In the paper, a possibility to develop the digital models of the seismic vibrations parameters is analyzed. To reach this goal, the observations at seismic station LUWI (Indonesia) were processed applying the statistical procedures. In fact, the biggest attention was given to the introduction of the Doppler effect expression and the employment of the theory of covariance functions. The trend in vectors of vibrations intensities values was detected and estimated upon using the least-squares method and polynomial approximation. In addition, by this technique, the random errors were eliminated partially. The self-developed computer programs based on Matlab programming package procedures were applied.

1. Introduction

Earthquake is one of the most costly, devastating, and deadly natural hazards. Every disaster damages thousands of buildings and displaces tens of thousands of people. The comprehensive knowledge of the earthquake nature and its behavior is extremely important. Here the main task for scientists is to constrain the suitable mathematical methods to analyse the earthquakes action, and most importantly to develop the earthquake model to predict its spread and to forecast its occurrence. The latest developments could be noted in [1–7], where the biggest efforts were taken for mathematical descriptions of wide earthquakes occurrence areas trying to construct the Ground Motion Prediction Equations. Deep analysis of different aspects of passive seismic methods like a horizontal to vertical spectral ratio, which is often used to describe the earthquake site, could be found in [8–14]. Some characteristics of concrete earthquake’s sites from world’s seismic zones are presented in [9, 15–22]. In some papers, the stress was done on significance of three-dimensional modelling of seismic waves propagation [18, 23–27].

Various data sources were applied to investigate the phenomena of eruptions [28–30]. Great achievements are done using modern techniques like InSAR interferometry [31–34], GNSS [35–38], and satellite imagery [39–42]. Several studies are dedicated to one of the most destructive eruptions—the 2018 Indonesia Sulawesi magnitude 7.5 Palu earthquake [34, 37, 41, 42].

For Indonesia, from which the practical example in this paper is given, some research results could be found in [43–50]. Indonesia has a high seismicity rate, which is related to complex interaction of several tectonic plates [51–63]. It should be especially noted that Indonesia’s seismic region is an area of highest magnitude (more than 6.0) eruptions [37, 60, 64–70].

What deals with scientific techniques and methods to investigate earthquakes application has InSAR technology, which enables detecting surface slips and Earth surface deformations [34, 37, 41]. For example, the 4–7 m surface slip in the area of Palu earthquake was detected [37] and the maximum horizontal deformation was from 1.8 m till 3.6 m [41], when ALOS-2 interferogram showed a peak slip of 6.5 m located at the south of Palu city [34]. GNSS plays a great role in the research of earthquakes giving very precise metrical parameters to improve the crustal deformation field and 3D geometric complexities of the faults in total [38, 53, 60, 65, 68]. Certainly, the main techniques to detect the technical parameters of earthquakes are seismograms and the combinations of some techniques as well [34, 67]. So,
from broadband regional seismograms, it was revealed that the 2018 Palu earthquake is a supershear rupture event from early on with an average rupture velocity of 4.1 km/s, and the total seismic moment of $2.64 \times 10^{20}$ Nm (equivalent to Mw 7.55) was released within 40 s [34].

In this paper, we will show how the Doppler effect expression and the application of the theory of covariance functions could be employed for seismic waves modelling. The practical calculations were executed using the two fragments of the observations data of the intensity $\varphi$ of the Earth’s seismic field, which were chosen from LUWI seismic station (Sulawesi, Indonesia, latitude: $-1.04180$, longitude: 122.77170, elevation: 6.0 m): first on August 05, 2018, within one hour (11:30–12:30), and second on November 11, 2018, within two hours (5:00–7:00). At these periods, the seismic stations around the world have registered unusual vibrations of low frequencies. Wide basic information on Palu earthquake could be found in specialized portals [71, 72].

The observations data were expressed by vectors $N$ (North), $E$ (East), and $Z$ (Zenith). The time series views of the centered vectors $N$, $E$, and $Z$ for both abovementioned periods are presented in Figures 1 and 2. In both figures, the time series views of the components $E$ and $N$ are similar. It looks like the influence of the unusual low frequencies vibrations in Figure 2 is possibly low. The systematic component of low frequency could be eliminated applying the 6-degree polynomial approximation. It is presented in Figure 3. The accuracy of vectors $N$, $E$, and $Z$ extracted from LUWI station data on August 05, 2018, is described by standard deviations $S_\varphi = (19480, 15926, 15810)$ cnt. These numbers show that the accuracies of components of seismic vectors presented in Figure 1 are approximately the same. The accuracy of vectors $N$, $E$, and $Z$ extracted from LUWI station on November 11, 2018 (if the systematic component is not eliminated), is described by the vector of standard deviations $S_\varphi = (1260, 559, 41)$ cnt. It shows that the accuracy of observations is slightly higher at this period. The accuracy of vectors $N$, $E$, and $Z$ extracted on November 11, 2018 (if the systematic component is eliminated), is described by the vector of standard deviations $S_\varphi = (215, 257, 41)$ cnt. In this case, the obtained accuracy of processed observation data is considerably higher.

Mathematical-statistical methods are widely applied for data processing in geophysics, geodesy, and other Earth sciences [73–75]. To predict and develop the model of the spread of seismic vibrations, first of all, we assume that seismic waves from the quake hypocenter spread as harmonic vibrations of decreasing amplitudes in all the directions. So, we can assume also that the core structures of seismic observations at the tracking stations mounted in short distances from the hypocenter and at those more distant are possibly very similar. For mathematical treatment of the seismic observations, the covariance functions and the theory of Doppler effect were applied. The correlations between changes of intensities of seismic waves spreading in time and space were detected by introducing the variations of covariances of the seismic vibrations intensities vectors. Some equations were derived to obtain the estimates of covariation matrices and autocovariances and cross-covariances of seismic field intensities vectors based on seismic observations data. The accuracies of corresponding calculated parameters were obtained also.

The background of the mathematical model of observations data treatment is concept of a stationary random function and especially paying attention to statement that the errors of seismic vibrations observations are random and possibly are near the same precision. So we assume that the mathematical average of random errors $M\Delta = \text{constant} \rightarrow 0$, its dispersion $D\Delta = \text{constant}$, and the covariances of the observations depend on the difference of the arguments only, so practically from the quantised intervals on the time scale.

![Figure 1: Time series of vectors N, E, and Z (LUWI station, August 05, 2018).](image1)

![Figure 2: Time series of vectors N, E, and Z (LUWI station, November 11, 2018).](image2)
2. Modelling of Seismic Vibrations

The observation data registered by the seismic station had been previously examined and processed upon reaching a goal to eliminate both random and possibly systematic errors. The most reliable values of the trend in the seismic vibrations arrays were detected employing the least-squares method. Application of least-squares technique gives a possibility to eliminate the random errors partially. While treating the big volumes of observations, the least-squares technique produces the asymptotically efficient values of the derived parameters also in case when a statistical distribution of the observations errors is not normal.

Any vector of seismic vibrations intensities could be treated as a random function, which involves the random errors of observations. By employing a least-squares technique to treat the vector of intensities \( \varphi \), we can detect the most reliable value \( \bar{\varphi} \) of the trend. A parametric equation of a single vector’s element \( \varphi_i \) will look like the following:

\[
\epsilon_i = \varphi_i - \bar{\varphi},
\]

(1)

where \( \epsilon_i \) is a random error of the vector’s element, \( \varphi_i \) is the value of the vector’s element, and \( \bar{\varphi} \) is vector’s trend.

The expression in matrix form of equation (1) will be as follows:

\[
\epsilon = \varphi - \bar{\varphi},
\]

(2)

where \( \epsilon \) is vector of random errors, \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T \) is vector of seismic field intensities, and \( \bar{\varphi} \) is vector of units (\( n \times 1 \)).

The most reliable value of vector \( \varphi \) trend could be calculated by introducing the general condition of the least-square method:

\[
\Phi = \epsilon^T P \epsilon = \min,
\]

(3)

where \( P \) is diagonal (\( n \times n \)) of weights \( p_i \) of the values \( \varphi_i \)

Weights \( p_i \) could be detected according to simple formula:

\[
p_i = \frac{\sigma_0^2}{\sigma_{\varphi_i}^2},
\]

(4)

where \( \sigma_0 \) is the standard deviation of the observation \( \varphi_0 \), the weight of which is supposed to be equal to unit, that is, \( p_0 = 1 \).

Furthermore, we can write the following equation:

\[
u_i = \ln \varphi_i,
\]

(5)

and we further obtain

\[
\sigma_{\varphi_i} = \sigma_{\varphi} \varphi_i.
\]

(6)

From formula (6), we can see that the value of \( \sigma_{\varphi_i} \) pertains from the value of \( \varphi_i \). So, the components, which have the bigger values, are of a lower accuracy just because \( \varphi_i \gg \sigma_{\varphi_i} \).

Upon applying formula (4), we write

\[
p_i = \frac{\sigma_0^2}{\sigma_{\varphi_i}^2} = 5 \cdot \varphi_i^{-2} \cdot 10^4,
\]

(7)

where the accepted average value is \( \sigma_0^2/\sigma_{\varphi_i}^2 = 5 \cdot 10^4 \).

To find the extremum of function (3), let us calculate its partial derivatives according to trend \( \bar{\varphi} \). We can write and solve the equation:

\[
\frac{\partial \Phi}{\partial \bar{\varphi}} = 2 \left( \frac{\partial \epsilon}{\partial \bar{\varphi}} \right)^T P \epsilon = 0.
\]

(8)

Then we will obtain

\[
-\epsilon^T Pe = 0,
\]

\[
e^T Pe\bar{\varphi} - \epsilon^T P\varphi = 0.
\]

(9)

Thus, we will get the following solution:

\[
\bar{\varphi} = (e^T Pe)^{-1} e^T P\varphi = N^{-1} \omega,
\]

(10)

where \( N = (e^T Pe) \), \( \omega = e^T P\varphi \).

The accuracy of the trend could be detected by calculating its covariance matrix \( K_\varphi \):

\[
K_\varphi^{-1} = \frac{\sigma_{\varphi}^2}{\sigma_0^2} = \sigma_0^2 N^{-1},
\]

(11)

where \( \sigma_{\varphi}^2 \) is the estimate of the standard deviation \( \sigma_0 \). It is assessed by formula:

\[
\sigma_{\varphi}^2 = \frac{1}{n - 1} \epsilon^T Pe,
\]

(12)

The considerably high systematic component of the vibrations of the data of seismic station LUWI was eliminated upon applying a 6-degree polynomial approximation.

Now, it is possible to calculate cross-covariance and autocovariance functions of the seismic vibrations as well as
the shifts of the seismic vibrations, respectively, to each other by introducing the Doppler effect expression.

Let us take the formula for the parameter \( z \) [76–78]:

\[
z = \frac{f_e}{f_o} - 1 = \frac{f_e - f_o}{f_o}
\]

(13)

where \( f_e \) is frequency of emitted vibrations and \( f_o \) is frequency of observed vibrations.

We accept that the changes of changes of phases observed at tracking stations possibly correspond to the changes of the seismic vibrations intensities. Consequently, sum of the seismic vibrations intensities is proportional to the algebraic sum of the frequencies phases of vibrations accordingly; that is,

\[
\delta B \sim \delta \omega;
\]

(14)

here \( \delta B, \delta \omega \) are changes of vibrations intensities and vibrations frequencies phases, respectively.

We can write the expressions for changes of vibrations intensities and the sum of them as follows:

\[
\delta a(t) = A\delta \omega \cdot \cos \omega t,
\]

\[
\delta a_e(t) - \delta a_o(t) = A_e\delta \omega_e \cdot \cos \omega_e t - A_o\delta \omega_o \cdot \cos \omega_o t,
\]

(15)

where \( \omega_e = 2\pi f_e, 2\pi f_o \), and the initial phases \( \phi_0 \) are supposed to be equal to zero; \( \delta a \rightarrow \delta B \).

By employing the parameter \( z \) of the Doppler effect formula, we can express the strength of the seismic vibrations at the moment in time \( t_i \):

\[
z_i \equiv \frac{\delta B_{ei} - \delta B_{ao}}{\Delta B_{ao}} - 1
\]

(16)

where \( B_{ei} \) is the intensity of emitted seismic vibrations, \( B_{ao} \) is the intensity of observed seismic vibrations, \( B_{ei} \sim \omega_{ei} \), and \( B_{ao} \sim \omega_{ao} \).

In further developments, we employ the theory of covariance functions to detect the value of the argument \( z \) from the Doppler effect expression. Mathematical derivations are grounded on the conception of a stationary random function considering that errors of observations of seismic vibrations are random and possibly have similar precision.

It is possible to express a cross-covariance function of the straight algebraic sum \( \Delta B_{ei} = B_{ei} - B_{ao} \) of the two intensities \( B_{ei} \) and \( B_{ao} \) (emitted and observed) at a moment in time \( t_i \) and a separate intensity \( B_{ao} \) as follows:

\[
K(\Delta B_{ei}) = K(\Delta \delta B_{ao}) = M(\delta B_{ei} \cdot \delta B_{ao}) = M(K(\Delta B_{ei} - \delta B_{ao}))
\]

\[
= M(\delta B_{ei} \cdot Mz_i = \sigma^2_m Mz_i,
\]

(17)

where \( \Delta B_{ei} = B_{ei} - MB_{ao}, \Delta B_{ei} = B_{ei} - MB_{ei}, \delta B_{ei} = \Delta B_{ei} \), \( z_i + 1 \), \( MB_{ao}, MB_{ei} \) are the average values of vibrations intensities; \( \delta B_{ao} = \delta B_{ei} - \delta B_{ao}, \sigma_B \) are the standard deviations of the vibrations intensities. It is supposed that the standard deviations of the observed and registered seismic vibrations intensities are equal.

The average magnitude \( M_{zi} \) of an argument \( z \) of the Doppler formula could be expressed upon introducing the vibrations intensities at the moment in time \( t_i \) applying the following formula:

\[
M_{zi} = \frac{1}{\sigma^2_B} K(\Delta B_{ei}, B_{ao}).
\]

(18)

By employing the theory of the covariance functions, it is possible to express the cross-covariance functions of the corresponding seismic vibrations vectors taking into account the fact that every vector of vibrations intensities could be treated as a random function as follows [77, 79, 80]:

\[
K(\Delta B_{ei}, B_{ao}) = K_z(t) = M(\delta \Delta B_{ei}(u) \cdot \delta B_{ao}(u + \tau)),
\]

(19)

or

\[
K_z(t) = \frac{1}{T - \tau} \int_{\tau}^{T-\tau} \delta \Delta B_{ei}(u) \cdot \delta B_{ao}(u + \tau) du,
\]

where \( u \) is argument of any seismic vibrations vector, \( \tau = z \cdot \Delta \) is quantised interval, which is variable, \( s \) is number of quantised intervals, \( \Delta \) is the value of the accepted unit of observations, and \( T \) is the diapason of the fluctuations of seismic vectors elements.

By using the vectors of observations data, an estimation \( K'_z(t) \) of the cross-covariance function could be calculated according to the following formula:

\[
K'_z(t) = K'_z(s) = \frac{1}{n - s} \sum_{i=1}^{n-s} \delta \Delta B_{ei}(u_i) \cdot \delta B_{ao}(u_{i+s}),
\]

(20)

where \( n \) is number of vector elements.

Now, using formula (18) in the vector form, we get the formula to detect the mathematical average of the argument \( z \) of the Doppler effect expression:

\[
M(z) = \frac{K'_z(s)}{M \sigma^2_B} = \frac{K'_z(s)}{m.K_z(0)}
\]

(21)

where \( \sigma^2_B \rightarrow K'_z(0) \) is the estimate of the dispersion and \( m \) is number of cross-covariance values.

3. Analysis of the Experimental Results

The estimates of autocovariance and cross-covariance functions of the seismic vibrations intensities could be calculated employing formula (20). The values of the quantised intervals were assigned from 1 to \( n/2 \). Here, \( n = 144000 \) is the number of seismic vibrations vector components. The graphical images of autocovariance and cross-covariance functions were generated also. Some graphical images of covariance functions are shown in Figures 4–9.

Upon applying formula (21), the mathematical averages of the argument \( z \) of the Doppler expression were detected. The positive values of the argument \( z \) point out that the seismic vibrations recede from each other. The negative values of the argument \( z \) indicate that seismic vibrations approach each other. The calculated approximate reciprocal velocity of seismic vibrations, registered in vectors \( N \) and \( E \), is about \( v = 90 \) km/s.
Figures 4 to 9 depict images of the normed autocovariance and cross-covariance functions for vectors $N$, $E$, and $Z$. Each figure shows the correlation values across quantised intervals, with the x-axis representing the quantised intervals $k$ and the y-axis showing the values of correlations $k e$. The functions $k f r 1$, $k f r 2$, and $k f r 3$ correspond to different vector pairs, with $k f r 1$ indicating the function between $N$ and $E$, $k f r 2$ for $E$ and $Z$, and $k f r 3$ for $N$ and $Z$. The graphs illustrate the variations in correlation values across different intervals, providing insights into the relationships between the vectors.
Let us derive the estimates of the standard deviation of the argument \( z \). It could be done in two ways. Firstly, \( z \) values could be detected upon applying the frequencies of vibrations emitted from earthquake source and observed frequencies of vibrations at seismic station. Secondly, \( z \) values could be detected using the vibrations intensities. The formula to calculate the estimate of standard deviation of the argument \( z \) using formula (13) could be written as follows:

\[
\sigma_z^2 = \frac{\sigma_f^2}{f_0^2} + \frac{\sigma_f^2}{f_0^2} = \frac{\sigma_f^2}{f_0^2} + \frac{\sigma_f^2}{f_0^2} \\
= 2 \cdot 10^{-16} \left\{1 + (z + 1)^2\right\} = 10 \cdot 10^{-16},
\]

\[
\sigma_z^2 = 3 \cdot 10^{-6}.
\]

In formula (22), the estimate \( \sigma_z^2 \) of the standard deviation of the argument \( z \) was detected assuming \( \sigma_f = \sigma_{f_0} = 1 \cdot 10^9 \), and \( \sigma_f/\sigma_{f_0} = 1.4 \cdot 10^{-8} \). So, the ratio of the mathematical average of the argument \( z \) is \( \sigma_z/\bar{z} = 3 \cdot 10^{-6} \).

Let us derive the accuracy of the argument \( z \) of the Doppler effect expression using the seismic vibrations intensities. Upon applying formulas (15), (16), and (22), we have

\[
\sigma_{z,a}^2 = \sigma_{B,\omega}^2 \left\{1 + (z + 1)^2\right\} = \frac{\sigma_{\omega}^2}{\omega_o^2} \cdot \left\{1 + (z + 1)^2\right\},
\]

\[
\sigma_{z,a} = 2.2 \frac{\sigma_{\omega}}{\omega_o} = 2.2 \cdot 1.4 \cdot 10^{-8} = 3 \cdot 10^{-8},
\]

and the above was calculated upon considering \( \sigma_{B,\omega} = \sigma_{B,\omega}/\sigma_{f_0} = \sigma_{\omega}/\omega_o \), and \( z = 1.0 \). The ratio of \( z_B \) will be \( \sigma_{z,a}/\bar{z}_a = 3 \cdot 10^{-8} \).

The results of the calculations demonstrate that the detected accuracies of the argument \( z \) of the Doppler effect expression are nearly the same in both cases: when registered phases of vibrations frequencies or intensities (strengths) of them are used in the calculation procedures.

Let us calculate the accuracy of the estimates of the motion speed \( v = z \cdot c \). Upon using the equation

\[
\ln v = \ln z + \ln c,
\]

we can write the equation of the ratio as follows:

\[
\sigma_v^2 = \sigma_z^2 + \sigma_c^2
\]

and \( (\sigma_v/v) = 3 \cdot 10^{-9} \), when \( (\sigma_c/c) = 3 \cdot 10^{-9} \).

Introducing the above used data, we find that \( (\sigma_v/v) = 3 \cdot 10^{-6} \) km/s, upon supposing that \( v = 1000 \) km/s.

4. Conclusions

(1) It was shown that employment of the Doppler effect expression and the application of the theory of covariance functions could be used to develop the digital models of the seismic vibrations. A method was suggested to detect the values of argument \( z \) from the Doppler effect formulas upon employing the expression of the cross-covariance function of the sum of the intensities of seismic vectors components and the intensities of separate seismic vector components.

(2) For LUWI seismic station, the expressions of the normed autocovariance functions of seismic vibrations vectors \( N, E, \) and \( Z \) are slightly different. The autocovariance of the components of seismic vector \( Z \) has a maximum value of correlation \( r = 0.4 \). The autocovariance of seismic vibrations vectors components is deep \( r \rightarrow 1.0 \) at little values of quantised interval only, when \( k \rightarrow 0(\tau_k \rightarrow 0) \). The probabilistic dependence of the vector’s \( Z \) elements gradually decreases to \( r \rightarrow 0 \) at \( k \rightarrow 50000(\tau_k \rightarrow 2500) \). The autocovariance functions of the vectors \( N \) and \( E \) have the low variable positive and negative values.

(3) The correlation values \( r \) of the normed cross-covariance functions of components of vectors \( N, E, \) and \( Z \) of tracking station are varying in a narrow range \( r \rightarrow (-0.2; 0.4) \) along the whole quantised diapason. The values \( r \) of normed cross-covariance functions of components of vectors \( N, E, \) and \( Z \) are close to zero.

(4) The speed of reciprocal motion of seismic vector \( E \) and \( N \) components was calculated. Its approximate value \( v = 90 \) km/s.

Data Availability

All data used during the research are available in a repository online in accordance with funder data retention policies. Practically, data for this research were taken from the EIDA and GEOFON Data Archives (http://eida.gfz-potsdam.de/webdc3/).

Disclosure

The research was done as a part of employment at Vilnius Gediminas Technical University, Lithuania.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

References


Advances in Civil Engineering


