Research Article

Overlapping Decentralized Control Strategies of Building Structure Vibration Based on Fault-Tolerant Control under Seismic Excitation

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The vibration control system of a building structure under a strong earthquake can be regarded as a large complex system composed of a series of overlapping subsystems. In this paper, the overlapping decentralized control of building structure vibration under seismic excitation is studied. Combining the overlapping decentralized control method, \( H_\infty \) control algorithm, and passive fault-tolerant control method, a passive fault-tolerant overlapping decentralized control method based on the \( H_\infty \) control algorithm is proposed. In this paper, the design of robust \( H_\infty \) finite frequency passive fault-tolerant static output feedback controller for each subsystem is studied. The fault matrix of the subcontroller is expressed by a polyhedron with finite vertices. In order to reduce the influence of external disturbance on the controlled output, the finite frequency \( H_\infty \) control is adopted and the Hamiltonian matrix is avoided. In this paper, the passive fault-tolerant overlapping decentralized control method based on \( H_\infty \) control algorithm is applied to the vibration control system of the four-story building structure excited by the Hachinohe seismic wave. One drive is set on each layer of the structure, and a total of four drives are set. Select the driver fault factor of 0.5 or 1 and the frequency band \([0.3, 8]\) Hz. The overlapping decentralized control scheme and 16 fault-tolerant fault matrices are designed, and the numerical comparison results are given. The results show that both overlapping decentralized control strategy and multi-overlapping decentralized control strategy have achieved good control results. Due to the different number of subsystems and overlapping information, the overlapping decentralized control scheme increases the flexibility of controller setting and reduces the computational cost.

1. Introduction

Structure vibration control strategies have been widely used in the seismic field of civil structures. A large number of microsmart drivers and sensors in the control process does not guarantee the eternal function. Each component is vulnerable to partial or total failure. Because the building structure is a large-scale multidegree-of-freedom structure, the research work mainly focuses on the model order reduction theory, parameter uncertainty control theory, decentralized control theory, etc. However, these theories all need to be considered the robustness of the control system.

In the application of structure vibration control technology in the building structure control system, the control device is usually installed on the floor of the structure, and the corresponding control force is exerted on the structure through the device so as to realize the vibration control of the building structure. The vibration control unit consists of a controller, sensors, and actuators. The controller calculates the corresponding control force according to the structural
response collected by the sensor, and the actuator is responsible for exerting the calculated control force to the structure to reduce the structural response under external excitation. However, in the actual construction structure engineering, the control device installed on the structure floor is vulnerable to the influence of factors such as delayed maintenance and damage caused by the external load, and there will be actuator (driver) or sensor failure or even lead to complete failure. In complex engineering systems, safety and reliability are very important. Fault-Tolerant Control (FTC) provides a new way to solve the reliability problem of large complex control systems.

Therefore, when designing the control system, it should be considered that the fault of all or part of the components can be recovered, and the fault-tolerant control method can effectively solve this problem. Fault-tolerant control has the function of making the system feedback insensitive to a fault. Fault-tolerant control can be divided into passive fault-tolerant control and active fault-tolerant control, each of which has its own characteristics. Passive fault-tolerant control can be regarded as traditional robust control without online identification and treats faults as an uncertainty. Passive fault-tolerant control has limited fault-tolerant capacity. However, compared with active fault-tolerant control, passive fault-tolerant control has the advantage of not requiring accurate actuator fault information. In addition, passive fault-tolerant control ensures system stability and expected control performance in the event of actuator failure. Active fault-tolerant control can be understood as generalized robust control, and the control law of the controller needs to be readjusted after the occurrence of a fault, and it can overcome the characteristics of passive fault-tolerant control.

Structural active control had used sensors and actuators to modify and enhance the resistance of the structure to the external environment [1]. Due to the increasing awareness of seismic risk and the challenges to structures in extreme environments, active control technology has received high attention in the past decades [2]. In the control strategy, if the system is far away from the position of the equilibrium state, it is particularly important to choose higher system stiffness. If the system returns to equilibrium, the stiffness value should be set to a lower value. Ramaratnam and Jalili [3] have studied the conversion stiffness method for structural vibration control. Jalili and Knowles IV [4] have controlled structural vibration by using active shock isolation devices. Moon et al. [5] has applied linear magnetostrictive actuator to structural vibration control. Fallah and Ebrahimnejad [6] have used piezoelectric actuators in active vibration control of building structures. Reithmeier and Leitmann [7] have applied Lyapunov stability theory to a structural vibration control system subject to control force constraints and arbitrary control inputs. However, the above control methods are all based on the infinite frequency domain control methods, and compared with the finite frequency domain control theory, they are more conservative. In fact, more and more studies have focused on the control theory of the finite frequency domain in practical problems [8]. Chen et al. [9] has studied the H∞ control problem of structural vibration under seismic excitation in the finite frequency domain, designed the state feedback controller to reduce the structural response, and proved the asymptotic stability of the closed-loop system. According to relevant literature, although the seismic wave is in the infinite frequency domain, only the frequency spectrum whose amplitude exceeds 0.4 can cause greater damage to the building structure is limited [10]. In recent years, according to the system with overlapping decomposition structure, the overlapping controller is designed by the inclusion principle, and the positive nature of the distributed controller is maintained in the design process. The overlapping decomposition method based on the inclusion principle has been applied in many fields and has effectively solved the control problems of various complex, large-scale systems, such as mechanical structure [11], applied mathematics [12], power system [13], automatic highway system [14], and aerospace engineering [15]. At the same time, an overlapping decentralized control strategy has been applied in the civil engineering field [16–21]. In other words, remarkable control effects can be achieved by suppressing seismic waves in a certain frequency domain. Therefore, the finite frequency domain controller can improve the seismic performance of the building structure.

However, in many practical engineering applications, not all states are used for controller design due to the cost of the sensor. In this case, only the measured output can be used to build a closed-loop system. That is, the output feedback control will save cost. At present, there are little researches on fault-tolerant overlaps and decentralized control methods to solve the vibration problem of multidegree-of-freedom structure building structures. The overlapping decentralized control strategy is to divide the whole structure vibration control system into several overlapping subsystems according to certain rules, and each subsystem uses local information of subsystem to control independently.

To study the control effect of overlapping decentralized control strategy to solve the vibration problem of building structure under earthquake excitation. Firstly, this paper introduces the control problem of a linear building structure system with n degrees of freedom. Secondly, this paper proposes a passive fault-tolerant overlapping decentralized control method based on the H∞ control algorithm by combining the overlapping decentralized control method, H∞ control algorithm, and passive fault-tolerant control method. Finally, the failure factor of the driver is selected as 0.5 or 1, and the frequency band [0.3, 8] Hz. The passive fault-tolerant overlaps and decentralized control method based on the H∞ control algorithm is applied to the vibration control system of a four-story building under the excitation of Hachinohe seismic wave. Four overlaps and decentralized control schemes and 16 fault-tolerant fault matrices were designed, and the numerical comparison results were given. Overlapping decentralized control strategy provides a new way to solve the vibration control problem of building structures. Since the number of subsystems and overlapping information is different, the overlapping decentralized control scheme increases the flexibility of the controller setting.
2. Description of Structural Control System Problems

For the linear building structural system with \( n \) degrees-of-freedom shown in Figure 1, under the action of earthquake ground motion, the motion equation can be formulated as follows:

\[
M \ddot{q}(t) + C \dot{q}(t) + K q(t) = T_u u(t) + E w(t),
\]

where \( M, C \) and \( K \in \mathbb{R}^{m \times m} \) are the mass matrix, damping matrix, and stiffness matrix of the building structural system, respectively; \( T_u \) is the position matrix of control force; \( E \) is the position vector of seismic excitation; \( q(t), \dot{q}(t), \) and \( \ddot{q}(t) \) are the displacement, velocity, and acceleration vectors of each floor of the structure relative to the ground, respectively.

The second-order ordinary differential equation in formula (1) can be converted to the first-order ordinary differential equation as follows [22]:

\[
\dot{x}_1(t) = A_1 x_1(t) + B_1 u(t) + E_1 w(t),
\]

where \( x_1 = [q(t); \dot{q}(t)] \in \mathbb{R}^{2n \times 1} \) is the state vector; \( A_1 \in \mathbb{R}^{2n \times 2n}, B_1 \in \mathbb{R}^{2n \times m}, \) and \( E_1 \in \mathbb{R}^{2n \times m} \) are system control, and excitation matrices, respectively:

\[
A_1 = \begin{bmatrix}
0_{2n 	imes n} & I_{2n 	imes n} \\
-M^{-1}K & -M^{-1}C
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0_{2n 	imes m} \\
M^{-1}T_u
\end{bmatrix},
\]

\[
E_1 = \begin{bmatrix}
0_{2n 	imes 1} \\
-1_{2n 	imes 1}
\end{bmatrix}.
\]

At this point, we can define a new state vector:

\[
x(t) = \varphi x_1(t),
\]

where

\[
\varphi = \begin{cases}
\varphi_{1,1} = 1, & \varphi_{2,2j+1} = 1, \\
\varphi_{2i-1,j} = -1, & \varphi_{2i-1,j+1} = 1, \ 1 < i \leq n, \\
\varphi_{2i,j+1} = -1, & \varphi_{2i,j} = 1, \ 1 < i \leq n, \\
\varphi_{j,j} = 0, & \text{otherwise}.
\end{cases}
\]

By defining state vector \( x = \varphi x_1 \), substitute \( x_1 = \varphi^{-1} x \) into equation (2) and \( \varphi \) multiply the equation left, the first-order motion equation of the following form can be written as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t) + E w(t),
\]

where, \( A = \varphi A_1 \varphi^{-1}, B = \varphi B_1, E = \varphi E_1 \) and

\[
x(t) = \begin{bmatrix}
q_1(t), \dot{q}_1(t), (q_2(t) - q_1(t)), (\dot{q}_2(t) - \dot{q}_1(t)), \\
\vdots, (q_n(t) - q_{n-1}(t)), (\dot{q}_n(t) - \dot{q}_{n-1}(t))
\end{bmatrix}^T.
\]

3. Passive Fault-Tolerant Control


Considering the equation (7) of the continuous time linear system of the building structure, this section introduces the passive fault-tolerant control technique of the driver, and the possibility of failure of the driver is taken into account. Thus, the state feedback control law can be expressed as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t) + E w(t),
\]

\[
y(t) = C_x x(t),
\]

\[
z(t) = C_z x(t) + D_z u(t),
\]

where \( x(t) \) is the state vector of the system; \( u(t) \) is the control input; \( w(t) \) is interference input; \( z(t) \) is the control output. The matrices \( A, B, E, C_x, C_z \) and \( D_z \) are constant matrices of appropriate dimensions.

\[
M_f = \text{diag} \{ M_{f_1}, \ldots, M_{f_n} \},
\]

where \( M_{f_i} \) is the fault matrix used to describe a drive fault. Therefore, the assumed fault matrix \( M_f \) can be written as follows:

\[
u(t) = M_f G y(t),
\]
drive is free of any failure. If $M_{fi}$ is between 0 and 1, the corresponding drive is partially failing. A complete failure of the drive indicates that the controller does not have any control. In this case, the system is opened loop. For the convenience of controller design, it can be assumed according to the actual situation,

$$0 \leq M_{fi} \leq M_{fi} \leq \bar{M}_{fi} \leq 1, \ (i = 1, \ldots, n). \quad (10)$$

Since $M_{fi}$ is a function of time between $[M_{fi}, \bar{M}_{fi}]$, the fault matrix $M_f$ can be described as follows:

$$M_f(\alpha) = \sum_{j=1}^{2^n} a_j M^2_f; \sum_{j=1}^{2^n} a_j = 1, \ a_j \geq 0, \quad (11)$$

where

$$M^2_f = \text{diag} \{M_{fi}, \ldots, M_{fn}\}, \ldots, M^2_f = \text{diag} \{\bar{M}_{fi}, \ldots, \bar{M}_{fn}\}$$

are constant matrices.

By substituting the fault-tolerant control law shown in equation (8) into system equation (7), a closed-loop system can be obtained:

\[
\begin{cases}
\dot{x}(t) = \bar{A}(\alpha)x(t) + Ew(t), \\
z(t) = C_zx(t) + D_zw(t),
\end{cases}
\]

where $B(\alpha) = BM_f(\alpha); \bar{A}(\alpha) = A + B(\alpha)GC_y$. For a deterministic $\alpha$, the transfer function from the disturbance $w(t)$ to the control output $z(t)$ can be expressed as follows [23]:

$$T_{zw}(s) = C_z(sI - \bar{A})^{-1}E + D_z. \quad (13)$$

Finite frequency static output feedback H∞ control can determine the gain matrix $G$ such that the closed-loop system equation (12) is asymptotically stable and satisfies the following inequality:

$$\sup_{\omega_1 \leq \omega \leq \omega_2} \|T_{zw}(j\omega)\|_\infty < \gamma. \quad (14)$$

Therefore, the aim is to design a feedback controller equation (8) with the possibility of driver failure and make the closed-loop system equation (12) asymptotically stable and satisfy the H∞ control condition equation (14) in the finite frequency domain. If the designed controller satisfies the above conditions, it is called a fault-tolerant feedback controller.

3.2. H∞ Control Theory in Passive Fault-Tolerant Finite Frequency Domain. Because H∞ control in finite frequency domain involves complex Hamilton matrix [24]. In this section, a passive fault-tolerant H∞ control method based on linear matrix inequality (LMI) is deduced according to the relevant lemma.

**Lemma 1.** Assuming $\bar{A}$ and $E$ are both real matrices and $\Theta$ are symmetric matrices, $\Phi \in \mathbb{S}_2$ and $\Psi \in \mathbb{H}_2$ can define curves in a complex plane. And, the following two statements are equivalent [25]:

(1) If $\Lambda \in \Lambda(\Phi, \Psi)$, then the following inequality is satisfied for all nonzero $(u, v) \in N_A(\bar{A}(\alpha), E)$:

$$\begin{bmatrix} u \\ v \end{bmatrix}^H \Theta \begin{bmatrix} u \\ v \end{bmatrix} < 0. \quad (15)$$

(2) For all nonzero $(u, v) \in N_A(\bar{A}(\alpha), E)$, there are symmetric matrices $P$ and $Q$ that satisfy

$$\begin{bmatrix} \bar{A}(\alpha) & E \\ I & 0 \end{bmatrix}^T (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} \bar{A}(\alpha) & E \\ I & 0 \end{bmatrix} + \Theta > 0, \quad (16)$$

where a complex plane can be defined as follows:

$$\Lambda \in \Lambda(\Phi, \Psi) = \left\{ \Lambda \in \mathbb{R} \left| \begin{bmatrix} \Lambda^H & \Lambda \end{bmatrix} = 0 \right. \right\}. \quad (17)$$

If $\Lambda \neq \infty$, then,

$$N_A(\bar{A}(\alpha), E) = \{ (u, v) \in \mathbb{R}^n \times \mathbb{R}^m \mid (\Lambda I - A)u = Ev \}. \quad (18)$$

If $\Lambda \neq \infty$, then, $N_A(\bar{A}(\alpha), E) = \{ 0 \} \times \mathbb{R}^b$.

From Lemma 1 and the transfer function considering the relevant parameters of the system, the following lemma can be established.

**Lemma 2.** With respect to the transfer function $T_{zw}(s)$, given a symmetric matrix $\Omega$, the following two statements are equivalent:

(1) When $\omega_1 \leq \omega \leq \omega_2$, the inequality holds in the finite frequency domain:

$$\begin{bmatrix} T_{zw}(j\omega) \\ I \end{bmatrix}^H \Omega \begin{bmatrix} T_{zw}(j\omega) \\ I \end{bmatrix} < 0. \quad (19)$$

(2) The existence of symmetric matrices $P$ and $Q_p$ makes the following inequality true:

$$Q_p > 0, \left\{ \begin{bmatrix} P & Q_p \end{bmatrix} \begin{bmatrix} C_z & D_z \end{bmatrix}^T \right\} < 0, \quad (20)$$

where

$$\Gamma[P, Q_p] = \begin{bmatrix} \bar{A}(\alpha) & E \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q_p & P + j\omega Q_p \\ P - j\omega Q_p & -\omega_1 \omega_2 Q_p \end{bmatrix} \begin{bmatrix} \bar{A}(\alpha) & E \\ I & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \text{sym}(D_z^T \Pi_{12}) + \Pi_{22} \end{bmatrix}, \quad (21)$$
where \( \omega_c = (\omega_1 + \omega_2)/2 \); \( \Omega_{12} \) and \( \Omega_{22} \) are the upper right and lower left positions of block matrix \( \Omega \), respectively.

Considering the finite frequency domain condition equation (14), the following lemma can be established.

**Lemma 3.** For the transfer function of the actual data system matrix, if there are symmetric matrices \( P \) and \( Q_p \) that make

\[
\Gamma_1 [P, Q_p] = \begin{bmatrix} \overline{A}(\alpha) & E \end{bmatrix}^T \begin{bmatrix} -Q_p & P + j \omega_s Q_p \\ P - j \omega_s Q_p \end{bmatrix} \begin{bmatrix} \overline{A}(\alpha) & E \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\gamma^2 I \end{bmatrix}. \tag{23}
\]

As the system matrix \( \overline{A}(\alpha) \) and positive definite matrix \( Q_p \) are coupled together in Lemma 3, projection lemma is introduced to understand the coupling.

**Lemma 4.** (projection lemma [26]): For a given real symmetric matrix \( \Pi \) and two real matrices \( \Lambda_1 \) and \( \Lambda_2 \), and considering a matrix \( \Xi \) of appropriate dimensions, the following inequality holds:

\[
\Pi + \Lambda_1 \Xi \Lambda_2 + (\Lambda_1 \Xi \Lambda_2)^T < 0. \tag{24}
\]

The above equation can solve the matrix \( \Xi \) if and only if the correlation matrix satisfies the following:

\[
\begin{align*}
N^{T}_\Lambda_1 \Pi N^{T}_{\Lambda_1} < 0, \\
N^{T}_\Lambda_2 \Pi N^{T}_{\Lambda_2} < 0.
\end{align*} \tag{25}
\]

**Theorem 1.** For a given positive real number \( y \) and a known matrix \( G \), the closed-loop system equation (12) is asymptotically stable and the Hoo control performance in the finite frequency domain can be satisfied if there are matrices \( P = PT \), \( Q_p = QT > 0 \), and \( K = KT > 0 \):

\[
K \overline{A}(\alpha) + \overline{A}^T(\alpha)K < 0, \tag{26}
\]

\[
\begin{bmatrix}
-Q_p & \Lambda_5 & 0 \\
\Lambda_6 & 0 & 0 \\
C_z^T & C_z & 0 \\
\end{bmatrix} < 0, \tag{27}
\]

where

\[
\Lambda_5 = P + j \omega_s Q_p - K^T; \quad \Lambda_6 = -\omega_s Q_p + K \overline{A}(\alpha) + \overline{A}^T(\alpha)K.
\]

**Proof 1.** By using Schur’s complement lemma, equation (27) can be rewritten as follows:

\[
\Pi_1 + \text{sym}(\Lambda_{11} \Xi \Lambda_{21}) < 0, \tag{28}
\]

where the inequality equation (22) valid, then the condition equation (14) in the finite frequency domain can be proved.

\[
\begin{bmatrix}
\Gamma_1 [P, Q_p] & [C_z, D_z]^T \\
* & -I
\end{bmatrix} < 0, \tag{22}
\]

where

\[
\begin{align*}
\Pi_1 &= \begin{bmatrix}
-Q_p & P + j \omega_s Q_p & 0 \\
* & -\omega_s Q_p + C_z^T C_z & C_z^T D_z \\
* & * & -\gamma^2 I + D_z^T F \omega
\end{bmatrix}, \\
\Lambda_{11} &= \begin{bmatrix}
-I & \overline{A}(\alpha) & E \\
0 & 0 & I
\end{bmatrix}, \\
\Lambda_{21} &= \begin{bmatrix}
0 & I & 0
\end{bmatrix}.
\end{align*} \tag{29}
\]

The null Spaces of the matrices \( \Lambda_{11} \) and \( \Lambda_{21}^T \) are, respectively,

\[
N_{\Lambda_{11}} = \begin{bmatrix}
\overline{A}(\alpha) & I & 0 \\
E & 0 & 0
\end{bmatrix}, \tag{30}
N_{\Lambda_{21}^T} = \begin{bmatrix}
I & 0 & 0 \\
0 & 0 & I
\end{bmatrix}.
\]

According to the projection theorem, the inequality equation (28) holds if the following inequality can be satisfied:

\[
N^{T}_{\Lambda_{11}} \Pi_1 N_{\Lambda_{11}} < 0, \quad N^{T}_{\Lambda_{21}^T} \Pi_1 N_{\Lambda_{21}^T} < 0. \tag{31}
\]

To prove that the closed-loop system equation (12) is asymptotically stable, the following Lyapunov function can be chosen:

\[
V(x(t)) = x^T(t)Kx(t). \tag{32}
\]

According to literature [27], inequality equation (26) can be held. In the two-step controller design approach, a fault-tolerant state feedback controller equation (33) can first be designed so that the system equation (7) is stable.

\[
u(t) = M_f G_{sf} x(t). \tag{33}
\]

By substituting the state control law equation (33) into system equation (7), a closed-loop system can be obtained,

\[
\dot{x}(t) = \overline{A}_{sf}(\alpha) x(t) + E \omega(t), \tag{34}
\]

where \( \overline{A}_{sf} = A + B(\alpha) G_{sf} \).

For the closed-loop system equation (34), the state feedback gain can be calculated according to the following theorem.

\[ \square \]
Theorem 2. The closed-loop system equation (34) is asymptotically stable if $Q_2 = Q^T_2 > 0$ and $M_{sf}$ exist, so that equation (35) holds.

$$AQ_2 + Q_2 A^T + B^T M_{sf} + (B^T M_{sf})^T < 0,$$  \hspace{1cm} (35)

where $B^T = B M^T_j, j = 1, \ldots, 2^n$. If equation (35) is solvable, then the state feedback gain $G_{sf}$ can be as follows:

$$G_{sf} = M_{sf} Q^{-1}_2.$$  \hspace{1cm} (36)

Proof 2: Equation (35) can be simplified as follows:

$$AQ_2 + Q_2 A^T + B(a) M_{sf} + (B(a) M_{sf})^T < 0.$$ \hspace{1cm} (37)

For the unforced closed-loop system equation (34), consider the following Lyapunov function:

$$V(x(t)) = x^T(t) Q_2^{-1} x(t),$$  \hspace{1cm} (38)

where $Q_2$ is a symmetric positive definite Lyapunov weight matrix.

The unforced closed-loop system equation (34) is asymptotically stable if the following inequality holds:

$$Q^{-1}_2 (A + B(a) G_{sf}) (A + B(a) G_{sf})^T Q_2^{-1} < 0.$$  \hspace{1cm} (39)

Once the state feedback gain $G_{sf}$ can be obtained, the second step in the controller design is to determine the static output feedback gain $G$. \hfill □

Theorem 3. For a given positive real number $\gamma$, the closed-loop system equation (12) is asymptotically stable and the $\Hinf$ control performance equation (14) in the finite frequency domain can be satisfied if there are matrices $P = P^T$, $Q_p = Q^T_p > 0$, $K = K^T > 0$, $K_{of}$ and $N$,

$$\begin{bmatrix} \Lambda_7 & \Lambda_4 \\ \asterisk & -N - N^T \end{bmatrix} < 0,$$  \hspace{1cm} (40)

$$\begin{bmatrix} -Q_p & \Lambda_5 \\ \asterisk & -\omega_1 \omega_2 Q_p + \Lambda_7 & KE & C^T_z & \Lambda_8 \\ \asterisk & \asterisk & -\gamma^2 I & D^T_z & 0 \\ \asterisk & \asterisk & \asterisk & -I & 0 \\ \asterisk & \asterisk & \asterisk & \asterisk & -N - N^T \end{bmatrix} < 0,$$  \hspace{1cm} (41)

where $\Lambda_7 = KA + KB^T G_{sf} + (KA + KB^T G_{sf})^T; \Lambda_8 = KB^T + (K_{of} C_p - N G_{sf})^T$. The feedback gain $G$ can be obtained by calculating equation $G = N^{-1} K_{of}$.

Proof 3: Applying the subdivideable space aggregate performance, equations (40) and (41) mean that

$$\begin{bmatrix} \Lambda_9 & \Lambda_{10} \\ \asterisk & -N - N^T \end{bmatrix} < 0,$$  \hspace{1cm} (42)

$$\begin{bmatrix} -Q_p & \Lambda_5 \\ \asterisk & -\omega_1 \omega_2 Q_p + \Lambda_7 & KE & C^T_z & \Lambda_8 \\ \asterisk & \asterisk & -\gamma^2 I & D^T_z & 0 \\ \asterisk & \asterisk & \asterisk & -I & 0 \\ \asterisk & \asterisk & \asterisk & \asterisk & -N - N^T \end{bmatrix} < 0,$$  \hspace{1cm} (43)

where $\Lambda_9 = KA + KB (a) G_{sf} + (KA + KB (a) G_{sf})^T; \Lambda_{10} = KB (a) + (K_{of} C_p - N G_{sf})^T$. By defining a new variable $S = GC_y - G_{sf}$, equation (27) can be written as follows:

$$\begin{bmatrix} -Q_p & \Lambda_5 \\ \asterisk & -\omega_1 \omega_2 Q_p + \Lambda_7 & KE & C^T_z & \Lambda_8 \\ \asterisk & \asterisk & -\gamma^2 I & D^T_z & 0 \\ \asterisk & \asterisk & \asterisk & -I & 0 \\ \asterisk & \asterisk & \asterisk & \asterisk & -N - N^T \end{bmatrix} < 0,$$  \hspace{1cm} (44)

where $\Lambda_{11} = -\omega_1 \omega_2 Q_p + K (A + B (a) G_{sf}) + (K (A + B (a) G_{sf}) + (K (A + B (a) G_{sf})^T$. According to the new variable $S$, equation (44) can be expressed as follows:

$$\begin{bmatrix} -Q_p & \Lambda_5 \\ \asterisk & -\omega_1 \omega_2 Q_p + \Lambda_7 & KE & C^T_z & \Lambda_8 \\ \asterisk & \asterisk & -\gamma^2 I & D^T_z & 0 \\ \asterisk & \asterisk & \asterisk & -I & 0 \\ \asterisk & \asterisk & \asterisk & \asterisk & -N - N^T \end{bmatrix} < 0.$$  \hspace{1cm} (45)

where

$$\begin{bmatrix} -Q_p & \Lambda_5 \\ \asterisk & -\omega_1 \omega_2 Q_p + \Lambda_7 & KE & C^T_z & \Lambda_8 \\ \asterisk & \asterisk & -\gamma^2 I & D^T_z & 0 \\ \asterisk & \asterisk & \asterisk & -I & 0 \\ \asterisk & \asterisk & \asterisk & \asterisk & -N - N^T \end{bmatrix} < 0.$$  \hspace{1cm} (46)

The right orthogonal complement of matrix $\Gamma^T$ is as follows:

$$N_{\Gamma^T} = [0 \ 0 \ 0 \ 0 \ -I].$$  \hspace{1cm} (47)

If the feedback regular is a state where the feedback is controlled, the variable $S$ is equal to a zero matrix. The left orthogonal complement of the matrix $\Gamma_0$ is as follows:

$$N_{\Gamma_s} = [0 \ 0 \ 0 \ 0 \ I].$$  \hspace{1cm} (48)
By using the projection lemma, if equation (45) exists, then equation (43) can be proved to hold. Therefore, the proof of equation (42) ensures that equation (26) can be obtained. □

**Corollary 1.** The static output feedback H∞ control problem can be expressed as the following convex optimization problem:

\[
\begin{align*}
&\text{minimize } \gamma^2, \\
&\text{Satisfaction Equations (40) and (41), } j = 1, \ldots, 2^n.
\end{align*}
\]

(50)

3.3. Fault-Tolerant Overlapping Decentralized Control. In this section, the overlapping decentralized control strategy is combined with the passive fault-tolerant control method, and the structure overlapping decentralized fault-tolerant control method based on the H∞ control algorithm is proposed. The calculation steps of the proposed control method are described as follows:

(1) The motion equation of the shear model of the n-story building structure is shown in equation (1). According to the inclusion principle and decomposition principle in reference [28] and considering the control output \( z(t) = C_x x(t) + D_x u(t) \), the first-order continuous-time state model equation (7) of the entire building structural vibration control system is extended and decoupled into a series of overlapping subsystems \( \tilde{S}^{(i)} (i = 1, 2, \ldots, L) \):

\[
\tilde{S}^{(i)}: \left\{ \begin{array}{l}
\dot{x}_i(t) = \tilde{A}_i x_i(t) + \tilde{B}_i u_i(t) + \tilde{E}_i \bar{u}_i(t), \\
\bar{y}_i(t) = (\tilde{C}_i)_j x_i(t), \\
\bar{z}_i(t) = (\tilde{C}_z)_{ij} \tilde{x}_i(t) + (\tilde{D}_z)_{ij} \bar{u}_i(t), \\
i = 1, 2, \ldots, L; 2 \leq L \leq n - 1.
\end{array} \right.
\]

(51)

(2) Use the passive fault-tolerant control method to calculate the feedback gain matrix \( \tilde{G}^{(i)} (i = 1, 2, \ldots, L) \) of each subsystem. For each overlapping subsystem in equation (51), the following iterative algorithm is used in this section to solve the convex optimization problem in equation (50) to obtain the feedback gain matrix of each subsystem.

Step 1: Calculate the initial state feedback gain \( \tilde{G}^{(i)}_{sf,0} \) by using Theorem 2;

Step 2: When \( k = 0 \), the initial static output feedback gain \( G^{(i)}_{sf,0} \) and H∞ performance \( \gamma_0^2 \) can be obtained by using the initial state feedback gain \( \tilde{G}^{(i)}_{sf,0} \) and solving the convex optimization equation (50) of Corollary 1;

Step 3: while \( k < N_{\text{max}} \) do,

(1) \( k = k + 1 \)

(2) \( G^{(i)}_{sf,k} = G^{(i)}_{sf,k-1} (\tilde{C}_y)_k \)

(3) Solve the convex optimization problem of Corollary 1 to obtain the initial static output feedback gain \( \tilde{G}^{(i)}_{sf,k} \) and H∞ performance \( \gamma_k^2 \), if \( \gamma_k^2 < \gamma_k^2 < \epsilon \), then,

Exit the loop end while

Step 4: The static output feedback gain \( \tilde{G}^{(i)}_D \) (i = 1, 2, \ldots, L) of each subsystem can be obtained by this algorithm.

(3) According to the feedback gain \( \tilde{G}^{(i)}_D \) (i = 1, 2, \ldots, L) calculated in the above steps, the diagonal extended feedback gain matrix of equation (51) of the extended decoupled system is formed,

\[
\tilde{G}_D = \text{diag}\left[ \tilde{G}^{(1)} D, \tilde{G}^{(2)} D, \ldots, \tilde{G}^{(L)} D \right].
\]

(52)

(4) According to the contraction principle and linear transformation [28], the extended controller \( \tilde{G}_D \) is shrunk into an overlapping controller:

\[
G_p = Q \tilde{G}_D V.
\]

(53)

4. Control Schemes Design and Numerical Simulation

In order to verify the proposed structure overlapping decentralized passive fault-tolerant control method based on H∞ control algorithm, this section takes the four-story shearing building structure (see Figure 2) as an example, and its mass, stiffness, and damping parameters are [29–31]:

- \( m_1 = 450 \times 10^3 \text{ kg} \), \( m_2 = 345 \times 10^3 \text{ kg} \);
- \( k_1 = 18.05 \text{ kN/m} \), \( k_2 = 340 \times 10^3 \text{ kN/m} \);
- \( k_3 = 326 \times 10^3 \text{ kN/m} \);
- \( c_1 = 280 \times 10^3 \text{ kN/} \text{s} \), \( c_2 = 26.17 \times 10^3 \text{ kN/} \text{s} \), \( c_3 = 490 \times 10^3 \text{ kN/} \text{s} \), \( c_4 = 467 \times 10^3 \text{ kN/} \text{s} \), \( c_5 = 410 \times 10^3 \text{ kN/} \text{s} \). The seismic load was Hachinohe seismic wave (Magnitude 8.3) on May 16, 1968, the peak acceleration was 2.250 m/s², the duration was 36 s, and the sampling step was 0.01 s. In Figure 2, \( a_i \) (i = 1, 2, 3, 4) are the control devices. The Hachinohe seismic wave is shown in Figure 3. Based on the principle of the wavelet transform, the energy distribution diagram of the input time history on the natural frequency-time plane can be obtained (Figure 4). The overlapping decentralized control scheme of building a structural vibration system is shown in Figure 5.

In the control output setting, only the feedback of velocity of the structural floor is considered, and the feedback of floor displacement is not considered. A drive is set in each layer of the structure, and a total of four drives are set. The fault matrix \( M_f \) of equation (10) over time is shown in Figure 6. Driver failure factor is 0.5 or 1. Wherein, the coefficient 0.5 is expressed as half failure of the drive, and the coefficient 1 is expressed as no failure of the drive. The failure matrix has the following 16 cases. Namely,
$M_j^f = \text{diag}\{a_j, a_2, a_3, a_4\}, \ a_j = 0.5 \text{ or } 1; \ j = 1, 2, 3, 4.$ can be specifically expressed as follows:

\begin{align}
M_j^1 &= \text{diag}\{0.5, 0.5, 0.5, 0.5\}, \ M_j^2 = \text{diag}\{1, 0.5, 0.5, 0.5\}, \ M_j^3 = \text{diag}\{0.5, 1, 0.5, 0.5\} \\
M_j^4 &= \text{diag}\{0.5, 0.5, 1, 0.5\}, \ M_j^5 = \text{diag}\{0.5, 0.5, 0.5, 1\}, \ M_j^6 = \text{diag}\{1, 1, 0.5, 0.5\} \\
M_j^7 &= \text{diag}\{1, 0.5, 1, 0.5\}, \ M_j^8 = \text{diag}\{1, 0.5, 0.5, 1\}, \ M_j^9 = \text{diag}\{0.5, 1, 1, 0.5\} \\
M_j^{10} &= \text{diag}\{0.5, 1, 0.5, 1\}, \ M_j^{11} = \text{diag}\{0.5, 0.5, 1, 1\}, \ M_j^{12} = \text{diag}\{0.5, 1, 1, 1\} \\
M_j^{13} &= \text{diag}\{1, 0.5, 1, 1\}, \ M_j^{14} = \text{diag}\{1, 1, 0.5, 1\}, \ M_j^{15} = \text{diag}\{1, 1, 1, 0.5\} \\
M_j^{16} &= \text{diag}\{1, 1, 1, 1\}. \tag{54}
\end{align}
When the frequency band \[0.3, 8\] Hz is selected, an iterative algorithm is applied to solve the structural vibration control inequality. The gain matrix of state feedback obtained from the first iteration of Case1 in Figure 5 can be obtained:

\[
G_{sf, Case1} = 10^8 \times \begin{bmatrix}
\end{bmatrix}
\]
Figure 7 shows the controller singular values of Case1 scheme under consideration of 16 fault-tolerant fault matrices. As can be seen from the figure, the optimal singular value \( y_{\text{Case1}} = 7.8225 \), and the corresponding static output feedback control gain matrix is as follows:

\[
G_{\text{Case1}} = 10^8 \times \begin{bmatrix}
4.7434 & 4.9433 & 5.7701 & -2.4312 \\
8.1733 & -2.2761 & -8.8142 & 1.6732 \\
-6.9247 & -5.4721 & 3.2535 & -6.3904 \\
-2.323  & 3.8482 & -1.3565 & 8.8640
\end{bmatrix}.
\]  

(56)

When considering the Case2 overlapping decentralized control scheme in Figure 5, the vibration control system of the whole four-story building structure can be divided into two overlapping subsystems \( \bar{S}^{(1)} \) and \( \bar{S}^{(2)} \) according to equation (51) in Section 3.3. Static output feedback gains \( \bar{G}^{(1)}_c \) and \( \bar{G}^{(2)}_c \) of subsystems \( \bar{S}^{(1)} \) and \( \bar{S}^{(2)} \) can be obtained from Section 3.3. Considering 16 fault tolerance matrices \( M^f_i \), \( i = 1, 2, \ldots, 16 \), the static feedback gain matrix \( G_{\text{Case2}} \) of Case2 scheme and the corresponding optimal singular value \( y_{\text{Case2}} = 8.6592 \) can be obtained according to equations (52) and (53). The singular values of the controller under the Case2 scheme are shown in Figure 8.

\[
G_{\text{Case2}} = 10^8 \times \begin{bmatrix}
-3.9331 & -2.7410 & 0 & 0 \\
-3.9327 & -1.8275 & 6.2847 & -3.9142 \\
0 & -1.8273 & -9.3248 & 8.43035 \\
0 & 1.3750 & 4.2053 & 2.2391
\end{bmatrix}.
\]  

(57)

Considering the Case3 multi-overlapping decentralized control scheme in Figure 5, the vibration control system of the whole four-story building structure can be divided into three overlapping subsystems \( \bar{S}^{(i)} \) \( i = 1, 2, 3 \). According to equation (51) in Section 3.3, static output feedback gains \( \bar{G}^{(1)}_c \), \( \bar{G}^{(2)}_c \), and \( \bar{G}^{(3)}_c \) of the three subsystems can be obtained by considering the 16 fault-tolerant fault matrices \( M^f_i \), \( i = 1, 2, \ldots, 16 \) in Section 3.3. According to equations (52) and (53), the static feedback gain matrix \( G_{\text{Case3}} \) of Case3 scheme and the corresponding optimal singular value \( y_{\text{Case3}} = 9.48249 \) can be obtained. The singular values of the controller under the Case3 scheme are shown in Figure 9.

\[
G_{\text{Case3}} = 10^8 \times \begin{bmatrix}
-1.7968 & -4.6898 & 0 & 0 \\
-6.5322 & -1.5132 & -5.5350 & 0 \\
0 & -6.1810 & 1.6297 & 3.0554 \\
0 & 0 & 1.050 5 & 5.7050
\end{bmatrix}.
\]  

(58)

When the building structure is externally excited by Hachinohe seismic wave, and 16 fault-tolerant fault situations are considered. The interstory displacement of the structure is shown in Table 1.

It can be seen from Table 1 that the overlapping decentralized passive fault-tolerant control method of building structure based on H∞ norm proposed in this paper is applied to the vibration control system of building structure under earthquake excitation and achieves a good control effect. Among them, the centralized control (Case1) scheme has the best control effect. Compared with the control effect of Case1 to Case3 in Table 1, the control effect is getting worse and worse with the increase of overlapping.
and dispersing times of the building structure. However, on the whole, both the overlapping decentralized control strategy (Csae2) and the multioverlapping decentralized control strategy (Csae3) achieve better control effects.

The time history of the control force installed on the drivers of each floor is shown in Figures 10–12. In the time history diagram of the control force, \( u_i \) (i = 1, 2, 3, 4) is the time history of the control force of corresponding digital floors.

5. Discussions

(1) The corresponding controller gain matrices are calculated according to different overlapping decentralized control strategies. The Maximum Singular Values \( c \) under the consideration of 16 fault-tolerant fault matrices become larger and larger as the degree of overlap and decentralization of structural vibration control schemes increases. However, the design of the controller can be calculated independently by the frequency of the structure itself when solving the vibration control problem of the actual engineering structure.

(2) According to the maximum interlayer displacement values in Table 1, the control rate is between 56.08% and 63.92% under the centralized control strategy (Case1). Under the overlapping decentralized control strategy (Case2), the control rate ranged from 47.77% to 52.50%. Under the multioverlapping decentralized control strategy (Case3), the control rate ranges from 43.95% to 47.92%. Therefore, in the building structural vibration control strategy, the control effect of the centralized control strategy is better than the overlapping decentralized control strategy and the multioverlapping decentralized control strategy.

(3) MATLAB software is used to program and calculate the different control strategies of the four-story building structural vibration control system. Among

<table>
<thead>
<tr>
<th>Stories</th>
<th>No control</th>
<th>Case1 (cm (%)</th>
<th>Case2 (cm (%))</th>
<th>Case3 (cm (%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.803</td>
<td>0.744 (58.74)</td>
<td>0.856 (52.50)</td>
<td>0.939 (47.92)</td>
</tr>
<tr>
<td>2</td>
<td>1.480</td>
<td>0.650 (56.08)</td>
<td>0.773 (47.77)</td>
<td>0.798 (46.10)</td>
</tr>
<tr>
<td>3</td>
<td>1.580</td>
<td>0.570 (63.92)</td>
<td>0.816 (48.35)</td>
<td>0.862 (45.47)</td>
</tr>
<tr>
<td>4</td>
<td>1.545</td>
<td>0.650 (57.93)</td>
<td>0.794 (48.61)</td>
<td>0.866 (43.95)</td>
</tr>
</tbody>
</table>
them, the running time of the centralized control strategy (Case1) is 1 hour, the running time of overlapping decentralized control strategy (Case2) is 20 minutes, and the running time of multi-overlapping decentralized control strategy (Case3) is 30 seconds. The overlapping decentralized control strategy (Case2) and the multi-overlapping decentralized control strategy (Case3) reduce the calculation cost.

6. Conclusions

The overlapping decentralized control is an effective method to solve complex, large-scale vibration control systems with overlapping information constraints. In this paper, an overlapping decentralized fault-tolerant control method based on H∞ control algorithm is proposed for the vibration control system of a 4-story building structure. Select the driver failure factor of 0.5 or 1 and the frequency band [0.3, 8] Hz. Using MATLAB program to solve and calculate the motion equation and linear matrix inequality in this paper, the results show the following.

(1) In this paper, finite frequency H∞ control strategy is adopted in the design of overlapping subcontrollers, and a two-stage method is proposed to solve the derived bilinear matrix inequality.

(2) In the passive fault-tolerant control system with 16 fault-tolerant fault matrices, the interstory displacement of the four-story building structure is effectively controlled. Among them, the centralized control (Case1) scheme has the best control effect. Compared with the control effect of Case1, the control effect is getting worse and worse with the increase of overlapping and dispersing times of building structures. However, on the whole, both the overlapping decentralized control strategy (Case2) and the multi-overlapping decentralized control strategy (Case3) achieve better control effects.

(3) When the building structure is subjected to seismic load excitation, the number of subsystems should be divided according to the actual situation. Because the number of subsystems and overlapping information is different, the overlapping decentralized control scheme increases the flexibility of the controller setting.

(4) In this paper, the computational efficiency of vibration control of building structures under seismic excitation is studied, and an interstory actuator with fault-tolerant control is designed. In the future, the theory can be combined with structural health detection technology to facilitate the real-time monitoring and control of structural vibration.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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