

Research Article

Minimum Flexural Reinforcement Steel Ratios of High-Strength Concrete Beams

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Considering that most studies depend on theoretical equations to determine the minimum reinforcement ratio, only a few studies on this ratio are available. Therefore, a more defined limit should be suggested to design codes by performing additional investigations and experimental studies on this limit. This study examines the behavior of high-strength concrete (HSC) beams with low reinforcing steel ratios to establish a limit for the lowest flexural reinforcement ratio that will ensure ductility. Experiments were performed on 12 reinforced HSC beams with a concrete compressive strength of 99 MPa, which were divided into three categories depending on their size. Each category comprised four beam reinforcement ratios (0%, 0.13%, 0.33%, and 0.65%), and two main parameters (beam size and reinforcement ratio) were investigated. Furthermore, to ensure flexural failure at the middle-span, adequate web reinforcing was used in all the beams and tested under a four-point load until they exhibited failure. Based on regression analysis, an equation was proposed for the rupture modulus of reinforced beams. The findings suggest that, in addition to the yielding strength of the reinforcements and the compressive strength of the concrete, the depth of the beams should be considered when computing the minimum flexural reinforcement of beams.

1. Introduction

Reinforced concrete, which is composed of concrete and reinforcement, is one of the most widely used materials in the field of construction. However, to guarantee the safety, ductility, and sufficient precaution of reinforced concrete members, the ratio between concrete and reinforcement should be balanced and controlled while considering the economic aspect. The main reinforced concrete members have reinforcement limitations (upper and lower), and the reinforcement ratio should be between these limits. If the ratio is not within these limits, it could influence the overall behavior of the beams in terms of safety, failure mode, ductility, stability, and durability.

Owing to the advancements in material technology, it is now possible to produce high-strength concrete (HSC). The definition of HSC varies significantly in the literature and mainly depends on the period and geographic region of use. According to ACI363R-10 [1], HSC is a type of concrete with

a compressive strength exceeding 55 MPa. In contrast to normal-strength concrete (NSC), HSC provides superior engineering properties, such as higher compressive and tensile strengths, higher stiffness, and better durability [2].

All design codes define the minimum reinforcement ratio [3–7], which is a significant limit and provides adequate precaution and ductility before the structure exhibits failure. Flexural members with a reinforcement ratio lower than the minimum limit, such as plain concrete members, may experience a sudden failure by a single localized crack without sufficient precaution, which is an undesirable failure mode according to all design codes. Structural designers prefer ductile failure in all structural members and try to eliminate sudden failure. Therefore, minimum reinforcement can control the crack width in a serviceability state, thereby having a positive effect on the durability and life of the structure. Šmejkal and Procházka [8] derived an equation that can control the crack width in reinforced concrete members. Fantilli et al. [9] concluded that the crack width is

directly proportional to the reinforcement ratio for lightly reinforced concrete LRC beams. Ductility is one of the essential properties of structural members, especially for seismic resistance structures. Therefore, one of the main requirements of these structures is to ensure minimum reinforcement ratio to prevent sudden failure. However, the contravention topic is determining the minimum reinforcement ratio for flexural members, considering design codes have different recommendations on determining the minimum reinforcement ratio. Furthermore, researchers prefer different equations of minimum reinforcement ratio, which turn depend on different reinforced concrete parameters. Concrete compressive strength and reinforcement yield strength are the main preferred parameters according to design codes (Eurocode 2, ACI 318-19) [3, 7]. However, the section size of members (depth) is also one of the effective parameters that significantly affects the limit (NS 3473E) [6, 10], where the depth of the beams is inversely proportional to the minimum reinforcement. According to Fantilli et al. [11], the diameter of the reinforcement bars is also an effective parameter in the minimum reinforcement ratio of flexural members owing to the bond behavior between steel and concrete. The concrete-steel bond effect on minimum reinforcement is also approved by Ozbolt and Bruckner [12]. According to Gerstle et al. [13], this minimum reinforcement ratio in beams can be controlled by the modulus of elasticity of both concrete and steel.

To date, only few studies have been conducted on reinforced HSC beams. Considering that most studies depend on theoretical equations to determine the minimum reinforcement ratio, only a few studies on this ratio are available. Therefore, a more defined limit should be suggested to design codes by performing additional investigations and experimental studies on this limit.

This study examines the behavior of HSC beams with low reinforcing steel ratios to establish a limit for the lowest flexural reinforcement ratio that will ensure ductility.

2. Literature Review

2.1. Code Provisions and Proposed Models. Almost all codes of practice provide reinforcement ratio boundaries for structural members such as columns, beams, shear walls, footings, and slabs, which control both the upper and lower reinforcement limits. Usually, each structural member has reinforcement limits for different stresses, such as the flexural reinforcement limit, shear reinforcement limit, or torsional reinforcement limit. The minimum reinforcement area of a flexural member is specified by codes and standards to guarantee the safety and ductility of the members (Table 1). Although there is a significant difference between the design codes, the majority specify a minimum amount of reinforcement. Furthermore, codes of practice equations for minimum reinforcement ratio of flexural area are experimented equations developed from investigations and do not include all effective parameters. Hence, codes provide equations and minimum reinforcement ratio values that differ from one another owing to different influential parameters. Moreover, it should be noted that most of these

TABLE 1: Equations of the design codes.

Code	Equations
ACI 318-19 ⁷	$A_{s,\min} = 0.25 \sqrt{f'_c/f_y} b_w d \geq 1.4/f_y b_w d$ (1)
CSA A23.3-04 ¹⁴	$A_{s,\min} = 0.2 \sqrt{f'_c/f_y} b_t h$ (2)
Eurocode 2-04 ³	$A_{s,\min} = 0.26 f_{ctm}/f_{yk} b d \geq 0.0013 bd$ (3)
JSCE-07 ⁵	$\rho_{\min} = 0.058 (h/d)^2 f_c t^{2/3}/f_y$ (4)
IS 456-2000 ⁴	$A_{s,\min} = 0.85/f_y b_w d$ (5)
NS 3473 E-03 ⁶	$A_{s,\min} = 0.35 k_w f_{ctk,0.05}/f_{yk} A_c$ (6)
	where $k_w = 1.5 - h/h_1 > 1.0$

$f_{ctk,0.05}$ is the lower characteristic tensile strength of concrete. A_c is the gross cross-sectional area; h is the beam depth in meter, and $h_1 = 1.0$ m.

equations were derived for NSC, and the validity and availability of these limited data results for HSC beams may be questioned.

The American Concrete Institute (ACI 318-19) [7] set a minimum reinforcement area $A_{s,\min}$ at each section where tension reinforcement is required by analysis, as shown in equation (1). The equation depends on the compressive strength and reinforcement yield strength of the concrete. However, ACI 318 up to 1995 provided an equation for computing the minimum required flexural reinforcement, which was dependent solely on the reinforcement yield strength and did not consider the grade of the concrete. From 1995 onwards, ACI 318 added another equation to consider the compressive strength of the concrete. The Canadian Standard Association (CSA A23.3-04) [13] set a limit for minimum flexural reinforcement based on the cracking moment and reinforcement moment capacity, as shown in equation (2), wherein the reinforcement capacity should be 20% greater than the cracking moment capacity. Although this equation is very similar to the ACI's equation with reference to the format and variables, ACI is more conservative. The Eurocode 2 and fib Model Code 2013 [3, 14] applied two limitations on the minimum flexural reinforcement, one for the serviceability limit states that control crack width and crack distribution, and one for ultimate limit states that control the ductility and failure mode of the member. For the ultimate limit state, Eurocode 2 [3] specified the lower limit of longitudinal tension reinforcement by equation (3).

A tension reinforcement ratio of 0.2% of the concrete area was provided by the Japan Society of Civil Engineers 2007 (JSCE) [5], where the characteristic compressive strength of concrete was equal to or smaller than 30 MPa and the design yield stress of reinforcement was approximately 350 MPa. However, when a higher concrete strength is used, the minimum reinforcement ratio is given as shown in equation (4), where h/d is the ratio of the total depth of the member to the effective depth. The JSCE equation for the lower bound of flexural reinforcement does not depend only on concrete and the reinforcement properties but also the size of the member for identifying this limit. In contrast to the other codes of practice, the Indian Standard (IS 456 2000) [4] stated that the minimum reinforcement ratio does not depend on both the concrete compressive strength and

member dimensions but only on the reinforcement yield strength. In many codes, the limit depends on both the concrete and the reinforcement properties, whereas some cases also consider the member size. The IS 456 2000 equation (5) is based on experiments conducted on concrete with normal strength; hence, it is not applicable to HSC members [15]. The Norwegian standard (NS 3473E 2003) [6] controls the minimum area of reinforcement in a flexural member according to equation (6), which demonstrates that the member size effect should be considered when calculating the minimum reinforcement ratio. Beams with a total depth of over 500 mm require a fixed amount of reinforcement. However, as the beam depth decreases by over 500 mm, the minimum reinforcement area increases. This size effect is not considered in most of the practice codes, except for the Norwegian and Japanese standards.

Figure 1 shows a comparison between the minimum reinforcement ratio and the flexural member total depth, assuming the compressive and reinforcement yield strengths of concrete to be 60 MPa and 420 MPa, respectively. The graph shows that only the Norwegian and Japanese standards consider the member size when calculating the minimum steel ratio. According to the Japanese standard, the minimum steel ratio is inversely proportional to the depth. Likewise, the Norwegian standard specifies that the minimum reinforcement ratio decreases as the member depth increases. However, the minimum reinforcement ratio decreases up to the depth of 500 mm and remains constant thereafter, and the member size no longer contributes to the reinforcement ratio. Nevertheless, none of the other codes depend on the depth; hence, the lines remain constant as the member depth varies.

Table 2 shows the proposed equations in the literature, their origin of derivation, and the parameters considered. While some only considered the compressive strength of concrete and the yielding strength of steel reinforcing bars, others also considered the height of the beams and recommended it be included in the design codes.

Table 3 shows the summary of the existing literature, presenting the number of specimens and their sizes, range of reinforcement ratios, and the concrete grades. In these studies, the beam specimens of various sizes and concrete strengths were tested. The height and compressive strength of the beams ranged from 100 to 500 mm and 30 to 102 MPa, respectively. Furthermore, reinforcing steel bars with f_y in the range 389–659 MPa and reinforcement ratios in the range 0.043–5.31% were used.

Only a few studies have been conducted on lightly reinforced, high-strength RC beams. The available minimum reinforcement equations in the design codes were validated using normal strength concrete data. Therefore, to use these codes for HSC members, they need to be further studied and verified. Subramanian [15] suggested that further studies should be conducted on HSC flexural members to investigate the longitudinal and shear reinforcement limits. Considering the experimental data of concrete strength under 40 MPa was utilized to develop reinforcement limits in flexural members in almost all code provisions, extrapolation of concrete properties from NSC to HSC may be

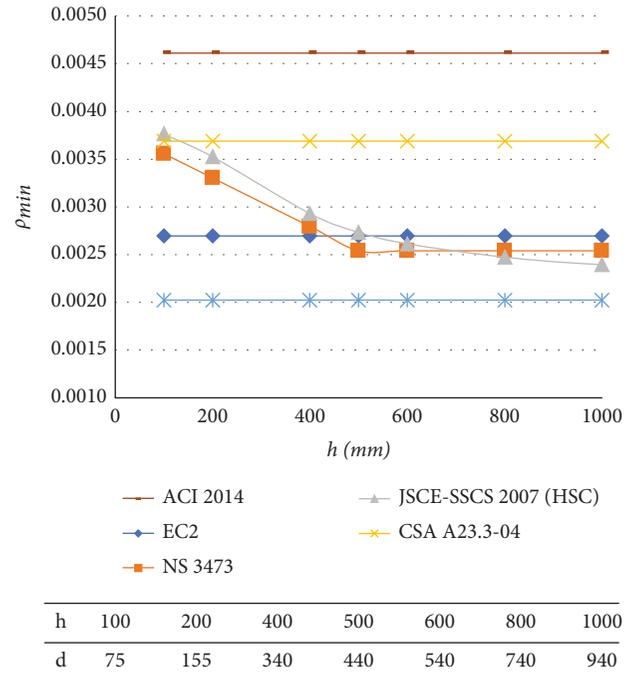


FIGURE 1: Comparison of the minimum reinforcement ratio vs beam depth between code provision, for $f'_c = 60$ MPa, $f_y = 420$ MPa.

erroneous. Furthermore, increasing the strength improves concrete brittleness and produces a more homogenous microstructure.

Ozcebe et al. [26] studied high-strength six-reinforced concrete T-beams and concluded that the ratio between reinforcement yield capacity and concrete cracking moment should be 1.5 for beams with a concrete compressive capacity of less than 80 MPa. However, the given ratio is not applicable for beams with a concrete strength equal to or higher than 80 MPa.

2.2. Theoretical Models. Design equations for minimum steel reinforcement in concrete beams are developed using various approaches, such as theoretical, empirical, and numerical studies, which generally lead to different results. Flexural concrete members with a low reinforcement ratio should have a design moment capacity M_d that exceeds the cracking moment M_{cr} to confirm adequate deflection and visible cracks [30]. The role of reinforcement steel becomes essential after concrete cracks and tension force transfer from concrete to steel reinforcement. In this instance, if the reinforcement has more capacity than the cracking moment, the beam will behave in a ductile manner and provide sufficient deflection after the first crack. However, if the design moment capacity is less than the cracking moment, the member will fail directly after the first crack, without any precautions. In this case, the beam can be classified as a brittle member. In addition, only a localized crack develops, considering the plain concrete in bending is a brittle material and fails suddenly after the first crack develops. Based on this concept, the equation for rectangular beams is expressed as:

TABLE 2: Summary of previously proposed equations.

Source	Proposed equation	Origin	Parameters
Mohammed et al. [16]	$A_{s,\min} = \sqrt{f'_c/3} f_y b d$	Empirical	Material properties
Al Sebai and Al Toubat [17]	$A_{s,\min} = K_{\min}^2 / 2n(1 - K_{\min}) b_w d_s$	Analytical	Material properties
Carpinteri et al. [18]	$K_{\min} = \max \left\{ \begin{array}{l} 3 - \sqrt{9 - 4n_c n_n^2} / 2 \\ 1 - C_n (\sqrt{3} \sin \alpha - \cos \alpha) \end{array} \right.$ $A_{s,\min} = 0.27 f_{ctm}^{0.7} K_{IC}^{0.3} / f_{yk} b_w h^{0.85}$	Numerical algorithm—nonlinear fracture mechanic	Material properties and member size
Bosco and Carpinteri [10]	$A_{s,\min} = (0.1 + 0.0023 f_{cm}) K_{IC} / f_y h^{0.5} A$	Numerical algorithm—nonlinear fracture mechanic	Material properties and member size
Cadamuro et al. [19]	$\rho_{\min} = 0.26 \sigma_u^{0.71} K_{IC}^{0.29} / \sigma_y h^{0.15}$	Numerical algorithm—nonlinear fracture mechanic	Material properties and member size
Shehata et al. [20]	$\rho_{s,\min} = 0.05 f_{ck}^{0.67} / f_y (1 + 1.5 (h/100))^{0.7} / (h/100)^{0.7}$	Analytical	Material properties and member size
Appa Rao et al. [21]	$\rho_{\min} = (-0.01 + 40.1/d) f_{ck}^{1.14} / f_{yk}^{0.57}$	Analytical method FMC	Material properties and member size
Baluch et al. [22]	$\rho_{\min} = 1.9134 K_{IC}^{0.82} E_c / f_y^{0.9922} (1.7 - 2.6 c_s / D)$	Unstable crack propagation	Material properties and member size
Gerstle et al. [23]	$\rho_{\min} = E_c / E_s \sqrt{\sqrt{0.0081 + 0.0148 \beta} - 0.09}$ $\beta = f'_t h / E_c COD_{cr}$		Material properties and member size

$1K_{IC}$ Concrete fracture toughness. σ_u Average tensile strength of concrete, σ_y tensile yield strength of steel. 2β is scale parameter of concrete, COD_{cr} is critical crack opening displacement, f'_t is concrete ultimate tensile strength.

TABLE 3: Summary of the literature empirical studies on minimum flexural reinforcement limit beams.

Source	No. of beams	Reinforcement ratio (%)	Beam size (mm)	Concrete cube strength (MPa)	Steel yield strength (MPa)	Notes
Mohammed et al. [16]	6	0.165–0.29	150 × (255–340)	30, 60, 90	659	$A_{s,\min} = \sqrt{f'_c/3} f_y b d$
Fantilli et al. [9]	36	0.1–0.4	100 × 200 200 × 400	30, 45, 60	450	
Appa Rao et al. [21]	12	0.15–1.0	50 × 100 100 × 200 200 × 400	30	389–637	The minimum reinforcement decreases as the beam depth increases. The optimum ductility number was observed to be 0.20 in RC beams with a compressive strength of 30 MPa.
Rashid and Mansur [24]	16	1.25–5.31	250 × 400	42–126 Cylinder	460–537	The stresses generated by shrinkage of concrete and the creep associated with it can significantly affect the cracking moment.
Ashour [25]	9	1.18, 1.77, 2.37	200 × 250	48, 78, 102 Cylinder	530	Cracking moment obtained by f_r over estimates actual cracking moment by 1.5–2.0 times.
Ozcebe et al. [26]	6	0.32–0.7	180 × 270 T-beams	60–90 Cylinder	387, 486	Recommend the ratio of yield load to cracking load to be taken as 1.5.
Carpinteri et al. [27]	45	0.12–2	100 × 100 100 × 200 200 × 400	48.2	518–643	Minimum reinforcement ratio is inversely proportional to the beam depth.
Bruckner and Eligehausen [28]	6	0.15	300 × 125 300 × 250 300 × 500	31.8, 33.6	580	Minimum reinforcement ratio is directly proportional to the beam depth.
Bosco et al. [29]	30	0.043–1.0	150 × 100 150 × 200 150 × 400	91.2	441–637	Minimum reinforcement ratio is inversely proportional to the beam depth.

$$M_d \geq M_{cr}. \quad (7)$$

The Canadian standard association (CSA A23.3-04) [13] specifies the cracking moment to design moment ratio to be greater than 1.2 for flexural members at every section where tensile reinforcement is required by analysis. Ozcebe et al. [26] experimentally investigated lightly reinforced concrete T-beams and suggested that the ratio of the cracking moment to the design yield moment should be considered as 1.5 (for $f'_c \leq 80$ MPa).

From Figure 2, the following equation can be obtained:

$$\emptyset A_s f_y \left(d - \frac{a}{2} \right) \geq \frac{f_r I_g}{C}, \quad (8)$$

where I_g is the moment of inertia of the beam section. For the rectangular beam, the formula is given as:

$$\emptyset A_s f_y \left(d - \frac{a}{2} \right) \geq \frac{f_r b h^3 / 12}{h/2}. \quad (9)$$

Both I_g and C are for the rectangular beams. However, other geometries like T-section and L-section can provide different results.

$$A_{s, \min} \geq \frac{f_r b h^2}{\emptyset f_y (d - a/2) 6}. \quad (10)$$

Assuming $\emptyset = 0.9$ and $a = 0.05 d$, we obtain the following.

$$A_{s, \min} \geq \frac{f_r b d}{5.13 f_y} * \left(\frac{h}{d} \right)^2. \quad (11)$$

For the rectangular flexural members, h/d varies in the range 1.05–1.2. In this equation, we substitute 1.2.

$$\rho_{\min} \geq \frac{f_r}{3.56 f_y}. \quad (12)$$

The minimum reinforcement (12) is not applicable to all beam sections, considering that different beam geometries result in different I_g , C , and h/d , as mentioned earlier (for example I_g of T beams is assumed to be 1.5 times of the rectangular beams). Consequently, 3.56 under the root of (12) is not accurate for all the sections. It is worth mentioning that the design codes generally provide a safety margin for the flexural members to have sufficient ductility.

2.3. Size Effect. Among all parameters that affect the minimum flexural reinforcement ratio in beams, the effect of the member size is considered the most controversial by researchers and design codes. Generally, design codes do not consider the beam depth as the driving parameter for the minimum reinforcement equation, although a limited number of codes include the depth of the member in their equation. When deriving the equation, the ratio of the total depth-to-effective depth ratio (h/d) of a member cross section is replaced by a constant number, as shown in equation (11), owing to which there is no size effect factor in the equations, thereby triggering this contradiction.

Bruckner and Eligehausen [28] studied the effects of beam depth on member ductility. The experimental results confirmed that as the beam size increased, the ductility of beams decreased. Hence, higher minimum reinforcement ratios are required for larger beams. The authors of these studies supported Ozbolt's [31] ideas and numerically conclusive results. Subsequently, Ozbolt and Bruckner [12] summarized their work and obtained the same results as their previous research based on their numerical and experimental results. In addition to size effect, their new study confirmed that there are many other factors that influence the required minimum reinforcement ratio, such as brittleness of the concrete, bond relation of steel and the surrounding concrete, and type and amount of distributed reinforcement.

In addition to linear stress versus crack opening displacement, Gerstle et al. [23] investigated the cracking behavior and minimum steel ratio for flexural members using the fictitious crack model, which was introduced by Hillerborg et al. [32]. The analysis derived a relation between normalized moment versus normalized crack length for different values of β "scale parameter for concrete" and α , which corresponded to the minimum reinforcement ratio. Furthermore, they proposed a new equation (see Table 2) wherein the size of the member influenced the minimum reinforcement ratio of the beams. Based on their minimum reinforcement equation, the ratio in a flexural member increased as the height of the members increased. Conversely, the reinforcement yield strength f_y effect was disregarded, while most of the proposed equations consider the relationship between the reinforcement yield strength and the minimum reinforcement ratio to be inversely proportional. According to Baluch and his colleagues [22], the depth of a member is directly proportional to the minimum reinforcement ratio. Furthermore, they controlled unstable crack propagation by developing a criterion to forecast the maximum flexural reinforcement, and guaranteed the capacity of the reinforced section before the propagation was larger than the unreinforced concrete cracking capacity. As a result, the following equation was proposed:

Carpinteri et al. [27] investigated the scale effect of RC beams in flexure through theoretical, numerical, and experimental analyses. Theoretically, they utilized the LEFM (Linear Elastic Fracture Mechanics model), where the reinforcement reaction was exerted on the crack surface, and the relation between crack opening displacement and reinforcement was locally under compatibility condition. This principle of transition condition "yielding of reinforcement simultaneously with the first crack of concrete" was used to calculate the size effect on lightly RC flexural members and calculate the minimum amount of reinforcement. In the numerical study, all beams in the experimental work were simulated using finite element (FE) models with nonlinear softening law. The beams were simulated using the SBETA program with different mesh sizes. For the crack modelling, the smeared crack model was used. The relation between ρ_{\min} and size is shown below in Figure 3 according to previous research.

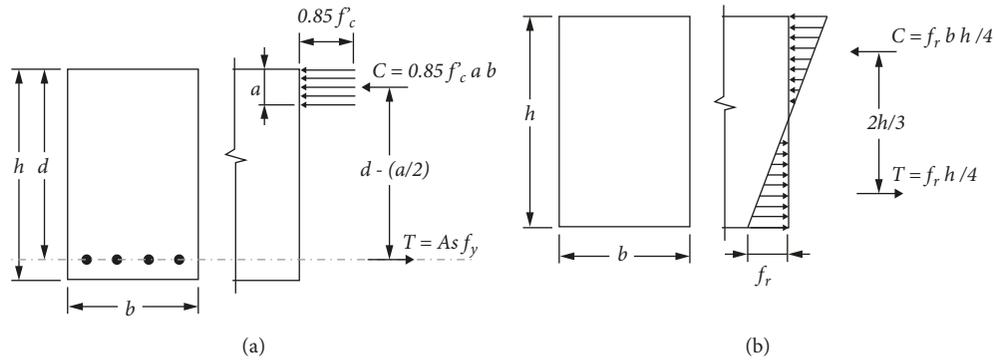


FIGURE 2: Flexural stress distribution of the beam. (a) Equivalent stress block for the reinforced beam and (b) elastic stress for the concrete beam before crack.

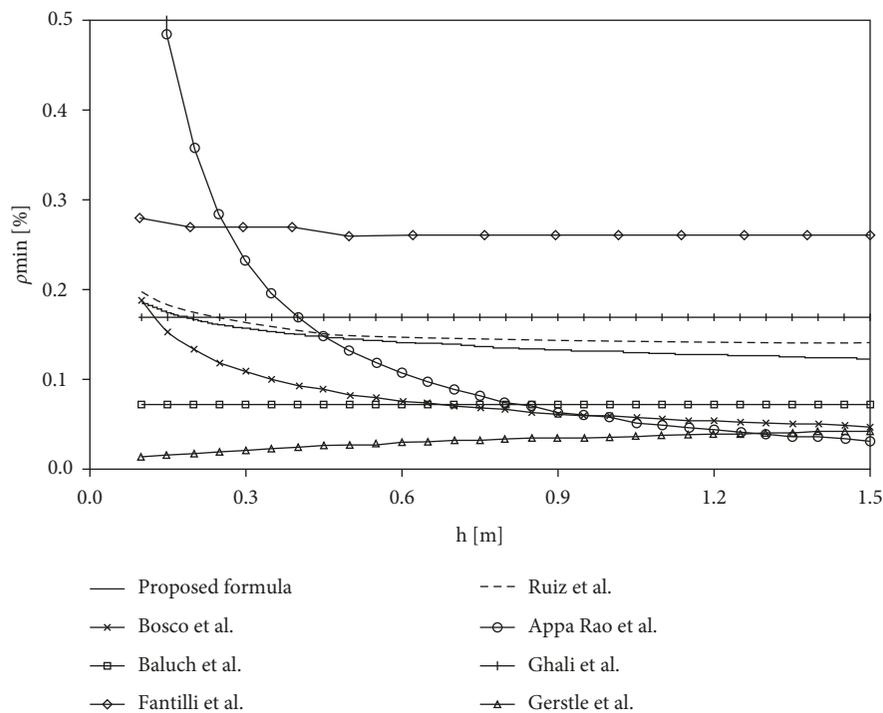


FIGURE 3: Proposed minimum reinforcement equation models versus beam depth, $f'_c = 36.9$ MPa, $f_y = 450$ MPa [19].

According to both the numerical and experimental results, the minimum reinforcement ratio is size-dependent, considering that the beam depth increased as the minimum reinforcement ratio decreased. In addition, the critical brittleness number [10] N_{pc} in (14) was satisfactory for the separation between the stable and unstable crack propagations, as well as calculating the minimum reinforcement ratios. According to the equations provided by design codes such as Eurocode 2-04, Japanese code JSCE-07, and fib model code-13 [3, 5, 14], the size of the member influences its flexural tensile strength. Based on these equations, members with a larger depth have smaller flexural tensile resistance, although in Eurocode 2, this effect is disregarded for depths beyond 600 mm, as shown in Figure 4. Appa Rao et al. [21] studied the behavior of normal concrete beams

with different cross-sectional sizes and confirmed that the modulus of rupture decreased as the member size increased.

$$N_p = \frac{f_y h^{0.5}}{K_{IC}} \frac{A_s}{A}, \quad (13)$$

$$N_{PC} = 0.1 + 0.0023 f_{cm}. \quad (14)$$

Theoretically, the equation for the lower limit of reinforcement in beams was derived by equating the cracking moment with the design moment, as explained previously. The cracking moment capacity has a relationship with the flexural tensile strength $f_{ctm,fl}$ (modulus of rupture), which is more representative of the actual flexural failure mode rather than the mean axial tensile strength f_{ctm} . Despite the

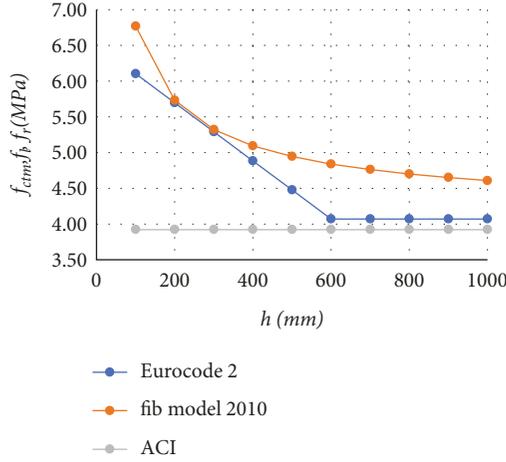


FIGURE 4: Effect of member depth on the flexural tensile strength of concrete for concrete $f_{ck} = 40$ MPa.

influence of the height of the member on the flexural tensile strength, most of the available design equations were substituted by a tensile equation without considering the size effect. Consequently, the design equations provided by the majority of the code's provisions disregarded the height of the member. For instance, in equation (3), direct axial tensile strength was used to develop the minimum flexural reinforcement equation by Eurocode 2-04 and fib model code-13 [3, 14]. It was recommended that flexural tensile strength would be used, considering the flexural tensile strength in these two codes depends on member depth, as shown in (8)(9) for Eurocode2 and fib model code 2010, respectively. This is one of the reasons why the size effect does not influence the abovementioned equations. In contrast, both ACI 318-19 and ACI 363R-10 [1,7] report modulus of rupture for NSC and HSC, irrespective of the member size. The flexural tensile strength ranges between 0.62 and 0.99 $\sqrt{f'_c}$.

$$f_{ctm,fl} = \max \left\{ \left(1.6 - \frac{h}{1000} \right) f_{ctm}; f_{ctm} \right\}, \quad (15)$$

$$f_{ctm,fl} = \frac{1 + 0.06 h^{0.7}}{0.06 h^{0.7}} f_{ctm}, \quad (16)$$

where the mean axial tensile strength f_{ctm} is equal to the following equations according to Eurocode 2 [3] and fib model code [14]:

$$\begin{aligned} f_{ctm} &= 0.3 f_{ck}^{2/3} & f_{ck} \leq 50 \text{ MPa}, \\ f_{ctm} &= 2.12 \ln(1 + 0.1(f_{ck} + \Delta f)) & f_{ck} > 50 \text{ MPa}, \end{aligned} \quad (17)$$

where h is the total depth of the member, f_{ctm} is mean axial tensile strength of concrete, and Δf is equal to 8 MPa.

Shehata et al. [20] studied the flexural, shear, and torsional minimum reinforcement of beams. For flexural minimum reinforcement, the cracked moment capacity was used with linear elastic stress distribution and tensile strength of concrete. Equation (16) was used as concrete

flexural tensile strength to derive the lower limit of flexural reinforcement, which depends on the member size and concrete compressive strength. Accordingly, equation (18) was proposed for flexural minimum reinforcement. The authors concluded that when the beam reached cracking moment, the load could be transmitted to the longitudinal tension reinforcement.

$$\rho_{s, \min} = 0.05 \frac{f_{ck}^{0.67}}{f_y} \frac{(1 + 1.5(h/100))^{0.7}}{(h/100)^{0.7}}. \quad (18)$$

Fayyad and Lees [33] investigated the behavior of RC beams by using a fracture mechanism-based model, which is used to investigate different local phenomena of tensile and compressive concrete softening. Both researchers summarized the literature studies of size effect on the minimum reinforcement ratio. Furthermore, they compared some design codes and the researcher's supposition on this point. As a result, the effect of the beam size on ductility was proved, that is, the member size was inversely proportional to the minimum reinforcement ratio, which contradicts most of the prevailing codes and standards.

Table 4 illustrates the differences in judgements about the size effect on the reinforcement limit in flexural members. They are divided into three groups: Some believe that the minimum reinforcement is independent of the member size, whereas some propose that the minimum reinforcement ratio should increase as the beam depth increases. Conversely, a group of researchers showed contradictory views by stating that the minimum reinforcement ratio decreases as the member size increases.

2.4. Ductility. Ductility is defined as the ability of a member to undergo loading and large plastic deformations without loss of strength. Each design code considers the importance of ductility in design because if a structure is ductile, its ability to absorb energy without critical failure increases. Ductility is one of the required structural properties that allow sufficient precaution before failure. Generally, ductility is described as material ductility, member ductility, or structural ductility. Structures exhibit ductility against loading that depends on the structural members, and hence, it is important to ensure the ductility of the members to ensure sufficient ductility to the structure. When using ductile members to construct a structure, the structure can undergo a loading process, resulting in large deformations that lead to failure while providing warning and precaution. Generally, there are three main ductility modes: shear ductility, axial ductility, and flexural ductility. This section focuses on flexural ductility. Reinforced concrete beams are designed for underreinforcement, that is, yielding of reinforcement followed by concrete crushing after considerable displacement and ultimate failure.

The New Zealand structural design standards NZS3101-06 [35] classified structures into four types based on their ductility factors: brittle (structures with insufficient ductility factor and sudden failure occurrence), nominal ductile, limited ductile, and ductile structures. Structures designed

with a ductility factor of $\mu = 1.25$ or less, $\mu = 3$ or less, and $\mu = 6$ or less are defined as nominal ductile, limited ductile, and ductile structures, respectively. Therefore, the maximum allowable ductility factor is 6. Ductility can be evaluated as ductility factor. Kwan et al. [36], who studied 20 reinforced concrete beams for flexural strength and ductility derived the below equation to determine the ductility of the reinforced concrete beams. When evaluating ductility, the maximum deflection is the most important parameter to be considered as it indicates maximum deformation of the member before failure:

$$\mu = \frac{\Delta_u}{\Delta_y}. \quad (19)$$

Furthermore, they discovered that the main parameters acting on ductility are reinforcement ratio, actual reinforcement to balanced reinforcement ratio, and the concrete grade.

3. Materials and Methods

3.1. Materials. In the concrete mix, as shown in Table 5, ordinary portland cement (OPC) conforming to ASTM C150, manufactured by a local company (Tasluja cement plant), was used as the specimen [37]. Fine aggregate was obtained from the local rivers. The grading was specified according to ASTM(C-33) [38]. Crushed gravel was taken from local rivers, with a nominal maximum size of 12.5 mm and a bulk specific gravity (SSD) of 2.49. The coarse aggregate was graded according to ASTM (C-33) [38]. Potable drinking water was used for both the mixing operation and curing. A silica fume type (Force 10,000) was used in the mix to achieve higher strength and durability for the concrete. Sika ViscoCrete-5930, a high-Performance Superplasticizer concrete admixture, was used to increase workability and reduce water, thereby leading to increased concrete density and strength.

An 8 mm normal steel bar was used for all vertical stirrups and as longitudinal compression reinforcement on top of the beam, whereas 8 mm and 12 mm steel rebars were used for longitudinal tension reinforcement. The steel bars were tested according to ASTM A-615 [39], as shown in Table 6.

3.2. Beam Specimens. We constructed and casted 12 beam specimens designed according to ACI 318-19 [7]. The beams were divided into three main groups (S, M, and L) according to their beam size (200×210 , 200×400 , and 200×575), as shown in Table 7 and Figures 5–7, respectively. Each group comprised 4 beams that were constructed according to various variable parameters and various reinforcement ratios, while maintaining a concrete strength of 99 MPa. In all beams, a clear cover of 20 mm was placed. Each subgroup comprised four beams depending on the variable reinforcement ratio (0%, 0.15%, 0.33%, 0.65%) to investigate the effects of reinforcement ratio on the strength of beams. ϕ 8 mm was used for all vertical stirrups with three different spacings according to the beam dimensions (75 mm,

150 mm, and 250 mm), and ϕ 8 mm was used for top beam compression reinforcement. However, for tension reinforcement, ϕ 8 mm and ϕ 12 mm were used according to their reinforcement ratio. After 24 h of casting, the molds of the beams and control specimens were stripped. The beams were cured continuously for 7 d, after which they were placed outdoors (moisture weather) until the testing day. In addition, for each group, the control specimens were cast with beams. A minimum of 12 cylinders (150×300) mm were cast to determine the compression strength, splitting strength, and modulus of elasticity (six for compression, three for splitting, and three for modulus of elasticity). In addition, five prisms ($500 \times 100 \times 100$) mm were cast to determine the flexural strength.

3.3. Instrumentation and Test Procedure. The test specimens were instrumented to measure the applied load, mid-span deflection, strain in concrete, and reinforcement bars. Five electronic strain gauges were used to measure the strain in both the longitudinal and stirrup bars. Strain gauges were fastened to the bars. Before testing, the beam specimens were colored white with emulsion paint to monitor crack growth and propagation. Then, the LVDT (Linear Variable Differential Transformers) deflection measurements and concrete strain gauges were installed in the required locations on the beam using super glue. Furthermore, the beam was located on a self-supporting loading machine with a maximum capacity of 800 kN, which in turn was loaded on the beam using two load cells located at a distance of 600 mm from each other, center to center, each with a maximum capacity of 300 kN. The load cells were connected to the loading machine. Concrete strain gauges were established on the mid span of the beam, and the steel strain gauges were installed on the predefined locations, as shown in Figure 8. An automatic data acquisition system was used to record data. All of them were connected to a 16-channel data logger that simultaneously read all the data.

4. Results and Discussion

4.1. Behavior of Tested Beams. Considering there were variable parameters such as the beam size and reinforcement ratio, the beams behaved differently under load. All beams were designed to fail in tension mode, such as flexural cracking and reinforcement rupture. During the testing, some similar behavior was observed in the beam specimens under loading, such as developing the first flexural crack at the mid span of the flexural member and an increase in the number of cracks as the load increased. However, different behavior, such as failure modes, was also observed.

The mode of failure for all the tested beams was tension control failure (TCF) as shown in Table 8, which indicates that the reinforcement yields and cracks appear at the bottom of the beams (tension side) before concrete crushing. However, some of the beams fail owing to concrete crushing in the final loading stage. This cannot be accounted for as compression-controlled failure considering reinforcement yields in prior stages. Generally, two main failure modes

TABLE 4: Effect of member depth on the minimum reinforcement ratio according to various models.

Source	No. of beams	Notes
ACI318 M-19 [7]	Independent	(i) Employs the criteria $M_{cr} = M_y$ (equating the moment capacity associated with steel yielding and the cracking moment) (ii) Uses empirical relationships in the derivation (iii) Does not consider size effects (iv) Assumes that the concrete tensile strength after cracking is zero (conservative assumption)
Eurocode 2-04 [3]	Independent	(i) Uses the same underlying approach as that of ACI318-19
Ozbolt and Bruckner [12]	Increase after reaching a critical beam size	(i) Employs the criteria $M_{cr} = M_y$ but allows strain-softening within a finite element framework (ii) Uses a finite element framework, making it difficult to be considered among the simple analytical models
Gerstle et al. [23]	Increase	(i) Based on nonlinear fracture mechanics (NLFM) (ii) Considers the equilibrium of the tensile and compressive forces with concrete tensile softening (iii) Defines the minimum reinforcement as that at which the crack propagation becomes stable (iv) Does not consider the interaction between concrete and steel or the softening in the concrete
Appa Rao et al. [21]	Increase	(i) Uses the same approach as Gerstle et al. [23]
Bosco et al. [29]	Decrease	(i) Is a linear fracture mechanics approach (ii) Proposes a brittleness number, which is a function of the reinforcement ratio and the beam size (iii) Does not consider the fracture process zone in concrete (strain softening) (iv) Does not consider the interaction between concrete and steel
Ruiz et al. [34]	Decrease	(i) Is a nonlinear fracture mechanics (cohesive model) approach (ii) Employs numerical modelling—uses an effective slip (iii) Model for reinforcement and concrete interaction (iv) Is difficult to apply as it includes many numerical parameters
Carpinteri et al. [18]	Decrease	(i) Uses a numerical approach based on nonlinear fracture mechanics (ii) Does not model the interaction between steel and concrete along the reinforcement bar

TABLE 5: Composition of concrete mix.

Materials	Mix
Concrete mixes	1 cum
Cement (kg/m^3)	500
Silica fume (kg/m^3)	50
Fine aggregate (kg/m^3)	960
Coarse aggregate (kg/m^3)	840
Water (kg/m^3)	137
w (cm)	0.25
HRWRA (g)	6875
HRWRA%	1.50%
Compressive strength f'_c (MPa)	98.9

were observed, the brittle failure (sudden tension failure) of plain concrete beams (CS00, CM00, and CL00) and TCF, which can be further divided into two main types. First is the sudden tension failure (steel rupture) that is caused by low reinforcement ratios such as CS15, CM15, CL15, and CS33, and the second is concrete crushing at the top cord of the beams (CS65, CM33, CM65, CL33, and CL65).

4.2. Crack Patterns. It was observed that the crack pattern and crack distribution of the beam specimens were dissimilar owing to the variable parameters (size and

reinforcement ratio). The initial crack was formed in the flexural area near the mid-span (between the point loads) considering that the beams were designed for flexural failure. The cracks developed from the bottom of the beams were propagated vertically upward towards the compression area. As the load increased gradually, additional flexural cracks appeared within the moment and shear regions. In addition, the cracks propagated towards the beam compression area under point loads, and the crack width and intensity increased as the load increased until failure, see Figures 9–11.

All the beams behaved similarly until the appearance of the initial crack, after which the beams behaved differently. Depending on the number of cracks in the middle zone (between the two point loads which are 600 mm apart), the beams were classified into three different groups (see Table 9). The first group comprised lightly cracked beams (LCB) with 1–5 cracks distributed purely in the moment region (flexural area) between the two-point loads. One of these cracks grew wider and propagated until failure. The second group comprised moderately cracked beams (MCB) with 6–10 cracks distributed moderately in the moment region. The last group comprised highly cracked beams (HCB with over 11 cracks distributed between the two-point loads). It was observed that when the flexural reinforcement ratio increased, the crack intensity increased and new cracks

TABLE 6: Properties of the tested reinforcement bars.

Steel tensile test—summary				
Steel bar (mm)	Diameter (mm)	f_y (MPa)	f_u (MPa)	Elongation (%)
Φ 8	7.68	332	510	22.72
Φ12	11.78	538	681.6	15.67

TABLE 7: Details of the beam specimens.

Size	ρ (%)	Beam ID
S	0.000	CS00
	0.130	CS15
	0.310	CS33
	0.620	CS65
	0.000	CM00
M	0.126	CM15
	0.300	CM33
	0.600	CM65
	0.000	CL00
L	0.128	CL15
	0.300	CL33
	0.600	CL65

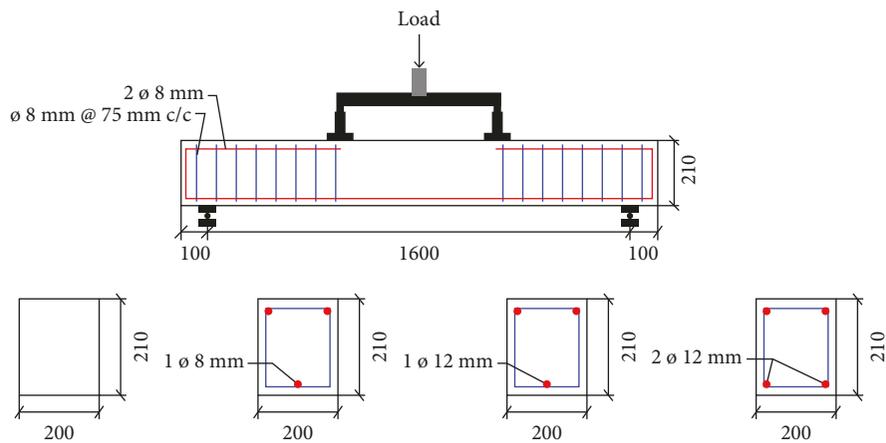


FIGURE 5: Small beams reinforcement detail.

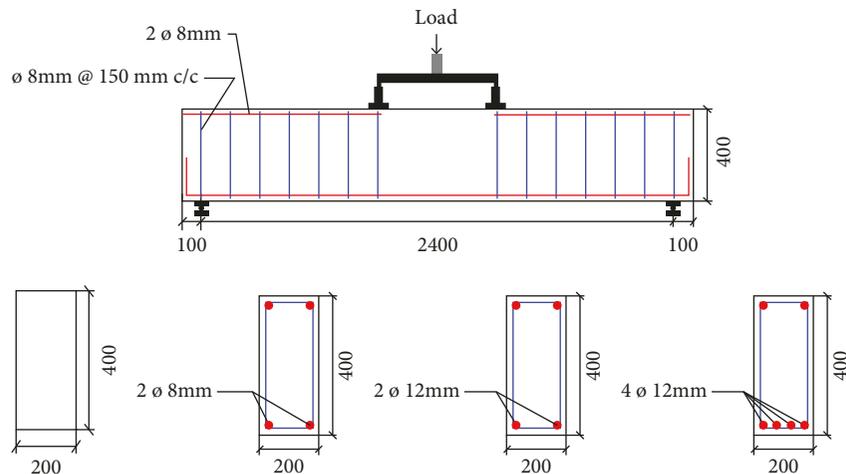


FIGURE 6: Large beams reinforcement detail.

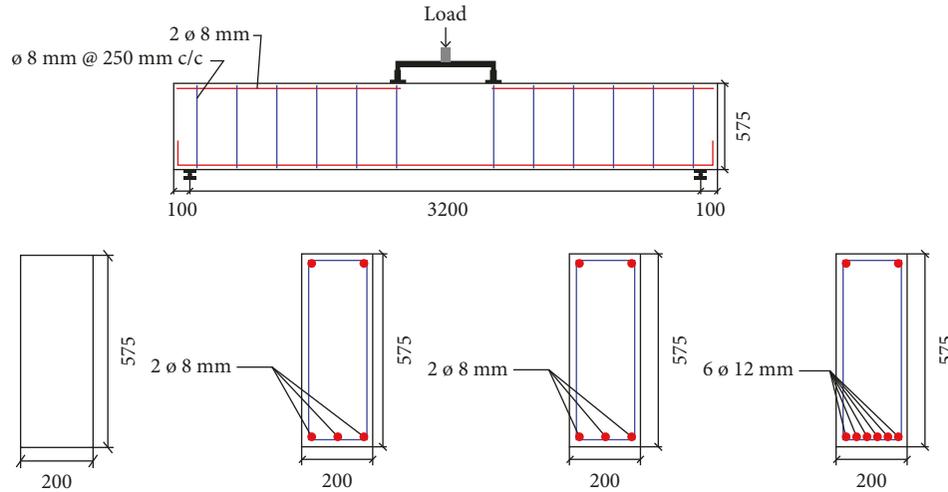


FIGURE 7: Medium beams reinforcement detail.

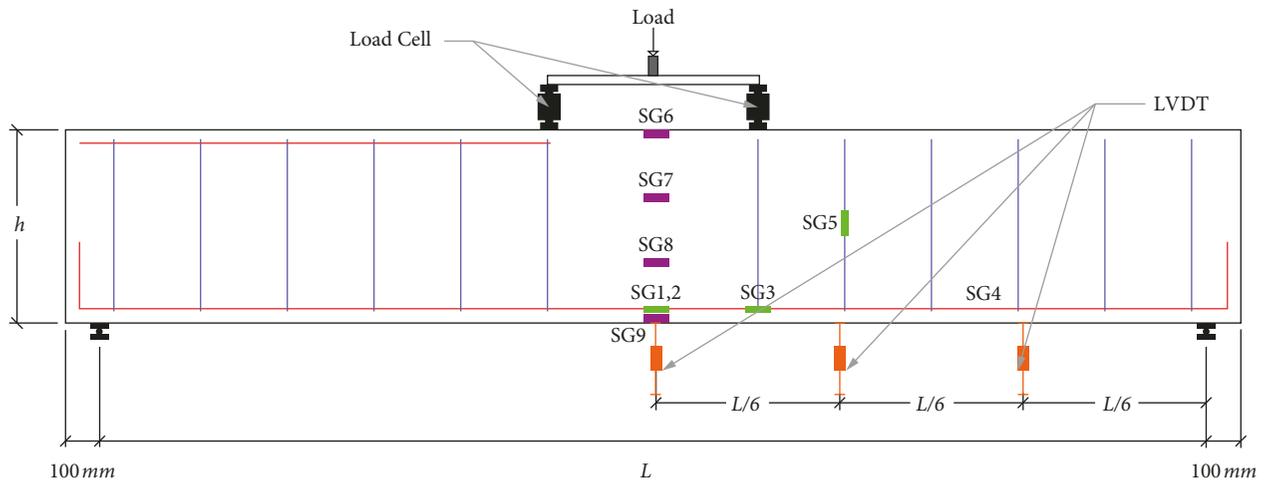


FIGURE 8: Typical location of LVDT and strain gauges on beam specimens.

TABLE 8: Results of the tested beams.

G	Beam ID	p_{cr} (KN)	p_y (KN)	p_u (KN)	Δ_y (mm)	Δ_u (mm)	Failure mode
C	CS00	16.20	N	N	N	0.34	Brittle failure
	CS15	15.70	11.13	18.62	2.98	35.71	TCF (steel rupture)
	CS33	22.81	38.00	51.13	4.96	80.42	TCF (steel rupture)
	CS65	25.00	75.57	99.56	6.65	54.58	TCF (concrete crushing)
	CM00	34.14	N	N	N	0.76	Brittle failure
	CM15	42.65	29.00	41.73	1.81	29.58	TCF (steel rupture)
	CM33	50.01	83.10	107.61	4.95	59.38	TCF (concrete crushing)
	CM65	51.83	169.85	198.12	10.00	53.15	TCF (concrete crushing)
	CL00	46.61	N	N	N	0.65	Brittle failure
	CL15	65.00	45.90	57.59	2.10	41.67	TCF (steel rupture)
	CL33	71.28	126.23	148.68	6.03	78.59	TCF (concrete crushing)
	CL65	72.20	254.66	274.07	11.26	45.22	TCF (concrete crushing)

were formed progressively nearer to the supports. No crack was observed in all the plain concrete beams (CS00, CM00, and CL00). CS15, CS33, CM15, CL15, and CL33 are pure crack beams, CS65 and CL65 are moderately cracked beams, and CM33 and CM65 are well-cracked beams.

4.3. First Crack Moment. Reinforcement ratio is one of the characteristics that affect the crack load. For the small-sized beams, adding ρ by 0.15% decreases the cracking moment by a small ratio. Furthermore, increasing ρ (0.33%, 0.65%) increases the cracking moment. For medium-sized beams,



FIGURE 9: Crack propagation of small beams.

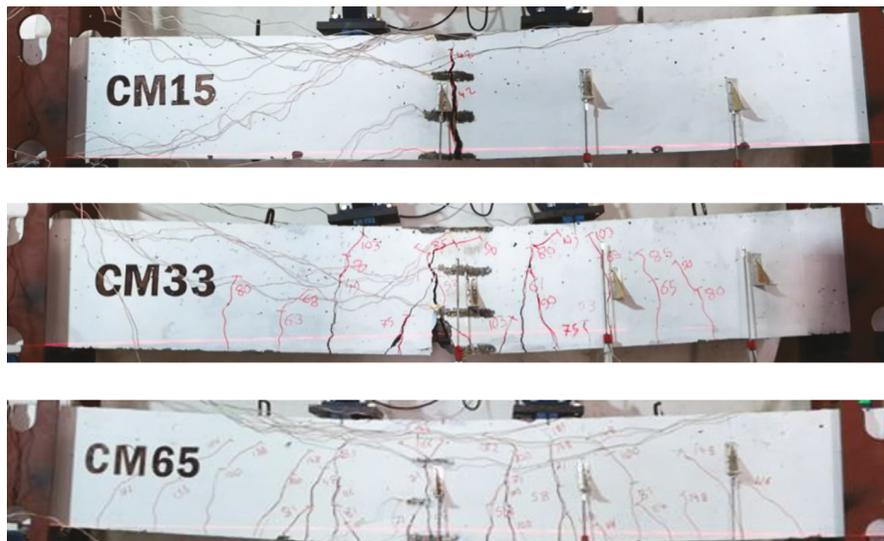


FIGURE 10: Crack propagation of medium beams.

adding ρ by 0.15% results increases the cracking moment reasonably. Enriching ρ to 0.33% further increases the cracking moment. Simultaneously, the cracking moment increases by enriching ρ to 0.65%. Large-sized beams exhibit a sudden increase in the cracking moment when ρ is increased by 0.15%. However, enriching ρ to 0.33% further increases the cracking moment, whereas a slight increase is observed when ρ is increased to 0.65%, as shown in Figure 12. Focusing on the overall results of all the groups, we can conclude that increasing the reinforcement ratio increases the cracking moment by a small ratio for the small beams. In other words, lifting the reinforcement ratio does significantly improve the cracking moment. However, for medium beams, increasing the reinforcement ratio,

especially from 0% to 0.33%, increases the cracking moment by a good ratio. Regarding the other intervals, increasing the reinforcement ratio does not significantly affect the cracking moment. However, for large beams, adding reinforcement by 0.15% significantly enhances the cracking moment.

Flexural strength is an important concrete characteristic that is derived from the first crack. As shown in Figure 13, when the beam depth changes from 100 to 210 mm, f_r decreases dramatically by a high ratio. However, increasing the beam depth from 210 mm to 400 mm and 575 mm had no clear effect on f_r . To investigate the effects of beam depth on flexural strength, the experimental results of tested beams were averaged and compared with the equations of f_r in ACI 318-19 [7], Eurocode 2-04 [3], and fib model code-13 [14]:

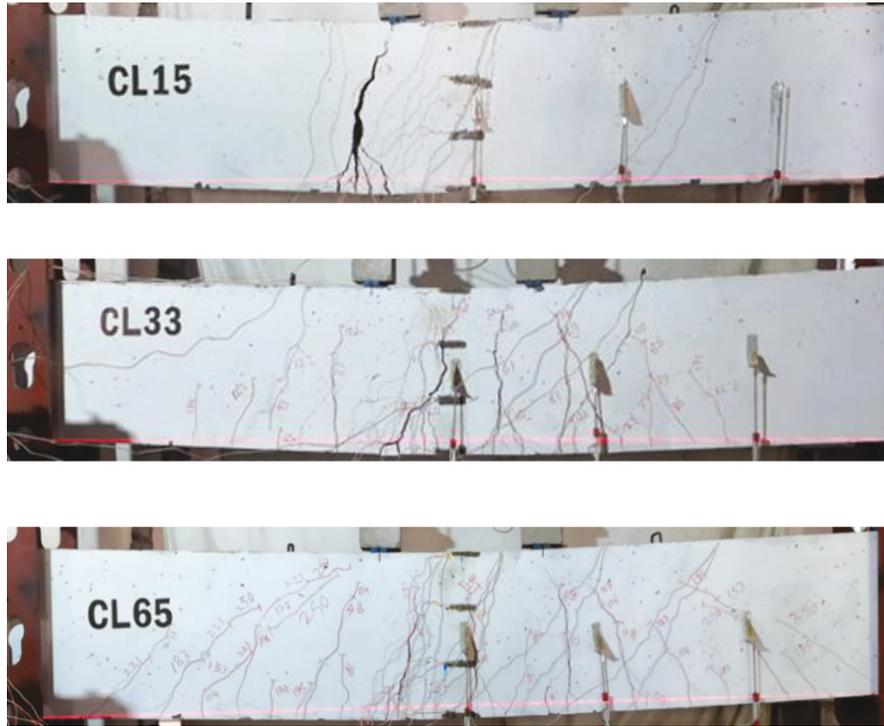


FIGURE 11: Crack propagation of large beams.

TABLE 9: Number of cracks.

Beam ID	Crack no. (length)	Crack no. (60 cm)	Crack distribution
CS00	—	—	—
CS15	3	2	LCB
CS33	10	5	LCB
CS65	13	9	MCB
CM00	—	—	—
CM15	1	1	LCB
CM33	17	13	LCB
CM65	23	12	LCB
CL00	—	—	—
CL15	4	1	LCB
CL33	16	5	LCB
CL65	24	7	MCB

Based on the results and Figure 13, it can be concluded that for the prism when depth = 100 mm, the experimental result was between the ACI code and Eurocode results. However, when the depth changed to 210 mm, the experimental result decreased dramatically, whereas the ACI result remained constant. Therefore, there is a significant difference among the experimental results with ACI, considering the ACI equation is derived from prisms and the size effect of members is neglected. As a result, f_r is independent of the beam depth. However, Eurocode and fib model code graphs showed that increasing the beam depth from 100 to 210, 400, or 570 mm decreases f_r moderately. Conversely, the experimental graph showed a dramatic decrease when the beam depth was increased from prism to 210 mm; however, increasing the beam depth over 210 mm results in the steady augmentation of f_r . Therefore, we can conclude that the

experimental results do not establish the same trend with codes. In addition, the experimental results provided a lower value than all code results, except in prism size. Shrinkage is one of the factors that influenced the decrease in f_r as the beam size increased, considering the beams with a larger size led to the induction of more microcracks owing to autogenous, plastic, and dry shrinkages. As a result, the larger sized beams were more disposed to micro cracks, resulting in a reduced flexural tensile strength of the beam.

4.4. Load Deflection. One way to describe the behavior of the beam specimens under loading is by using load-deflection curves. Generally, there are four main load-deflection curves for examined beams as shown in Figure 14. The first case is for plain concrete, where the beam fails suddenly after

reaching a cracking moment owing to the unavailability of reinforcement in the tension zone to transmit the tensile stress from the concrete. The second case is the beams with $\rho < \rho_{\min}$, the flexural moment capacity of the reinforcement is less than the concrete cracking moment ($M_y < M_{cr}$), resulting in a sudden decrease in the moment capacity after the cracking moment. As a result, the beam fails after reaching the cracking moment, such as the beams with ($\rho = 0.15\%$). The third case is the beams with $\rho = \rho_{\min}$, where there is sufficient amount of reinforcement to provide flexural moment capacity equivalent to the cracking moment ($M_y = M_{cr}$). These curves start linearly with a steep slope up to the first crack, after which the steel yields with the same value of cracking loads. Then, the curve continues almost horizontally until failure. This reinforcement ratio is the minimum amount sufficient to provide an acceptable ductility to some of the beams ($\rho = 0.33\%$). The last case is the beams with a flexural moment capacity larger than the cracking moment ($M_y > M_{cr}$) that can carry loads after cracking and provide sufficient precaution before failure. These curves start linearly with a steep slope up to the appearance of the initial crack. After which, there is a sudden change in the curve and increases up to yielding. Then, the curve continues with the lowest slope until failure, such as beams with $\rho = 0.65\%$.

4.5. Proposed Equation Based on $M_{cr} = M_y$. A suitable method to determine an appropriate equation for the minimum reinforcement ratio is by utilizing f_r of the reinforced beams. It should be noted that there is a difference between the f_r equations of plain and reinforced beams. However, considering all members are reinforced, using (f_r) of reinforced beams to determine the minimum reinforcement ratio equation is considered a suitable approach. It can be noted that all the reinforced beams loaded experimentally, the concrete resists the load up to the appearance of the initial crack, after which the load is transmitted to the reinforcement. The appearance of the initial crack in the reinforced beams is represented by M_{cr} , which is utilized to determine f_r of the reinforced beams. In addition, the founded f_r was based on the average of three different reinforcement ratios (0.13%, 0.33%, and 0.65%).

Herein, there is a need for an (f_r) equation based on the regression analysis of the current test data. Many trials have been conducted to construct a relationship between the

modulus of rupture (f_r), beam size, and the concrete compressive strength that has a linear relationship with the compressive strength. Therefore, the relationship was found to have the following form:

$$f_r = \mathbf{a} + \mathbf{b} \frac{\sqrt{f'_c}}{h^2} + \mathbf{c} h. \quad (20)$$

Furthermore, by using regression analysis, we obtained (20) for the modulus of rupture of reinforced concrete beams, that is, the best average and provides the maximum R^2 for all beam types.

$$f_r = 2.45 + 3917 \frac{\sqrt{f'_c}}{h^2} + 0.0033h, \quad (21)$$

where f'_c is measured in MPa and h in mm.

The beam depth (h) was used in the equation individually considering the results clearly showed enlargement of the beam depth, which makes the effect of reinforcement ratio on M_{cr} more evident, thereby indicating that adding reinforcement to plain beams improves M_{cr} significantly in large sections compared to the small section from (0% to 0.15%), as shown in Figure 12.

Finally, based on the derived equation, (f_r) values of the tested beams and the derived equation were plotted according to the beam depth and compared. It can be seen that there is a close match in results of (f_r) with a good R^2 , as shown on the graphs in Figure 15.

The same procedure of deriving the minimum reinforcement ratio equation of plain beams was applied to reinforced beams by using the f_r equation. Therefore, using (21) for rectangular solid section, the first crack moment (M_{cr}) is given by:

$$M_{cr} = \frac{bh^2}{6} f_r, \quad (22)$$

$$M_{cr} = \frac{bh^2}{6} \left(2.45 + 3917 \frac{\sqrt{f'_c}}{h^2} + 0.0033h \right). \quad (23)$$

To obtain the accurate value of the minimum steel ratio and avoid sudden failure, it is recommended to use the criteria ($M_{cr} = M_y$) by equating with (23).

$$\frac{bh^2}{6} \left(2.45 + 3917 \frac{\sqrt{f'_c}}{h^2} + 0.0033h \right) = 0.95 f_y \rho b d^2, \quad (24)$$

$$\rho_{\min} = \frac{h^2}{2.32 f_y d^2} + 687.2 \frac{\sqrt{f'_c}}{f_y d^2} + \frac{h^3}{1727.27 f_y d^2}. \quad (25)$$

To derive the equation of minimum reinforcement ratio for reinforced beams, the same procedure is applied by equating 23 and ($M_{cr} = M_y$).

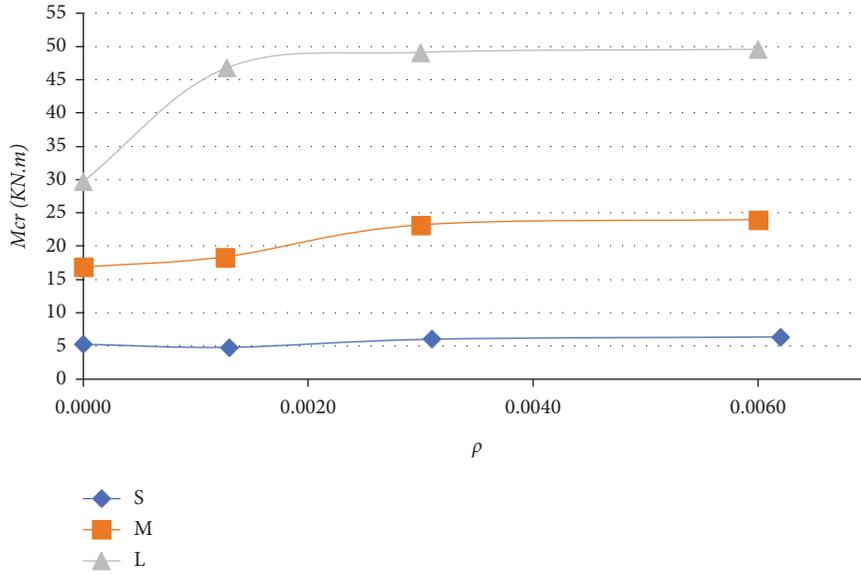


FIGURE 12: Effect of reinforcement ratio on the cracking moment.

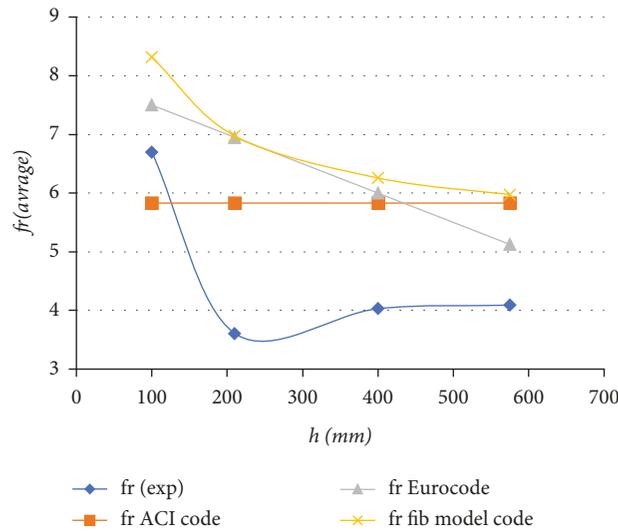


FIGURE 13: Effect of beam depth on modulus of rupture (f_r) comparing with codes of practice.

$$\frac{bh^2}{6} \left(2.45 + 3917 \frac{\sqrt{f'_c}}{h^2} + 0.0033h \right) = \rho bd f_y \left(d - \frac{\rho bd f_y}{1.7 f'_c b} \right), \quad (26)$$

$$\rho_{\min} = 0.85 \frac{f'_c}{f_y} \left(1 - \sqrt{1 - \frac{1535.4 \sqrt{f'_c} + 0.96h^2 + 0.0013h^3}{f'_c d^2}} \right). \quad (27)$$

Finally, we can conclude that owing to the vicinity of the results and to avoid the complexity, (25) with (27) for reinforced beams should be utilized.

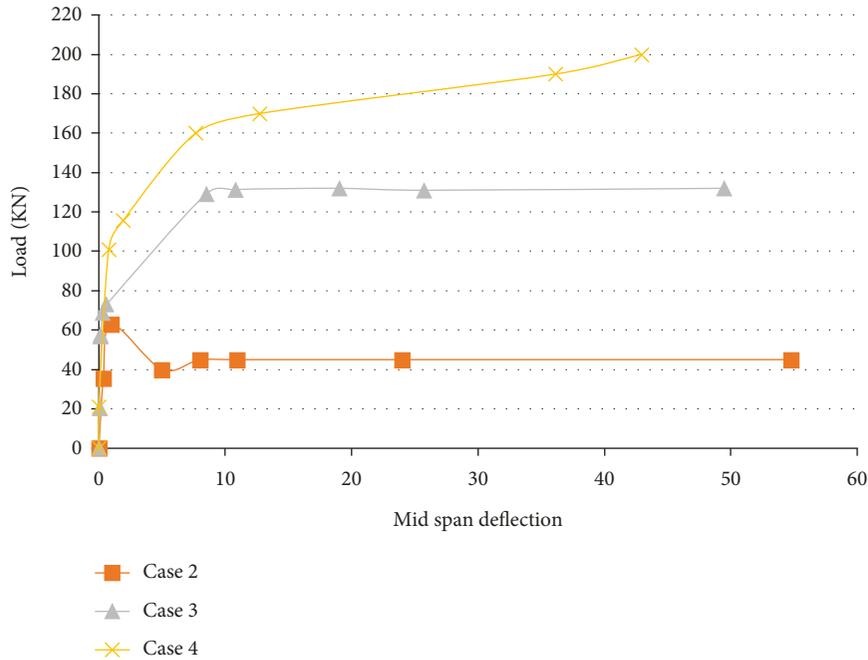


FIGURE 14: Load-deflection graph types according to the reinforcement ratio.

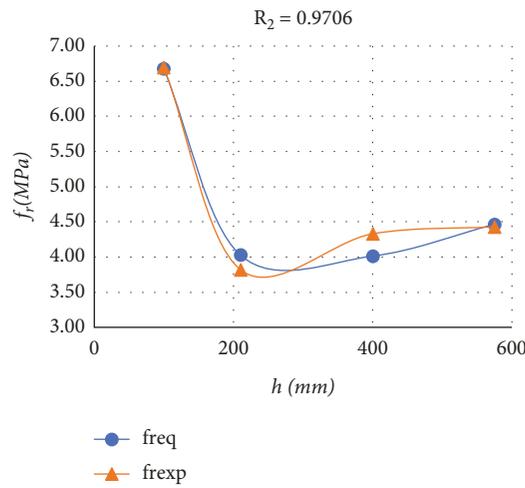


FIGURE 15: Comparison between modulus of rupture obtained by experiments and the derived equation in accordance with the beam depth with relative R^2 .

5. Conclusions and Recommendations for Future Studies

5.1. Conclusions. Based on the experimental and theoretical results obtained in this study, the following conclusions are drawn:

- (1) It was found that f_r decreases as the depth increases (100–210 mm), thereby resulting in a reduced minimum reinforcement ratio, which depends on the f_r equation.
- (2) The proposed equation in this study has been developed based on testing HSC specimens, which provide less conservative results compared with the

provided equations by the codes which are primarily based on testing NSC specimens.

- (3) In the experiments of this study, all the specimens failed in tension-controlled failure, except for the plain-concrete specimens which failed suddenly in a brittle mode.
- (4) In the lightly reinforced concrete beams ($\rho = 0.13\%$), it was observed that one or two main cracks developed from the bottom side of the beams near the midspan, then propagated towards the top of the specimens
- (5) For heavily reinforced beam specimens, the number of cracks ranged between 8 and 21 and 14 and 30 cracks for beams with ρ of 0.33% and 0.65%, respectively. The cracks initiated at the tension-side at the mid-span and then propagated towards the compression side. At the final stage of loadings, further cracks developed at the side of beams close to the supports. This high-intensity distribution of cracks ensured ductile behavior with sufficient warnings and deformability before failure.
- (6) Adding reinforcement to plain concrete beams increases the cracking moment capacity (M_{cr}), thereby increasing f_r . This increase in cracking moment has been observed in all beam sizes, especially in large beams.
- (7) In this study, four various failure modes have been observed.
 - (a) In plain concrete beams, sudden brittle failure occurred without warning.
 - (b) For beams with low-reinforcement ratio ($\rho = 0.15\%$) ($M_{cr} > M_y$), sudden failure occurred almost exactly at the cracking of the concrete.
 - (c) For beams with $\rho = 0.33\%$, concrete cracking and steel yielding happened simultaneously.
 - (d) For beams with $\rho = 0.65\%$, ($M_{cr} < M_y$), reinforcement yields after concrete cracking that provides gradual and ductile failure
- (8) From the equations derived from $M_{cr} = M_y$ (derived from f_r of reinforced beam specimens), it can be concluded that increasing the beam depth will decrease ρ_{min} up to the depth of 400 mm, and increasing the beam depth beyond 400 mm can lead to the enrichment of ρ_{min} by a smaller ratio.

5.2. Recommendations

- (1) Further studies on the minimum reinforcement ratio should be conducted to relate NSC and HSC by testing a larger number of specimens, investigating the differences, and deriving a valid equation for both NSC and HSC.
- (2) It is recommended to further investigate the size effect by testing a larger number of specimens having different sizes, such as depths of 300, 500, and 600 mm.

- (3) Most previous researchers concluded that the size influences both f_r and ρ_{min} . However, in this study, there is a dramatic decrease in f_r when the beam depth changes from 100 to 210 mm, which is not the case when the beam size changes from 200 mm to 400–575 mm. This dramatic decrease in f_r might be due to the member length rather than the depth; as the beam depth changes from 100 to 210 mm, the length increases simultaneously. The autogenous shrinkage and the formation of extra mini cracks in the beams with longer spans can decrease f_r . Therefore, it is highly recommended to study f_r of prisms and beams by considering various lengths while keeping the depth constant, which can directly impact the derived minimum reinforcement ratio equation.
- (4) It is recommended to investigate the effect of aggregate type (such as crushed stone) on the minimum reinforcement ratio of flexural members, which can directly influence concrete bonds, as well as crack propagation and f_r .

Abbreviations

A_c :	Concrete cross-sectional area (mm ²)
$A_{s, min}$:	Minimum reinforcement area (mm ²)
A_s :	Reinforcement area (mm ²)
E_c :	Concrete modulus of elasticity (MPa)
E_s :	Steel modulus of elasticity (MPa)
M :	Moment (kN.m)
M_{cr} :	Cracking moment (kN.m)
M_d :	Design moment (kN.m)
M_u :	Ultimate moment (kN.m)
M_y :	Yielding moment (kN.m)
N_p :	Brittleness number
N_{pc} :	Critical brittleness number
b_t :	The width of the tension zone of the section considered (mm)
b_w :	Web width of the member (mm)
f_c' :	Concrete Compressive strength (MPa)
f_{ck} :	Characteristic compressive cylinder strength of concrete at 28 days (MPa)
f_{cm} :	Mean value of concrete cylinder compressive strength (MPa)
$f_{ctm, fl}$:	Flexural tensile strength of concrete (modulus of rupture) (MPa)
f_{ctm} :	Mean value of axial tensile strength of concrete (MPa)
f_{ctk} :	Characteristic axial tensile strength of concrete (MPa)
f_r :	Concrete flexural strength (Modulus of rupture) (MPa)
f_t :	Concrete tensile strength (MPa)
f_t' :	Concrete ultimate tensile strength (MPa)
f_y :	Reinforcement yield strength (MPa)
f_{yk} :	Characteristic yield strength of reinforcement (MPa)
I_g :	Moment of inertia of beam section (mm ⁴)
k_{IC} :	Concrete fracture energy (N/mm ^{3/2})
k_w :	Size effect ratio

p_{cr} :	Cracking load (kN)
p_u :	Ultimate load (kN)
p_y :	Yielding load (kN)
Δ_u :	Maximum deflection (mm)
Δ_y :	Deflection at yielding (mm)
σ_u :	Average tensile strength of concrete
σ_y :	Tensile yield strength of steel
ρ :	Reinforcement ratio
ρ_{min} :	Minimum reinforcement ratio
\emptyset :	Reduction factor
μ :	Ductility factor (unitless)
a :	Depth of compression area (mm)
b :	Width of the member (mm)
C :	Distance from extreme compression fibre to neutral axis (mm)
d :	Distance from extreme compression fibre to centroid of longitudinal tension reinforcement (effective depth of member) (mm)
h :	Total depth of the member (mm)
β :	Scale parameter for concrete
ρ :	Reinforcement ratio
ϕ :	Reinforcement bar diameter
COD_{cr} :	Critical crack opening displacement (mm)
K_w :	Matrix of the coefficients of influence for the nodal displacements
c :	Concrete cover to longitudinal reinforcement (mm)
β :	Scatter factor
ACI:	American Concrete Institute
ACI363R-10:	American concrete institute-report on high strength concrete
HSC:	High strength concrete
NSC:	Normal strength concrete
LRC:	Lightly reinforced concrete
NS:	Norwegian standard
CSA:	Canadian standard association
JSCE:	Japan society of civil engineers
IS:	Indian standard
RC:	Reinforced concrete
LEFM:	Linear elastic fracture mechanics
SBETA:	Crack modelling program
NLFM:	Nonlinear fracture mechanics
NZS:	New Zealand structural design standards
OPC:	Ordinary portland cement
ASTM:	American standard for testing material
SSD:	Saturated surface dry
LVDT:	Linear variable differential transformers
kN:	Kilo newton
TCF:	Tension control failure
LCB:	Lightly cracked beams
MCB:	Moderately cracked beams
HCB:	High cracked beams
FE:	Finite element.

Data Availability

All data are listed in the submitted article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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