Research Article

Prediction of Long-Term Prestress Loss and Crack Resistance Analysis of Corroded Prestressed Concrete Box-Girder Bridges

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Received 8 February 2022; Revised 17 March 2022; Accepted 24 March 2022; Published 9 April 2022

Academic Editor: Angelo Marcelo Tusset

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Reasonable assessment of long-term prestress loss and crack resistance is essential to ensure the service performance of prestressed concrete (PC) bridges. In this paper, a novel prediction model of long-term prestress loss considering the coupling effect of shrinkage and creep of concrete, prestressing steel relaxation, and presence of nonprestressing steel as well as the corrosion of prestressing and nonprestressing steel is proposed. Then, the probability analysis approach of long-term prestress loss considering the uncertainties of calculation parameters and models is introduced. Moreover, the assessment approach of crack resistance of local and overall structure is also developed, taking into account the effect of corrosion and time-dependent long-term prestress loss. The previously proposed approaches are applied to a three-span PC box-girder bridge. It is found that the prediction results of long-term prestress loss have high uncertainty, and the upper bound of long-term prestress loss at the confidence level of 95% is approximately 50% higher than the result obtained by deterministic analysis. The effect of corrosion on the long-term prestress loss is basically negligible, and the crack resistance of bottom slab and web is more sensitive to the coupled effect of corrosion and long-term prestress loss than that of top slab. Additionally, setting vertical prestressing tendons is conducive to improving the crack resistance level of web and whole section.

1. Introduction

Reasonable prediction of long-term prestress loss is the basis for durability assessment of prestressed concrete (PC) structures in service [1–3]. For structures whose long-term performance changes will significantly affect the overall performance, such as long-span PC box-girder bridges, long-term prestress loss is not only closely related to unexpected cracking and excessive deflection [4, 5] but also critical to the formulation of maintenance strategies. In addition, concrete cracking caused by prestress loss or environmental attack is one of the typical defects of PC box-girder bridge [6]. Concrete cracking not only significantly reduces the section stiffness and increases the bridge deflection but also accelerates the corrosion of prestressing and nonprestressing steels. These result in a reduction of service performance and durability of structure. Therefore, particular attention should be given to reasonable assessment of long-term prestress loss and time-dependent crack resistance of box-girder bridges.

Prediction of long-term prestress loss in current codes is mainly formed by the following three methods: (1) the time-step analysis methods were adopted by the ACI-209 code [7] and PCI Bridge Design Manual [8]; (2) the subitem superposition methods were used in the CEP-FIB 90 code [9], AASHTO LRFD specification [10], and JTG3362-2018 specification [11]; (3) the overall estimation methods were recommended by the AASHTO specification [12] and NCHRP 496 report [13]. Although these methods are widely used, most of them have more or fewer defects. For example, the effect of nonprestressing steel on long-term prestress loss is neglected in the CEB-FIP 90 code. The coupled effect of concrete shrinkage and creep is not considered in the JTG3362-2018 specification. Therefore, some scholars have actively explored other methods to overcome the shortcomings of the aforementioned methods. Considering the
effects of concrete shrinkage and creep, Cao et al. [14] improved the prestress loss prediction model caused by prestressing steel relaxation in Chinese code JTG D62-2004 [15]. Additionally, different prediction models of long-term prestress loss are proposed based on the internal force equilibrium and strain compatibility conditions [16, 17] or the finite element method [18, 19]. However, existing methods do not analyse the coupled effect of corrosion, concrete shrinkage and creep, prestressing steel relaxation, and presence of nonprestressing steel and also ignore the uncertainty of time-dependent prestress loss.

Since the reliability-based assessment method of long-term service performance of PC structures can effectively deal with the time-dependent and uncertainty of influencing parameters, it has gradually become a research hotspot in this field. Pillai et al. [20] developed a time-dependent predict method of service reliability (mainly related to crack resistance) of posttensioned (PT) segmental concrete bridges under different corrosion environments and load models. Stewart and Mullard [21] developed a probability calculation model of severe cracking of concrete surface under different durability design specifications, and the corresponding research results provide better guidance for the design of concrete structures. Based on the definition of the limit state that the main tensile stress of concrete in the web reaches the specified allowable stress value, Chen [22] established a probability assessment model of web cracking of box-girder and carried out the sensitivity analysis of web crack resistance. However, these studies ignore the coupled effect of concrete shrinkage and creep, time-dependent prestress loss and corrosion, and the variation regularity of local and overall crack resistance of structure, which are also not involved. A few scholars use the probabilistic finite element analysis method to make up for the shortcomings of the above research. For example, Guo et al. [23, 24] developed a reliability assessment of PC box-girder bridges under the combined action of creep, shrinkage, and corrosion based on an advanced probabilistic finite element method. Based on the finite element grillage model, Tu et al. [25] proposed a computational probabilistic framework for time-dependent reliability analysis of widened concrete highway bridges considering shrinkage and creep of concrete. Tong et al. [26] estimated the coupled effects of concrete shrinkage, creep, and cracking on the long-term behavior of postconnected prestressed steel-concrete composite girders by using the three-dimensional finite element model. Although the probabilistic finite element method can consider the complex spatial stresses, the coupling effect of various factors, and uncertain parameters, its computational cost is very high, and the rationality verification of the finite element model is also difficult. Therefore, it is urgent to develop a theoretical assessment method of crack resistance of PC structure under the multivariable coupling effects.

The objective of this study is to propose a novel prediction model of long-term prestress loss under the coupled effect of multiple factors and then to assess the reliability degradation of cracking resistance of box-girder bridges considering the effect of prestress loss on concrete stress. First, the well-established step-by-step approach is adopted to estimate long-term prestress loss in combination with the coupled effect of shrinkage and creep of concrete, pre-stressing steel relaxation, and presence of nonprestressing steel as well as corrosion of prestressing and nonprestressing steel. Then, the uncertainty analysis approach of long-term prestress loss is introduced. After that, the assessment approach of crack resistance of local and overall structure considering the effect of corrosion and long-term prestress loss is also proposed. Finally, the long-term prestress loss, effective prestressing force, and crack resistance of a three-span PC box-girder bridge are analysed.

2. Prediction Model of Long-Term Prestress Loss

2.1. Derivation of Prediction Model. To predict long-term prestress loss more reasonably, the well-established step-by-step method (denoted as SSM) with higher accuracy provided in [27, 28] is adopted to perform superposition calculation of long-term prestress loss, taking into account the coupled effect of shrinkage and creep of concrete, prestressing steel relaxation, and the presence of non-prestressing steel. In addition, the effect of corrosion on the cross-sectional area of prestressing and nonprestressing steel is also integrated into the calculation process.

In the calculation process, the assumptions of plane cross-section, linear behavior, and uncracked state of concrete, perfect bond performance between concrete and prestressing tendons and nonprestressing steels in time, as well as nonprestressing steels are used. The age of concrete at loading, \( t_0 \), to the calculation time of prestress loss, \( t \), is divided into \( w \) time intervals, and each time interval has the same length of \( (t - t_0)/w \). The parameters, such as stress and cross-sectional area of prestressing and nonprestressing steel, are assumed to be constant in a small-time interval. For any time interval \([t_{i-1}, t_i]\), when the external load remains constant, the cross section of members must satisfy the equilibrium equation of internal force increment, compatibility equation of strain increment, and the corresponding constitutive equation of each material.

2.1.1. Equilibrium Equation of Internal Force Increment. The relationship between the concrete compressive force increment \( \Delta F_c(t_{i-1}, t_i) \), the internal force variation of nonprestressing steels \( \Delta F_s(t_{i-1}, t_i) \), and the tensile force loss of prestressing tendons \( \Delta F_p(t_{i-1}, t_i) \) in time interval \([t_{i-1}, t_i]\) can be expressed as

\[
\Delta F_c(t_{i-1}, t_i) + \Delta F_s(t_{i-1}, t_i) + \Delta F_p(t_{i-1}, t_i) = 0. \tag{1}
\]

Then, the stress changes of concrete at the centroid of prestressing tendons and nonprestressing steels in time interval \([t_{i-1}, t_i]\) can be further derived, which are
\[
\Delta \sigma_{c,p}(t_{i-1}, t_i) = -\frac{1}{A_c(t_i)} \left[ \Delta \sigma_p(t_{i-1}, t_i) A_p(t_i) \left( 1 + \frac{e_p^2(t_i) A_c(t_i)}{I_c(t_i)} \right) + \Delta \sigma_s(t_{i-1}, t_i) A_s(t_i) \left( 1 + \frac{e_p(t_i) e_s(t_i) A_c(t_i)}{I_c(t_i)} \right) \right],
\]
\[
\Delta \sigma_{c,s}(t_{i-1}, t_i) = -\frac{1}{A_s(t_i)} \left[ \Delta \sigma_s(t_{i-1}, t_i) A_s(t_i) \left( 1 + \frac{e_p^2(t_i) A_s(t_i)}{I_s(t_i)} \right) + \Delta \sigma_p(t_{i-1}, t_i) A_p(t_i) \left( 1 + \frac{e_p(t_i) e_s(t_i) A_s(t_i)}{I_s(t_i)} \right) \right],
\]

where \( A_c(t_i) \) and \( I_c(t_i) \) are the area and inertia moment of net cross section of concrete at time \( t_i \), respectively. \( A_p(t_i) \) and \( A_s(t_i) \) are the cross-sectional areas of prestressing tendons and nonprestressing steels at time \( t_i \), respectively. \( \Delta \sigma_p(t_{i-1}, t_i) \) and \( \Delta \sigma_s(t_{i-1}, t_i) \) are the stress changes of prestressing tendons and nonprestressing steels in time interval \([t_{i-1}, t_i]\), respectively. \( e_p(t_i) \) and \( e_s(t_i) \) are the eccentricity from the centroid of prestressing tendons and nonprestressing steels to the centroid of concrete cross section at time \( t_i \), respectively.

### 2.1.2. Compatibility Equation of Strain Increment.

Figure 1 shows the strain increment of a typical cross section of the bonded PC box-girder member. According to the strain increment compatibility between the prestressing tendons, nonprestressing steels, and concrete at the same height of box-girder section, the strain increment equation in time interval \([t_{i-1}, t_i]\) is expressed as

\[
\Delta \varepsilon_{c,p}(t_{i-1}, t_i) = \Delta \varepsilon_{f,c,p}(t_{i-1}, t_i) + \frac{\Delta \sigma_{c,p}(t_{i-1}, t_i)}{E_c(t_i)} = \frac{\sigma_{c,p}(t_0)}{E_c(t_0)} \left[ \varepsilon_0(t_0) - \varepsilon_{e,sh}(t_0) \right] + \left( \sum_{j=1}^{i-1} \frac{\Delta \sigma_{c,p}(t_{j-1}, t_j)}{E_c(t_j)} \right) \left[ \varepsilon(t_j, t_j) - \varepsilon(t_{j-1}, t_j) \right]
\]
\[
+ e_{sh}(t_i, t_0) - e_{sh}(t_{i-1}, t_0) + \frac{\Delta \sigma_{c,p}(t_{i-1}, t_i)}{E_c(t_i)}.
\]

When \( i = 1 \), the above formula can be simplified as

\[
\Delta \varepsilon_{c,p}(t_0, t_1) = \Delta \varepsilon_{f,c,p}(t_0, t_1) + \frac{\Delta \sigma_{c,p}(t_0, t_1)}{E_c(t_1)}
\]
\[
= \frac{\sigma_{c,p}(t_0)}{E_c(t_0)} \left( \varepsilon_0(t_0) + e_{sh}(t_1, t_0) \right) + \frac{\Delta \sigma_{c,p}(t_0, t_1)}{E_c(t_1)},
\]

where \( \sigma_{c,p}(t_0) \) is the initial stress of concrete at the centroid of prestressing tendons (nonprestressing steels) generated by the effective prestressing force and external load at time \( t_0 \). \( E_c(t_0) \) is the elasticity modulus of concrete, and it is expressed as [29]

\[
E_c(t_i) = \frac{E_c(t_0)}{1 + \chi(t_i, t_0) \phi(t_i, t_0)},
\]

where \( \chi(t_i, t_0) \) is the concrete aging coefficient, and it can be calculated by (10) [9].

\[
\chi(t_i, t_0) = 1 - 0.91 e^{-0.666 \phi(t_i, t_0)} - \frac{1}{1 - 0.91 e^{-0.666 \phi(t_i, t_0)}}
\]

### 2.1.3. Constitutive Equations of Materials.

Since the strain of concrete at the upper and lower edge of uncracked box-girder section is relatively small under the effect of prestressing force and external load, the prestressing and
nonprestressing steel as well as concrete can be considered in elastic deformation stage. That is, their corresponding constitutive equations can adopt the linear elastic model for simplicity. It is worth noting that the prestressing steel relaxation does not cause the corresponding strain increment of prestressing tendons, and the stress relaxation effect should be reasonably reduced by the relaxation coefficient because of the length change of prestressing tendons. Thus, considering the effect of stress relaxation and reasonable reduction, the stress-strain relationship of the prestressing steel can be expressed as

$$\Delta \varepsilon_p(t_{i-1}, t_i) = \left[ \Delta \sigma_p(t_{i-1}, t_i) - \chi_i \Delta \sigma_{pr}(t_{i-1}, t_i) \right] / E_p,$$

(11)

where $E_p$ is the elastic modulus of prestressing tendons, $\chi_i$ is the reduced relaxation coefficient, and its value can be taken as 0.75 \cite{17, 30}. $\Delta \sigma_{pr}(t_{i-1}, t_i)$ is the inherent relaxation loss in time interval $[t_{i-1}, t_i]$ and can be calculated as \cite{10}

$$\Delta \sigma_{pr}(t_{i-1}, t_i) = \log\left[ \frac{24(t_i - t_{i-1})}{45} \right] \left[ \frac{\sigma_p(t_{i-1})}{f_{py}} - 0.55 \right] \sigma_p(t_{i-1}).$$

(12)

where $\sigma_p (t_{i-1})$ is the stress value of prestressing tendons at time $t_{i-1}$ and $f_{py}$ is the yield stress of prestressed tendons.

By substituting (4) and (11) into (2) and the linear stress-strain relationship of nonprestressing steel and (5) into (3), the strain changes of prestressing tendons, nonprestressing steels, and concrete at their centroids can be deduced as follows:

$$\Delta \varepsilon_p(t_{i-1}, t_i) = \Delta \varepsilon_p(t_{i-1}, t_i)$$

$$= \frac{\Delta \varepsilon_{f,p}(t_{i-1}, t_i) (M + m_p) - m_p \Delta \varepsilon_{f,p}(t_{i-1}, t_i) - M/E_p N \Delta \varepsilon_{pr}(t_{i-1}, t_i) [m_p + 1/M (m_p m_s - m_p^2)]}{M + M/N m_p + m_s + 1/N (m_p m_s - m_p^2)},$$

(13)

$$\Delta \varepsilon_c(t_{i-1}, t_i) = \Delta \varepsilon_c(t_{i-1}, t_i)$$

$$= \frac{\Delta \varepsilon_{f,c}(t_{i-1}, t_i) (N + m_p) - m_p \Delta \varepsilon_{f,c}(t_{i-1}, t_i) - m_p/E_p \Delta \varepsilon_{pr}(t_{i-1}, t_i)}{N + m_p + N/M m_s + 1/M (m_p m_s - m_p^2)},$$

(14)

where $M = A_c(t_i) E_c(t_i) / [A_p(t_i) E_p]$ and $N = A_c(t_i) E_c(t_i) / [A_p(t_i) E_p]$. $E_c$ is the elastic modulus of nonprestressing steel, $m_p(t_i) = 1 + \varepsilon_p(t_i) A_c(t_i) / I_p(t_i)$, and $m_p = 1 + \varepsilon_p(t_i) A_c(t_i) / I_p(t_i)$.

For the cross section of actual PC box-girder, the longitudinal nonprestressing steels are often constructed according to the design code. As a result, their centroid is often very close to the centroid of the concrete section \cite{17}.
Therefore, the value of $e_i$ can be taken as 0, and the corresponding values of $m_i$ and $m_{pi}$ are 1. Then, (13) and (14) can be rewritten as

$$
\Delta \varepsilon_p (t_{i-1}, t_i) = \Delta \varepsilon_c (t_{i-1}, t_i) = \frac{\Delta \varepsilon_{cf,p} (t_{i-1}, t_i)(M + 1) - \Delta \varepsilon_{cf,s} (t_{i-1}, t_i) - (M + 1)m_p - 1/E_p N\Delta \sigma_{pr} (t_{i-1}, t_i)}{M + 1 + (M + 1)m_p - 1/N}.
$$

$$
\Delta \varepsilon_s (t_{i-1}, t_i) = \Delta \varepsilon_{c1} (t_{i-1}, t_i) = \frac{\Delta \varepsilon_{cf,s} (t_{i-1}, t_i)(N + m_p) - \Delta \varepsilon_{cf,p} (t_{i-1}, t_i) - 1/E_p N\Delta \sigma_{pr} (t_{i-1}, t_i)}{N + m_p + N + (m_p - 1)/M}.
$$

Finally, the prestress variation in time interval $[t_{i-1}, t_i]$ can be further obtained according to the constitutive equation, and then the corresponding prestress loss in this period and the effective prestress value at time $t_i$ are obtained. The above calculation results are all used as the initial value in the next time interval for stepwise calculation. The prestress variation of the prestressing tendons in time interval $[t_{i-1}, t_i]$ and the effective prestress at time $t_i$ can be calculated as

$$
\Delta \sigma_p (t_{i-1}, t_i) = E_p \Delta \varepsilon_p (t_{i-1}, t_i) + \Delta \sigma_{pr} (t_{i-1}, t_i),
$$

$$
\sigma_p (t_i) = \sigma_{p0} - \sum_{k=1}^{6} \Delta \sigma_p (t_k,t_k),
$$

where $\sigma_{p0}$ is the initial prestress of prestressing tendons.

2.2. Time-Dependent Corrosion Model. Corrosion is the main cause of the deterioration of concrete structure [31–33], and one of its direct effects is the cross-sectional area loss of prestressing and nonprestressing steels [24]. Here, the most used pitting model proposed in [34] is adopted to calculate the cross-sectional area of non-prestressing steel after corrosion. The cross-sectional area loss model of prestressing steel strands proposed in [35] is also adopted because of its simplicity and rationality. Therefore, the net cross-sectional area of a 7-wire prestressing steel strand at time $t$ can be expressed as [36]

$$
A_p (t) = \sum_{q=1}^{6} A_{ow,q} (t) p_{M} (t, i_{corr}, t_c, l) + A_{iw} (t, p_{M} (t, i_{corr}, t_c, l), l),
$$

where $A_{ow,q} (t, p_{M})$ and $A_{iw} (t, p_{M})$ are, respectively, the cross-sectional areas of the $q$th outer wire and inner wire of prestressing steel strand at time $t$, which are assumed to be calculated using the pit model proposed in [34]. $p_{M} (t, i_{corr}, t_c, l)$ and $p_{M} (t, i_{corr}, t_c, l)$ are the maximum pit depths of the $q$th outer wire and inner wire, respectively. They can be simulated as the extreme-value distribution (type I) when the corrosion current density $i_{corr}$, corrosion initiation time $t_c$ (years), and wire length $l$ (mm) are given. $t_c$ is related to corrosion environment and concrete cover. The corresponding probability model of maximum pit depth is shown in [25].

2.3. Model Verification. After deducting all the instantaneous prestress loss produced in tension stage, the value of effective prestress at any time can be obtained by bringing the initial effective prestress into the prediction model of long-term prestress loss. To verify the validity of the prediction model, the long-term prestress loss considering and not considering the effect of corrosion is predicted in turn, and the prediction results are compared with the experimental results.

The measured data of time-dependent prestress loss of seven bonded PC members under the noncorrosion condition and laboratory environment in [17] are selected here to verify the validity of the proposed method. Due to the length limitation of the paper, the relevant test parameters are not listed here and can be found in [17]. Their long-term prestress loss is calculated by the proposed prediction model, and the time-dependent results are compared with that of JTG3362-2018, CEB-FIP 90, and AASHTO LRFD specifications, as shown in Table 1.

Due to the lack of measured value of prestress loss under the effect of corrosion, an indirect method is used to verify the validity of the prediction model of long-term prestress loss. That is to say, it is verified by comparing the effective
Table 1: Comparison of experimental and theoretical values of long-term prestress loss.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Initial prestressing force of each tendon (kN)</th>
<th>Quantity and layout form of prestressing tendon</th>
<th>Measured value (MPa)</th>
<th>Theoretical value (MPa)</th>
<th>JTG3362-2018 (MPa)</th>
<th>CEB-FIP 90 (MPa)</th>
<th>AASHTO LRFD (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>100.1</td>
<td>1 (curve)</td>
<td>48.0</td>
<td>45.0</td>
<td>52.8</td>
<td>66.5</td>
<td>67.4</td>
</tr>
<tr>
<td>PC3</td>
<td>102.3</td>
<td>1 (curve)</td>
<td>45.7</td>
<td>45.4</td>
<td>52.8</td>
<td>66.5</td>
<td>67.4</td>
</tr>
<tr>
<td>PC4</td>
<td>135.9</td>
<td>1 (curve)</td>
<td>70.6</td>
<td>66.3</td>
<td>65.6</td>
<td>78.6</td>
<td>80.1</td>
</tr>
<tr>
<td>PC5</td>
<td>108.7</td>
<td>1 (straight)</td>
<td>55.2</td>
<td>53.8</td>
<td>59.6</td>
<td>69.5</td>
<td>69.4</td>
</tr>
<tr>
<td>PC6</td>
<td>101.4</td>
<td>2 (straight)</td>
<td>53.3</td>
<td>56.2</td>
<td>56.5</td>
<td>62.4</td>
<td>68.8</td>
</tr>
<tr>
<td>PC7</td>
<td>102.6</td>
<td>1 (straight)</td>
<td>46.8</td>
<td>48.1</td>
<td>50.4</td>
<td>65.6</td>
<td>64.1</td>
</tr>
<tr>
<td>PC8</td>
<td>106.0</td>
<td>1 (straight)</td>
<td>49.0</td>
<td>51.2</td>
<td>55.1</td>
<td>66.0</td>
<td>67.6</td>
</tr>
</tbody>
</table>

As indicated in Table 1, the calculation results of the prediction model proposed in this paper and the JTG3362-2018 specification are closer to the experimental values than the CEB-FIP 90 model and AASHTO LRFD refined method. However, compared with the proposed prediction method, the prediction method in JTG3362-2018 specification ignores the interaction between shrinkage, creep of concrete, and prestressing steel relaxation. Moreover, the calculated values of effective prestressing force are in good agreement with the experimental values in Table 2, which indirectly verifies the validity of the proposed prediction model. In conclusion, the proposed model considering the coupled effect of multifactor is applicable to the prediction of long-term prestress loss.

3. Uncertainty Analysis of Long-Term Prestress Loss

As indicated in the previous section, the proposed prediction model of long-term prestress loss involves a large number of calculation parameters, including the material, structural dimensions, environmental parameters, load effects, and corrosion-related parameters [39]. Since the values of these parameters are usually obtained by the statistical analysis of limited data, their uncertainties should be considered in the prediction of long-term prestress loss.

Additionally, the uncertainty of relevant calculation models in the prediction process of long-term prestress loss should also be considered. The shrinkage and creep models of concrete in the existing codes are mostly empirical or semiempirical and semitheoretical models based on experimental data, and there are inevitable errors in their calculation results compared with the actual effect of shrinkage and creep of concrete. Therefore, the uncertainty coefficients of concrete shrinkage and creep model are incorporated within the prediction process. According to the statistical analysis results in [40], the uncertainty coefficients of creep and shrinkage models in CEB-FIP 90 code are subject to the standard normal distribution, in which the mean values of both are 1, and the coefficient of variation is 0.339 and 0.451, respectively. Moreover, the maximum pit depth of prestressing steel strand also has great uncertainty under the effect of corrosion. The maximum pit depth at different times can be simulated as a random variable subject to the extreme-value type I distribution, and the probability model proposed in [36] is also incorporated within the prediction process.
4. Reliability Analysis of Crack Resistance of Box-Girder Bridge

According to the provisions of JTG3362-2018 [11], the normal tensile stress of concrete at the edge of normal section and inclined section of PC box-girder members under the effect of long-term load should satisfy \( \sigma_b \leq \sigma_{pc} \) and \( \sigma_{tp} \leq 0.4 f_{ik} \), respectively. Moreover, the maximum compressive stress of concrete at the compression zone of the normal section also needs to satisfy \( \sigma_{kc} + \sigma_{pt} \leq 0.5 f_{ik} \), in which \( \sigma_b \) is the normal tensile stress of concrete at the bottom edge of the section under the effect of long-term load, and \( \sigma_{pc} \) is the precompression stress of concrete at the bottom edge of the section under the effect of effective prestressing force. \( \sigma_{tp}, \sigma_{kc} \), and \( \sigma_{pt} \) are principal tensile stress, normal compressive stress, and tensile stress of concrete under the combined effect of long-term load and effective prestressing force, respectively, \( f_{ik} \) and \( f_{ik} \) are standard values of axial tensile and compressive strength of concrete, respectively.

Based on the previous provisions relating to concrete cracking and reliability theory, the limit state functions of concrete cracking at the bottom slab, top slab, and web of PC box-girder can be expressed in turn as

\[
g_b(t_i) = \frac{F_{tp}(t_i)}{A_c(t_i)} + \frac{F_{tp}(t_i) e_p(t_i) y_b(t_i)}{I_c(t_i)} - \frac{\left[ M_{d(t_c(t_i))} + M_{f(t_c(t_i))}\right] y_b(t_i)}{I_c(t_i)}
\]

\[
g_t(t_i) = 0.5 f_{ck} - \sigma_{tp}(t_i)
\]

\[
g_w(t_i) = 0.4 f_{ck} - \sigma_{tp}(t_i),
\]

where \( g_b(t_i), g_t(t_i), \) and \( g_w(t_i) \) are the value of crack resistance limit state functions of the bottom slab, top slab, and web at time \( t_i \), respectively. \( F_{tp}(t_i) \) is the total effective prestressing force of this section, and \( y_b(t_i) \) is the distance from the bottom edge to the centroid of the concrete section. \( M_{d(t_c(t_i))} \) and \( M_{f(t_c(t_i))} \) are the bending moment values of the section under dead and live load effect, respectively. \( \sigma_{tp}(t_i) \) is the principal tensile stress of concrete in web, and it can be calculated by the simplified two-dimensional stress model provided by JTG3362-2018 specification [11]. It is worth noting that the time-dependent effect of model parameters is further considered here.

\[
\beta_{l,z}(t_i) = \Phi^{-1}\left[1 - P_{f_{iz}(t_i)}\right] = \Phi^{-1}\left[1 - P\left(g_z(t_i) < 0\right)\right], \quad z = b,t,w,
\]

\[
\beta_{as}(t_i) = \Phi^{-1}\left[1 - P_{f_{jz}(t_i)}\right] \left[1 - P_{f_{iz}(t_i)}\right] \left[1 - P_{f_{jw}(t_i)}\right],
\]

where \( \beta_{l,z}(t_i) \) and \( P_{f_{iz}(t_i)} \) are the reliability index of crack resistance and cracking probability of local area, respectively. The subscript \( z \) denotes that \( b, t, \) and \( w \), respectively, correspond to the above two indexes of bottom slab, top slab, and web. \( \Phi^{-1}() \) is the inverse of the cumulative distribution function of standard normal distribution, \( P\left(\right) \) is the
probability function, and $\beta_m(t)$ is the reliability index of crack resistance of the whole section.

5. Application Example

5.1. Bridge Description. The proposed prediction model of long-term prestress loss and crack resistance analysis approach is applied to a three-span PC box-girder bridge located in a marine environment. The span layout of the superstructure of the main bridge is $80 \pm 150 \pm 80$ m. The superstructure is made of C55 concrete, of which the 28-day standard cubic compressive strength is 50 MPa and the initial elasticity modulus $E_{ci}$ is $3.55 \times 10^4$ MPa. Considering that the bottom slab of the midspan section is easy to crack and the crack also has a great impact on the deflection of the bridge, the midspan section is selected as the research object. Two hot-rolled ribbed steel bars of the JL32 type are arranged as the vertical prestressing steel in webs on both sides of the midspan section. The corresponding tension control stress is 706.5 MPa, and the average value of the initial elastic modulus $E_p$ is $2 \times 10^5$ MPa. Twenty-four (excluding one pair of the spare prestressing tendon) and two longitudinal prestressing tendons (PTs) are equipped in the bottom and top slab, respectively. The corresponding tension control stress, nominal tensile strength, and cross-sectional area of longitudinal prestressing steel strands are 1395 MPa, 1860 MPa, and $140 \text{ mm}^2$, respectively. The HRB400 grade hot-rolled ribbed steel bar and the HPB300 grade hot-rolled plain steel bar are, respectively, used for longitudinal nonprestressing steel with a diameter greater than 12 mm and less than 12 mm, and the corresponding average values of initial elastic modulus are $2.0 \times 10^5$ MPa and $2.1 \times 10^5$ MPa, respectively. The initial total cross-sectional area $A_0 = 9.09 \times 10^4 \text{ mm}^2$, initial section inertia moment $I_{0} = 1.3485 \times 10^7 \text{ mm}^4$, and the external perimeter of section is 30020 mm. The initial total cross-sectional area of longitudinal PTs at the bottom and top slab is $A_{tp0} = 5.04 \times 10^4 \text{ mm}^2$ and $A_{tp0} = 4.2 \times 10^5 \text{ mm}^2$, respectively. The initial total cross-sectional area of longitudinal nonprestressing steels $A_{0} = 3.7486 \times 10^4 \text{ mm}^2$. The initial distance from the centroid of all longitudinal PTs to the centroid of the concrete section and bottom edge is $e_{p0} = 1593.6 \text{ m}$ and $y_{p0} = 368.5 \text{ mm}$, respectively. In addition, the bending moment values generated by the dead and live load are obtained by Midas Civil finite element analysis, which are $7.65 \times 10^4 \text{ kN-m}$ and $4.02 \times 10^5 \text{ kN-m}$, respectively. The overall layout of the case bridge, cross-section dimensions, and arrangement of longitudinal PTs in the midspan section is shown in Figure 2, and the random parameters related to the calculation are shown in Table 3.

The coefficient of variation (COV) of long-term prestress loss decreases first, then remains unchanged, and finally stabilizes above 0.24 after 20 years of service (Figure 4(d)). This also shows that the uncertainty of parameters and calculation models will lead to the significant variability of long-term prestress loss. After 10, 30, and 100 years of service, the upper bound of long-term prestress loss of CT1 at the confidence level of 95% is 51.04%, 51.12%, and 50.64% higher than the results obtained by the deterministic analysis, respectively. The corresponding results of CB1 are 50.25%, 49.38%, and 49.60%, respectively. This indicates that the final effective prestressing force of CT1 and CB1 will be 7.60% and 4.2% lower than that of the deterministic analysis when the degradation and uncertainty of the cross-sectional area of prestressed tendons are not considered. Moreover, with the increase of long-term prestress loss, number of uncertain parameters, or uncertainty strength of each parameter, the upper bound of long-term prestress loss is all significantly higher than the result obtained by deterministic analysis. Therefore, it is recommended to estimate the long-term prestress loss at a reasonable confidence level and select
the upper bound of confidence interval as the most unfavorable value of long-term prestress loss from the perspective of preventing early cracking.

5.2.2. Effect of Corrosion on Long-Term Prestress Loss. It also can be concluded from Figures 4(b) and 4(c) that the mean value of the final long-term prestress loss of CB1 and all PTs considering the effect of corrosion is, respectively, only increased by 1.36% and 1.55% compared with the corresponding value without considering the effect of corrosion. Even if the uncertainty of parameters and models is considered, the effect of corrosion on the upper and lower bounds of prestress loss at the confidence level of 95% is still small. In addition, Figure 4(d) shows that the effect of corrosion on the coefficient of variation (COV) of long-term prestress loss is also negligible. The reason for the above results is that the total cross-sectional area of prestressing and longitudinal nonprestressing steels is far less than that of concrete section (the former is about 1% of the latter in this case study). Even if the prestressing and longitudinal nonprestressing steels are seriously corroded, the relative change of cross-sectional area and the concrete strain change caused by the previous area loss are very minor, which only result in a slight change of long-term prestress loss. Therefore, in practical engineering calculation, the reduction of the sectional area of steel bars caused by corrosion has a negligible impact on long-term prestress loss.

Figure 2: Bridge configuration. (a) Overall layout of the bridge. (b) Cross-section dimensions of midspan section (unit: cm). (c) Arrangement of longitudinal PTs of midspan section (unit: cm).
Table 3: Statistical characteristics of parameters related to long-term prestress loss prediction.

<table>
<thead>
<tr>
<th>Classification of parameters</th>
<th>Description of parameters</th>
<th>Distribution</th>
<th>(Mean, COV)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material and dimension parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial elastic modulus of concrete, $E_{c0}$ (MPa)</td>
<td>Normal</td>
<td>$(3.55 \times 10^4, 0.04)$</td>
<td>[41]</td>
<td></td>
</tr>
<tr>
<td>Concrete cover</td>
<td>Normal</td>
<td>$(1.0178^*, 0.0496)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial elastic modulus of prestressing steel strand, $E_p$ (MPa)</td>
<td>Normal</td>
<td>$(1.95 \times 10^5, 0.04)$</td>
<td>[41]</td>
<td></td>
</tr>
<tr>
<td>Initial elastic modulus of HRB400 grade nonprestressing steel (MPa)</td>
<td>Normal</td>
<td>$(2 \times 10^5, 0.04)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial elastic modulus of HPB300 grade nonprestressing steel (MPa)</td>
<td>Normal</td>
<td>$(2.1 \times 10^5, 0.04)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength of HRB400 grade nonprestressing steel (MPa)</td>
<td>Normal</td>
<td>$(434, 0.0791)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength of HPB300 grade nonprestressing steel (MPa)</td>
<td>Normal</td>
<td>$(324.6, 0.1211)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial compressive strength of concrete, $f_{c0}$ (MPa)</td>
<td>Normal</td>
<td>$(55.2, 0.137)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Environmental parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual relative humidity, RH (%)</td>
<td>Normal</td>
<td>$(76.9, 0.061)$</td>
<td>CMA</td>
<td></td>
</tr>
<tr>
<td>Annual average temperature, $T$ (°C)</td>
<td>Normal</td>
<td>$(21.7, 0.251)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty coefficient of concrete creep model, $\Psi_c$</td>
<td>Normal</td>
<td>$(1, 0.339)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty coefficient of concrete shrinkage model, $\Psi_s$</td>
<td>Normal</td>
<td>$(1.0148^*, 0.0431)$</td>
<td>[41]</td>
<td></td>
</tr>
<tr>
<td>Load effect parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Live load effect (bending moment), $M_{bl}$ (kN-m)</td>
<td>Normal</td>
<td>$(0.7995^*, 0.0862)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear force, $V_s$ (kN)</td>
<td>Normal</td>
<td>$(0.0148^*, 0.0431)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial tension control stress, $\sigma_{con}$ (MPa)</td>
<td>Normal</td>
<td>$(1^*, 0.04)$</td>
<td>[42]</td>
<td></td>
</tr>
<tr>
<td>Surface chloride concentration, $C_s$ (kg/m³)</td>
<td>Normal</td>
<td>$(3.5, 0.5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold chloride concentration, $C_{thr}$ (kg/m³)</td>
<td>Normal</td>
<td>$(0.9, 0.2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chloride diffusion coefficient, $D_0$ (cm²/year)</td>
<td>Log normal</td>
<td>$(0.631, 0.2)$</td>
<td>[24]</td>
<td></td>
</tr>
<tr>
<td>Corrosion current density, $i_{corr}$ (μA/cm²)</td>
<td>Normal</td>
<td>$(1, 0.2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Penetration ratio, $R$</td>
<td>Normal</td>
<td>$(6.0, 0.18)$</td>
<td>[39]</td>
<td></td>
</tr>
</tbody>
</table>

*Note. *$^*$The ratio of actual value to nominal value, $D$ = design data, and CMA = China Meteorological Administration.

Table 4: Mean value of effective prestress after deducting all instantaneous losses.

<table>
<thead>
<tr>
<th>Prestressing tendons</th>
<th>CB1</th>
<th>CB2</th>
<th>CB3</th>
<th>CB4</th>
<th>CB5</th>
<th>CB5'</th>
<th>CB6</th>
<th>CB7</th>
<th>CB8</th>
<th>CB9</th>
<th>CB10</th>
<th>CT1</th>
<th>N138</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial tension control stress (MPa)</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>1395</td>
<td>706.5</td>
</tr>
<tr>
<td>Effective prestress (MPa)</td>
<td>1204.9</td>
<td>1243.3</td>
<td>1261.4</td>
<td>1271.4</td>
<td>1269.9</td>
<td>1269.9</td>
<td>1264.2</td>
<td>1254.1</td>
<td>1244.9</td>
<td>1235.9</td>
<td>1227.3</td>
<td>1261.8</td>
<td>626.5</td>
</tr>
</tbody>
</table>

*Note. CB = PTs in bottom slab, CT = PTs in top slab, and N = vertical PTs.

Figure 3: Probability density of long-term prestress loss of CT1 and CB1 at different time. (a) CT1. (b) CB1.
5.2.3. Effective Prestressing Force. The long-term prestress loss and the reduction of the cross-section area of the tendon will reduce the effective prestressing force, which will cause the change of concrete stress and structural crack resistance. Thus, the variation of effective prestressing force also needs to be analysed. As indicated in Figure 5, the effective prestressing force of CT1 is only slightly reduced in service, and the mean value of effective prestressing force at 100 years of service is only decreased by about 12.7% compared with the initial value. While the effective prestressing force of CB1 and whole section is greatly reduced when considering the effect of corrosion, the corresponding mean values are only 29.2% and 33.8% of the initial mean value, respectively. This is mainly because the corrosion of CT1 does not occur during the service period due to the protection of the larger concrete cover and the deck pavement larger than 17 cm, but the cross-section area loss of CB1 and all longitudinal PTs gradually increases with the development of corrosion process. Therefore, it can be considered that the reduction of the cross-sectional area of prestressed tendons caused by corrosion is the main reason for the decrease in effective prestressing force.

Additionally, Figures 5(b) and 5(c) also show that the variation of effective prestressing force of CB1 and all longitudinal PTs in this section can be divided into three phases. In the first phase (the first year of service), the significant increase of long-term prestress loss is the main reason for the rapid reduction of effective prestressing force of CB1 and the whole section. In the second phase (about 1 to 9 years of service), the time-dependent prestress loss caused by concrete shrinkage and creep is the main reason for the decrease of effective prestressing force, and the pace of decline is obviously slower than before. This is because the PTs have not been corroded and the prestress loss caused by

![Figure 4: Time-dependent curve related to long-term prestress loss. (a) CT1. (b) CB1. (c) Average loss of all longitudinal PTs. (d) Coefficient of variation.](image-url)
prestressing steel relaxation is very minor in this phase. In the third phase (after about 9 years of service, i.e., PTs have been corroded), the cross-sectional area loss of all longitudinal PTs becomes the primary reason for the decrease of effective prestressing force, and its degradation rate increases first and then slows down as the corrosion turns worse.

5.2.4. Crack Resistance. The reliability index of crack resistance of the midspan section is shown in Figure 6. As indicated, the reliability indexes of crack resistance of all local areas gradually decrease with the increasing service time. After about 9 years of service, the decreasing rate of reliability index considering the effect of corrosion is much higher than that without considering. When the corrosion effect is not considered, only the long-term prestress loss leads to the decrease of effective prestressing force. At this time, the concrete at the bottom slab edge and three calculated fibers at the web still maintain highly precompressive stress, and the increase of compressive stress of concrete at the top slab is relatively low. Accordingly, the bottom slab, top slab, and web of the section all maintain good performance of crack resistance during the whole service period, and the whole structure will not crack. When the corrosion effect is considered, the reliability indexes of crack resistance of the bottom slab and web reduce to the target reliability of 1.5 (failure probability is about 0.067) in the 27th and 38th years, respectively. At this time, the concrete in the above areas should be repaired. For the top slab, the change of effective prestressing force has little effect on the compressive stress growth of concrete at the edge of the top slab, and the discreteness of the compressive stress of concrete at the top slab is also small in this case. As a result, the reliability index of crack resistance of top slab after 100 years of
service is only 8.11% lower than the initial value, and its value is 4.11 (Figure 6). As such, the crack resistance of concrete at bottom slab and web is more sensitive to the coupled effect of corrosion and long-term prestress loss than that of top slab.

For the concrete at the edge of the bottom slab, its initial compressive stress is greatly improved by the application of prestressing force, and the mean and lower bound of the confidence interval are up to 7.41 MPa and 5.19 MPa, respectively. In addition, the standard deviation of the initial compressive stress is also relatively small. As such, the bottom slab has good crack resistance performance at first, and its initial reliability index of crack resistance is 6.55. After that, the variation of crack resistance of the bottom slab can also be divided into three phases similar to the change of effective prestressing force. In the first stage, the rapid decrease of effective prestressing force (Figure 5(c)) leads to a significant reduction of compressive stress of concrete at the bottom slab. The corresponding reliability index of crack resistance also decreases rapidly, with a decrease of 7.86% in the first year. In the second phase, since the degradation rate of concrete compressive stress caused by the total effective prestressing force of the section is minor, the reliability index of crack resistance decreases slowly. The final reliability index is only 10.36% lower than the initial value. In the following service period (i.e., the third phase), the significant reduction of total effective prestressing force causes the rapid decrease of concrete compressive stress. This eventually leads to tensile stress or cracking of concrete at the edge of the bottom slab after the precompression stress is completely eliminated. The variations of crack resistance of the top slab and web during the service period are very similar to that of the bottom slab (Figure 6); thus, they are not described in detail here.

### 5.2.5. Effect of Vertical PTs on Crack Resistance

The variation of crack resistance of web and whole section with and without vertical PTs is shown in Figures 7 and 8. As indicated in Figure 8, the initial reliability index of web is only 4.83 without setting vertical PTs due to the lack of vertical compressive stress reserve of web concrete, which is much lower than that of setting vertical PTs. In this case, the concrete fibers at three calculation points are all subjected to principal tensile stress, in which the maximum value of principal tensile stress is 0.41 MPa. With the increase of prestress loss and corrosion degree, the reliability index of web decreases to the threshold value in about 25 years, which is 13 years (accounting for 34.2%) ahead of the corresponding value when setting vertical PTs. For sections near support or at 1/4 span, the web crack resistance is worse, and the cracking time of web is also greatly advanced due to the greater shear force and shear stress of these sections. Therefore, the vertical PTs should be reasonably arranged, and the prestress loss should be minimized to ensure that web concrete has sufficient compressive stress reserve.
As indicated in Figure 8, the time when the reliability index of the whole section decreases to the threshold value is about 14.8% earlier than that of setting vertical PTs. In addition, according to the comprehensive analysis of Figures 6 to 8, the section is more prone to web cracking before 54 years of service without setting vertical PTs. At this time, the reliability index of the whole section has already fallen below the reliability threshold value. This indicates that the overall crack resistance performance of the section under this condition is mainly determined by the crack resistance performance of web. When the web is provided with vertical PTs, the cracking position is more likely to be at the top slab in the first 13 years of service, and it will change to the bottom slab during subsequent service. Accordingly, based on the previous evaluation results of local crack resistance, the targeted protection or repair measures for concrete in different regions and times can be formulated by the bridge management department to improve the durability and safety of the bridge in service.

6. Conclusions and Recommendations

This paper uses the well-established step-by-step approach to estimate long-term prestress loss in combination with the expressions estimating the cross-sectional area loss of nonprestressing and prestressing steels caused by corrosion. In addition, an assessment approach of crack resistance reliability of box-girder is developed considering the coupled effect of corrosion and time-dependent long-term prestress loss. The proposed approach is applied to a three-span PC box-girder bridge. The following conclusions are drawn:

1. Considering the coupled effect of shrinkage and creep of concrete, prestressing steel relaxation, presence of nonprestressing steel, and corrosion, a prediction model of long-term prestress loss is established by using the well-established step-by-step method, and its validity is also verified by the existing test data.

2. The prediction results of long-term prestress loss have high uncertainty, and the upper bound of long-term prestress loss at the confidence level of 95% is approximately 50% higher than the result obtained by deterministic analysis. It is recommended to select the upper bound of confidence interval as the most unfavorable value of long-term prestress loss from the perspective of preventing early cracking.

3. The effect of corrosion on the long-term prestress loss is basically negligible, but it is closely related to the effective prestressing force and crack resistance of the section. In addition, the crack resistance of bottom slab and web is more sensitive to the coupled effect of corrosion and long-term prestress loss than that of top slab.

4. Setting vertical PTs can greatly improve the crack resistance of web and whole section. In this case study, the time when the reliability index of crack resistance of web without setting vertical PTs decreases to the reliability threshold value is at least 36.8% earlier than that of setting vertical PTs.

The limitation of this study is that the effect of corrosion on bond between strand and concrete is not considered. In future research, the effect of bond degradation due to strand corrosion on prestress loss needs to be further considered. Moreover, a series of monitoring tests of long-term prestress loss of corroded PC structure need to be carried out so as to obtain more measured data to improve the applicability of the proposed prediction method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (Grant no. 52108135), the Hunan Provincial Natural Science Foundation of China (Grants nos. 2022JJ40024, 2021JJ50153, and S2022JSSLH0137), and the Research Foundation of Education Bureau of Hunan Province (Grants nos. 21B0723, 18A401, 21C0671, and 21B0721).

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