Pasternak Model-Based Tunnel Segment Uplift Model of Subway Shield Tunnel during Construction

Xiang-Ling Tao,1,2 Yun-He Su,4 Qi-Yin Zhu,4 and Wen-Long Wang3,4

1School of Mines, China University of Mining and Technology, Xuzhou 221116, China
2College of Innovation and Entrepreneurship, Jiangsu Vocational Institute of Architectural Technology, Xuzhou 221116, China
3Anhui Huizhou Geology Security Institute Co., Ltd., Hefei 230000, China
4State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou 221116, China

Correspondence should be addressed to Qi-Yin Zhu; qiyin.zhu@gmail.com

Received 15 July 2021; Accepted 31 August 2021; Published 9 September 2021

Academic Editor: Jian Ji

Copyright © 2021 Xiang-Ling Tao et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Segment uplift is a frequent problem during the construction of shield tunnels. The phenomenon of segment uplift in Xuzhou Metro Line 1 is investigated. The segment uplift magnitude in the shale layer is quite smaller than that in the clay layer. A tunnel segment uplift calculation model based on the Pasternak foundation beam model is proposed, which considers the hardening process of the grouted body with time. The finite difference method is adopted to calculate the magnitude of segment uplift during different tunnel construction stages, and a numerical solution of segment uplift on the longitudinal direction is obtained. The applicability of the numerical solution is studied by comparing with the field test results and parametric analyses are also performed to investigate the effects of different factors on segment uplifting. The results show that the coefficient of subgrade modulus, shear stiffness of the shear layer, and grout pressure all influence the segment uplifting of the tunnel. The influence of the coefficient of subgrade modulus and grout pressure on segment uplift is more significant.

1. Introduction

China is building a large-scale urban underground infrastructure. The shield method is widely used in the construction of subway tunnels because of the less impact on the surrounding environment. However, one disadvantage of this method is that the diameter of the tunnel boring machine is larger than the tunnel lining, so there is a certain gap between the lining and the surrounding soil which can release the constraint of the surrounding soil. Thus, the gap is simultaneously filled by grouting which needs a relatively long time to solidify. Meanwhile, since the tunnel boring machine is continuously advancing, there are always certain lengths of tunnel lining located in the unsolidified grout. Therefore, the buoyancy is induced by the unsolidified grout and may be greater than the gravity of the lining, causing the segment uplifting tendency. The excessive segment uplift not only damages the segment but also destructs the tunnel's waterproof facilities, leading to deterioration in the safety and serviceability of the tunnel structure [1–4].

Previous studies have shown that segment uplift is caused by many factors, such as engineering geology, hydrogeology, grouting quality, and shield posture [5–11]. Based on the experimental investigations, Shirlaw et al. [12] believed that the slurry performance and grouting pressure have a great influence on the floating of pipe segments. Hashimoto et al. [13, 14] came to conclusion that the pressure of the lining is related to the dissipation rate of the grouting pressure. Zhou and Ji [15] concluded that the chamber pressure, advance rate, and total thrust have little influence on segment uplift, but the grouting pressure and grout composition are the most sensitive influencing factors on segment uplift. In addition, the numerical method is favored for studying the mechanism of segment lift, for example, the two-dimensional simulations conducted by
Zhang et al. [16] and three-dimensional simulations presented by Kasper and Meschke [17]. Do et al. [18], Mo and Chen [19], Chen et al. [20], and Kasper and Meschke [17] pointed out that the segment uplift increases with the increasing of grouting pressure. Bezuijen et al. [21] indicated that the hardenability of the grout affects the antifloating capacity of segments. Even though the influence of some factors such as grouting pressure, advance rate, and chamber pressure has been investigated, the relationship between the tunnel segment uplift and the solidification process of the grout has not been studied. The lining-grout-soil interaction mechanism and lining uplift behavior are not yet fully understood.

In this study, the phenomenon of segment uplift in Xuzhou Metro Line 1 is investigated and a tunnel segment uplift model based on the Pasternak foundation beam model is proposed, which considers the hardening process of grouting body with time. The finite difference method is used to calculate the magnitude of segment uplift during different tunnel construction stages, and a numerical solution is obtained. The applicability of the numerical solution is studied by comparing with the field test results and parametric analyses are also performed to investigate the effects of different factors on segment uplifting.

2. Segment Uplift of Subway Shield Tunnel

The segment uplift of the subway shield tunnel during the construction is quite notable in various soil areas of China. Figure 1 shows the route of 900 ring shield segments in a section of Xuzhou Metro Line 1. The segments are buried at a depth of 9.8–19.5 m. The ground characteristics of the soil layers are listed in Table 1. The soil layers are mainly Quaternary Late Pleistocene clay and fully weathered shale. As shown in Figure 1, the first tunnel section of 450 segments is constructed in the shale layer while the next 450 segments are constructed in the clay layer. However, the segment uplift magnitudes in the two layers are quite different (Figure 2). The average, maximum, and minimum uplift magnitudes of the tunnel constructed in the shale layer are 42 mm, 74 mm, and 5 mm, respectively. For segments in the clay layer, the corresponding magnitudes are 80 mm, 129 mm, and 44 mm, respectively.

3. Segment Uplift Model

3.1. Mechanical Model. The segment uplift behavior is not affected by the advancing rate of the shield machine according to the study by Zhou and Ji [15]. The reasonable explanation is that the segment uplifting occurs while the shield machine stands still for segment installing. Figure 3 depicts the working condition of the shield tunnel during the construction stage. The segments of lining are subjected to the grout pressure and jacking force of the shield machine and constrained by the foundation and the shield tail. Load and constraints on the segment are assumed to be a certain pattern owing to the periodic process of tunnel advancing. As presented in Figure 4, the segment, grout pressure, and constraints are all taken into consideration.

Considering the time-dependent phase transition from liquid to solid of grout, the segment uplift is divided into five stages according to the uplift characteristics as shown in Figure 4: (I) the ungrouted stage, (II) the fast uplift stage, (III) the slow uplift stage, (IV) the equilibrium stage, and (V) the grout solidification stage. In the ungrouted stage, the segment is in the shield tail, and the grouting has not yet started. After the completion of grouting, the segments will float upward under the buoyancy of slurry and water. Meanwhile, the soil on the top of the lining will exert less reaction force on the segment, which will quickly float up. When the soil on the top is squeezed by the segments and begins to produce upward displacement, the segment uplifting rate reduces owing to the counterforce of the soil. When the resistance of the topsoil to the segments can completely overcome the buoyancy, the segment slowly uplifts. That is, the segments are in the equilibrium stage. As the slurry gradually solidifies, the buoyancy gradually disappears. When the position of the segment is fixed, it is in the stage of grout complete solidification.

The theoretical derivation in this study is based on the following basic assumptions and simplified processing: (1) the uplift speed of the segments is slow and the segment is assumed to be in the static equilibrium state. (2) The transverse and cross-sectional deformations of the tunnel segment are small and can be neglected. (3) The infiltration of grout in the soil is ignored. (4) The soil is linear elastic isotropy and small deformation. (5) The grouting liquid exerts uniform pressure on the surrounding strata.

3.2. Segment Uplift Equations Based on the Pasternak Foundation Beam Model. Following the five-stage definition of segment uplift, $T_1$, $T_2$, $T_3$, $T_4$, and $T_5$ are defined as the durations of the ungrouted (I), fast uplift (II), slow uplift (III), equilibrium (IV), and grout solidification (V) stages, respectively. The total duration of the five periods is divided into $n$ segments, as shown in Figure 5. Assuming that the speed of the shield advancing is constant, the lengths of the segments passing through space at each time interval, denoted by $l$, are equal. Note that the segments divided by the time interval from the beginning to the end are 0, 1, 2, 3, ..., $r−1$, $r$, $r+1$...$s1$, $s$, $s+1$...$n−1$, $n$, where $T_1$ includes nodes 0 to $r$; $T_2$, $T_3$, and $T_4$ include nodes $r$ to $s$, and $T_5$ includes nodes $s$ to $n$. Then, the total uplifting magnitude of the tunnel from the ungrouted stage to complete grout solidification is equal to the sum of the floating amounts in each period time. The value of buoyancy in each period time is related to the force state, the boundary conditions of the tunnel segment.

In this study, the shield tail is taken as the origin to establish a reference system, the reverse direction of the shield advancing is taken as the $x$-axis, and the vertical direction is taken as the $y$-axis to establish a plane coordinate system. As shown in Figure 5, because segment A is separated from the shield tail, the position relative to the shield machine changes as the shield machine advances (in fact, segment A is stationary relative to the geodetic coordinate system and the shield tail). Then, segment A goes through a process from $x=0$ to $x=\infty$, and the total segment uplift
The magnitude of segment A is equal to the sum of the uplift at each x position. Therefore, the foundation beam model can be used to simulate the uplift of the segment [22].

The Pasternak foundation beam model is adopted to establish the uplift deflection model of the tunnel. The Pasternak model adds a shear layer G to the Winkler model, which can simulate the shear stiffness between soil masses and calculate the deflection differential equation based on a two-parameter foundation model, as expressed by

$$EI \frac{d^4y}{dx^4} - GB \frac{d^2y}{dx^2} + Ky = P(x),$$

where EI is the flexural stiffness of the foundation beam, y is the uplifting of the segment, K is the coefficient of subgrade modulus, G is the shear stiffness of the subgrade, B is the width of the tunnel (m), and P(x) is the buoyancy.

As shown in Figure 5, when segment A separated from the shield tail, there is no influence from the grouting pressure in the ungrouted section, and the buoyancy is zero. The gap of the shield tail exists around the segments. At this stage, the subgrade modulus and the shear stiffness of the subgrade against the segment are very small, denoting K_1 and G_1 here, and the shield tail is considered at the fixed end owing to the effect of the shield tail jack. When the grout is completely solidified, the segment will no longer float. At this stage, the tunnel is practically stable. The maximum subgrade modulus K_3 and shear stiffness G_3 are taken and the boundary condition can be regarded as the fixed end. During the segment uplift period, the K_2 and G_2 of subgrade increase gradually. To simplify the calculation, the K_2 and G_2 are assumed to change linearly from K_1 and G_1 to K_3 and G_3, respectively. The uplifting pressure in node r is the largest, which changes linearly up to node s, where the uplifting pressure is zero.
From the above introduction, the stage transformation between (I) and (II) and (IV) and (V) can be judged by whether there is uplifting. However, it is difficult to distinguish accurately the stage transformation between stages (II, III, and IV) because the segment uplifting in three stages occurs continuously. Thus, the segment uplifting calculation of stages (II, III, and IV) needs to be considered together. Then, based on (I), the deflection equations for the tunnel at the ungrouted (I), uplifting (II, III, and IV), and complete solidification (V) stages are

\[ \frac{d^4 y}{dx^4} - G_1 B \frac{d^2 y}{dx^2} + K_1 By = 0, \]  

\[ \frac{d^4 y}{dx^4} - G_2(x) B \frac{d^2 y}{dx^2} + K_2(x) By = P_{(x)}, \]

with

\[ G_{2(x)} = \frac{G_3 - G_1}{s - r} \cdot x + \frac{sG_1 - rG_3}{s - r}, \]

\[ K_{2(x)} = \frac{K_3 - K_1}{s - r} \cdot x + \frac{sK_1 - rK_3}{s - r}, \]

\[ P_{(x)} = \frac{P_m}{r - s} \cdot x - \frac{s \cdot P_m}{r - s}, \]

where \( P_m \) denotes the buoyancy at segment \( r \).

### 4. Model Parameters and Numerical Solution

#### 4.1. Determination of Parameters

The magnitude and distribution of the buoyancy force, the equivalent bending
stiffness of the beam, and the parameters of the foundation and their reduction should be determined for the segment uplift.

4.1.1. Magnitude and Distribution of the Buoyancy Force. According to Ye et al. [23], the main forces causing the uplift of segments are the static uplift caused by slurry and groundwater and the dynamic uplift caused by grouting. The static buoyant force can be calculated as follows:

\[ F_J = \pi R^2 \gamma I, \]  

where \( F_J \) is the static buoyant force, \( R \) is the peripheral radius of the tunnel, and \( \gamma I \) is the unit weight of fluid.

During grouting, in addition to static buoyancy, dynamic grouting pressure will also affect the stress state of segments. The dynamic action of grouting pressure on the segment changes with the grouting position. When the grouting point is directly below the tunnel, the grouting pressure makes the segment bear an additional maximum upward load, as shown in Figure 6. At this condition, the buoyant force caused by the grouting pressure on the segment is

\[ P_D = 2 \int_0^\theta pR \cos \alpha \, d\alpha = 2pR \sin \theta, \]  

where \( P_D \) is the dynamic buoyant force, \( p \) is the grouting pressure, and \( \theta \) is the angle between the boundary of the grout distribution range and the vertical direction.

4.1.2. Equivalent Bending Stiffness of the Beam. Because the tunnel consists of multiple segments joined by bolts, the bending stiffness of the whole tunnel is nonuniform. The equivalent bending stiffness of tunnels is usually to analysis. In this study, the equivalent flexural stiffness of the beam proposed by Shiba [24] is adopted, which is expressed as

\[ EI_{eq} = \frac{\cos^3 \phi}{\cos \phi + ((\pi/2) + \phi)} \cdot \sin \phi \cdot E_s I_s, \]  

with \( \phi + \cot \phi = \pi \left(1 + \frac{uK_b}{E_s A_s l}\right) \).

where \( \phi \) is the position of the neutral axis, \( K_b \) is the linear stiffness of the joint bolt and \( K_b = E_b A_b / h_b \), \( E_b \) is the elastic modulus of the bolt, \( A_b \) is the cross-sectional area of the bolt, \( h_b \) is the bolt length, \( u \) is the number of longitudinal bolts, \( I_s \) is the longitudinal moment of inertia of the tunnel, \( E_s \) is the elastic modulus of tunnel section, \( A_s \) is the cross-sectional area of the tunnel, and \( l \) is the distance between the center lines of two adjacent pipe segment rings.

4.2. Numerical Solution. When segment A in Figure 5 is separated from the shield tail, the shield tail constrains its vertical displacement and angle, leading to node 0 being at the fixed end. Because the grout around segment A has solidified during the T5 period time, the far end can be regarded as fixed. Therefore, the vertical displacement and angle of the node 0 at the shield tail and the node \( n \) at the far end are zero, and the boundary conditions are

\[ y_0 = 0, \ y'_0 = 0, \ y_n = 0, \ y'_n = 0. \]

Before node 0 and after node \( n \), two virtual nodes \( -1 \) and \( n + 1 \) are added for calculation, respectively. Based on the standard finite difference principle, the finite difference form of equations (2)–(4) can be written as follows:

\[ \frac{d^4 y}{dx^4} = \frac{6y_i - 4(y_{i+1} + y_{i-1}) + (y_{i+2} + y_{i-2})}{l^4}, \]

\[ \frac{d^2 y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{l^2}. \]

By substituting (10) and (11) into (2)–(4) and (9), the finite difference form of the deflection line can be obtained, and the matrix expression method is used as follows:

\[ [K][y] = [P], \]

where \( [K] \) is the stiffness coefficient matrix, \( [K] = [K_1] + [K_2] + [K_3] \), \( [y] \) is the deflection matrix, and \( [P] \) is the load matrix, with
Table 2: The design parameters of segments.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>External diameter of tunnel, $D$</td>
<td>6200 mm</td>
</tr>
<tr>
<td>Thickness of segment, $t$</td>
<td>350 mm</td>
</tr>
<tr>
<td>Width of segment, $L$</td>
<td>1200 mm</td>
</tr>
<tr>
<td>Connecting format of segments</td>
<td>M30 (16 in circular seam, 12 in longitudinal joint)</td>
</tr>
<tr>
<td>Strength of concrete</td>
<td>C50</td>
</tr>
</tbody>
</table>

\[
[K] = \begin{bmatrix}
5 & -4 & 1 \\
-4 & 6 & -4 & 1 \\
1 & -4 & 6 & -4 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & -4 & 6 & -4 \\
1 & -4 & 5
\end{bmatrix}
\]

\[
[K_2] = -\frac{GB}{sl}
\begin{bmatrix}
-2 & 1 \\
1 \times 2 & -2 \times 2 & 1 \times 2 \\
1 \times 3 & -2 \times 3 & 1 \times 3 \\
\vdots & \vdots & \vdots & \vdots \\
1 \times (n-2) & -2 \times (n-2) & 1 \times (n-2) \\
1 \times (n-1) & -2 \times (n-1)
\end{bmatrix}
\]

\[
[y] = \begin{bmatrix} y_1 \ y_2 \ y_3 \ \cdots \ y_{n-2} \ y_{n-1} \end{bmatrix}^T,
\]

\[
[P] = \begin{bmatrix} 0 \ 0 \ \cdots \ 0 \ \frac{p_m(s-r-1)}{s-r} \ \cdots \ \frac{p_m(s-r-j)}{s-r} \ \cdots \ \frac{p_m}{s-r} \ \frac{0}{s-r} \ \cdots \ \frac{0}{s-r} \end{bmatrix}^T
\]

(12) can be solved to obtain $y$, which is the solution of the static uplifting magnitude at the longitude direction of the tunnel.

5. Model Validation and Parameter Analysis

5.1. Model Validation. The validation was performed in terms of the segment uplift of the Xuzhou Line 1 metro tunnel, as shown in Figure 2. The basic parameters of the tunnel lining are shown in Table 2. The model parameters are listed in Table 3. The foundation modulus and shear stiffness are determined by a simplified method by Attewell [25] and Liang [26], as

\[
K = \frac{1.3E_0}{D(1-\mu^2)} \sqrt{E_0D^4} ET,
\]

\[
G = \frac{E_0H_t}{6(1+\mu)}
\]

where $\mu$ is the Poisson ratio, which is assumed constant ($= 0.3$) in this study, $E_0$ is the elastic modulus of soils, $H_t$ is the thickness of the shear layer, and $H_t = 2.5D$ is suggested by Xu [27]. The calculated segment uplift of the 40 segments is presented in Figure 7. In the ungrouted stage, the segment uplifts rapidly. The maximum uplift point occurs at the beginning of the fast uplift stage. After that, the uplift magnitude of the segment gradually decreases. The
Table 3: The model parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight of grouting and water, $\gamma$</td>
<td>15 kN·m$^{-3}$/10 kN·m$^{-3}$</td>
</tr>
<tr>
<td>Grouting pressure, $p$</td>
<td>0.6 MPa</td>
</tr>
<tr>
<td>Static buoyancy, $F_I$</td>
<td>378 kN</td>
</tr>
<tr>
<td>Dynamic buoyancy, $F_D$</td>
<td>1860 kN</td>
</tr>
<tr>
<td>Unit length gravity of segment, $W$</td>
<td>154 kN</td>
</tr>
<tr>
<td>Equivalent bending stiffness, $E_{Iq}$</td>
<td>$6.9 \times 10^7$ kN·m$^{-2}$</td>
</tr>
<tr>
<td>Coefficient of subgrade modulus, $K$</td>
<td>35 MPa·m$^{-1}$</td>
</tr>
<tr>
<td>Shear stiffness, $G$</td>
<td>$1.53 \times 10^5$ kN·m$^{-1}$</td>
</tr>
<tr>
<td>Natural density of soil, $\rho$</td>
<td>19 g·cm$^{-3}$</td>
</tr>
</tbody>
</table>

Figure 7: The predicted segment uplift of the tunnel with the proposed model.

Figure 8: Influence of coefficient of subgrade modulus on the segment uplift: (a) variation of segment uplift under the different values of $K$; (b) relationship between the maximum segment uplift and $K$.  

calculated maximum uplift magnitude is 122 mm, a little smaller than the monitoring maximum value of 129 mm. From the validation, it is indicated that the proposed method adopted in the analysis can provide a rapid and effective way for evaluating segment uplift during tunnel construction. Thus, the safety of segments can be judged by comparing the calculated value of segment uplift with the allowable value, which can be used to guide the next shield construction.

5.2. Parameter Analysis. A series of parametric studies are carried out to gain a greater understanding of the effects of the different factors on the segment uplift, including the coefficient of subgrade modulus, the shear stiffness of shear layer, and the grout pressure. For a direct comparison with the corresponding factors, the other parameters in Tables 2 and 3 are adopted.

5.2.1. Influence of Coefficient of Subgrade Modulus. The coefficient of subgrade modulus is used to express the compression properties of soils. For this reason, the coefficient of subgrade modulus is taken as 20, 25, 30, 35, and 40 MPa to study its influence on the segment uplift of the lining. The calculation result is shown in Figure 8. It can be seen from the figure that the coefficient of subgrade modulus has a significant effect on the magnitude of the segment. With the increase of the coefficient of subgrade modulus, the maximum uplift magnitude of the segment decreases.
rapidly. This is reasonable because a large modulus means that the soil is not easily compressed.

5.2.2. Influence of the Shear Stiffness. The shear stiffness of soil in the shear layer soil affects its shear resistance. For this reason, the shear stiffness is chosen to be 0.5, 1.0, 1.5, 2.0, and 2.5 × 10^5 kN/m, respectively, to study its influence on the segment uplift. Substituting the above shear stiffness into the proposed model, the calculation result is shown in Figure 9. It can be seen from the figure that as the shear stiffness increases, the uplift magnitude of the segment decreases. However, compared with the influence of K on the segment uplifting, the influence of the shear stiffness is small. Due to the increase in the shear stiffness of the soil, the shear strength between the soil particles increases. When the soil produces uneven deformation, it is necessary to overcome a greater shear resistance, which leads to an increase in the resistance to suppress the upward trend of the segment.

5.2.3. Influence of Grout Pressure. The grouting pressure directly affects the buoyancy of the tunnel. The grouting pressures p = 0.2, 0.4, 0.6, 0.8, and 1.0 MPa are used to study its influence on the uplift of the segment. Adopting the above grouting pressures into the calculation of the proposed model, the result is shown in Figure 10. It can be seen that as the grouting pressure increases, the uplift magnitude of the segment increases, but the increasing rate decreases.

6. Conclusions

The phenomenon of segment uplift of the tunnel passing through two different strata of Xuzhou Metro Line 1 is investigated. A segment uplift model of the tunnel is proposed based on the Pasternak foundation beam model. The finite difference method is used to calculate the uplifting of segments at different stages and then the numerical solution is obtained. The applicability of the model is calibrated by comparison with field test results. Parametric analyses are also performed to investigate the effects of different factors on segment uplift. The following conclusions can be drawn:

1. The segment uplift magnitude during construction of the Xuzhou Metro Line 1 tunnel when passing through two different soil layers is quite different. The average, maximum, and minimum uplift magnitudes of the tunnel constructed in the shale layer are 42 mm, 74 mm, and 5 mm, respectively. For the segments in the clay layer, the corresponding magnitudes are 80 mm, 129 mm, and 44 mm, respectively.

2. Segment uplift during shield tunneling is divided into five stages: (I) the ungrouted stage, (II) the fast uplift stage, (III) the slow uplift stage, (IV) the equilibrium stage, and (V) the grout solidification stage. The Pasternak foundation beam model is used to develop the segment uplift model considering the mechanical actions of buoyancy, grout pressure, and constraints on the lining. The numerical solution of the model is obtained through the finite difference method.

3. The application of the model is validated by measurement comparison. Parametric analyses show that the coefficient of subgrade modulus, shear stiffness of shear layer, and grout pressure all influence the segment uplifting of the tunnel. The influence of the coefficient of subgrade modulus and grout pressure on segment uplift is more significant.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors acknowledge the support received from the Natural Science Fund for Colleges and Universities in Jiangsu Province (No. 20KJA560003), China Postdoctoral Science Foundation (No. 2020M681769), and Guidance Project of Housing and Urban-Rural Development Department of Jiangsu Province (2019ZD079 and 2017ZD094). This work was also supported by Qing Lan Project and Postdoctoral Workstation of Hefei.

References


