

Research Article

A Simple Three-Dimensional Failure Criterion for Jointed Rock Masses under True Triaxial Compression

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The joint configuration and the intermediate principal stress have a significant influence on the strength of rock masses in underground engineering. A simple three-dimensional failure criterion is developed in this study to predict the true triaxial strength of jointed rock masses. The proposed failure criterion in the deviatoric and meridian planes adopts the elliptic and hyperbolic forms to approximate the Willam–Warnke and Mohr–Coulomb failure criterion, respectively. The four parameters in the proposed failure criterion have close relationships with the cohesion and the internal friction angle and can be linked with the joint inclination angle using a cosine function. Two suits of true triaxial strength data are collected to validate the correctness of the proposed failure criterion. Compared with other failure criteria, the proposed failure criterion is more reasonable and acceptable to describe the strength of jointed rock masses.

1. Introduction

Rock mass strength is an extremely important parameter in predicting the stability of geoenvironment such as rock slopes, dam foundation, and deeply buried tunnels [1]. Due to the existing joints, rock mass strength is anisotropic [2–8]. In addition, true triaxial stress state ($\sigma_1 > \sigma_2 > \sigma_3$) is more universal with the increasing depth of engineering. The influence of σ_2 on the compressive strength of jointed rock masses has been investigated extensively in the experimental tests [9–14]. Therefore, establishing a strength criterion considering both the joint and σ_2 dependency is essential for better designing the layout and construction of underground engineering.

A large number of research papers are concentrated on rock strength. Among others, the Mohr–Coulomb failure criterion [15] and the Hoek–Brown failure criterion [16] are most widely used because of their simplicities. However, these two failure criteria ignore the σ_2 effect. Based on the true triaxial strength data, the authors in [17–21] have

proposed the three-dimensional failure criteria which can describe the variation in the rock strength with increasing σ_2 very well. There are also some popular true triaxial failure criteria [22–24]. However, these failure criteria are applicable for intact rock and do not consider the joint effect.

To develop a failure criterion considering the joint effect under true triaxial compression, Tiwari and Rao [14] introduced the joint effect into the generalized von Mises theory [20]. In addition, enormous research studies [25–27] have extended the Hoek–Brown failure criterion into the three-dimensional form to predict rock mass strength. Moreover, Singh and Singh and Zhang et al. [28, 29] established a modified Mohr–Coulomb criterion for the polyaxial strength of rock masses, and Rafiai [30] proposed an empirical criterion for rock mass strength under multi-axial state on the basis of a comprehensive experimental database. Based on the research in [7, 10, 30], the modified nonlinear criteria are presented to determine the strength of rock masses.

Most of the abovementioned failure criteria can only consider the effect of the joint or σ_2 on the prediction of rock mass strength. In addition, some aforementioned failure surfaces in three-dimensional principal stress space are not absolutely continuous, which could present difficulties in numerical calculations. To overcome these problems, the true triaxial failure criterion proposed in this paper adopts the modified Willam–Warnke yield criterion and hyperbolic function in the deviatoric and meridian planes, respectively. Moreover, the adopted hyperbolic function takes the Mohr–Coulomb criterion as the asymptote in the meridian plane. To valid the correctness of the developed failure criterion, it is employed to fit two suits of true triaxial strength data of rock masses.

2. A Failure Criterion for Jointed Rock Masses

Within this paper, tensile stress is considered positive and compressive stress is negative. A general form of a failure criterion suggested in [31] can be expressed in a quadratic function:

$$F(\tau_{\text{oct}}, \sigma_{\text{oct}}, \theta_{\sigma}) = \alpha \sigma_{\text{oct}}^2 + \beta \sigma_{\text{oct}} + \gamma + \bar{\sigma}_+^2 = 0, \quad (1)$$

where $\bar{\sigma}_+ = \tau_{\text{oct}}/g(\theta_{\sigma})$; θ_{σ} is the Lode angle ($\theta_{\sigma} = 1/3 \cos^{-1}(3\sqrt{3}J_3/2J_2^{3/2})$), where J_2 ($J_2 = 1/6[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$) and J_3 ($J_3 = (\sigma_1 - \sigma_{\text{oct}})(\sigma_2 - \sigma_{\text{oct}})(\sigma_3 - \sigma_{\text{oct}})$) are the second and third invariants of the deviatoric stress tensor, respectively; τ_{oct} and σ_{oct} are the octahedral shear and normal stresses ($\tau_{\text{oct}} = 1/3 \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ and $\sigma_{\text{oct}} = (\sigma_1 + \sigma_2 + \sigma_3)/3$); and α , β , and γ are material constants. First, the dependency of τ_{oct} on θ_{σ} in the deviatoric plane perpendicular to the hydrostatic axis is determined; namely, $g(\theta_{\sigma})$ is obtained. Then, the influence of σ_{oct} on $\bar{\sigma}_+$ can be investigated in the meridian plane.

2.1. True Triaxial Failure Criterion in the Deviatoric Plane.

The failure curve in the deviatoric plane displays the following significant characteristics of rock failure: (i) continuous, smooth, and convex; (ii) a closed curve or vertex; and (iii) symmetric with respect to three principal stress axes. For this purpose, an elliptical form developed in [32] is adopted here to express the relationship between $g(\theta_{\sigma})$ and θ_{σ} :

$$g(\theta_{\sigma}) = \frac{R + (2K - 1)\sqrt{2R \cos \theta_{\sigma} + 5K^2 - 4K}}{2R \cos \theta_{\sigma} + (1 - 2K)^2}, \quad (2)$$

where $R = 2(1 - K^2)\cos \theta_{\sigma}$; K is the ratio ($K = \tau_{\text{oct,ae}}/\tau_{\text{oct,ac}}$) between the strength ($\tau_{\text{oct,ae}}$) for axisymmetric extension and the strength ($\tau_{\text{oct,ac}}$) for axisymmetric compression; $0^\circ \leq \theta_{\sigma} \leq 60^\circ$ and $\theta_{\sigma} = 0^\circ$ ($g(\theta_{\sigma}) = K$) for axisymmetric extension; and $\theta_{\sigma} = 60^\circ$ ($g(\theta_{\sigma}) = 1$) for axisymmetric compression.

It is notable that $g(\theta_{\sigma})$ in (2) unconditionally satisfies the abovementioned properties of a failure criterion in the deviatoric plane for the range $0.5 \leq K \leq 1$. Therefore, taking θ_{σ} as the independent variable, $g(\theta_{\sigma})$ is only a function of K .

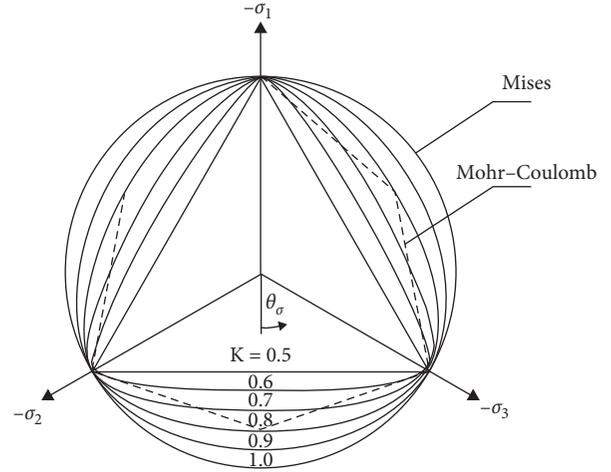


FIGURE 1: The Willam–Warnke failure criterion in the deviatoric plane with different K values.

Figure 1 shows that the shapes of the failure curves in the deviatoric plane changes with K . More specifically, as K varies from 0.5 to 1, failure surfaces gradually change from a triangle to a circle. It can be seen that the Willam–Warnke failure criterion is more appropriate than the Mohr–Coulomb failure criterion, and the Willam–Warnke criterion can change into the Mises criterion when $K=1$. Here, to present the joint effect on the failure function in the deviatoric plane, the parameter K depends on both the joint inclination angle β and σ_{oct} , namely, $K = K(\sigma_{\text{oct}}, \beta)$. If K is constant at different σ_{oct} , $g(\theta_{\sigma})$ is only a function of β . Thus, the joint inclination angle β can describe the influence of the joint on the failure surfaces in the deviatoric plane.

2.2. True Triaxial Failure Criterion in the Meridian Plane.

As pointed out by equation (1), once $g(\theta_{\sigma})$ is determined using equation (2), a true triaxial failure criterion can be obtained if the relationship between $\bar{\sigma}_+$ and σ_{oct} is known. Here, a hyperbolic function (as shown in Figure 2) is adopted because of its simplicity and continuity to present the failure criterion in the meridian plane. Thus, equation (1) can be defined by

$$F(\tau_{\text{oct}}, \sigma_{\text{oct}}, \theta_{\sigma}) = \left(\frac{\sigma_{\text{oct}} - d}{a}\right)^2 - \left(\frac{\bar{\sigma}_+}{b}\right)^2 - 1 = 0, \quad (3)$$

where a , b , and d are material constants. Figure 3 exhibits a typical failure surface of the proposed criterion in the principal stress space.

2.3. Determination of the Parameters in the Failure Criterion.

Generally, equation (3) has four parameters: K , a , b , and d . As mentioned above, K can be determined by the strength ratio under axisymmetric extension and compression at different β and σ_{oct} and be given by

$$K = \frac{\tau_{\text{oct,ae}}(\sigma_{\text{oct}}, \beta)}{\tau_{\text{oct,ac}}(\sigma_{\text{oct}}, \beta)}. \quad (4)$$

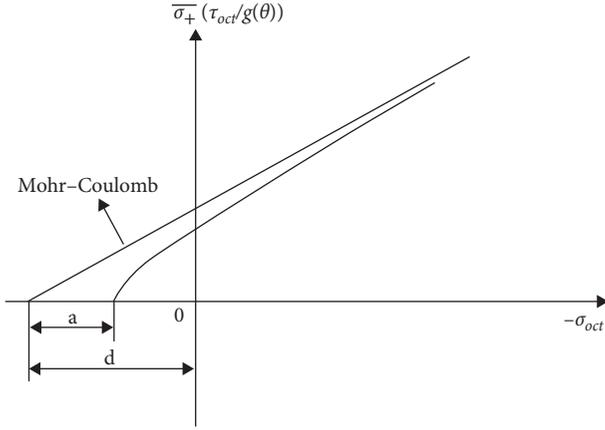


FIGURE 2: The hyperbolic function to approximate the Mohr–Coulomb failure criterion for axisymmetric compression in the meridial plane.

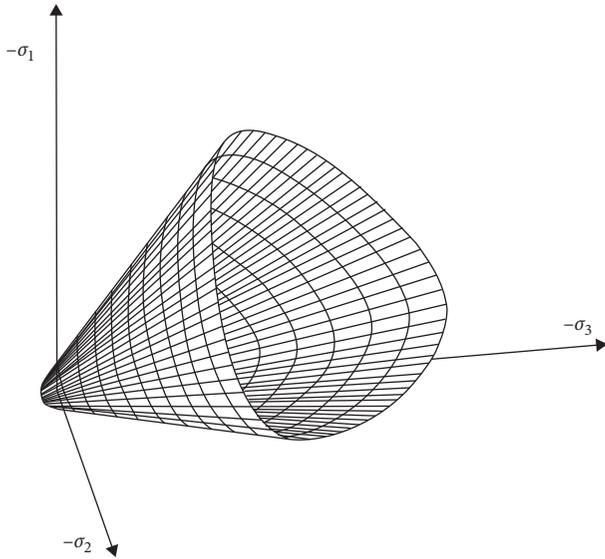


FIGURE 3: The typical failure surface in the principal stress space.

To determine the values of the other parameters, the hyperbolic function in equation (3) approximates the Mohr–Coulomb failure criterion for axisymmetric compression ($g(\theta_\sigma = 60^\circ) = 1$) in the meridial plane (as shown in Figure 2). The Mohr–Coulomb failure criterion at $\theta_\sigma = 60^\circ$ can be expressed by

$$F(\tau_{oct}, \sigma_{oct}, \theta_\sigma) = \bar{\sigma}_+ + \frac{2\sqrt{2}\sin\varphi}{3 - \sin\varphi}\sigma_{oct} - \frac{2\sqrt{2}c\cos\varphi}{3 - \sin\varphi} = 0, \quad (5)$$

where c and φ are the cohesion force and internal friction angle. Taking the Mohr–Coulomb line as the hyperbolic asymptote, the parameters a , b , and d are then obtained as follows:

$$\frac{b}{a} = \frac{2\sqrt{2}\sin\varphi}{3 - \sin\varphi}, \quad (6)$$

$$d = c \cot\varphi.$$

The parameter a can be fitted by a series of simple experimental tests under axisymmetric compression. As the parameter a tends to be 0, equation (3) is getting closer to the Mohr–Coulomb failure surfaces in the meridial plane. In addition, $g(\theta_\sigma)_{MC}$ in equation (5) is given by

$$g(\theta_\sigma)_{MC} = \frac{3 - \sin\varphi}{2\sqrt{3}\cos((\theta_\sigma - \pi)/6) - 2\sin\varphi\sin((\theta_\sigma - \pi)/6)}. \quad (7)$$

Thus, the strength ratio K_{MC} under axisymmetric extension and compression can be calculated using the Mohr–Coulomb failure criterion.

$$K_{MC} = g(\theta_\sigma = 0^\circ)_{MC} = \frac{3 - \sin\varphi}{3 + \sin\varphi}. \quad (8)$$

If equation (4) is simply substituted by equation (8), four unknown parameters (K , a , b , and d) in equation (3) become three (a , c , and φ) using equations (6) and (8). It is noted that the joint inclination angle β affects the unknown parameters ($a(\beta)$, $c(\beta)$, and $\varphi(\beta)$).

3. Validation of the Proposed Failure Criterion

The validity of the proposed failure criterion in this study is discussed using two following examples. The prediction accuracy is also analyzed and compared with other strength criteria.

3.1. Example 1. The true triaxial strength data for the first example are derived from the experimental results of [9]. This test was conducted on jointed marble which contains a natural joint plane with a rectangular prismatic size ($50 \times 50 \times 100 \text{ mm}^3$). Table 1 lists the derived test data.

The proposed failure criterion is used to model these selected strength data, and the calculated results are also listed in Table 1. Figure 4 presents the comparisons of the predicted strength σ_{1cal} with the experimental data σ_{1exp} . The reference line ($\sigma_{1cal} = \sigma_{1exp}$) is also demonstrated in Figure 4 to exhibit the calculation accuracy. All of the data points are concentrated around the reference line, which indicates that the predictions by the proposed failure criterion are generally consistent with the experimental data.

The linear Mogi–Coulomb failure criterion [17] was adopted in [9] to fit the test data and is defined by

$$\tau_{oct} = \frac{2\sqrt{2}}{3}c\cos\varphi + \frac{2\sqrt{2}}{3}\sin\varphi\sigma_{m,2}, \quad (9)$$

$$\sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{2},$$

where $\sigma_{m,2}$ is the effective mean stress. Here, the fitting results by these two failure criteria are compared in Figure 5, and the relationship between σ_{1cal} and σ_{1exp} can be expressed by

$$\begin{aligned} \sigma_{1cal} &= 1.0049\sigma_{1exp} \quad (R2 = 0.9042), \\ \sigma_{1cal} &= 1.0059\sigma_{1exp} \quad (R2 = 0.9189), \end{aligned} \quad (10)$$

TABLE 1: True triaxial compression strength of experimental and predicted results of jointed marble (data from [9]; here, compressive stress is considered positive).

σ_3 (MPa)	σ_2 (MPa)	σ_{1exp} (MPa), test data at joint inclination angle β (°)						σ_{1cal} (MPa), calculated data at joint inclination angle β (°)					
		0	20	40	60	80	90	0	20	40	60	80	90
0	0	158.59	100.79	73.76	77.57	112.21	141.07	146.23	110.16	81.81	84.31	118.00	144.63
10	60	279.94	296.43	272.40	240.12	242.96	258.62	276.72	254.21	220.28	202.30	224.88	240.92
30	60	332.25	361.55	330.96	305.12	338.63	357.16	354.39	339.66	319.75	307.68	300.75	304.06
30	90	409.76	387.18	363.12	337.69	311.01	297.24	395.24	384.82	363.43	337.62	330.11	331.26
30	120	377.51	366.23	350.79	333.50	316.97	309.78	427.12	421.00	397.62	357.13	350.11	349.71
K	—	—	—	—	—	—	0.7196	0.7221	0.7011	0.6651	0.7084	0.7272	—
a	—	—	—	—	—	—	22.64	23.38	5.58	12.21	23.07	25.38	—
b	—	—	—	—	—	—	17.98	19.34	4.84	10.37	17.15	17.24	—
d	—	—	—	—	—	—	40.96	30.27	17.54	20.26	38.97	55.32	—

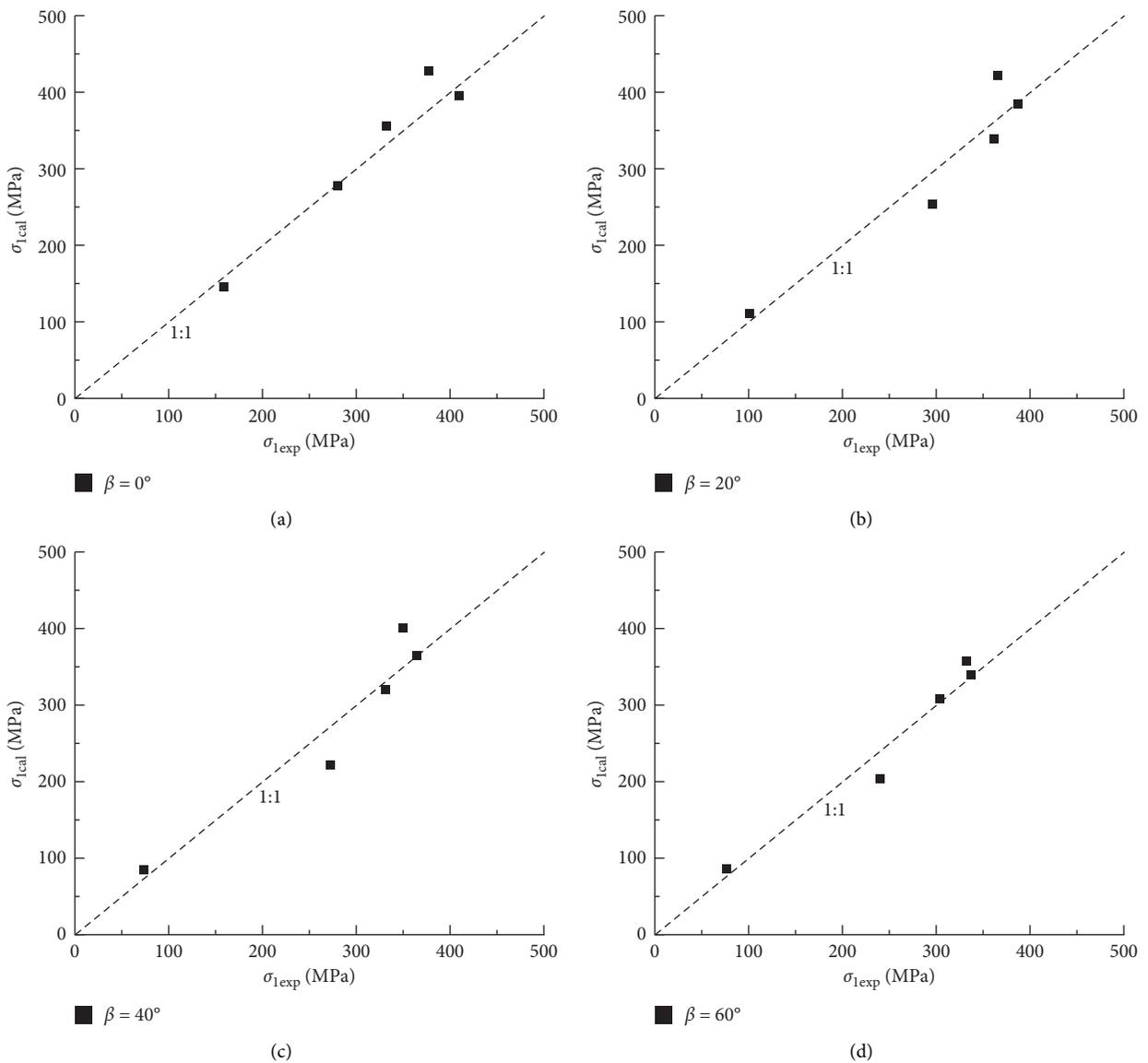


FIGURE 4: Continued.

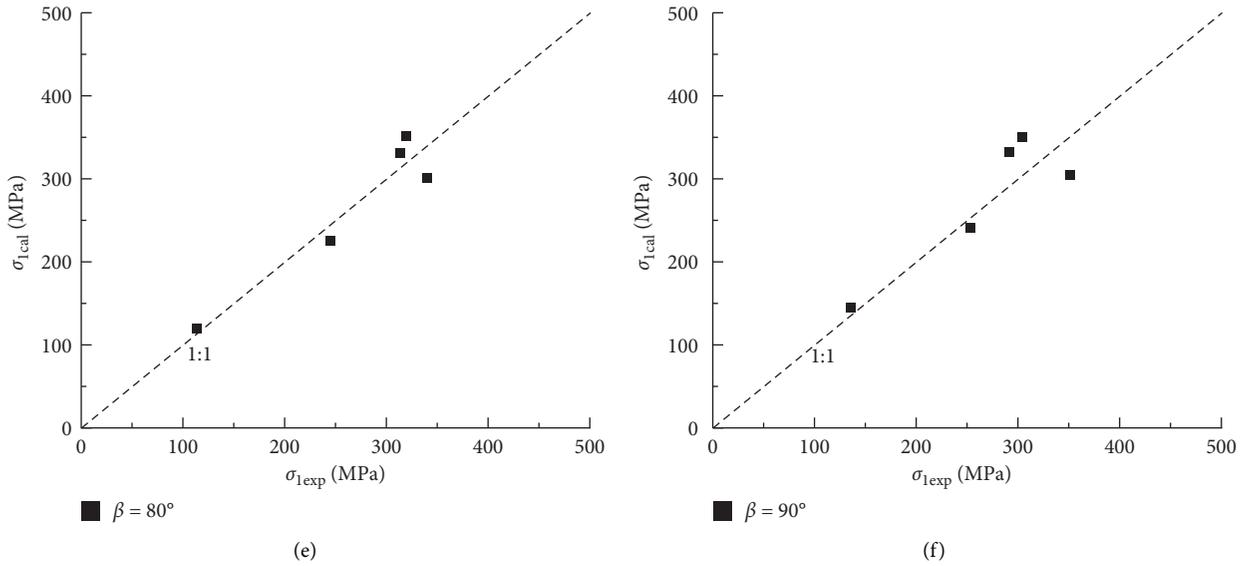


FIGURE 4: The comparison results of the calculated and experimental strength of jointed marble for different joint inclination angles: (a–f) $\beta = 0^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ,$ and 90° .

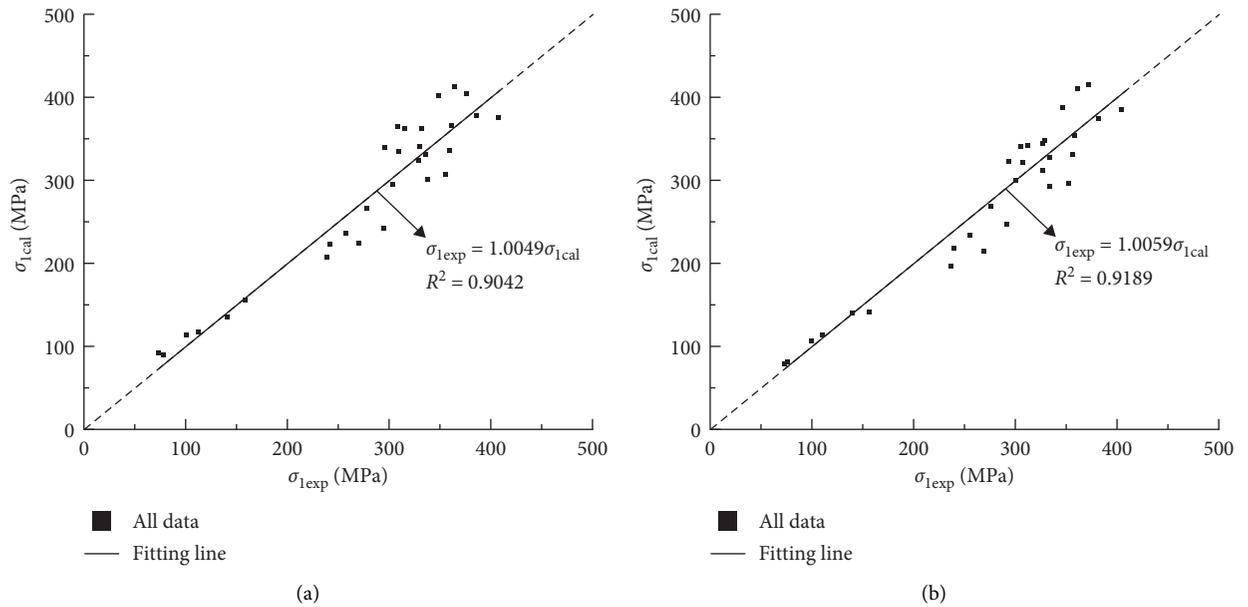


FIGURE 5: The fitting results of all the experimental strength of jointed marble: (a) the linear Mogi–Coulomb failure criterion and (b) the proposed failure criterion.

where equation (10) refers to the fitting results by the linear Mogi–Coulomb and proposed failure criteria, respectively. Table 2 also shows the percentage error in predicting strength data of jointed marble. Obviously, the proposed failure criterion has higher accuracy than the Mogi–Coulomb criterion. Therefore, the proposed failure

criterion is reasonable and applicable to predict the true triaxial strength of jointed rock masses.

3.2. Example 2. The strength data for example 2 are collected from the test results of [14]. This series of true triaxial tests

TABLE 2: Percentage error in predicting strength data of jointed marble.

σ_3 (MPa)	σ_2 (MPa)	Percentage error in predicting strength data at joint inclination angle β (°)					
		0	20	40	60	80	90
0	0	7.79	9.30	10.91	8.69	5.16	2.52
10	60	1.15	14.24	19.13	15.75	7.44	6.84
30	60	6.66	6.05	3.39	0.84	11.19	14.87
30	90	3.54	0.61	0.08	0.02	6.14	11.45
30	120	13.14	14.95	13.35	7.09	10.46	12.89
Average of this paper		6.46	9.03	9.37	6.48	8.08	9.71
Average of the Mogi-Coulomb criterion [17]		4.88	10.43	11.80	8.52	9.17	11.57

TABLE 3: True triaxial compression strength of experimental and predicted results of rock mass models (data from [14]; here, compressive stress is considered positive).

σ_3 (MPa)	σ_2 (MPa)	σ_{1exp} (MPa), test data at joint inclination angle β (°)						σ_{1cal} (MPa), calculated data at joint inclination angle β (°)					
		0	20	40	60	80	90	0	20	40	60	80	90
0.31	0.31	6.81	4.23	2.92	1.84	4.91	7.28	6.81	4.35	2.96	1.87	5.08	7.04
0.31	0.59	7.78	5.01	5.32	2.85	6.76	8.68	7.42	5.05	3.85	2.27	5.71	7.66
0.31	0.95	7.65	6.05	7.34	3.67	7.69	8.34	7.87	5.78	4.89	2.64	6.39	8.11
0.31	1.22	9.32	7.41	7.96	4.24	8.27	7.33	8.09	6.26	5.62	2.86	6.84	8.35
0.31	1.62	9.32	8.68	8.54	7.53	8.54	9.18	8.33	6.90	6.67	3.09	7.44	8.59
0.78	0.78	9.76	7.23	6.26	3.54	7.73	9.54	9.76	6.99	6.18	3.47	7.39	10.05
0.78	1.22	10.6	9.01	9.38	4.99	8.59	10.65	10.68	8.08	7.59	4.10	8.36	10.99
0.78	2.24	11.45	10.6	11.91	7.66	10.86	11.97	11.67	10.03	10.52	5.08	10.13	12.00
1.22	1.22	12.51	9.32	9.13	4.93	9.36	13.13	12.52	9.45	9.18	4.96	9.55	12.87
K	—	—	—	—	—	—	—	0.6109	0.7475	0.8433	0.8309	0.7666	0.6085
a	—	—	—	—	—	—	—	0.001191	0.001393	0.001269	0.001199	0.001474	0.002107
b	—	—	—	—	—	—	—	0.001073	0.001193	0.001185	0.000753	0.001179	0.001917
d	—	—	—	—	—	—	—	0.9252	0.5677	0.1452	0.3420	0.9118	0.9330

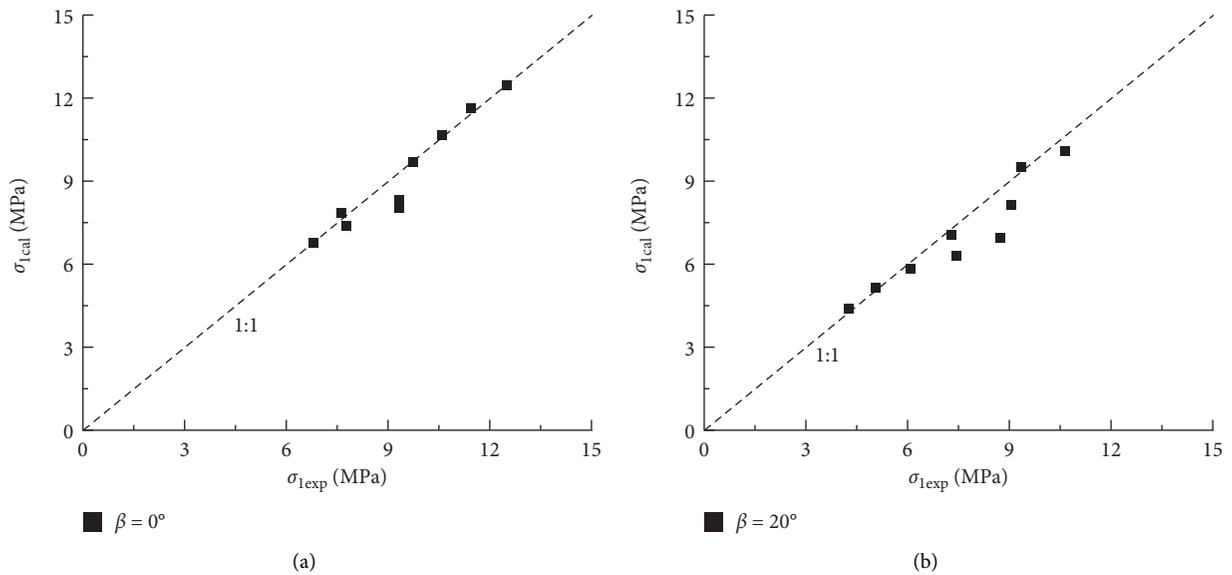


FIGURE 6: Continued.

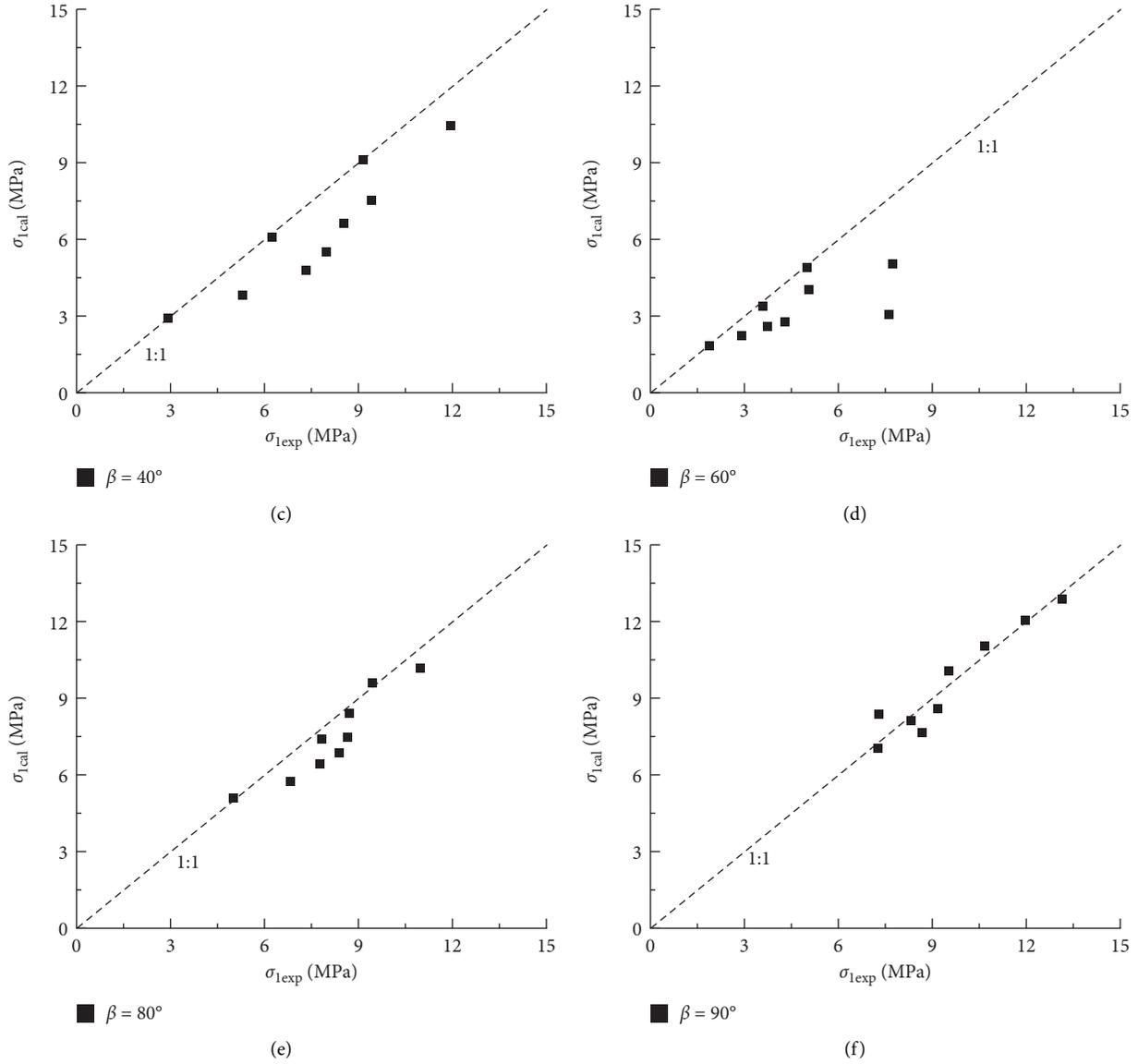


FIGURE 6: The comparison results of the calculated and experimental strength of jointed blocky mass for different joint inclination angles: (a–f) $\beta = 0^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ,$ and 90° .

were carried out on rock mass models with different joint inclination angles, which is made using similar material and contains three joint sets. Table 3 summarizes the strength data of both experimental and calculated results.

Figure 6 shows the comparison results of the calculated and experimental strength of jointed blocky mass. The data points except the joint inclination 60° are almost around the reference line. Thus, Figure 7 gives the fitting results of all the data and the data except the joint inclination 60° , and the relationship between σ_{1cal} and σ_{1exp} can be given by

$$\begin{aligned} \sigma_{1cal} &= 0.9247\sigma_{1exp} (R2 = 0.8738), \\ \sigma_{1cal} &= 0.9400\sigma_{1exp} (R2 = 0.9907), \end{aligned} \tag{11}$$

where equation (11) refers to the fitting results using all data and all data except the joint inclination 60° , respectively. In addition, two similar relationships suggested in [10, 28] are defined as follows:

$$\begin{aligned} \sigma_{1cal} &= 1.0150\sigma_{1exp} (R2 = 0.9383), \\ \sigma_{1cal} &= 1.0706\sigma_{1exp} (R2 = 0.8268). \end{aligned} \tag{12}$$

Table 3 also exhibits the percentage error in predicting strength data of rock mass models using these three strength criteria. Generally, compared with the modified Mohr–Coulomb failure criterion [28] and the empirical strength criterion [10], the proposed failure criterion in this

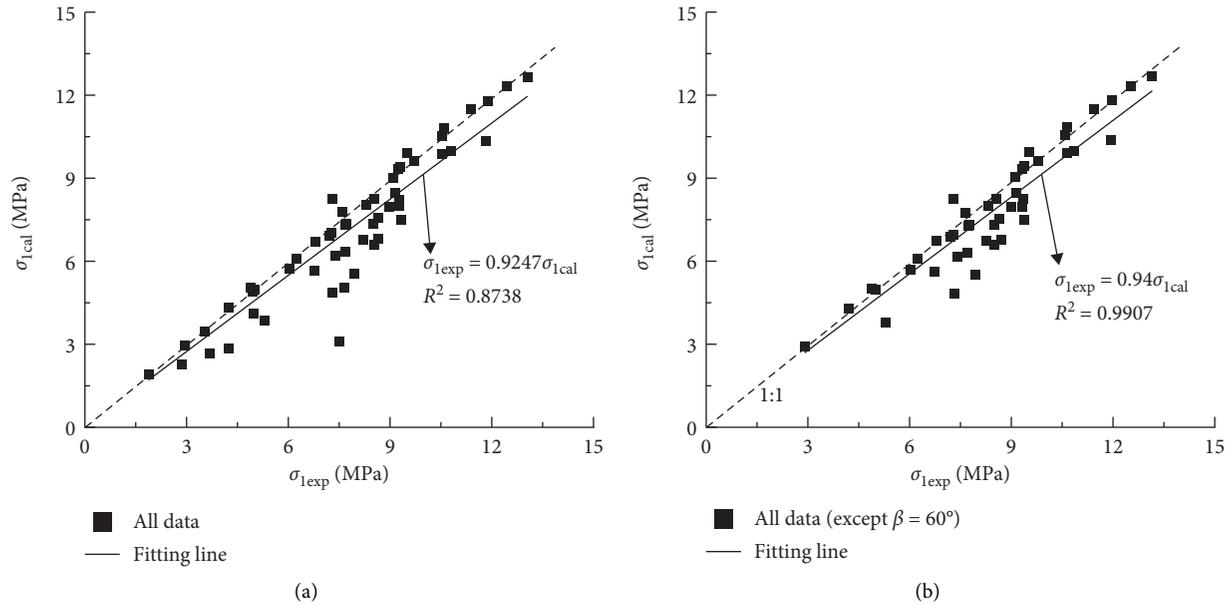


FIGURE 7: The fitting results of all the experimental strength of jointed blocky mass: (a) all the data and (b) all the data except $\beta = 60^\circ$.

TABLE 4: Percentage error in predicting strength data of rock mass models.

σ_3 (MPa)	σ_2 (MPa)	Percentage error in predicting strength data at joint inclination angle β ($^\circ$)					
		0	20	40	60	80	90
0.31	0.31	0.06	2.90	1.52	1.77	3.52	3.35
0.31	0.59	4.62	0.77	27.55	20.33	15.56	11.79
0.31	0.95	2.83	4.42	33.43	28.01	16.94	2.72
0.31	1.22	13.15	15.50	29.41	32.63	17.32	13.89
0.31	1.62	10.59	20.49	21.95	58.92	12.89	6.40
0.78	0.78	0.01	3.35	1.35	2.01	4.41	5.35
0.78	1.22	0.75	10.35	19.07	17.79	2.68	3.19
0.78	2.24	1.88	5.42	11.68	33.63	6.73	0.25
1.22	1.22	0.05	1.44	0.56	0.68	2.00	1.97
Average of this paper		3.77	7.18	16.28	21.75	9.12	5.43
Average of [14]		8.70	11.30	15.30	49.60	13.30	11.80
Average of [10]		17.80	8.36	15.60	29.70	7.70	12.50

study has higher correlation coefficient and smaller average percentage error, as shown in Table 4, which indicates that the proposed failure criterion is acceptable.

4. Conclusions

To investigate the influence of the joint orientation and σ_2 on the strength characteristics of jointed rock masses, a three-dimensional failure criterion is developed and is validated by two examples of true triaxial test results. Some important conclusions can be drawn as follows:

- (1) The proposed failure criterion in the deviatoric plane adopts the elliptic form based on the Willam–Warnke failure criterion. The joint effect is considered by only one parameter, strength ratio K .

- (2) The proposed failure criterion in the meridian plane uses the hyperbolic function to approximate the Mohr–Coulomb failure criterion so that the parameters can be linked with the cohesion and the internal friction angle.
- (3) The proposed failure criterion in the presented study can achieve small percentage error and high correlation coefficient to predict the true triaxial strength data in two examples.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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