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Review Article

Design Optimization of Reinforced Concrete Cantilever Retaining Walls: A State-of-the-Art Review

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The booming growth of computational abilities in the 21st century has led to its assimilation and benefit in all horizons of engineering. For civil engineers, these advancements have led to groundbreaking technologies such as BIM, automation, and optimization. Unfortunately, even in an era of dwindling resources and dire need for sustainability, optimization has failed to attract implementation in practice. Despite an exponential growth as an area of research interest, the optimization of engineering structures such as reinforced concrete (RC) is still a complex task that requires multidisciplinary knowledge, hindering its practicability. Although past review papers have delved into this topic, they have only been able to cover the breadth of information available by covering broader aspects of optimization of structures. This study on the other hand aims to cover this topic in depth to uncover problem specific trends and issues, by focusing only on optimization of RC cantilever retaining walls. Although there is an abundance of research studies on this topic, there is an absence of any critical review to tie them up, and concurrently with its broader scope, it suffers the same lack of applicability in the field. The in-depth review presents a summarization of all the online publications including research articles, conference papers, and theses to the best of authors' knowledge on the topic of RC cantilever retaining wall optimization. Geographical trends, regional developments, and prominent journals have been identified. The design codes, problem formulation, objectives, constraints, variables, and their optimization techniques are tabulated for ease of understanding. Unique areas of development investigated by the different researchers have been highlighted. Lastly, comprehensive recommendations for future works have been detailed with a focus on improving its applicability and assimilation into the construction industry.

1. Introduction

Concrete is the most common construction product in the world; in fact in terms of consumption it is second only to water [1]. This rise is directly related to the success of reinforced concrete (RC) which offers better durability, strength, resilience, and insulation in an affordable cost range compared to other construction materials [2]. However, it also carries certain disadvantages that are drastically exacerbated as the global concrete production reached to an all-time high of 10 billion m^3. The construction sector is responsible for 40% of global energy consumption and 30% of all greenhouse gases emissions [3, 4]. Cement has a massive role to play in these negative effects as 85% of the CO_2 emissions in construction are related to cement production [5]. This sector is also responsible for 15% of total industrial energy used [6]. It is suffice to say that to achieve sustainability moving forward is not possible unless the most used material in construction is somehow improved. Intensive research is being carried out on green materials and alternatives to make construction sector more environment-friendly; however, the current trends suggest that there has been a growing interest in computational design optimization [7]. The focus is to replace the conventional iterative design practice with the optimized designing as the conventional design is largely dependent on the experience of the designer and does not guarantee the most economical design.

Optimization is the selection of the best possible solution under a certain set criterion; it is to be performed for specific objective which is selected by the user. The process of optimization is defined by formulation of the problem in a mathematical form, defining the objective of problem and the variables, and then application of feasible ranges and constraints according to provisions of construction [8]. After the mathematical modeling, optimization algorithms are applied, which can be of deterministic nature requiring continuous solutions or heuristics that are of probabilistic nature. Although the first case of optimization of RC structures dates back to 1960's, it had to face a standstill due to the complex nature of variables involved in these real world problems [9]. It was not until the advent of high level processors and improvements in their computing power that optimization of objectives became much easier.

Consequently, since the last decade extensive work has been carried out on optimization in civil engineering. Milajić et al. [10] provide a brief review of optimization of RC structures and critique the difficulties in implementation. Rajput and Datta [11] summarize the optimization techniques utilized for material's blend and structure design. Evins [12] covers optimization but from the prospect of sustainable building systems design. The review of Dede et al. [13] provides a broad review of optimization within the different fields of civil engineering. Rahmanian et al. [14] provided a detailed review of optimization of RC beams, and Afzal et al. [15] provide a critical review regarding optimization of all RC structures. These literature reviews show that although significant work has been done, there is little progress in the practical application of optimization in design practice. Hence, it is imperative that a more in-depth critical review should be conducted so that this new subject area can reach its full potential. For this purpose, the sole focus of this paper is the optimization of RC cantilever retaining walls.

Although there are multiple types of Earth retaining structures, the cantilever retaining wall is the most commonly used type because of its economic potential in the height range of up to 10 m [16]. Due to this reason, this study only focuses on cantilever walls. Several researchers have worked on optimizing the design of RC cantilever retaining walls. One of the first such studies is by Pochtman et al. [17] who performed optimization of an anchorage cantilever retaining wall using random search algorithm. Dembicki and Chi [18] tried to optimize the shape of cantilever retaining wall using the coordinates as variables using Monte-Carlo simulation of developed Pareto optimal equations. However, the novel study was of Sarıbaş and Erbatur [19] which solidified the direction taken by researchers for problem formulation of RC cantilever retaining walls and is the most common reference study used by subsequent researchers. Since then multiple theses (Medhekar [20], Purohit [21], Naeem [22], Rahbari [23], and Schmied and Karlsson [24]), conference papers (Bhatti [25], Ahmadi-Nedushan and Varaee [26], Villalba et al. [27], Pei and Xia [28], Papazafeiropoulos et al. [29], Uray and Tan [30], Al Sebai et al. [31], Srivastavaa et al. [32], and Yücel et al. [33]), and journal articles (Ceranic et al. [34], Chau and Albermani [35], Babu and Basha [36], Yepes et al. [37], Khajehzadeh et al. [38], Ghazavi and Bonab [39], Kaveh and Abadi [40], Khajehzadeh et al. [41], Camp and Akin [42], Khajehzadeh and Eslami [43], Sable and Patil [44], Sable and Patil [45], Kaveh and Behnam [46], Kaveh et al. [47], Kaveh and Khayatazad [48], Khajehzadeh et al. [49], Sheikholeslami et al. [50], Talatahari and Sheikholeslami [51], Gandomi et al. [52], Kaveh and Mahdavi [53], Singla and Gupta [54], Bekdaş et al. [55], Kaveh and Farhoudi [56], Sheikholeslami et al. [57], Temür and Bekdas [58], Aydogdu [59], Kaveh and Laien [60], Gandomi et al. [61], Gandomi et al. [62], Kumar and Suribabu [63], Rahbari et al. [64], Ukritchon et al. [65], Kayhan and Demir [66], Mohammad and Ahmed [67], Kalateh-Ahani and Sarani [68], Moayyeri et al. [69], Öztürk and Türkeli [70], Uray et al. [71], Dagdeviren and Kaymak [72], Kaveh et al. [73], Kaveh et al. [74], Kayabekir et al. [75], Konstandakopoulou et al. [76], Mergos and Mantoglou [77], Kalemci et al. [78], Kayabekir et al. [79], Hoang and Cong [80], Millán-Paramo et al. [81], Kashani et al. [82], Uray et al. [83], Ravichandran et al. [84], Yücel et al. [85], Kaveh et al. [86], Sharma et al. [87], Mevada et al. [88], Uray and Çarbaş [89], Tousi et al. [90], Eroğlu et al. [91], Uray et al. [92], Linh et al. [93], Dodigović et al. [94], Tutuş et al. [95], Uray et al. [96], Yücel et al. [97], Tutuş et al. [98], Temür [99], Shakeel et al. [100], Khajehzadeh et al. [101], Uray et al. [102], and Khajehzadeh et al. [103]) have been written on this topic. Despite ample work conducted that demonstrates the potential of optimization in this field, its acceptance in practical works is still little to none [100, 104, 105]. The subject also suffers from a lack of an extensive review that effectively briefs the plethora of works conducted and guides the future researchers towards the crux of difficulties that hinder its acceptance in construction industry.

Consequently, the aim of this study is to provide an exhaustive literature review on the optimization of RC cantilever retaining walls. This paper summarizes the works conducted into a concise yet perspicuous manner so that meaningful information can be extracted for future research. The thought process for collection of data and the regional trends is detailed in Section 2. Section 3 details the problem formulation, i.e., the objective functions, the variables, the constraints, and the different algorithms used by each study. All the data is tabulated to convey the bigger picture and reveal the diversity of works conducted till date. Section 4 summarizes the scope of all the research works published with a highlight on their novelties. Lastly, Section 5 presents the gaps in research conducted and recommendations for future works.

2. Methodology

There are two purposes of this detailed review. First is to summarize all the research that has been conducted on this topic in a manner of ease of comprehension. The second objective is to identify key deficiencies and potential research scopes for the advancement of research and acceptance in industry in the field of RC cantilever retaining wall optimization. To achieve a comprehensive database, a structured methodology was adopted. All research studies, to the best of authors' knowledge, that were accessible online have been considered. The databases used to gather the literature include Google Scholar, Springer, Taylor and Francis, ASCE, and Scopus. A collection of 87 research articles, conference papers, and theses on the specific topic of cantilever retaining walls and their optimization is obtained. A vast variety of regions, journals, objectives, optimization techniques, and problem formulation methodology is utilized in all the studies. It is necessary to split them in a manner where the data can be converted into meaningful information. This is accomplished by a detailed tabulation, to convey progress and complexity of works conducted. Firstly, the regions, journals, and building codes of active research are presented. Secondly, a breakdown of problem formulation components, i.e., objectives, variables, constraints, optimization techniques, and software applications used, are presented. Lastly, the gaps in literature and recommendations for future work are elaborated to promote research in the area and increase its practicability.

2.1. Regional Trends. A total of 86 research articles, conference papers, and theses matching the exact requirements mentioned in methodology are gathered. Roughly 84% of the database or a total of 73 papers are research articles, 65 of which have been published after 2010 and onwards. This trend is graphically visible in Figure 1, which shows a drastic rise in publications in the past 5 years (2016–2021). A record high of 18 papers published in 2021 shows the growing interest as an area of research.

Majority of research papers (20) have been published in Springer, followed by ASCE (7), Elsevier (5), MDPI (5), and Techno-Press (4). In terms of countries in which works have been published, USA (10), UK (9), Germany (9), Turkey (9), Switzerland (8), Iran (6), and India (7) are identified as major contributors. The share of contribution of the above countries and the rest of the contributors can be visualized from Figure 2.

However, in terms of authors/institutional origin, there is a clear hegemony of Turkish institutes with 24 papers from their country, followed by 22 from Iran, 10 from India, and 10 from the USA. This certain trend is due to certain authors and their massive contribution in the field of RC cantilever retaining wall optimization throughout the decade. Another factor could be the earthquake activity in these countries. Iran, Turkey, USA, and India all lie on active seismic faults. This can lead to increase in construction of RC cantilever retaining walls in such countries and that they are designed for earthquake forces. These forces can lead to heavier designs which in turn could have more potential for objective based optimization, hence, the greater research interest in optimization of retaining walls in these specific countries. The share of contribution of authors' countries of origin is shown in Figure 3.

2.2. Journals of Interests. Another area investigated is the top journals in which work has been published on the topic of cantilever retaining wall optimization. Multiple civil engineering journals exist whose aims and scopes are specifically related to optimization. The "Journal of Structural and Multidisciplinary Optimization," "Engineering Optimization," "International Journal of Optimization in Civil Engineering," and "Structural Engineering and Mechanics" are a few such journals. As optimization covers the aim of sustainability as well, journals relating to that topic are also under the umbrella of published works. Apart from that, journals with multidisciplinary subject considerations such as computer science and civil engineering, mathematics and civil engineering, and artificial intelligence and civil engineering are also key areas for publication. Table 1 summarizes the top 10 journals in which the research work under consideration has been published.

3. Problem Formulation

Optimization must be defined by formulation in a mathematical form; Arora [8] defines it as an objective function $f(x)$ calculated using the design variables x_n as shown in

$$min_{\overrightarrow{x}} \text{ or } min_{\overrightarrow{x}}:\ f(x) = f(x_1, x_2, x_3 \ldots x_n). \tag{1}$$

This function is subject to various conditions as shown in equation (2). Equality or inequality constraints are given by $h(x)$ and $g(x)$, respectively, where p and m are number of constraints to be applied, x_iU are upper bounds, and x_iL are lower bounds on ith variable (x).

$$\% \text{ subject to} \begin{cases} \sum_{j=1}^{p} h_j(x) = 0 \\ \sum_{k=1}^{m} g_k(x) \le 0 \\ x_iL \le x_i \le x_iU, i = 1, \ldots, n. \end{cases} \tag{2}$$

Their ranges are defined by requirements pertaining to architectural restraints. Some bounds are given by design codes, such as the minimum and maximum reinforcement ratios, and some bounds are derived by experience, like for concrete sections as described by Sarıbaş and Erbatur [19] and Bowles [106]. The former study especially has been crucial to problem formulation in optimization of RC cantilever retaining walls and has also often been the subject of comparative analysis for later studies. After the development of problem formulation, algorithms are applied.

3.1. Building Codes. RC retaining walls are designed to mostly resist lateral loads exerted by the retained Earth; however, considerations for seismic loads can also be made. The decision of different loads applied is based on multiple factors such as site conditions, function of wall, ground water conditions, service life, and serviceability limits. These limits and their respective safety factors for uncertainty in loads are defined by building codes. These codes ensure adequate capacity has been obtained against any type of failure in each portion of the retaining wall, i.e., stem, heel, and toe, by treating them as individual members. Provision of resultant, constrained within the middle third portion of the base and design checks for overturning moment, sliding moment, and bearing capacity are applied. The codes also

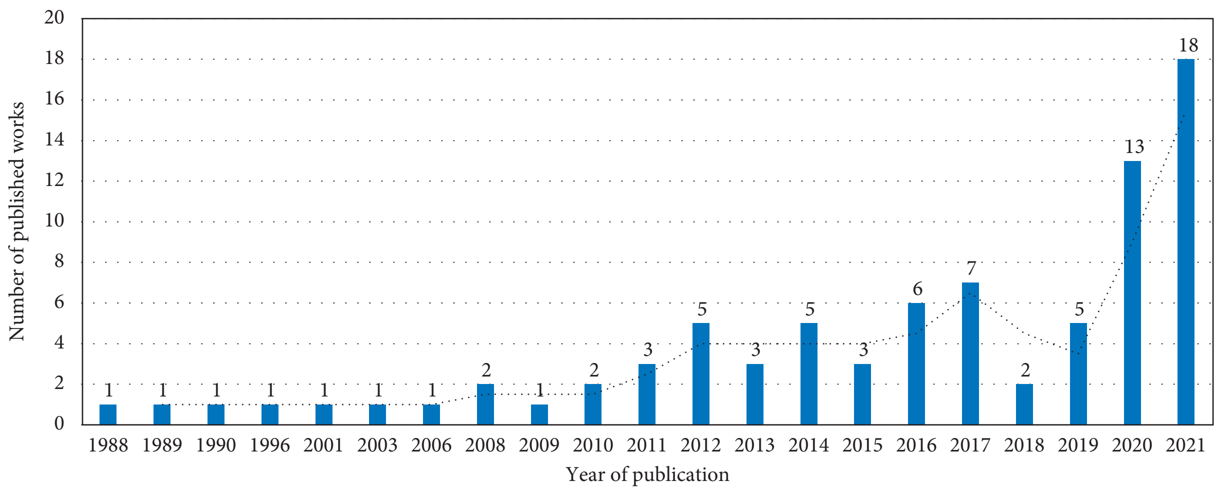

Figure 1: Trend of research publication on RC cantilever retaining wall optimization by year.

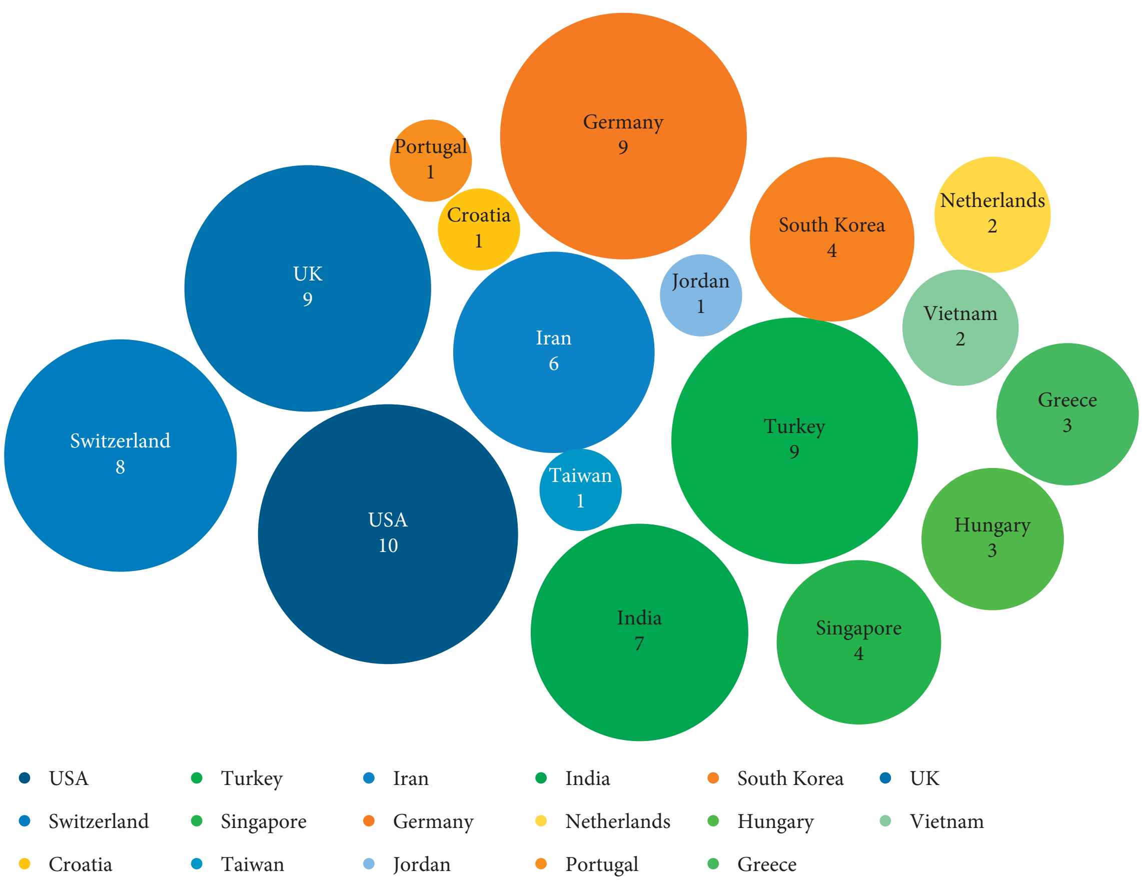

Figure 2: Countries of origin of research publishers.

define the limiting geometric and reinforcement values for construction of a safe retaining wall. The ACI code dominates with 53 studies using it, followed by IS-456 with 10 studies and Euro code with 9 studies. All the building codes and a breakdown of ACI codes used are shown in Figure 4. Apart from building codes, highway codes like AASHTO and TDOK and seismic codes like EC-7, TBEC-18, and DBYBHY have also been used.

3.2. Objective Function. Traditionally, minimum weight of structure had been the chosen objective as this function bodes well for steel structures or plain concrete structures. However, RC is a composite material and the weight contribution is not an effective measure to develop a fitness function to achieve the most economical section. For reinforced concrete, the cost based function provides the best results [9]. The function takes into account the total

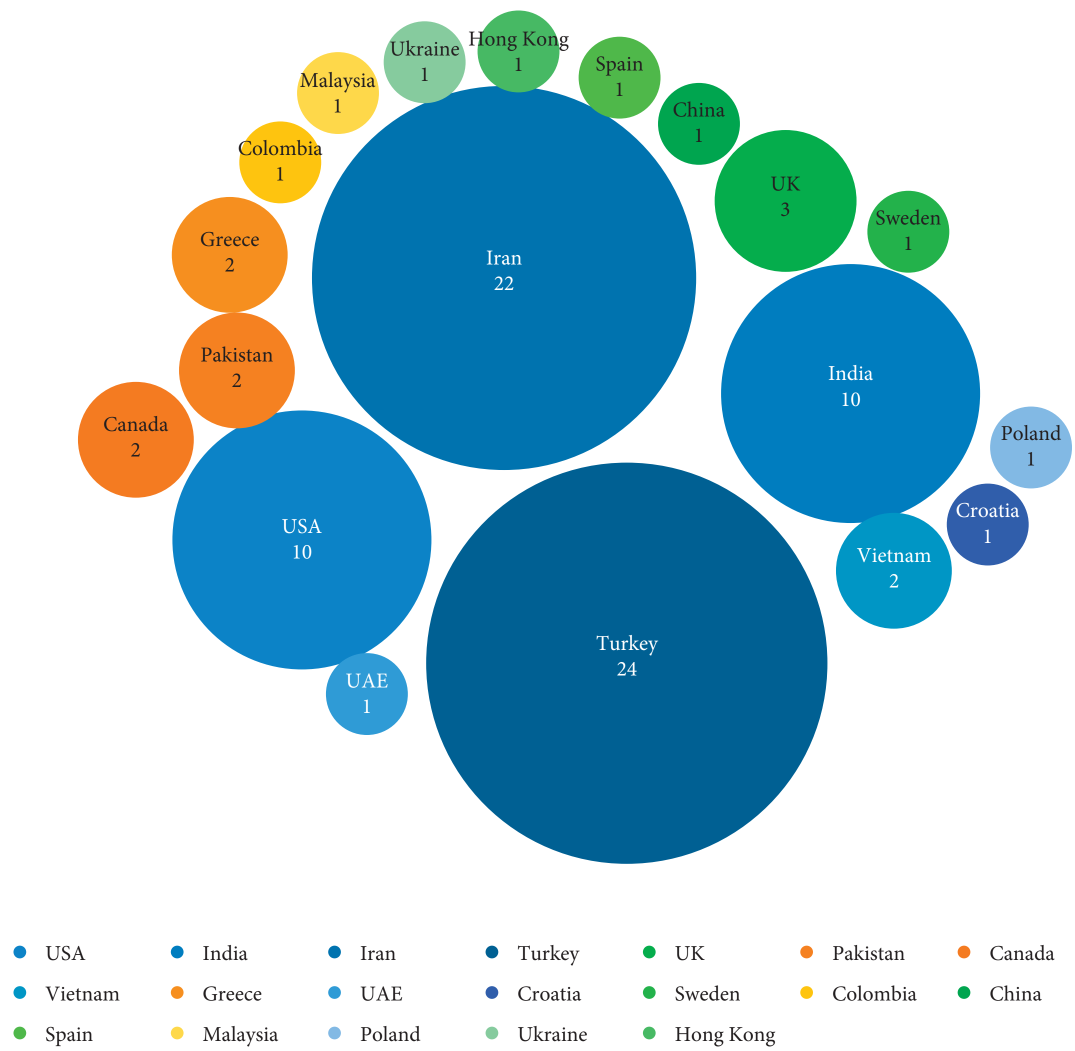

FIGURE 3: Countries of origin of authors of published research.

TABLE 1: Journals with most works published on optimization of cantilever retaining walls.

Journal	Publisher	Journal country	Studies
Structural Engineering and Mechanics	Techno-Press	South Korea	Kaveh et al. [47]; Temür and Bekdas [58]; Khajehzadeh et al. [103]
KSCE Journal of Civil Engineering	Korean Society of Civil Engineers	Germany	Talatahari and Sheikholeslami [51]; Sheikholeslami et al. [57]; Ukritchon et al. [65]
Structural and Multidisciplinary Optimization	Springer	Germany	Gandomi et al. [61]; Dagdeviren and Kaymak [72]; Mergos and Mantoglou [77];
Periodica Polytechnica Civil Engineering	Budapest University	Hungary	Kalateh-Ahani and Sarani [68]; Kaveh et al. [73]; Khajehzadeh et al. [101]
Challenge Journal of Structural Mechanics	TULPAR Academic Publishing	Turkey	Uray et al. [71]; Kayabekir et al. [79]; Eroğlu et al. [91]
International Journal of Engineering Research and Technology	Irphouse	India	Sable and Patil [44]; Sable and Patil [45]; Millán-Paramo et al. [81]
Engineering Structures	Elsevier	UK	Yepes et al. [37]; Gandomi et al. [52]
Journal of Structural Engineering	ASCE	USA	Chau and Albermani [35]; Camp and Akin [42]
Mathematics	MDPI	Switzerland	Moayyeri et al. [69]; Uray et al. [102]
Sustainability	MDPI	Switzerland	Kayabekir et al. [75]; Yücel et al. [85]

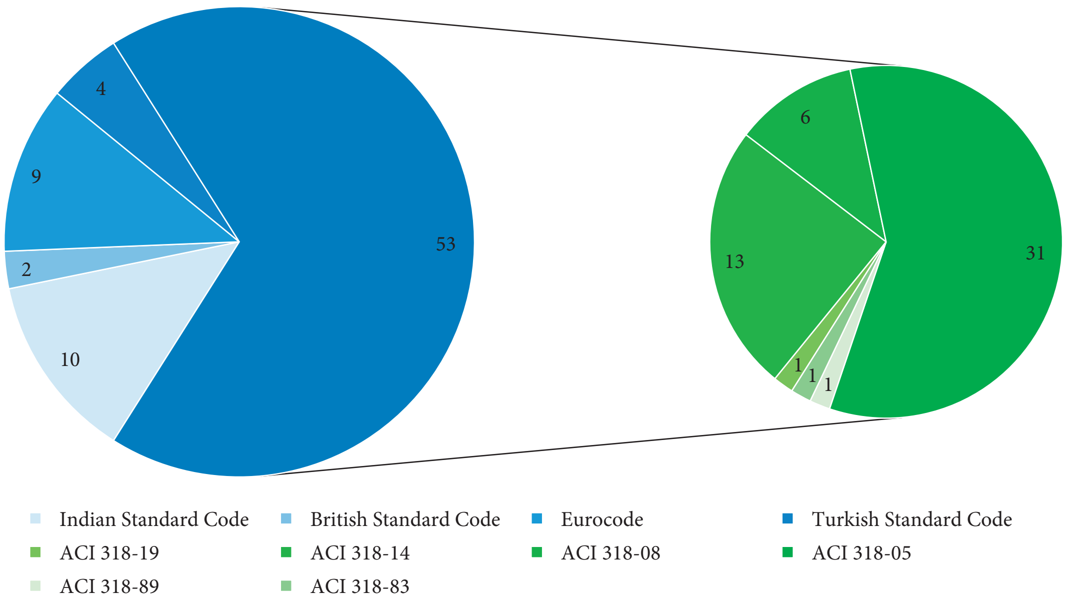

Figure 4: Distribution of building codes used in optimization of retaining walls.

volume of concrete (V_c) while for steel it accounts for the total weight (W_{st}). These values can then be multiplied by unit costs (C_c for cost of concrete and C_s for cost of steel) to obtain total costs, by unit emission rates to obtain total carbon emission or added (by first converting volume of concrete to weight) to obtain the total weight of structure.

The paper of Afzal et al. [15] shows an evolution of objective functions which boils down to material efficiency (minimum weight) as shown in equation (3), material and cost efficiency (minimum cost) as shown in equation (4), environmental performance (minimum carbon emission) as shown in equation (5), safe design (maximum factor of safety (FOS) or minimum displacement), and sustainable design (multiobjective design). The study illustrates that most work has been done on RC frame structures and signifies a dire need for research of multiobjective optimal design of structures. The analysis of literature only on retaining walls also shows a similar pattern. The most used categories are cost optimization, weight optimization, carbon emission optimization, and factor of safety optimization. However, cost optimality is the premier function with 74 studies using it for the development of their optimization problem. A detailed breakdown of objective function taken by research studies is illustrated in Figure 5.

$$f\,(\text{weight}) = W_{st} + 100\,V_c\gamma_c, \tag{3}$$

$$f\,(\text{cost}) = C_s W_{st} + C_c V_c, \tag{4}$$

$$f\,(\text{emission}) = C_c\,(co_2)V_c + C_s\,(co_2)W_{st}. \tag{5}$$

The cost of concrete can be expanded to include the cost of formwork, transportation, labor, vibration, Earth removal, and cost of backfill as done by Naeem [22], Villalba et al. [27], Al Sebai et al. [31], Ceranic et al. [34], Yepes et al. [37], Camp and Akin [42], Sable and Patil [45], Kaveh et al. [47], Talatahari and Sheikholeslami [51], Kaveh and Farhoudi [56], Temür and Bekdas [58], Mohammad and Ahmed [67], Moayyeri et al. [69], Konstandakopoulou et al. [76], Mergos and Mantoglou [77], Tousi et al. [90], and Dodigović et al. [94]. The cost of varying concrete and steel strength can also be used for optimization as done by Villalba et al. [27], Yepes et al. [37], Kaveh et al. [47], Kalateh-Ahani and Sarani [68], Konstandakopoulou et al. [76], Tousi et al. [90], and Shakeel et al. [100]. The research of Mohammad and Ahmed [67] has used cost ratios to simplify the results and lessen the effect of local currency on optimal results. The second most used objective is weight minimization. It is simply an amalgamation of weight of concrete sections and weight of reinforcement. In case of key the weight of key is also included in the formulation as done by Camp and Akin [42], Sable and Patil [45], Gandomi et al. [52], Gandomi et al. [61], Kalemci et al. [78], Millán-Paramo et al. [81], Kashani et al. [82], Sharma et al. [87], and Uray et al. [102]. Recently the carbon emission minimization objective has also been an area of interest; optimization with such an objective has been paired under the umbrella of sustainable design. Schmied and Karlsson [24], Villalba et al. [27], Khajehzadeh et al. [49], Öztürk and Türkeli [70], and Kayabekir et al. [75] have tried to optimize the carbon emissions of retaining walls. Their methodologies involve taking values of unit carbon emissions for concrete and steel from reputable databases and multiplying them with total volume or weight of concrete to obtain total carbon emissions. This value can be then set as objective to be optimized.

The multiobjective optimization is still a developing concept and has been applied by Purohit [21], Kaveh et al. [47], Khajehzadeh et al. [49], Rahbari [23], Rahbari et al. [64], Kalateh-Ahani and Sarani [68], Kayabekir et al. [75], Ravichandran et al. [84], Uray et al. [92], Dodigović et al. [94], and Tutuş et al. [95]. The study of Kayabekir et al. [75] has also attempted to vary the cost of concrete and steel to

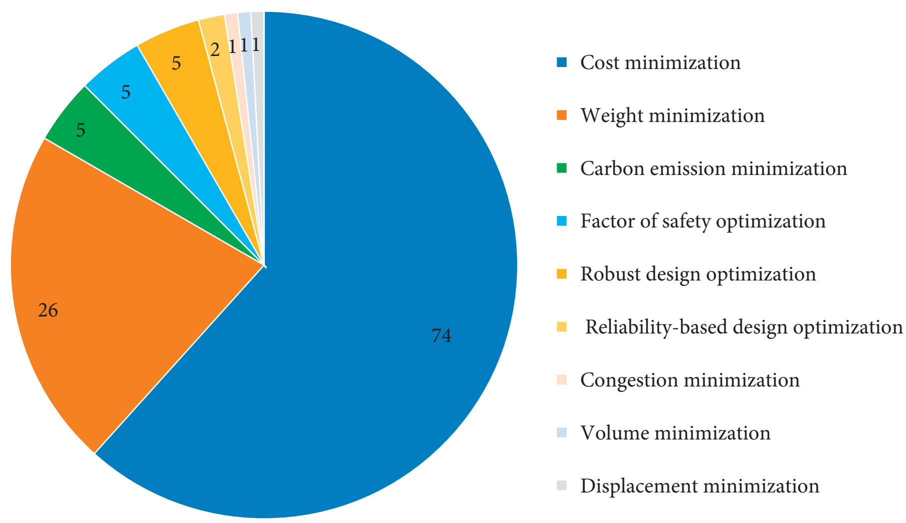

FIGURE 5: Distribution of objective functions used in optimization of retaining walls.

study their effects on optimization with respect to economical and sustainable design. Only 11 research studies have investigated the multiobjective optimization aspect; a breakdown is provided in Figure 6.

The factor of safety objective is often analyzed as a multiobjective problem. The problem can be modeled to maximize safety factors to ensure a safe design in high seismicity prone regions (Purohit [21] and Dodigović et al. [94]) or to minimize to indirectly create an economical design (Uray and Tan [30] and Uray et al. [92]). However, one of the first studies to investigate this objective was Dembicki and Chi [18]. Their study aimed to minimize the weight of wall while maximizing its stability by deriving nonlinear Pareto solutions using every point that describes the geometry of the retaining wall (coordinates of the wall).

3.3. Design Variables. In RC cantilever retaining wall optimization, variables are categorized in three groups: concrete geometric sections, steel reinforcement bars/areas and their spacing, and material strengths. All these variables must be provided with their upper and lower bounds; otherwise an infeasible section may be obtained. A penalty function must be applied while using continuous variables without bounds to ensure the algorithm rejects the infeasible answer and moves on to a better solution as done by Sarıbaş and Erbatur [19], Medhekar [20], Srivastava et al. [32], Camp and Akin [42], Khajehzadeh and Eslami [43], Sheikholeslami et al. [50], Kaveh and Laien [60], Kumar and Suribabu [63], Moayyeri et al. [69], Kayabekir et al. [75], and Temür [99]. The placement and bar diameter of steel plays a vital role and has been neglected by many studies making it an area of interest. Consequently, studies of Schmied and Karlsson [24], Villalba et al. [27], Al Sebai et al. [31], Yepes et al. [37], Camp and Akin [42], Kaveh et al. [47], Gandomi et al. [61], Gandomi et al. [62], Bekdaş et al. [55], Kayhan and Demir [66], Moayyeri et al. [69], Öztürk and Türkeli [70], Kalemci et al. [78], Tousi et al. [90], Tutuş et al. [95], Temür [99], and Uray et al. [102] have tried to optimize steel areas while keeping in mind the practicality of available rebars and their spacing. This is achieved by generating a pool of predefined steel bars and indexing them in tabular form while programming. A basic RC cantilever wall with the generic variables defined is shown in Figure 7.

3.4. Constraints. Constraints are ranges for parameters which must be specified to obtain values that conform to structural requirements. They are specified by regional building codes and ensure that the structure remains within the limit states to maximize safety and comfort. Constraints are categorized into three groups: geotechnical requirements/external stability, structural requirements/internal capacities, and geometric feasibility. They are usually applied as inequality equations to ensure the optimization algorithms do not violate the preset conditions. The following section broadly explains the type of constraints usually used for optimization.

3.4.1. External Stability/Failure. The first constraint is to avoid any type of failure in retaining walls and it is ensured by applying sufficient FOS on the obtained values of resistances. Overturning failure is caused when overturning moments (M_O) due to lateral loads are larger than stabilizing moments (M_R) due to vertical loads applied on the wall. These resistive forces include gravity load of soil and overburden, self-weight of wall, and toe of the wall. The factor of safety for overturning (FS_o) about the toe is defined as in the following equation:

$$FS_o = \frac{\Sigma M_R}{\Sigma M_O}. \tag{6}$$

Sliding failure is caused by pressure applied by backfill soil and surcharge. The horizontal components of pressure forces are taken as the total applied horizontal sliding force (F_R). They tend to push the wall away from soil and are resisted by driving forces (F_D) formed by a combination of weight of soil, self-weight, and soil on the passive side. In

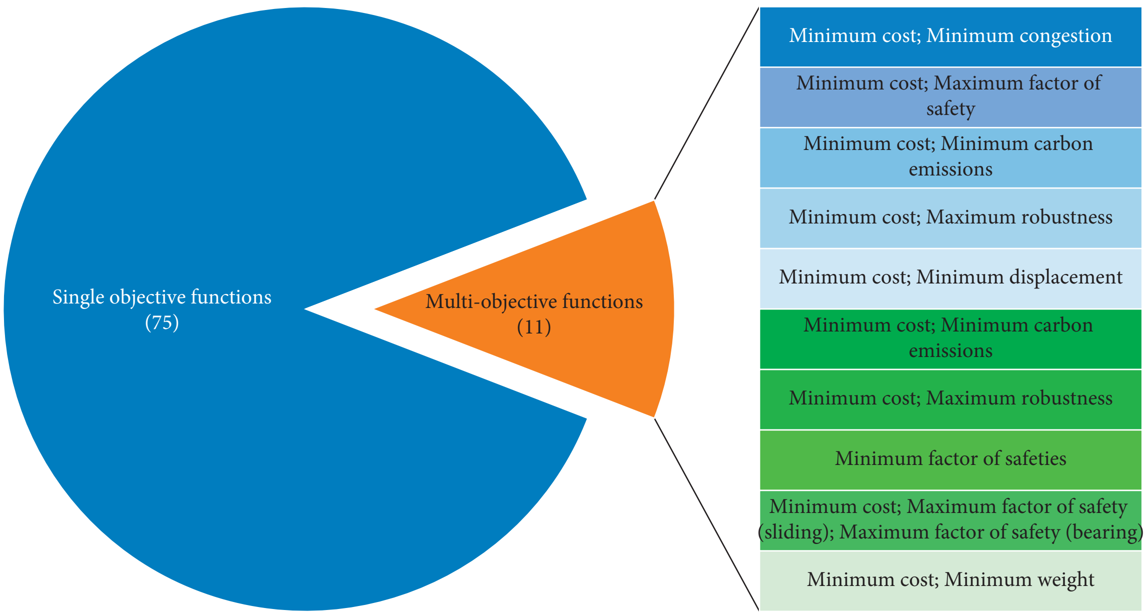

Figure 6: Distribution of single and multiobjective functions used in optimization of retaining walls.

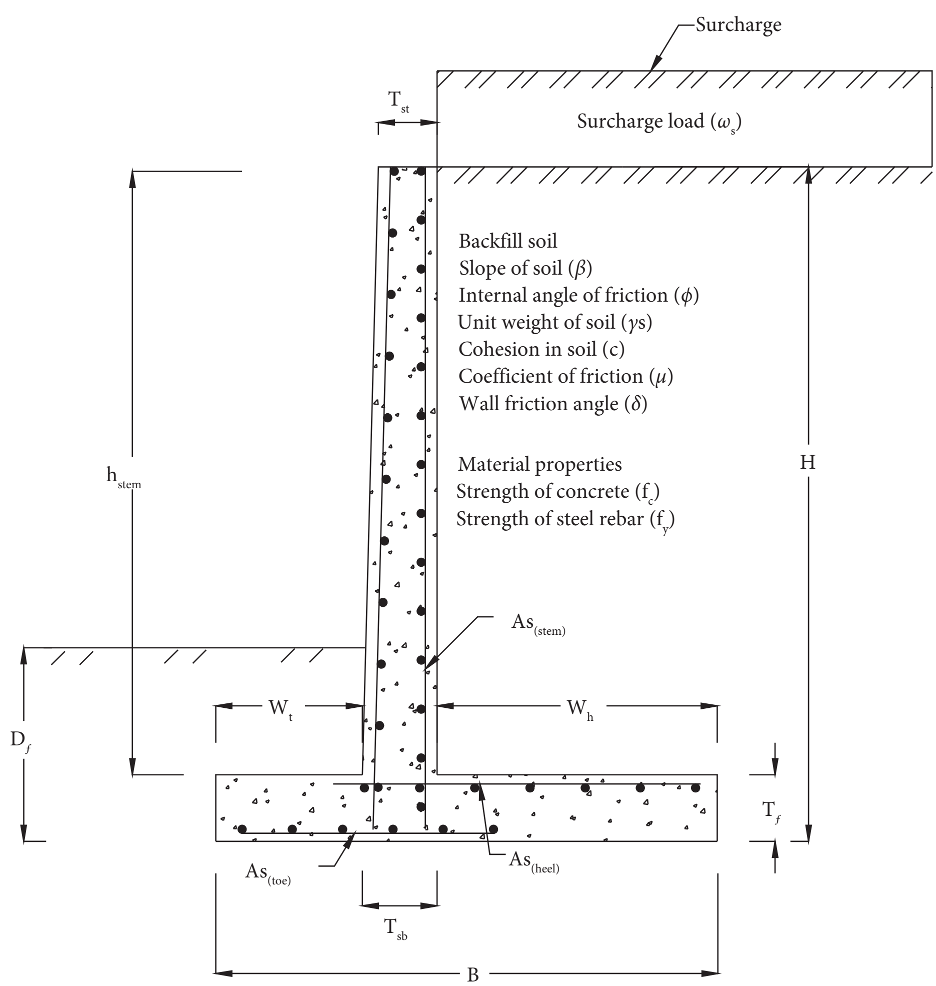

Figure 7: RC cantilever retaining wall with prominent variables defined.

case of drastic forces, a key can be provided for another block of passive soil resistance to horizontal forces. Factor of safety for sliding (FS_s) is defined as in

$$FS_s = \frac{\Sigma F_R}{\Sigma F_D}. \quad (7)$$

Bearing failure is a failure in which the maximum load on soil exerted by the base of the wall is greater than the capacity of the soil. There are two bearing pressures: q_{max} on toe due to more overturning moment applied on it and q_{min} on heel having little or negative pressure caused by uplift of the heel. It is determined by equation (8) and must satisfy the conditions in equation (9) as well:

$$q_{max\,/\,min} = \frac{V_{\text{total}}}{B}\left(1 \pm \frac{6e}{B}\right), \quad (8)$$

$$\text{subject to}\begin{cases} q_u \geq q_{max} \\ q_{min} \geq 0, \end{cases} \quad (9)$$

where V_{total} is the total vertical forces, B is the width of the base, e is the eccentricity of resultant forces, and q_u is the ultimate bearing capacity of soil. q_u is derived through soil investigations and different capacity calculation theories. The final check is to ensure that no uplift occurs at the heel of the wall for which there should be no negative bearing pressure at the heel, meaning its value must be greater than zero.

3.4.2. Internal Capacity of Sections. The capacity of sections in shear and flexure must be checked for all three arms of the cantilever retaining wall. The moments at each section are calculated and the shear checks are applied at d (effective depth of section) distance from the face of the stem and at the face for base slabs. The basic concept is that capacity (ϕMn and ϕVn) of the section must be more than the demand (Mu and Vu) at that section. For stem, heel, and toe, equations (10) and (11) must be true for the capacity check to be satisfied. In case a key is provided, its capacity check must also be done.

$$\phi Mn \geq Mu, \quad (10)$$

$$\phi Vn \geq Vu. \quad (11)$$

3.4.3. Geometric Requirements and Deflection Check. These bounds ensure that the geometric values of walls are within a feasible range and yield practical results. A constraint in equation (12) to ensure total base width (B) is equal to the sum of the heel (W_h), toe (W_t), and stem base (T_{sb}) widths and a constraint for resultant force (R) in equation (13) to be acting within the middle third portion have been applied. Such geometric feasibility checks have been incorporated by multiple studies to improve the practicability and constructability concerns of optimization [18, 25, 25, 28, 39, 42, 52, 58, 61, 62, 67, 70–72, 78, 80, 82, 87, 89, 92–94, 98–101]. Apart from these controlling constraints, settlement can also be calculated by Newmark's sliding block method to control design as performed by Konstandakopoulou et al. [76]. Settlement can also be calculated for a parametric investigation concerning the width of the retaining wall as done by Gandomi et al. [61]. Deflection check is another factor taken as a part of serviceability limit state of retaining wall. Extensive work was done by Yepes et al. [37] on deflection limits and they concluded that a value of 1/150 of stem height is sufficient for practical optimized design of retaining walls. A deflection check has been included by the studies of Villalba et al. [27], Al Sebai et al. [31], Yepes et al. [37], Khajehzadeh et al. [41], Khajehzadeh and Eslam [43], Kaveh et al. [47], Khajehzadeh et al. [49], and Ravichandran et al. [84].

$$B \geq W_h + T_{sb}, \quad (12)$$

$$R = \frac{\Sigma M_R - \Sigma M_O}{V_{Total}}. \quad (13)$$

3.4.4. Eccentricity As discussed earlier, due to large overturning moments, uplift can be generated at heel. To mitigate this, pressure at heel must not be negative and an additional check for eccentricity (e can be applied as done by various studies [17, 20, 21, 39, 41, 43, 47, 49, 54, 63, 68, 71, 80, 83, 88, 93, 101]. Generally it is defined as in equation (14); it should not be greater than one-sixth of the total base width (B.

$$e = \frac{B}{2} - \frac{(\Sigma M_R - \Sigma M_O)}{V_{Total}}. \quad (14)$$

3.4.5. Maximum and Minimum Reinforcement. The area of steel utilized must be bounded as well to ensure ductile failure of wall. The bounds for steel are provided by the building codes of the region. As reinforcement is provided in each arm of cantilever retaining wall, their bounds are also calculated separately. It can be either applied as reinforcement ratios or area of steel calculated. Equation (15) must be satisfied for stem, heel, and toe to attain a feasible design.

$$As_{min} \leq As \leq As_{max}. \quad (15)$$

According to ACI code limits of reinforcement are defined as in equations (16) and (17), where ρ is reinforcement ratio, b is unit length of wall, and d is effective depth:

$$A_{min} = \rho_{min} bd, \quad (16)$$

$$A_{max} = 0.75\rho_{max} b\, d. \quad (17)$$

3.4.6. Development and Hook Lengths. All sections of RC cantilever retaining wall are to be provided with development length (l_{db}) or hook lengths (l_{dh}) to develop full strength against applied forces. This can be applied as an additional constraint for each section in the mathematical model [42, 59, 61, 62, 69, 70, 78, 82, 90, 95, 98, 100, 102]. Applied in two phases, first development length according to the design code being utilized is checked and provided against available space; in case of insufficient space a hook is to be provided that satisfies all minimum hook development criteria. The heel is developed as top bar and toe as bottom bar. The hook can be provided with further reduction if sufficient cover has been provided in design. Equations (18)–(21) must be satisfied for a feasible design.

$$l_{db,stem} \leq T_f - \mathrm{Cov}_{st}, \tag{18}$$

$$l_{dh,stem} \leq T_f - \mathrm{Cov}_{st}, \tag{19}$$

$$l_{db,heel} \leq W_t + T_{sb} - \mathrm{Cov}_f, \tag{20}$$

$$l_{db,toe} \leq W_h + T_{sb} - \mathrm{Cov}_f. \tag{21}$$

T_f is footing thickness, Cov_f is footing cover, Cov_{st} is stem cover, T_{sb} is stem thickness at bottom, W_t is width of toe, and W_h is width of heel.

3.5. Optimization Techniques. A plethora of mathematical techniques exists to solve linear and nonlinear engineering problems. The advancements of computers has also enabled that thousands of such calculations can be done in a matter of seconds. But the complexities of real world problems are a multifaceted problem. Although all safety related issues can be modeled, when it comes to inducing practicability, the preexisting techniques do not bode well. Real world civil engineering structures have discrete, nonlinear, and nonconvex solution spaces. It means that gradient based nonlinear programming techniques do not always yield a global solution and often get stuck in local minima. In these conditions stochastic methods are the most feasible as they do not require continuous bounds or gradients and find solutions based on probabilistic methods with enough iterations that solution gets out of a local minima solution to find the global optimum solution. A summarization of the entire problem formulation structure of each study is presented in Table 2. Based on the element of randomness involved, two general approaches can be derived which are defined as follows.

3.5.1. Deterministic Approaches. These techniques utilize the problem function and its gradient information to search a continuous solution space for optimized solution. This process is hence possible with continuous variables and they also require an initial starting point from which it can move step by step until optimality condition is satisfied. Some popular techniques to search directions are the Broyden–Fletcher–Goldfarb–Shanno (BFGS) and Quasi-Newton method with line search. Gradient based techniques are fast and efficient as they rely on derivative information; they can have superlinear rate of convergence for second-order methods. The benefit of high speed is, however, offset by uncertainty in achieving global solution as the gradient methods can at best guarantee local optimality. Therefore, deterministic approaches are not the best for the optimal design of RC structures. Only a few studies [17, 19, 20, 25, 31, 36, 44, 65] have applied nonlinear programming (NLP) techniques as the vastly superior metaheuristic techniques have made their use redundant. However, studies are being done to combine metaheuristic techniques with NLP techniques to improve their search capabilities and computational speeds [31, 49, 93].

3.5.2. Metaheuristic Approaches. Heuristic means "to discover by trial and error" while meta means "beyond." Metaheuristics are advanced heuristic techniques that utilize best solution to get directions to search in right directions. Both these methods use probability factors to determine the next solution, making them stochastic in nature. Metaheuristic methods not only provide a global solution but also do not utilize a lot of computational time. They are best suited for nonlinear, complex problems with multiple variables. They also have the advantage of not requiring gradients, explicit constraints, and working in a discrete space. These methods utilize diversification and intensification to determine which solution is to be examined next and how it will be produced. Their probabilistic approach does not let them get stuck in a local minima solution and leads them to global optimal solution.

4. Research Scope and Novelties

This section details the scope of works conducted and tabulated above. The uniqueness of these research studies is presented along with their findings. The review expands upon the research conducted to shine light on the trend of works and identify gaps left in this field of study.

4.1. Classical Optimization Algorithms and Their Advancements. Two major types of algorithms exist to apply heuristic approaches, namely, population based or evolutionary and swarm based techniques. Evolutionary techniques treat a solution as a chromosome and through the concept of survival of the fittest, update the entire population of solution in consecutive iterations. On the other hand swarm based optimization treats the solution as a particle and updates its position in solution space. On the basis of behavioral patterns algorithms can be divided into four types, namely: biology inspired, art inspired, science inspired, and social inspired. Some of the major algorithms utilized for RC cantilever retaining wall are briefly discussed here.

4.1.1. Genetic Algorithm (GA). One of the more popular algorithms is an evolutionary based stochastic technique. Genetic algorithms developed by Holland [107] involve biological concepts of evolution and survival of the fittest. They are heavily influenced by the initial starting values and parameters. Two main operators, i.e., crossover and mutation, are applied on a solution which is converted into a binary code and termed as a chromosome. A group of chromosomes forms a population of solution to ensure diversification. The crossover ensures intermixing of

Table 2: Summarization of the formulation of RC cantilever retaining wall optimization problem of all available literature.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[17]	Pochtman et al., 1988	Minimum cost	5 variables (T_{st}, T_f, B, h_{anchor}, F_{anchor})	Constraints (internal stability and eccentricity)	Random search algorithm
[18]	Dembicki and Chi, 1989	Minimum weight; maximum factor of safety	8 variables (coordinates of wall)	17 constraints (external stability and geometric requirements)	Monte-Carlo simulation
[19]	Sarıbaş and Erbatur, 1996	Minimum cost; minimum weight	7 variables (B, W_t, T_{st}, T_f, As_{stem}, As_{heel}, As_{toe})	10 constraints (internal and external stability)	Nonlinear programming (NLP)
[20]	Medhekar, 1990	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	11 constraints (internal and external stability, slip circle, and eccentricity)	Interior penalty function
[21]	Purohit, 2014	Minimum cost; maximum factor of safety	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	Constraints (internal stability, external stability, and eccentricity)	NSGA-II
[22]	Naeem, 2016	Minimum cost	9 variables (W_h, W_t, T_{sb}, T_f, W_{key}, T_{key}, Cov_{st}, Cov_f)	9 constraints (internal and external stability)	GA
[23]	Rahbari, 2017	Minimum cost; maximum robustness	4 variables (W_h, W_t, T_{st}, T_f)	Constraints (internal and external stability)	NSGA-II
[24]	Schmied and Karlsson, 2021	Minimum cost; minimum carbon emissions	7 variables (T_{st}, T_{sb}, $T_{f\,(toe)}$, $T_{f\,(\text{heel})}$, $T_{f\,(\text{between stem an d slab})}$, W_h, W_t)	Constraints (internal and external stability, deflection, crack width, and minimum reinforcement spacing)	Pattern search (PS); GA
[25]	Bhatti, 2006	Minimum cost	5 variables (T_{st}, T_{sb}, B, W_t, T_f)	Constraints (internal stability, external stability, and geometric requirements)	Generalized reduced gradient (GRG) solver
[26]	Ahmadi-Nedushan and Varaee, 2009	Minimum cost; minimum weight	7 variables (B, W_t, T_{st}, T_{sb}, As_{stem}, As_{heel}, As_{toe})	10 constraints (external and internal stability)	Particle swarm optimization (PSO)
[27]	Villalba et al., 2010	Minimum cost; minimum carbon emissions	20 variables (T_{st}, T_{st}, W_h, W_t, 4 for f_c and f_y, 12 variables for primary, secondary, and shear reinforcement)	Constraints (external stability, internal stability, and deflection)	SA
[28]	Pei and Xia, 2012	Minimum cost	9 variables (T_{st}, T_{sb}, T_f, W_h, W_t, $As_{\text{stem (top)}}$, $As_{\text{stem (bot.)}}$, As_{heel}, As_{toe})	25 constraints (internal stability, external stability, and geometric requirements)	Complex method (CM); GA; PSO; SA
[29]	Papazafeiropoulos et al., 2013	Minimum volume of wall	6 variables (W_h, W_t, T_{st}, $T_{f\,(\text{heel})}$, $T_{f\,(toe)}$, Df)	Constraints (internal stability, uplift, undrained shear, and displacement)	GA

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[30]	Uray and Tan, 2019	Minimum factor of safety	4 variables (B, T_f, W_t, slope of base of retaining wall)	7 constraints (external stability, slope stability, and geometric requirements)	HS
[31]	Al Sebai et al., 2021	Minimum cost (reliability based design optimization)	10 variables (W_h, W_t, T_{st}, T_{sb}, T_f, Df, $As_{\text{stem (top)}}$, $As_{\text{stem (bot.)}}$, As_{heel}, As_{toe})	23 constraints (internal stability, external stability, deflection, and geometric requirements)	Covariance matrix adaptation-evolution strategy (CMS-ES); SQP
[32]	Srivastavaa et al., 2022	Minimum cost; minimum weight	12 variables (T_{sb}, T_f, W_t,W_h, W_{key}, T_{key}, As_{stem}, As_{heel}, As_{toe})	12 constraints (internal stability, external stability, and eccentricity)	PSO
[33]	Yücel et al., 2021	Minimum cost	5 variables (T_f, T_{st}, T_{sb}, W_h, W_t)	16 constraints (internal stability, external stability, and maximum and minimum reinforcement)	TLBO; JA; GA; PSO; DE
[34]	Ceranic et al., 2001	Minimum cost	7 variables (T_{st}, T_{sb}, W_h, W_t, T_f, W_{key}, T_{key})	Constraints (external stability)	Simulated annealing (SA)
[35]	Chau and Albermani, 2003	Minimum cost	3 variables (T_f, bar diameter, bar spacing)	Constraints (internal stability and crack width)	Genetic algorithm
[36]	Babu and Basha, 2008	Target reliability index (reliability based design optimization)	—	Constraints (external stability)	Method of Lagrange multipliers
[37]	Yepes et al., 2008	Minimum cost	20 variables (T_f, T_{st}, W_h, W_t, $f_{c\text{(stem)}}$, $f_{c\text{(footing)}}$, $f_{y\text{(stem)}}$, $f_{y\text{(footing)}}$, 12 variables for primary, secondary, and shear reinforcement)	Constraints (internal stability, external stability, and deflection)	SA
[38]	Khajehzadeh et al., 2010	Minimum cost	8 variables (W_h, W_t, T_{st}, T_{sb}, T_f, As_{stem}, As_{heel}, As_{toe})	Constraints (internal and external stability)	Particle swarm optimization with passive congregation (PSOPC)
[39]	Ghazavi and Bonab, 2011	Minimum cost; minimum weight	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	20 constraints (external stability, eccentricity, geometric requirements, and maximum and minimum reinforcement)	Ant colony optimization (ACO)
[40]	Kaveh and Abadi, 2011	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	9 constraints (internal and external stability)	Harmony search (HS); improved harmony search (IHS)
[41]	Khajehzadeh et al., 2011	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	11 constraints (internal and external stability, eccentricity, and deflection)	PSO; PSOPC; modified particle swarm optimization (MPSO)

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[42]	Camp and Akin, 2012	Minimum cost; minimum weight	12 variables (T_{st}, T_{sb}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development length, hook length, and maximum and minimum reinforcement)	Big bang–big crunch (BB-BC)
[43]	Khajehzadeh and Eslami, 2012	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	Constraints (internal and external stability, eccentricity, and deflection)	Gravity search algorithm (GSA)
[44]	Sable and Patil, 2012	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	21 constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	Interior point method (IPM)
[45]	Sable and Patil, 2012	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	21 constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	Interior point method (IPM)
[46]	Kaveh and Behnam, 2013	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability, and maximum and minimum reinforcement)	Charged system search (CSS); IHS
[47]	Kaveh et al., 2013	Minimum cost; minimum congestion	35 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, B_{key}, $f_{c(stem)}$, $f_{c(footing)}$, 15 for bar diameter and 11 for bar spacing)	Constraints (internal and external stability, eccentricity, and deflection)	Nonsorting genetic algorithm (NSGA-II)
[48]	Kaveh and Khayatazad, 2014	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Ray optimization (RO); PSO; CSS
[49]	Khajehzadeh et al., 2014	Minimum cost; minimum carbon emissions	8 Variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	11 constraints (internal and external stability, eccentricity, and deflection)	Adaptive gravitational search algorithm with pattern search (AGSA-PS); GSA; BB-BC
[50]	Sheikholeslami et al., 2014	Minimum cost	11 variables (T_{sb}, T_{st}, T_f, W_h, W_t, T_{key}, h_{top}, As_{stem}, As_{heel}, As_{toe}, As_{key})	Constraints (internal and external stability)	Improved firefly harmony search (IFA-HS); ACO

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[51]	Talatahari and Sheikholeslami, 2014	Minimum cost	11 variables (T_{sb}, T_{st}, T_f, W_h, W_t, T_{key}, h_{top}, $As_{stem(top)}$, $As_{stem(bot.)}$, As_{heel}, As_{toe})	Constraints (internal and external stability)	Enhanced charged system search (ECSS); hybrid big bang–big crunch (HBB-BC); CSS; PSO; BB-BC
[52]	Gandomi et al., 2015	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	Constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	Accelerated particle swarm optimization (APSO); firefly algorithm (FA); PSO; cuckoo search (CS)
[53]	Kaveh and Mahdavi, 2015	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Democratic particle swarm optimization (DPSO); colliding bodies optimization (CBO); PSO; IHS
[54]	Singla and Gupta, 2015	Minimum cost	3 variables (T_f, T_{sb}, W_t)	Constraints (internal and external stability, eccentricity, and maximum and minimum reinforcement)	Parametric equations
[55]	Bekdaş et al., 2016	Minimum cost	10 variables (T_{st}, T_f, W_h, W_t, 6 for bar diameter and bar spacing)	17 constraints (internal and external stability, spacing of bars, concrete cover, and maximum and minimum reinforcement)	Teaching learning based optimization (TLBO)
[56]	Kaveh and Farhoudi, 2016	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Dolphin echo location (DEO); HS; IHS; CSS
[57]	Sheikholeslami et al., 2016	Minimum cost	11 variables (T_{sb}, T_{st}, T_f, W_h, W_t, T_{key}, h_{top}, As_{stem}, As_{heel}, As_{toe}, As_{key})	Constraints (internal and external stability)	IFA-HS; IFA; HS; ACO
[58]	Temür and Bekdas, 2016	Minimum cost	11 variables (T_{st}, T_f, W_h, W_t, T_{sb}, 6 for bar diameter and bar spacing)	29 constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	TLBO; improved teaching learning based optimization (ITLBO); modified teaching learning based optimization (MTLBO); PSO; BB-BC; IHS
[59]	Aydogdu, 2016	Minimum cost	13 variables (B, W_t, W_{key}, T_{st}, T_{sb}, T_f, T_{key}, B_{key}, $As_{stem(top)}$, $As_{stem(bot.)}$, As_{heel}, As_{toe}, As_{key})	Constraints (internal and external stability, spacing of bars, development lengths, and maximum and minimum reinforcement)	Biogeography based optimization (BBO); biogeography based optimization with Levy flight (LFBBO)

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[60]	Kaveh and Laien, 2017	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Vibrating particle system (VPS); enhanced colliding body (ECBO); CBO
[61]	Gandomi et al., 2017	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	BBO; GA; differential evolution (DE); evolutionary strategy (ES)
[62]	Gandomi et al., 2017	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	Interior search algorithm (ISA)
[63]	Kumar and Suribabu, 2017	Minimum weight	7 variables (B, T_{st}, T_{sb}, W_t, As_{stem}, As_{heel}, As_{toe})	10 constraints (internal stability, external stability, and eccentricity)	Differential evolution algorithm (DEA); PSO
[64]	Rahbari et al., 2017	Minimum cost; maximum robustness	4 variables (W_h, W_t, T_{st}, T_f)	Constraints (internal and external stability)	NSGA-II
[65]	Ukritchon et al., 2017	Minimum cost	11 variables (W_h, W_t, T_{st}, T_{sb}, T_f, Df, As_{stem}, As_{heel}, As_{toe}, 2 for coordinates of wall)	22 constraints (internal stability, external stability, and slip circle)	NLP
[66]	Kayhan and Demir, 2018	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	15 constraints (internal and external stability and maximum and minimum reinforcement)	Differential genetic algorithm (DGA)
[67]	Mohammad and Ahmed, 2018	Minimum cost	6 variables (T_{sb}, T_f, W_h, W_t, As_{stem}, As_f)	20 constraints (internal stability, external stability, and geometric requirements)	Evolutionary algorithm (EA)
[68]	Kalateh-Ahani and Sarani, 2019	Minimum cost; minimum displacement	9 variables (B, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, $f_{c(stem)}$, $f_{c(footing)}$)	Constraints (internal and external stability, eccentricity, and maximum and minimum reinforcement)	NSGA-II

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[69]	Moayyeri et al., 2019	Minimum cost	12 variables (T_{st}, T_{sb}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	PSO
[70]	Öztürk and Türkeli, 2019	Minimum cost; minimum carbon emissions	12 variables (T_{st}, T_{sb}, T_f, B, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	25 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	Jaya algorithm (JA)
[71]	Uray et al., 2019	Minimum weight	4 variables (B, W_t, T_f, angle of footing)	Constraints (external stability and geometric requirements)	HS
[72]	Dagdeviren and Kaymak, 2020	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	12 constraints (internal stability, external stability, and geometric requirements)	Artificial bee colony (ABC); parametric equations
[73]	Kaveh et al., 2020	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Imperialist competitive algorithm (ICA); tug of war optimization (TWO); water evaporation optimization (WEO); cyclical parthenogenesis algorithm (CPA); BB-BC; TLBO; CS; CSS; RO; VPS; ABC
[74]	Kaveh et al., 2020	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	Constraints (internal and external stability)	Shuffled shepherd optimization algorithm (SSOA); BB-BC; CS; CSS; ICA; RO; TWO; WEO
[75]	Kayabekir et al., 2020	Minimum cost; minimum carbon emissions	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	16 constraints (internal and external stability, maximum and minimum reinforcement)	Flower pollination algorithm (FPA); HS
[76]	Konstandakopoulou et al., 2020	Minimum cost	4 variables (B, T_f, T_{st}, T_{sb})	Constraints (internal and external stability)	Parametric equations

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[77]	Mergos and Mantoglou, 2020	Minimum cost	6 variables (T_f, T_{st}, T_{sb}, W_h, W_t, T_{key})	Constraints (external stability, eccentricity, and maximum and minimum reinforcement)	FPA; PSO; GA
[78]	Kalemci et al., 2020	Minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_t, B, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	Grey wolf optimization (GWO); backtracking search algorithm (BSA); BB-BC; GA; DE; ES: PSO; APSO; FA; CS; ISA
[79]	Kayabekir et al., 2020	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	16 constraints (internal and external stability, maximum and minimum reinforcement)	JA
[80]	Hoang and Cong, 2020	Minimum weight	7 variables (T_{st}, T_{sb}, T_f, W_h, W_t, H, Df)	12 constraints (internal and external stability, eccentricity, and geometric requirements)	Differential evolution
[81]	Millán-Paramo et al., 2020	Minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_t, B, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	21 constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	Modified simulated annealing algorithm (MSAA)

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[82]	Kashani et al., 2020	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_h, W_t, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	PSO; MPSO; improved particle swarm optimization (IPSO); comprehensive learning particle swarm optimization (CLPSO); heterogeneous comprehensive learning particle swarm optimization (HCLPSO); extraordinary particle swarm optimization (EPSO); fractional-order Darwinian PSO (FDPSO); improved random drift PSO (IRDPSO); autonomous particle groups for particle swarm optimization (AGPSO); time varying acceleration particle swarm optimization (TACPSO)
[83]	Uray et al., 2020	Minimum weight	7 variables (B, T_{st}, T_f, W_t, W_{key}, T_{key}, $Batter\ slope$)	7 constraints (external stability, geometric requirements, and eccentricity)	ABC
[84]	Ravichandran et al., 2020	Minimum cost; maximum robustness	4 variables (B, W_t, T_{st}, T_f)	Target reliability and deflection	NSGA-II
[85]	Yücel et al., 2021	Minimum cost	5 variables (T_f, T_{st}, T_{sb}, W_h, W_t)	16 constraints (internal and external stability, maximum and minimum reinforcement)	Adaptive hybrid harmony search (AHHS); AHS; HS; GA; DE; PSO; FA; ABC; TLBO; FPA; GWO; JA
[86]	Kaveh et al., 2021	Minimum cost; minimum weight	7 variables (T_{st}, T_{sb}, T_f, T_{key}, W_h, W_t, h_{top})	14 constraints (internal and external stability, geometric requirements, and angle of footing)	Plasma generation optimization (PGO); CS; TLBO
[87]	Sharma et al., 2021	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_t, B, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	16 constraints (internal and external stability, geometric requirements, and maximum and minimum reinforcement)	Butterfly optimization algorithm (BOA); symbiosis organism search (SOS) algorithm; h-BOASOS

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[88]	Mevada et al., 2021	Minimum cost	8 variables (T_{st}, T_f, T_{batter}, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	10 constraints (internal and external stability, eccentricity)	FPA; NSGA
[89]	Uray and Çarbaş, 2021	Minimum cost	5 variables (T_f, T_{st}, T_{sb}, W_h, W_t)	9 constraints (external stability in static and dynamic case, geometric requirements)	HS; MABC; PSO
[90]	Tousi et al., 2021	Minimum cost; minimum weight	20 variables (B, T_{st}, T_{st}, T_f, W_t, 12 for As_{stem}, As_{heel}, As_{toe} (compressive and tensile) and f_y), 2 for $f_{c(stem)}$, $f_{c(footing)}$ and bar diameter)	23 constraints (internal and external stability, maximum and minimum reinforcement, development lengths, stem slope control, and minimum depth of footing)	Gases Brownian motion optimization algorithm (GBMOA); bacterial foraging optimization BFOA; ACO; NLP
[91]	Eroğlu et al., 2021	Minimum cost	7 variables (T_{st}, T_{sb}, T_f, W_h, W_t, h_{stem})	12 constraints (internal and external stability, maximum and minimum reinforcement)	JA
[92]	Uray et al., 2021	Minimum factor of safety	4 variables (B, T_f, W_t, batter slope)	7 constraints (external stability, slope stability, and geometric requirements)	HS
[93]	Linh et al., 2021	Minimum weight	5 variables (T_{st}, B, W_h, W_t, h_{stem})	12 constraints (internal and external stability, eccentricity, and geometric requirements)	DE-feasibility rule-based constraint-handling (FRBCH)
[94]	Dodigović et al., 2021	Minimum cost; maximum factor of safety (sliding); maximum factor of safety (bearing)	2 variables (B, Df)	4 constraints (external stability and maximum reinforcement)	NSGA-II
[95]	Tutuş et al., 2021	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_t, B, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	26 constraints (internal and external stability, geometric requirements, development lengths, spacing of reinforcement, maximum reinforcement, minimum reinforcement, and minimum cover)	CS

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[96]	Uray et al., 2021	Minimum weight (robust design optimization)	4 variables (B, T_f, W_t, batter slope)	7 constraints (external stability, slope stability, and geometric requirements)	Scatter search (SS)
[97]	Yücel et al., 2021	Minimum cost	5 variables (T_f, T_{st}, T_{sb}, W_h, W_t)	16 constraints (internal stability, external stability, and maximum and minimum reinforcement)	Flower pollination algorithm based artificial neural network (FPA-ANN)
[98]	Tutuş et al., 2021	Minimum cost; minimum weight	12 variables (T_{sb}, T_{st}, T_f, W_t, B, W_{key}, T_{key}, B_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key})	30 constraints (internal and external stability, geometric requirements, development lengths, spacing of reinforcement, maximum reinforcement, minimum reinforcement, and minimum cover)	Improved flower pollination algorithm (IFPA); FPA, PSO, DE; GWO
[99]	Temür, 2021	Minimum cost	24 variables (B, T_{st}, T_{sb}, T_f, W_t, B_{key}, W_{key}, T_{key}, *16* for primary and secondary reinforcement in stem, heel, toe, and key)	Constraints (internal and external stability, geometric requirements, and reinforcement spacing)	Hybrid teaching learning based optimization (HTLBO); TLBO; BB-BC; BBO; FPA; HS; PSO; GWO; JA; Rao-1; Rao-2; Rao-3
[100]	Shakeel et al., 2022	Minimum cost	10 variables (W_h, W_t, T_{st}, T_{sb}, T_f, As_{stem}, As_{heel}, As_{toe}, f_c, f_y)	23 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	EA
[101]	Khajehzadeh et al., 2022	Minimum cost	8 variables (T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe})	16 constraints (internal and external stability, eccentricity, and maximum and minimum reinforcement)	Particle swarm sine-cosine algorithm (PSSCA); sine-cosine algorithm (SCA); tunicate swarm algorithm (TSA); GSA; GWO

Table 2: Continued.

Ref. No	Study	Objective	Variables	Constraints	Optimization technique
[102]	Uray et al., 2022	Minimum cost; minimum weight (robust design optimization)	12 variables ($B, T_{st}, T_{sb}, T_f, W_t, B_{key}, W_{key}, T_{key}, As_{stem}, As_{heel}, As_{toe}, As_{key}$)	26 constraints (internal and external stability, geometric requirements, development lengths, hook lengths, and maximum and minimum reinforcement)	Taguchi method integrated hybrid harmony search algorithm (TIHHSA); BBO; GA; DE; ES
[103]	Khajehzadeh et al., 2022	Minimum cost	8 variables ($T_{sb}, T_{st}, T_f, W_h, W_t, As_{stem}, As_{heel}, As_{toe}$)	17 constraints (internal stability, external stability, and maximum and minimum reinforcement)	Sperm swarm optimization (SWO); adaptive sperm swarm optimization (ASSO); GSA; GWO; SCA

successful parent chromosome to obtain a better solution while the mutation factor randomly changes an individual component of the binary code, termed as gene, to again ensure diversification of population. The genetic algorithm (GA) is one of the most successful techniques used due to its simplistic design and ease of application. In case of retaining walls, multiple studies have either successfully applied the GA to obtain optimized designs or have used GA for comparative analysis (Purohit [21], Naeem [22], Rahbari [23], Schmied and Karlsson [24], Pei and Xia [28], Papazafeiropoulos et al. [29], Yücel et al. [33], Chau and Albermani [35], Kaveh et al. [47], Gandomi et al. [61], Rahbari et al. [64], Kayhan and Demir [66], Kalateh-Ahani and Sarani [68], Mergos and Mantoglou [77], Kalemci et al. [78], Ravichandran et al.[84], Mevada et al. [88], Dodigović et al. [94], and Uray et al. [102]). The above studies prove the effectiveness of GA in providing optimized results; however, it is seen that modern algorithms perform slightly better in terms of percentage of optimization and significantly better in terms of rate of convergence. The above studies do conclude GA as being superior to other classical techniques such as differential evolution (DE) and evolutionary strategy (ES). The improved form of GA, i.e., nonsorting genetic algorithm (NSGA-II), performs significantly better and comparable to modern algorithms. The NSGA-II is also able to handle multiple objectives which is the reason for its ever increasing popularity among multiobjective studies.

4.1.2. Harmony Search (HS). It is another population based evolutionary technique developed by Geem et al. [108]. They are art inspired algorithms mimicking the improvisation of a musician. The goal is to seek the fantastic harmony or the best function determined by aesthetic estimation. This is achieved by recombination of variable values stored in its harmony memory (HM) and tweaked by parameters such as the harmony memory size (HMS), the harmony memory considering rate (HMCR), and the pitch adjusting rate (PAR). The harmony search (HS) is also able to perform local and randomization searches and replace bad solutions with those of good harmony due to its memory function. It has also been used by several studies to optimize retaining wall design and has proven its effectiveness and robustness; see, for example, the papers of Uray and Tan [30], Kaveh and Abadi [40], Kaveh and Behnam [46], Sheikholeslami et al. [50], Kaveh and Mahdavi [53], Kaveh and Farhoudi [56], Sheikholeslami et al. [57], Temür and Bekdas [58], Uray et al. [71], Kayabekir et al. [75], Yücel et al. [85], Uray and Çarbaş [89], Uray et al. [92], Temür [99], and Uray et al. [102]. The HS has also been improved over the years by updating its memory considering rate, randomization, and local search capabilities. Kaveh and Abadi [40] researched on improved harmony search and evidenced improvement in results especially in convergence rate. Sheikholeslami et al. [50] and Sheikholeslami et al. [57] combined firefly algorithm (FA) with HS to form improved firefly algorithm-harmony search (IFA-HS) which had significantly better local search capacity and provided better results than improved firefly algorithm (IFA) and HS considered individually. Uray et al. [102] have tried to solve the computationally exhaustive process of metaheuristic algorithms by combining it with a statistical technique called Taguchi method. The Taguchi method utilizes orthogonal array for the parameters involved and calculation of signal to noise ratios (S/N ratio) to decrease number of trials required and the variance in the mean value. This drastically improves the performance of HS algorithm, which is evidenced by its superior performance against biogeography based optimization (BBO), GA, DE, and ES. Lastly, HS has also been utilized in multiobjective studies by Kayabekir et al. [75] and Yücel et al. [85], the later of which developed adaptive hybrid harmony search (AHHS) and compared it with adaptive harmony search (AHS) and HS. AHHS is a composite algorithm being a mixture of Jaya algorithm (JA) and HS. The result is a robust algorithm with a significantly decreased standard deviation rate and variance.

4.1.3. Particle Swarm Optimization (PSO). It is a swarm based optimization that mimics the social behavior and the movements of insects or animals in swarms. Developed by James and Eberhart [109], it considers each solution as a particle moving through the solution search space. Each particle is given a specific velocity and position in the search space. The particles are attracted to local best and the global best positions and move accordingly. It combines the local and global searches by inducing a weighting or inertia factor to the velocity to act as tradeoff between two searches. After iteration the positions of the particles are updated and the solution moves closer to global optima. Particle swarm optimization (PSO) is also a popular algorithm to optimize structural design and as opposed to evolution based algorithms like GA, PSO is a swarm based technique. It has been successfully employed by Ahmadi-Nedushan and Varaee [26], Pei and Xia [28], Srivastavaa et al. [32], Yücel et al. [33], Khajehzadeh et al. [38], Khajehzadeh et al. [41], Khajehzadeh and Eslami [43], Talatahari and Sheikholeslami [51], Gandomi et al. [52], Kaveh and Mahdavi [53], Temür and Bekdas [58], Kumar and Suribabu [63], Moayyeri et al. [69], Mergos and Mantoglou [77], Kalemci et al. [78], Kashani et al. [82], Uray and Çarbaş [89], Tutuş et al. [98], and Temür [99] to optimize RC cantilever retaining walls. The above studies evidence the speed of convergence of PSO and standard deviation is superior to other classical algorithms. Despite this, over the years research has been conducted to further improve upon the PSO. Khajehzadeh et al. [38] used an improved form of PSO using passive congregation, i.e., particle swarm optimization with passive congregation (PSOPC) with newly derived velocity equations. They concluded that PSOPC can significantly improve the convergence rate and improves search performance on objective functions when compared with PSO. Gandomi et al. [52] utilized accelerated form of PSO, i.e., accelerated particle swarm optimization (APSO), and compared it with other swarm techniques. In APSO, the particle's best position parameter is removed to accelerate the algorithms, and a new velocity vector equation is generated. They demonstrated that although APSO is quicker, its fast convergence affects its results and it performs poorly compared to other swarm techniques. Khajehzadeh et al. [41] further improved the PSOPC by providing an updated particle velocity equation by introducing a restriction factor. This velocity restriction factor is the base of the new modified particle swarm optimization (MPSO). Kaveh and Mahdavi [53] researched on optimization using democratic particle swarm optimization (DPSO) and compared it with PSO and other swarm algorithms. DPSO works by enhancing the performance of PSO by helping the agents receive information about good regions of the search space and letting some poor particles to be retained in swarm. For this purpose a new term is added to the velocity vector of PSO which represents this democratic effect. They concluded that DPSO performs better optimization than PSO by addressing its premature convergence problem. The latest study of Kashani et al. [82] provides perhaps the most comprehensive study on PSO algorithm for retaining wall optimization. It compares a plethora of PSO algorithms, i.e., PSO, MPSO, improved particle swarm optimization (IPSO), comprehensive learning particle swarm optimization (CLPSO), heterogeneous comprehensive learning particle swarm optimization (HCLPSO), extraordinary particle swarm optimization (EPSO), fractional-order Darwinian PSO (FDPSO), improved random drift PSO (IRDPSO), autonomous particle groups for particle swarm optimization (AGPSO), and time varying acceleration particle swarm optimization (TACPSO) in an effort to provide a comprehensive review and to discover the superior technique. Their results indicate all PSO algorithms are capable of solving the retaining wall optimization problem; however, in terms of mean and standard variation the superiority of HCLPSO and EPSO was evident.

4.1.4. Simulated Annealing (SA) Algorithm. The simulated algorithm (SA) was developed by Kirkpatrick et al. [110] and it mimics the process of annealing of metals. Annealing is a type of heat treatment of metals in which a metal is heated to specific temperature and then allowed to cool down till its lowest energy state. The recrystallization which occurs this way improves the properties of metal. The algorithm uses the same analogy to bring an objective function to a minimum (lowest energy state). A solution is randomly generated with high energy and temperature and monitored by Boltzmann factor. The temperature is decreased in each iteration by a factor called cooling coefficient and a number of iterations allowed at each step are called Markov chain. When the temperature drops to zero or is minimum an optimal solution has been obtained. It has been successfully employed for optimization of a retaining wall problem by Villalba et al. [27], Pei and Xia [28], Ceranic et al. [34], and Yepes et al. [37]. Recently, the study of Millán-Paramo et al. [81] proposed the modified simulated annealing algorithm (MSAA) which improves the performance of SA by introducing three stages of cooling and new parameters that improve the search capabilities of the algorithm.

4.2. Application of Advanced Metaheuristic Algorithms. Many studies have worked on developing methodologies to optimize the design of RC cantilever retaining walls by more advanced and newly developed algorithms. The success of newly developed algorithms is tested by comparative analysis and its robustness is tested through a sensitivity analysis. The objective is to find the most potent algorithm that can be easily applied or handle the difficulties of real world problem.

4.2.1. Gravitational Search Algorithm (GSA). Khajehzadeh and Eslami [43] developed methodology to apply gravitational search algorithm (GSA) that mimics the law of gravity and mass interactions on retaining walls. It is a swarm based optimization algorithm that considers individual solutions as agents with masses calculated using fitness functions. All of the objects attract each other by the gravity force and the heaviest masses (good solutions) have the biggest pull. Hence, after some time all agents will be attracted to a region of optimum solution. They compared it to PSO and GA and demonstrated its effectiveness in comparison to the classical algorithms. GSA has further been used by Khajehzadeh et al. [101] and Khajehzadeh et al. [103] for comparative analysis with other algorithms and improved upon in Khajehzadeh et al. [49] as adaptive gravitational search algorithm with pattern search (AGSA-PS). It combines an improved form of GSA called adaptive gravitational search algorithm (AGSA), which has improved global search capabilities due to inclusion of opposition-based learning. The improved AGSA is then combined with pattern search (PS), which is an NLP technique which improves its local search capabilities. The end result is an algorithm that has improved performance over GSA and big bang–big crunch (BB-BC) algorithm.

4.2.2. Dolphin Echo Location (DEO). Kaveh and Farhoudi [56] worked on a nature inspired algorithm called dolphin echo location (DEO). It has self-adaptive capabilities and it is based on the communication method of dolphins, i.e., echolocation (use of sound waves and their reflections to evaluate surroundings). In this optimization method user feeds a predefined convergence curve on which optimization needs to be performed based on convergence factor. The algorithm optimizes the curve and reduces search space in each loop until desired fitness is achieved. Comparisons with HS, improved harmony search (IHS), and charged system search (CSS) are detailed which show DEO achieved better results and higher convergence than its counterparts.

4.2.3. Teaching Learning Based Optimization (TLBO). Temür and Bekdas [58] optimized the design of a retaining wall using teaching learning based optimization (TLBO). It mimics the process of learning in a classroom by declaring the set of solutions as a class of students. The best solution is assigned as teacher and the class learns from it and then the students interact with each other and improve themselves further in learning phase. Based on changes in equations of the algorithm's parameters, modified TLBO (MTLBO) and improved TLBO (ITLBO) were also proposed. They concluded by demonstrating the effectiveness and robustness of the TLBO algorithms as compared to the traditional heuristic algorithms such as PSO, BB-BC, and IHS. It has also been used by studies of Yücel et al. [33], Bekdaş et al. [55], Kaveh et al. [73], Kaveh et al. [86], and Temür [99]. The study of Temür [99] proposes an improved version of TLBO which uses equations of different algorithms like flower pollination algorithm (FPA), JA, Rao-1, Rao-2, and Rao-3 for population generation and then applies TLBO for optimization of selected population. The end result is a more robust algorithm compared to TLBO, BB-BC, FPA, PSO, BBO, HS, grey wolf optimization (GWO), and Rao algorithms.

4.2.4. Jaya Algorithm (JA). Öztürk and Türkeli [70] formed a framework to apply the newly developed Jaya algorithm (JA) to minimize emissions of a retaining wall. The algorithm tries to achieve success by moving towards good solutions and avoids failure by moving away from bad solutions. Jaya algorithm is superior in the sense that it does not require any other parameters except population size and number of generations. They demonstrated JA as an effective tool to apply optimization by a parametric analysis. Further studies which have successfully employed the JA are Yücel et al. [33], Kayabekir et al. [79], Eroğlu et al. [91], and Temür [99].

4.2.5. Flower Pollination Algorithm (FPA). Mergos and Mantoglou [77] proposed a framework for a newly developed nature inspired algorithm, namely, flower pollination algorithm (FPA). The FPA follows the process of pollination by biotic process (global search) and abiotic pollination (local search) by pollinators. It also takes into effect the flower constancy meaning only specific flowers may be selected to extract nectar while ignoring others. The candidate solution is representation of a flower and solution space is formed by n number of flowers. The Levy flight factors are applied to generate movement of pollination process which leads to an optimum solution. A detailed parametric analysis was carried out for algorithm parameters of FPA to standardize a framework. Apart from that, comparisons with GA and PSO were drawn which showed that FPA outperforms GA and PSO in terms of variability. Kayabekir et al. [75], Mevada et al. [88], and Temür [99] also utilized FPA and evidence its capabilities and effectiveness. Yücel et al. [97] produced a hybridized algorithm by combining the metaheuristic technique of FPA with a machine learning technique called artificial neural network (ANN). It involves application of FPA and then ANN successively. FPA performs the optimization and ANN performs the predication process to improve the directionality of parameters and objective functions, reducing errors and improving solution search in each iteration. Lastly, Tutuş et al. [98] have developed an improved form of FPA, namely, improved flower pollination algorithm (IFPA). It improves upon the global search capabilities of FPA by combining it with DE algorithm. The IFPA is shown to have better statistical performance than FPA, DE, PSO, and GWO algorithms.

4.2.6. Shuffled Shepherd Optimization Algorithm (SSOA). Kaveh et al. [74] optimized the cost of a retaining wall using a newly developed shuffled shepherd optimization algorithm (SSOA). SSOA is a population based algorithm inspired by herding behavior of shepherds. The candidate solution is considered as a herd of sheep which is then ranked based on penalty functions and then divided into smaller herds. Each sheep (solution) is considered as a shepherd one by one and improved by the horses (better solutions) by moving their positions. Hence, it first randomly generates candidate solutions, ranks them in ascending order using penalized objective function, divides the solutions into subsets, determines a step size for each element, and applies exploitation and exploration on good and bad candidate solutions using factors. Again, elements are ranked but now are replaced by newer generation based on penalty functions. An extensive comparative study with multiple heuristic algorithms was performed. They concluded that SSOA has an effective optimization algorithm on par with other algorithms while having better standard deviations, average function value, and faster convergence rates.

4.2.7. Plasma Generation Optimization (PGO) Algorithm. Kaveh et al. [86] designed the most economical retaining wall using a recently developed population based algorithm, namely, plasma generation optimization (PGO) algorithm. Inspired by process of plasma generation it has two parallel phases; one is excitation/deexcitation phase and other is ionization phase. It considers the solution as atomic and free electrons with specific energy levels represent the quality of solution. The excitation phase is an intensification technique where electrons search near the higher energy electron determined through penalizing objective function. The deexcitation phase is an exploration phase that randomly shifts higher energy electrons to lower energy electrons based on deexcitation rate (DR). The other phase is ionization phase in which free electrons move to higher energy electrons based on step size obtained by multiplying Levy flight and energy difference between two compared electrons. This represents the diversification phase of PGO; both processes continue in parallel until the plasma with the highest density of ions is generated, i.e., the optimum solution. The authors performed a comparative study with TLBO and cuckoo search (CS) while also detailing a parametric analysis of all three algorithms. Their findings showed that PGO's effectiveness is on par with the other two algorithms with PGO giving slightly better optimization.

4.2.8. Hybrid-Butterfly Optimization Algorithm Symbiosis Organism Search (H-BOASOS). Sharma et al. [87] developed a methodology to combine two algorithms, namely, butterfly optimization algorithm (BOA) and symbiosis organism search (SOS), and form a novel hybrid algorithm, i.e., h-BOASOS. The BOA is a population based nature inspired algorithm while SOS is swarm based algorithm. The BOA mimics the behavior of butterflies in nature while SOS is applied through the ecosystem's interactive behavior with different organisms. BOA works by two phases. The first is global search where candidate solution, i.e., the butterflies, has a specific fragrance which all butterflies can sense. The butterfly with the most fragrance (good solution) attracts the rest of the butterflies towards it to create a social mobilization. The second phase is local search where butterfly cannot sense a fragrance and it randomly searches its surroundings. The SOS algorithm is based on interactions of organisms and is applied through three phases: mutualism phase (two organisms interact and both benefit), commensalism phase (two organisms interact and only one benefits), and parasitism phase (two organisms interact, one benefits, and the other is harmed). Each solution is updated by three phases and then evaluated by the fitness function. It can be seen from above explanation that BOA can perform better global search which affects exploration and SOS performs better local search but diversification worsens and premature convergence occurs. The authors combined the algorithms by performing BOA and then SOS to form h-BOASOS. They used the algorithm to optimize cost and weight of two different height retaining walls and compared it with the results of BOA, SOS, and Optimtool. Their findings demonstrated significant performance improvement and higher degree of optimization achieved through h-BOASOS.

4.3. Multiobjective Optimization and Novel Objectives. Traditionally cost and weight minimization had been the preferred objectives for the optimization process. However, some researchers have considered other objectives to achieve specific desired results. Generation of sustainable or low emission design structures has become top priority in recent years. Studies of Villalba et al. [27], Khajehzadeh et al. [49], Öztürk and Türkeli [70], and Kayabekir et al. [75] utilize the carbon objective function separately or in conjunction (multiobjective) to optimize the design and obtain a sustainable section. It is seen that the least emission section is not the most economical section. This is due to opposite fronts formed, i.e., steel minimization for cost and concrete minimization for emission objective; hence, in multiobjective a Pareto front or a compromise between two objectives is formed. Kaveh et al. [47] present a unique problem of cost and bar congestion minimization. The congestion objective is dependent on number of bars obtained from bar spacing divided by dimension along which bar is provided. Cost minimization prefers smaller diameter bars with smaller spacing which increases congestion. They concluded for their example that the minimum cost design decreased cost by 25% but increased congestion by 68.7% while minimum congestion problem increased cost by 32% but decreased congestion by 40%. Purohit [21] performed a detailed parametric study for a multiobjective optimization for cost minimization and safety maximization. The study tried to maximize factors of safety of overturning, sliding, bearing, and eccentricity while decreasing cost using NSGA-II. It was observed that there is a steady increase of cost till a factor of safety of 4.0 for bearing but after that FOS does not change appreciably with cost increase. It was also concluded that sliding failure was the controlling parameter for optimized design. Rahbari et al. [64] developed a methodology for design of retaining wall in high seismicity locations. They considered cost minimization and robustness maximization of the retaining walls as objective functions. The robustness index contained standard deviation and signal to noise ratio. Another Pareto front problem is formed and the design is optimized using NSGA-II. They discovered that decreasing the standard deviation or increasing signal to noise ratio tried to decrease risky design which yielded greater volume per unit length, which in turn yielded a more costly design. Kalateh-Ahani and Sarani [68] performed simultaneous optimization of cost and permanent displacement minimization. They considered permanent displacements in high seismic zones under AASHTO limit state constraints and calculated the deflections using Newmark sliding block method. The optimization is applied for three cases of walls: with toe and heel, without heel, and without toe. Two extreme cases were presented for each wall, high risk (minimum cost) and low risk (minimum displacement), and their design plans were drawn.

A new novel area of research is reliability based design optimization (RBDO). This type of optimization considers factors of safety as insufficient especially as they do not cover uncertainties in design parameters. Reliability based designs impart probability based linkage between design parameters and failure. In optimization process, this can be achieved by applying reliability based constraints that ensure that a solution moves from global optimum towards a reliable optimum. The study of Al Sebai et al. [31] performed cost based optimization using reliability constraints, which include 20 hard constraints and 3 soft constraints, all with their covariance defined as per degree of uncertainty. All 10 variables used have their covariance and marginal distribution functions defined as well. A target reliability index of 3 is fixed and Covariance Matrix Adaptation-Evolution Strategy (CMS-ES) is used to determine a feasible solution space. Within this feasible space, nonlinear based sequential quadratic programming (SQP) is used to determine the minimal cost satisfying the reliability criteria. The study of Babu and Basha [36] used the method of Lagrange multipliers for optimization. Their study includes 10 modes of failures (overturning, sliding, eccentricity, bearing, shear failure of toe, heel, and stem, and moment failure of toe, heel, and stem). Calculation of factors of safety is done, followed by verification of constraint violation. If no constraint is violated, first-order reliability method (FORM) is used to calculate reliability indices of each failure mode. These indices must be above the set target reliability index. Their study also develops charts between geometric proportions and target reliability index for 5% and 10% covariance. Their study indicates significant cost savings for target reliability index between 3 and 3.2 with lower covariance values.

Another novel area of research is robust design optimization (RBO). Although metaheuristic algorithms have proven their capability as an effective optimization technique they come with a complexity of their own. The algorithms have predefined value of specific parameters that are used for exploration of a solution space. The value of these parameters may become the difference between finding a global optimum solution and waste of computational effort. It is necessary to perform extensive number of trials to judge the effectiveness of undertaken parameters. However, RBO attempts to overcome this issue by minimizing the variations on performance caused by these parameters and consequently develop a robust optimization model. The studies of Rahbari [23], Rahbari et al. [64], and Ravichandran et al. [84] investigate the optimization of shredded tyre-filled backfill under seismic loadings. Due to the high level of variability involved in the type of loading and the type of backfill material, RBO is used. A dynamic finite element analysis is performed while keeping geometric dimensions of wall as variable and applying first ten seconds of the acceleration-time history of El Centro earthquake. This is done to obtain wall displacement and then the wall tip displacement history. To access the validity of this result, response surface method is applied using regression analysis and results are compared with the finite element analysis. Three statistical techniques are applied to measure the variance of data. After the validation of good results from these methods, this data was utilized in design optimization. The design optimization in all three papers has been modeled as a multiobjective problem, optimized using NSGA-II. The first objective is cost and the second objective is either minimization of standard deviation (SD) or maximization of signal to noise ratio (S/N ratio). A Pareto front is formed using optimization which can be used to extract the optimum solution. The studies of Uray et al. [96] and Uray et al. [102] also perform robust optimization using scatter search and harmony search algorithms. Using a statistical technique called Taguchi method, they evidence a boost in performance of algorithms while simultaneously decreasing the number of iterations required to obtain optimum results.

4.4. Optimization Tools and Parametric Equations. Optimization can be applied through multiple programming software; however, due to their complexities civil engineers often choose simpler programming languages such as MATLAB (Schmied and Karlsson [24], Pei and Xia [28], Al Sebai et al. [31], Srivastavaa et al. [32], Babu and Basha [36], Khajehzadeh et al. [38], Khajehzadeh and Eslami [43], Sable and Patil [44], Sable and Patil [45], Kaveh and Behnam [46], Kaveh et al. [47], Khajehzadeh et al. [49], Gandomi et al. [52], Kaveh and Mahdavi [53], Gandomi et al. [61], Öztürk and Türkeli [70], Uray et al. [71], Kalemci et al. [78], Kayabekir [79], Kashani et al. [82], Mevada et al. [88], Uray et al. [92], Tutuş et al. [95], Khajehzadeh et al. [101], and Khajehzadeh et al. [103]), Fortran (Villalba et al. [27] and Ukritchon et al. [65]), C#.NET (Linh et al. [93]), Python (Dodigović et al. [94]), and C++ (Dagdeviren and Kaymak [72]). Studies have also tried to combine analysis software such as ABAQUS, ANSYS, PLAXIS 2D, and GeoSlope (Rahbari [23], Papazafeiropoulos et al. [29], Uray and Tan [30], Rahbari et al. [64], and Ravichandran et al. [84]) with optimization. Research studies of Singla and Gupta [54], Dagdeviren and Kaymak [72], and Konstandakopoulou et al. [76] have tried to tackle the optimization problem by developing regression equations and eliminating the need of programming but with certain limitations. Research of Sarıbaş and Erbatur [19] developed RETOPT and Ceranic et al. [34] developed GENOD, which are computer programs to apply optimization to the design of RC retaining walls but none of the above programs are commercially available. Recent studies have used built-in or add-in optimization tools such as Solver of Excel or Maple (Bhatti [25], Ukritchon [65], and Mohammad and Ahmed [67]) and have proven them to be effective in achieving optimization for retaining wall design. Hoang and Cong [80] and Shakeel et al. [100] have built upon these solvers to develop user-friendly tools for designers to readily apply optimization on a design without dealing with the jargons of algorithms.

4.5. Comparative Analysis of Metaheuristic Algorithms. The ability of metaheuristic techniques in escaping local solutions and achieving global optimum solutions lies in their probabilistic nature. However, this ability also makes direct comparison difficult as the variability involved may result in a different solution in each optimization run. Apart from that, the complexities and differences in problem formulation modeling and the optimization problem modeling make a like-for-like comparison among the many metaheuristic algorithms ambiguous. The differences in optimization parameters, different types of parameters, and the stopping criteria may also cause slight variances. However, many researchers have attempted to perform a comparative investigation in order to determine the superlative algorithm for optimization among the many successful algorithms.

Pei and Xia [28] perform a comparative analysis between a random search technique called complex method (CM) and classical heuristic methods like SA, GA, and PSO. They optimize a retaining wall having 9 design variables (without a shear key) for minimum cost. In accordance with the literature, CM is unable to compete with other algorithms as it gets stuck in local minima for such large scale problems. Although the results evidence that no single heuristic algorithm outperforms the other, they do conclude PSO as their recommendation due to its lower number of computations with respect to time consumed in order to obtain similar results as SA and GA. Talatahari and Sheikholeslami [51] optimize a cantilever retaining wall for total material cost using 7 variables. They compare the best and average results of 20 optimization runs using enhanced charged system search (ECSS), hybrid big bang–big crunch (HBB-BC), BB-BC, and PSO algorithms. The optimized values evidence ECSS as having better convergence rate and percentage of optimization. However, no statistical analysis is performed to determine which algorithm obtained the better mean results. Gandomi et al. [52] performed a comprehensive comparison between PSO, APSO, FA, and CS for three examples of retaining walls (with and without shear key) using 12 variables. The analysis was conducted for 100 optimization runs for cost and weight of the retaining wall. The best, worst, mean, and standard deviations of all three examples for both objectives are presented. Although the results vary widely among the examples, APSO consistently performs faster convergence. In terms of worst, mean, and standard deviation, PSO and CS perform identically; on the other hand, APSO and FA perform comparatively worse. Kaveh et al. [73] present one of the most comprehensive comparisons of cost optimization of retaining walls by using 11 different metaheuristic algorithms. Their study considers varying levels of seismic loadings and utilized both the Coulomb method and the Rankine method for determining the lateral Earth pressures in its problem formulation. The optimization for all algorithms is fixed at 5000 maximum evaluations. The results indicate that all algorithms perform relatively well and can converge to quality optimum designs rapidly. However, slight differences exist when the best results, average results, standard deviations, and convergence histories are compared. For the best optimized cost using the Coulomb method, artificial bee colony (ABC), cyclical parthenogenesis algorithm (CPA), vibrating particle system (VPS), BB-BC, and TLBO give the best results, while for the best optimized cost, using the Rankine method, imperialist competitive algorithm (ICA), CPA, CSS, TLBO, and VPS give better results. However, when average optimized cost and standard deviations are considered, BB-BC, CSS, and TLBO have better performances.

Kaveh and Mahdavi [53] considered a retaining wall with 7 variables including a shear key and seismic loadings in the problem formulation. They compare the results of colliding bodies optimization (CBO), PSO, IHS, and DPSO for various backfill cases and soil lateral load estimation theories. Their results evidence the robustness of CBO and DPSO as better than the other two algorithms. Kaveh and Farhoudi [56] performed a comparative analysis between DEO, HS, IHS, and CSS for two different types of backfill soils. The results are compared for the best optimized value achieved. Their analysis indicates that DEO takes less than half the number of iterations of other algorithms to achieve the best result. Kaveh and Laien [60] extend the work done in their previous studies. They compare the results of IHS, DPSO, DEO, and CSS as reported in their previous studies [53, 56] with enhanced colliding body (ECBO), VPS, and CBO. The convergence histories show that the newly developed VPS algorithm performs better optimization in terms of speed of convergence among the 20 independent runs. VPS and ECBO also have a much smaller standard deviation than CBO for two different types of backfill soil cases. However, all algorithms perform identically with regard to the optimized value of the objective function. Another area of interest is comparing metaheuristic algorithms of the same category. Gandomi et al. [61] performed such a comparative investigation for evolutionary algorithms. They compare the results of DE, ES, BBO, and GA. Three different retaining walls are optimized for cost and weight using each algorithm and results of 100 runs are reported. The standard deviations, best, mean, and worst values are extracted and convergence rate plots are drawn. Their results indicate BBO as the superior algorithm in all categories. It is also noticed that the performance and degree of optimization of BBO increase with heavier designs which include a base shear key.

Some researchers have also compared the performance of hybridized algorithms with their root algorithms. The study of Sheikholeslami et al. [57] optimizes two examples of retaining wall for cost using IFA-HS and then compares their results with optimization using IFA and HS. The results indicate that the IFA-HS is comparatively more efficient, achieving an optimized result in just 4,200 evaluations compared to 6,700 for IFA and 4,700 for HS. Further tuning of IFA-HS is also conducted using a sensitivity analysis which reduces the number of evaluations to 4,180. Kaveh and Abadi [40] performed a comparative investigation between HS and IHS algorithms. The model contains 7 variables, optimized for the total cost of a retaining wall with a shear key. The updated form of the HS algorithm, i.e., IHS, is evidenced to have slightly better efficiency and robustness in its results, the main contributor being the HS parameter called pitch adjusting rate (PAR). The calculation of PAR is converted from a fixed value as used in HS to a dynamically generated value with each generation in IHS leading to a boost in performance. Khajehzadeh et al. [41] compare PSO, PSOPC, and MPSO for a retaining wall with 8 design variables for minimum cost. A detailed statistical comparison is made by running the algorithm 50 times and comparing the best, average, and worst results. Multiple statistical tests are applied to differentiate between the performances of the algorithms. First, the Kolmogorov-Smirnov test is applied to match results with the normal distribution curve. Afterwards, the Kruskal-Wallis non-parametric test and Mann-Whitney *U* test are applied to compare the mean ranks of each algorithm. The results evidence the mean rank of MPSO as the best among the compared algorithms. Apart from that, MPSO also had the fastest convergence rate and performed significantly better in earlier iterations. Kashani et al. [82] have published a detailed review on PSO algorithms and their variants used for the optimization of geotechnical structures. As a part of their study, they also ran simulations for optimizing a cantilever retaining wall for cost and weight. They compared PSO and its 11 other variants for 9 different seismic load combinations. The results of 100 runs are extracted and the best, mean, and standard deviations are compared. All PSO algorithms are successfully able to reach optimum results. However, HCLPSO and EPSO perform better than others, while RDPSO recorded the poorest performance. The Friedman statistical test is further used to rank the performance of each algorithm which also confirms the efficiency of HCLPSO and EPSO in optimization. Yücel et al. [85] also performed a comparative investigation of HS algorithm and its variants and compared it to 10 different metaheuristic algorithms. Optimization for the cost of a retaining wall with 5 geometric variables was applied for 4 different cases. Each case included differing values of optimization runs, heights, and wall parameters. The best cost, average cost, and standard deviations were compared. Variations of HS parameters and their effect on HS, AHS, and AHHS are also tabulated. It was noted that GA, DE, PSO, and GWO had high standard deviations while AHS, AHHS, JA, TLBO, and FA could converge to a minimum solution but with extremely minor values of standard deviation.

Lastly, testing out the capabilities of newly developed algorithms and comparing their results to popular metaheuristic algorithms is also a common trend in research. Kaveh et al. [74] performed a comparative analysis for the newly developed SSOA. They compared SSOA optimization with the results of 11 algorithms analyzed in a previous study [73]. Optimization was conducted by considering seismic loading cases, Rankine theory, and Coulomb theory separately. Among all cases, it was demonstrated that SSOA had more efficient results than any other compared algorithms. Kalemci et al. [78] also performed optimization using a newly developed algorithm, namely, GWO. They optimized two retaining walls for total weight and compared their results with other studies using the same examples. GWO was run 30 times and the best and mean optimized weight were obtained for all runs. The values were compared with optimized results of 12 other algorithms, taken from 5 other studies. It was concluded that GWO gave up to 1.5% lighter sections than other algorithms. However, the authors also pointed out that this could be due to discrepancies in the problem formulation and the number of optimization runs between the different studies. The research of Sharma et al. [87] tested three newly developed algorithms, BOA and SOS and a proposed hybrid algorithm (h-BOASOS). However, unlike previous studies, optimization was not tested on a case study retaining wall but on 35 different benchmark (modal and unimodal) functions. They compared the performance of these three algorithms with 10 other algorithms for 30 optimization runs with 10,000 maximum iterations. Performance evaluation is done through the statistical method of Friedman's rank test. According to the analysis, h-BOASOS has rank 1, ABC has rank 2, SOS has rank 3, and BOA has rank 4. After this, cost and weight optimization is applied on two examples of a retaining wall with 12 variables using BOA, SOS, and h-BOASOS. Comparisons are also made with a study from literature which used the Optimtool [45]. In all cases, the efficiency of h-BOASOS is evident over other algorithms.

4.6. Structural and Geotechnical Design Investigations. Multiple studies have performed a sensitivity analysis with regard to soil parameters involved in design of retaining walls (Yepes et al. [37], Camp and Akin [42], Sable and Patil [45], Singla and Gupta [54], Temür and Bekdas [58], Gandomi et al. [61], Mohammad and Ahmed [67], Millán-Paramo et al. [81], Uray et al. [83], Kaveh et al. [86], Tousi et al. [90], Uray et al. [92], Uray et al. [96], and Tutuş et al. [98]). The trend of all the studies is similar and indicates that increasing the height of wall, depth of soil on toe, unit weight of soil, backfill slope, and surcharge load is directly related to the cost, while increasing bearing capacity, cohesion, and internal angle of friction is indirectly related to the cost of wall.

Although the design codes advise coarse aggregates as prescribed material for backfill, it may not be feasible to provide them in all cases. Some studies have researched this aspect and tried to optimize the design for backfill with different types of soil with different cohesion, unit weight, bearing, and internal angle of friction values (Al Sebai et al. [31], Srivastavaa et al. [32], Yepes et al. [37], Kaveh and Abadi [40], Kaveh and Behnam [46], Kaveh and Laien [60], Konstandakopoulou et al. [76], Uray et al. [83], Uray and Çarbaş [89], and Dodigović et al. [94]). The backfill included coarse granular fill (gravel), granular soils with more than 12% of fines (GW, GS, SM, and SL), and fine soils with more than 25% of coarse grains (CL–ML). It is clearly evidenced that soils with better mechanical properties like gravel and sand perform better and are more economical.

Another important factor in terms of geotechnical parameters is the bearing capacity of the soil. In design optimization of retaining walls most studies have simply taken the value of safe bearing capacity as is provided in the undertaken design example. However, the research of Moayyeri et al. [69] is unique as it varied the bearing capacity equations used in literature. The study uses the theories of Meyerhof, Hansen, and Vesic to determine ultimate bearing capacity of soil. The study concluded that the Meyerhof method gives more optimization; however, the difference decreased with increase in height of wall. Apart from these theories Terzaghi's bearing capacity theory has also been used by the studies of Uray et al. [92], Tutuş et al. [98], and Uray et al. [102] to calculate the bearing capacity. Inclusion of a bearing capacity calculation in problem formulation instead of requiring a direct input of safe bearing capacity provides a more in-depth model capable of handling site specific complexities. Another unique research is of Gandomi et al. [61] which included the calculation of retaining walls settlement. The study optimized the wall for cost and weight and drew relation between settlement and base width of wall for different values of modulus of elasticity and Poisson ratio. It was demonstrated that increasing the width of wall increases the vertical load on it and consequently increases the settlement of wall. However, the increasing elastic modulus of soil decreases the total settlement.

The application of Coulomb or Rankine theory is another intricately investigated problem conducted by multiple studies (Kaveh and Mahdavi [53], Kaveh et al. [73], Kaveh et al. [74], Konstandakopoulou et al. [76], Kaveh et al. [86], Uray and Çarbaş [89], and Tousi et al. [90]). The difference is that Rankine theory considers the wall face on soil side as frictionless while Coulomb theory is more detailed and takes into account the wall friction angle which is dependent on soil's internal angle of friction. It is also due to this reason that all studies conclude that Rankine method gives a more expensive wall as it overestimates the pressure to simplify the design.

4.7. Effect of Seismic Loads. Earthquake loading as a part of optimized retaining wall design is another matter investigated by multiple researchers (Naeem [22], Kaveh and Mahdavi [53], Aydogdu [59], Kalateh-Ahani and Sarani [68], Kaveh et al. [74], Konstandakopoulou et al. [76], Ravichandran et al. [84], Kaveh et al. [86], Eroğlu et al. [91], Temür [99], Khajehzadeh et al. [101], and Khajehzadeh et al. [103]). The potential of optimization is significantly increased in case of heavier designs, developed to withstand large seismic loads. The seismic lateral pressures are calculated by the Mononobe–Okabe equation which is derived from Coulomb's sliding wedge theory. The equation is used to calculate the equivalent static loads which are then used in the design of retaining walls. In this method, the seismic inertia angle is calculated, which is then used to determine active (K_{ae}) and passive (K_{pe}) Earth coefficients due to earthquake. These coefficients are used to determine the active and seismic pressures. The value of seismic inertia angle is reliant on vertical earthquake acceleration (K_v) and horizontal earthquake acceleration (K_h). These values contain high degree of uncertainty and are dependent on multiple factors such as soil classification, peak ground acceleration (PGA), and frequency and duration of seismic waves. In terms of optimization, increasing seismic forces and cost of wall, their effects on factors of safety, provision of key, and weight of retaining wall have been analyzed. The results show that increased lateral loads lead to heavier and costly sections. In fact in case of weak clayey soils, the design becomes economically infeasible. It is also concluded that RC cantilever retaining walls are impractical for height above 7.5 m in a zone with PGA greater than 0.25 g. The provision of key is evidenced to improve safety factors and decrease cost and weight as compared to a T-shape retaining wall in case of increasing seismic loads.

4.8. Shear Key Effect and Sloped Footed Walls. The provision of shear key is also a topic of interest among multiple research studies. While some studies have simply modeled a retaining wall with a shear key [32, 84, 87, 95], others have performed comparative analysis between retaining walls with and without shear keys (Naeem [22], Ceranic et al. [34], Sable and Patil [45], Kaveh and Behnam [46], Sheikholeslami et al. [50], Gandomi et al. [52], Kaveh and Farhoudi [56], Gandomi et al. [61], Öztürk and Türkeli [70], Kaveh et al. [73], Kaveh et al. [74], Millán-Paramo et al. [81], Uray et al. [83], Sharma et al. [87], Tousi et al. [90], Tutuş et al. [98], and Temür [99]). The consensus on the findings is clear and it indicates that although the provision of key does increase the safety factors it also increases both cost and weight of a retaining wall. However, in regions of high seismicity or large surcharge loadings the provision of key gives better results as it provides extra passive resistance to the wall. Both cost of wall and weight of retaining wall can be significantly optimized with a shear key in regions of high seismic or lateral loads. In case of large lateral loads in retaining walls without shear keys, there is an increase in reinforcement area (increase in cost) and the thickness of footing (increase in weight) to counteract the increased forces. However, in case of walls with keys, the passive resistance provided by soil is often enough to resist these forces. These results are purely theoretical and the actual effectiveness of a shear key is dependent on other site related factors like soil disturbance, which have not been thoroughly investigated. Uray et al. [71] and Kaveh et al. [86] investigated a sloping retaining wall footing against retaining wall with key. Their findings suggest that, in case of low seismicity, a key is a more economical solution for cost and weight minimization problem. However, in case of high seismicity, a sloping footing is viable and has lower cost. However, even in this case it has larger weight than a wall with key. It was also evidenced that a sloped footed retaining wall has higher factor of safety in overturning, sliding, and slope stability.

4.9. Limitations. As detailed in length, a plethora of research work has been conducted on various topics regarding optimization of retaining walls. However, the majority of works have been conducted under some limiting factors that are necessary to be understood before integrating design optimization in a project. Design optimization is often performed with continuous variables which although providing a larger percentage of optimization will result in constructability concerns and will require an additional review by the engineer before on-site application. Otherwise an optimization model with discrete values and practical step size must be defined for each variable. It is also limited by the problem formulation modeled into it; in case of cost optimization, the total cost in most studies includes only the cost of concrete and steel. However, a more thorough model can have cost of formwork, labor, placement, vibration, and transportation included in it. Another limitation is reinforcement detailing, as most optimization studies have only modeled areas of reinforcement. The variables should contain variables for bar diameters, spacing, and development lengths or else they must be reviewed by an engineer before finalizing the design. Similarly, the effect of material strength needs to be modeled in the development stage and linked with resulting variation in cost to ensure practical variation and optimization occurs. The effect of seismic forces is mostly applied using the Mononobe–Okabe equation which is a conservative estimation method to determine seismic pressure. Better dynamic analysis methods can be applied to improve safety. Similarly, Earth pressure is calculated using Rankine or Coulomb theory. More accurate Earth pressures are calculated using finite element methods. Most studies simply input maximum bearing capacity of soil in the model; a more thorough model that is able to take soil parameters as input would be more suitable. Such models would be able to calculate data using whichever theory provides better optimization. In terms of limitations of algorithms, most algorithms can handle only single objective optimization. Specific algorithms have to be used to deal with multiobjective problems. As detailed above unless robust design is applied, there exists a vast variability in terms of algorithm parameters that may affect results. A problem specific, sensitivity analysis must be performed to decide the ranges for undertaken parameters. Traditional algorithms are also time consuming and perform thousands of iterations; hence sufficient computational capabilities are a must to smoothly run optimization algorithms. Lastly, algorithms can only work within the specified bounds; provision of effective lower and upper bounds would decrease computational efforts. The optimization is also dependent on initial population or position. A feasible initial population, if provided, would greatly increase the efficiency of the optimization algorithm.

5. Recommendations for Future Work

The detailed literature review mentioned above shows that although significant advancements have been made in the field of optimization, particularly of RC cantilever retaining walls, the task is still very complicated and challenging. There is a lack of optimization performed in the construction industry on real world cantilever retaining walls despite proven advantages. These difficulties can be attributed to the multidisciplinary nature of the optimization of RC structures. This makes it a complex task that requires in-depth knowledge before effective applicability. There are also some other complications and conundrums that must be addressed by future researchers before its acceptance and assimilation into the construction industry. The following section briefly mentions using the extracts from the literature review the topics of paramount importance that must be addressed moving forward.

5.1. Tool/Application Development and Integration. The most critical objective identified by the analysis of the literature is that there is a dire need for problem specific optimization tools. It is detailed in Section 4.4 that most work on optimization has been achieved through programming algorithms. This type of work requires extensive knowledge of the optimization algorithms and the ability to code in a modern computing language. Both these tasks require strenuous efforts along with prerequisite skills and knowledge. There are add-in tools to apply optimization algorithms like Solver of MS Excel (Frontline Systems Inc.), Evolver (Palisade Corporation), and the MATLAB's optimization tool box (The MathWorks Inc.). They also require extensive problem formulation in their respective compatible environments before application of optimization. Only the availability of user-friendly applications which can handle the mathematical jargons of the algorithms on the backend will make use of optimization more popular. As evidenced by building integrated modeling (BIM) and 3D computer aided modeling (CAD) software, only convenience of usability will make use of complex technologies norm in the practical fieldwork. Therefore, further investigation is required to make it widely accepted as a design tool for engineers. Another solution can be integration of optimization algorithms with finite element modeling (FEM) environment. Tools like Abaqus2Matlab [111], for example, are a step in the right direction. Abaqus2Matlab has the capability to connect an FEM software like Abaqus with the user-friendly environment of MATLAB. Users can model a structure and perform an in-depth analysis on Abaqus and then easily import their results in MATLAB, where optimization can be applied using built-in or programmed algorithms. This will drastically improve real time design optimization while also providing designer's flexibility and command over structural design and analysis. The visualization aspect of the optimized geometric dimensions with original designs will also greatly improve the value of optimization techniques, as they will lend the designer a conceptual rendition of the constructed wall.

5.2. Constructability Concerns. The optimization in above studies has mostly considered continuous variables to achieve a more cost-effective solution, but this is not always possible due to irregular section sizes. It is true that metaheuristic algorithms are able to deal with discrete variables, an ability not possible using nonlinear programming. They also drastically decrease the solution space, which leads to much lower optimization effort. Future researchers must balance these needs and provide options for setting step size when developing their framework so that feasible structural designs can be developed. These designs must be constructable on site or else the cost for modular formworks and labor may increase costs drastically. Another critical quandary between theoretical optimization and practical design is the provisions of reinforcement. Most studies utilized area of reinforcement as variable to be optimized but on site, this quantity is to be converted into number of reinforcement bars of certain diameters. Optimized shape of concrete along with later conversion from area of steel to bar can cause congestion of reinforcement. This can be mitigated by either taking reinforcement bar diameters and their spacing as variables or considering bar congestion as an objective function as well.

5.3. Material Blend and Strength Considerations. The effects of increasing strength of concrete and steel and their variability in cost have also not been studied in detail. As optimization process provides the designer to choose from infinite solutions, future researchers can focus on developing methodologies for mix design development for specific cases of retaining walls. This would mean problem specific strength of concrete or material blends for sustainable objectives, i.e., proportioning of secondary cementitious materials can be done with varying cost with ease. Effects of different type of cements can also be a topic of future discussion. Similarly, different yield strength of reinforcement and their effects on objectives and congestion must be studied in detail.

5.4. Parametric Investigations and Sensitivity Analysis. No study to the authors' knowledge has investigated the effects of layered soil, water table, and wind loads on the optimized design of a retaining structure. These site specific issues are best suited to be solved by an optimization algorithm owing to their difficulty in manual design. As optimization heavily benefits from advance computational capabilities, complexities like those of lateral pressures through trial wedge methods, development of slip circle calculations, use of multiple theories for lateral pressures, bearing pressures, settlement, and seismic loadings can also be easily investigated. This assimilation will optimize the entire design procedure for retaining wall design and can be extremely beneficial for site specific problems. Another popular topic is development of problem formulation and application of optimization for RC cantilever retaining walls using modern holistic algorithms. To test their dexterity and robustness sensitivity analysis are to be performed. However, most modern metaheuristic algorithms do not have significant advantage over another in terms of percentage of optimization achieved. Algorithms can be vastly superior to one another in terms of speed of optimization, their standard deviations, and variance from mean value. Hybridization and improvement of existing (traditional metaheuristic) techniques is another area of interest that can be probed by future researchers.

5.5. Advancements of Objective Functions and Pragmatic Approach. As the focus of global research has shifted towards sustainability, it is imperative that multiobjective optimization should be the top priority for future works as well. Sustainability is a complex objective that requires two or more functions to be fully defined. Multiple objectives can be experimented to prove what better constitutes sustainable RC cantilever retaining wall design. Cost, weight, carbon emissions, embodied energy, and cementitious material minimization can all contribute to this objective. Multiobjective studies are more complex and require specific algorithms that have nondominating sorting capabilities to form a Pareto front for solutions. Development of algorithms to solve multiobjective problems is also a task of imperative importance. Novel objectives like coordinates of wall, to optimize the entire shape of cantilever retaining wall, are another unique idea that requires further investigation. Lastly, no research compares the optimized designs of a retaining wall with an already built retaining wall. The cost minimization type of objectives would be best evidenced if compared with real world structures and their total costs. An ambitious project could be the physical construction of a prototype of the optimized retaining wall to demonstrate the effectiveness of objective based design in practice. Optimization techniques will be more approachable and acceptable for the construction industry only after they offer solutions to on-site problems, provide proof of effectiveness, and do not constitute mathematical and applicability complications.

6. Concluding Remarks

Optimization is sure to revolutionize the structural design process as the need for objective based complex structures becomes a priority. However, for optimization to reach at that level, it must gain the confidence of field professionals and become their go to tool for optimum design. To tackle these issues this review paper provides an in-depth review on optimization of reinforced concrete cantilever retaining walls. The review paper summarizes all the works conducted on this topic in a concise and effective manner. A clear trend towards advanced and hybrid metaheuristic algorithms is visible. These algorithms are more robust and require smaller computational efforts compared to their traditional counterparts. A significant lack of sustainable and multiobjective optimization has also been identified. There is a need to improve the practicability of algorithm based optimum designs. The use of discrete variables with a sufficient and practical step size would be beneficial for field adaptation. Similarly, constraints must include bar diameters, reinforcement spacing, checks for development lengths, and inclusion of material strength parameters to make the optimization model pragmatic. Lack of any case study of a real world retaining wall or a study model constructed with modern optimization techniques is also a cause of skepticism for engineers. A comparative study on the performance of a built optimized retaining wall would improve its credence. However, the most major hurdle currently is identified as lack of any tool or program that can directly apply optimization on an engineer's initial design estimate. Easing applicability of optimization and reducing its multidisciplinary complexities is the best way to improve its acceptance and use in construction industry. Lastly, utilizing the core capability of optimization, i.e., superior computational capabilities to present unique solutions to large scale complex and multifaceted problem with ease should be the objective for future research. Incorporating optimization with reliability based design, mix design, finite element modeling capabilities, and building integrated modeling techniques are some such unique applications that could make optimization the definitive tool for advanced structural design.

Abbreviations

As_f: Area of steel reinforcement in footing
$As_{(\text{heel})}$: Area of steel reinforcement in heel
As_{max}: Maximum area of steel reinforcement
As_{min}: Minimum area of steel reinforcement
$As_{(\text{stem})}$: Area of steel in stem
$As_{(\text{toe})}$: Area of steel in toe
B: Total width of footing
$B_{\text{ key}}$: Length from heel to face of key
b: Unit length of wall
C_c: Unit cost of concrete
$C_c\,(\text{co}_2)$: Unit carbon emissions of concrete
Cov_f: Cover of footing
C_s: Unit cost of steel reinforcement
Cov_{st}: Cover of stem
$C_s\,(\text{co}_2)$: Unit carbon emissions of steel
c: Cohesion of soil
Df: Depth of soil on toe
d: Effective depth of section
e: Eccentricity
F_{anchor}: Strength of anchor
F_D: Horizontal driving forces
F_R: Resisting forces
FS_o: Factor of safety (overturning)
FS_s: Factor of safety (sliding)
f_c: Compressive strength of concrete
f_y: Yield strength of steel
H: Total height of retaining wall
h_{anchor}: Height from top stem to anchor
h_{stem}: Height of stem
h_{top}: Height of top (slim) half of stem
l_{db}: Development length of bar
l_{dh}: Hook length of bar
M_O: Overturning moment
M_R: Righting moment
Mu: Factored moment
$q_{max\,/\,min}$: Maximum or minimum pressure on footing
q_u: Ultimate bearing capacity of soil
R: Point of resultant
T_f: Thickness of footing
T_{key}: Depth/thickness of key
T_{sb}: Thickness of stem bottom
T_{st}: Thickness of stem top
V_c: Volume of concrete
V_{Total}: Total vertical load
Vu: Factored shear
W_h: Width of heel
W_{key}: Width of key
W_{st}: Weight of steel reinforcement
W_t: Width of toe
β: Slope of backfill soil
γ_c: Unit weight of concrete
γ_s: Unit weight of soil
δ: Wall friction angle
ρ_{min}: Minimum allowable reinforcement ratio
ρ_{max}: Maximum allowable reinforcement ratio
μ: Coefficient of friction
ϕ: Internal angle of friction
ϕMn: Design moment capacity
ϕVn: Design shear capacity
ω_s: Surcharge load
T_{batter}: Thickness of battered portion of stem.

Data Availability

The data used to support this study are from previously reported studies and datasets, which have been cited at relevant places within the article as references. The methodologies for acquisition of these references have also been detailed in the text.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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