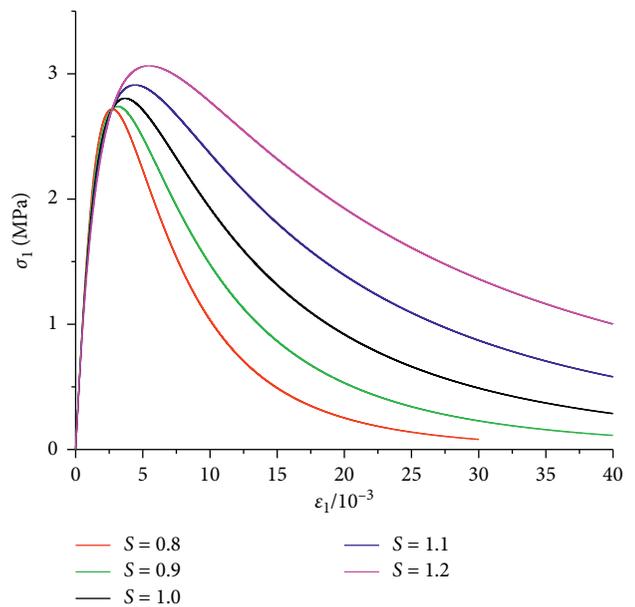


FIGURE 4: The sensitivity analysis of parameters S .

concentrated failure once the strain reaches the average value of the rock strain limit, causing brittle failure. In the case of large value of S , there is still a large part of the rock that has not failed when the strain value reaches the average strain limit, which results in the ductile failure of rock. This is consistent with the previous conclusion drawn by Figure 4.

5. Conclusion

- (1) Through theoretical derivation, this paper established a new statistical damage constitutive model of rock subjected to freeze-thaw cycles based on the lognormal distribution. The model is simple in expression, the parameters are easy to solve, and the stress-strain curve of the rock after the action of

different freeze-thaw cycles can be obtained, which has strong applicability.

- (2) The calculated theoretical curves by established model were compared with the experimental curves, which have similar trends and show a great coincidence, indicating that the statistical damage constitutive model is reasonable and valid. However, it cannot describe the compaction stage and the postpeak stage of the rock very well.
- (3) The sensitivity analysis of two lognormal distribution statistical parameters μ_ε and S have been carried out. The results show that μ_ε reflects the strength of the rock, which shows a positive relation. S represents the brittleness of the rock. When other parameters remain unchanged, the rock ductility increases while the rock brittleness decreases, as S increases.

Data Availability

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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