In this paper, the failure mode of the “deep foundation pit slope with high dip angle structural surface and cave development” is analyzed by numerical simulations, and an innovative numerical calculation method for calculating the stability of foundation pit slope based on the principle of upper limit analysis and the prediction method of rock slope surface is proposed. The lateral earth pressure acting on the support structure is further deduced, which can be used more widely than the traditional earth pressure calculation equations. Based on finite element analysis of a pole system, we developed an improved numerical calculation method of the force, and deformation of retaining piles is proposed to carry out the analysis of the deformation of structures under force. Our method avoids the need to artificially place earth pressure on either side of the embedded portions of the piles. Compared with the measured horizontal displacement of the retaining pile, this improved numerical calculation method can reflect the force and deformation law of retaining piles at certain construction stages. In this paper, the upper limit analysis method based on the stability analysis of slopes and earth pressure calculation equations is proposed, taking into account the working conditions of the karst cave in different locations, and the minimum supporting force required can be calculated more accurately while predicting the range of slope sliding surface. An improved numerical calculation method of the retaining piles is proposed in this paper, and the calculation results can reflect the deformation law of the retaining piles during the excavation of the rock foundation pit, and the whole analysis and calculation process has certain advantages compared with the traditional method.

1. Introduction

In traditional support design concepts, the stability of rock pit slopes is generally considered to be relatively easy to control compared to the stability of soil pit slopes. However, in actuality, the support design of rock foundation pit slopes is often more difficult. The fundamental reason for this is that the stability of rock slopes is mainly controlled by the presence of unfavorable geological structures, such as caves, joints, and weak interlayers. Thus far, many researchers have conducted in-depth studies on various complex geological rock slopes [1, 2]. Most of these studies have focused on natural large-scale landslides, but little attention has been paid to man-made slopes.

The stability analysis of slopes is generally based on limit equilibrium methods [3–5], limit analysis [6–8], and numerical analysis [9–12]. Although researchers have considered the influence of unfavorable geological conditions in their analyses, these analyses have been limited to the general reduction of the overall stability coefficient due to the presence of unfavorable geological structures [13–16], without focusing on the effects of specific characteristics, such as the distribution of karst caves, on the overall stability. This has led to a disconnection between the stability analysis of rock slopes and the actual operating conditions. For the treatment of adverse geological measures such as caves, scholars have proposed various methods [17–19], but considering the environmental impact, it is difficult to use...
traditional methods to construct many urban pits, except for large caves that are reinforced by grouting. And as a result, in rock support design, the elastic fulcrum method is usually adopted for the internal force analysis of foundation support structures [20–23]. The design process often relies on the designer's engineering experience regarding the laws of force and deformation of a pile, especially the calculation of the earth pressure in the embedded portion of the pile.

However, for complex rock slopes containing both karst caves and structural planes, using engineering experience as a design reference has a high degree of uncertainty in guiding the design. For this reason, some scholars have studied the distribution of lateral earth pressure on piles based on indoor tests [24, 25] or field tests [26], and based on this, earth pressure models have been proposed to describe the relationship between earth pressure and displacement [27–29], but this part of the research is limited to soil slopes. Therefore, to avoid subjectivity in the design process, it is necessary to develop methods for predicting the range of the sliding fractures and to support design calculation methods for different failure modes of sliding fractures for complex rock slopes containing karst caves and structural planes.

This paper proposes a stability and numerical calculation method for lateral earth pressure on piles based on the principle of upper limit analysis for different failure modes caused by karst caves and structural planes, and compares the calculation results with the traditional earth pressure calculation methods to determine its accuracy. At the same time, the traditional pile deformation calculation model will be improved and made more suitable for rock foundation pit deformation analysis, so that it can provide a reference for rock foundation pit design.

2. Failure Modes of Complex Rock Slopes

The stability of rock slopes with high dip angles is generally controlled by the structural planes. However, for rock slopes containing both karst caves and structural planes, the stability of the rock slopes is affected by the development of karst caves, and thus, the stability of the rock slopes is influenced by both structural planes and karst caves. From the perspective of the different positions and shapes of the caves and structures, the stability failure characteristics of such type of rock slopes are mainly divided into the three categories that follow Figure 1. (1) Sliding failure occurs along a particular structural plane, and the karst cave does not affect the overall stability of the slope. (2) The karst cave is penetrated, and the upper part of the fracture extends to the top surface of the slope, resulting in sliding failure characterized by a combination of curved and flat surfaces. (3) Karst cave penetration occurs between multiple structural planes, resulting in polyline-like sliding failure.

Of the above three failure modes, stability evaluation and lateral earth pressure calculation equations only exist for sliding failure along a single structural plane. There are currently no direct calculation equations for the other two failure modes.

3. Stability Analysis of a Rock Foundation Pit Based on the Upper Bound Analysis Method

For the failure mode due to penetration between karst caves described in Section 2, a simplified model was developed for foundation pit analysis with a single retaining pile. Based on the virtual power equation, the total internal energy dissipation rate of the system, $W_{\text{int}}$, is equal to the internal energy dissipation rate of the unstable rock mass on the velocity discontinuity surface, $W_{\text{pint}}$, plus the dissipation rate of the internal energy of the pile resistance force in the kinematically admissible velocity field, $W_{\text{pint}}$. The balance equation is

$$W_{\text{int}} = W_{\text{pint}} + W_{\text{pint}},$$

where $W_{\text{pint}}$ is the lateral pressure and the slip surface function of the rock, that is, the dissipation rate of the internal energy by the pile body's resistance force in a kinematically admissible velocity field, and $W_{\text{int}}$ is the internal energy dissipation rate of the unstable rock mass itself on the velocity discontinuity surface.

3.1. Rock-Soil Lateral Pressure Calculation Method Based on Complex Rock Slopes. In the upper bound analysis, it is possible to obtain the virtual power equation by setting the internal energy dissipation rate of the system equal to the power exerted by the external force [30].

$$W_{\text{int}} = W_{\text{ext}},$$

where the internal energy represents the internal energy dissipation rate of the unstable rock mass and the velocity discontinuity surface. The external power includes the equivalent load generated by the self-weight of the rock mass and the virtual power made in the kinematically admissible velocity field.

The final function of the sliding failure surface of the rock mass needs to be considered as a piecewise function. Specifically, the expression for the entire slope's surface is established in the following manner. The structure itself and the rock bridges between the structural planes, and the caves are expressed as linear functions, while the penetrated sections inside a cave and between the caves are expressed as logarithmic spirals.

As shown in Figure 2, the rock bridge between the karst cave and the structural planes is considered as individual segments. $MN$ is defined as the upper structural plane, and $GR$ is defined as the lower structural plane. The upper rock bridge $DE$ is a straight line. The lower rock bridge and the karst cave are approximated by a logarithmic helix through the failure section, denoted by $EF, OE$ is the starting point of the logarithmic spiral. $D$ is the starting point of the rock bridge facture. The horizontal line $DK$, which crosses vertical excavation face $AG$ at point $K$, is the constructed virtual working surface. $DK$ is subjected to the gravity of the upper rock mass $ACDK$. The gravity function of segment $DK$ corresponds to a uniform load with a length $L$ and amount $q$. Therefore, the force analysis of the entire analysis model can be divided into three parts: the logarithmic spiral surface.
composed of KDEO, OEF, and OFG. $H_C$ represents the depth of the lower structural plane on the slope surface. The angle between $DB$ and $DC$ is $45° + \varphi/2$.

$c_1$ and $\varphi_1$ are the shear strength indicators for the rock mass. $c_2$ and $\varphi_2$ are the shear strength indicators for the structural plane. If $k$ is the safety factor of the foundation pit, then the relationship between the strength parameters of the rock mass $(c_1, \varphi_1)$ and the reduced strength parameters $(c_{m1}, \varphi_{m1})$ and the relationship between the strength parameters of the structural plane $(c_2, \varphi_2)$ and the reduced strength parameters $(c_{m2}, \varphi_{m2})$ are as follows:

$$
\tan \varphi_{m1} = \frac{\tan \varphi_1}{K} c_{m1} = \frac{c_1}{K},
$$

$$
\tan \varphi_{m2} = \frac{\tan \varphi_2}{K} c_{m2} = \frac{c_2}{K}.
$$

The relationships between $v_0$, $v_1$, $v_2$, and $v$ can be expressed as

$$
v_0 = v_1 \cdot \cos \varphi_{m1},
$$

$$
v = v_1 \cdot e^{\theta \tan \varphi_{m1}},
$$

$$
v_2 = v_1 \cdot e^{\theta_1 \tan \varphi_{m1}}.
$$

The total work done by the internal force is equal to the sum of the work done on straight line $DE$, the logarithmic spiral slip surface $EF$, and the straight line $FG$, and can be expressed as

$$
E_{total} = E_{EF} + 2E_r + E_{FG},
$$

where $E_{total}$ is the total internal force of the work, $2E_r$ is the total internal force of the work of the logarithmic spiral, and $E_{EF}$ and $E_{FG}$ are the internal forces of the work on $EF$ and $FG$, respectively.

$$
T = e^{\theta_1 \tan \varphi_{m1}},
$$

and $E_{total} = c_{m1} \cdot v_1 \cdot \cos \varphi_{m1} \frac{DE}{r_0 \tan \varphi_{m1}} + c_{m1} \cdot v_1 \cdot \frac{v_0^2}{r_0 \tan \varphi_{m1}} \left( e^{\theta_1 \tan \varphi_{m1}} - 1 \right) + E_{FG},
$$

where $DE = L \cdot \tan \beta - \frac{r_0 T}{\cos (90 - \varphi_{m1} - \theta_1)} + \frac{h}{\cos \beta} + r_0 \cdot \sin \varphi_{m1}.

The total external force is the sum of the weight of the sliding soil, the earth pressure, the external force equivalent to the weight of the overlying soil, and the work done by the retaining force.

$$
W_{total} = W_{DEOK} + W_{OEF} + W_q + W_{OFG} + W_p,
$$

where $W_{DEOK} = y \cdot v_1 \cdot \cos \varphi_{m1} (\overline{OK} \cdot L + 1/2r_0^2 \sin \varphi_{m1} \cos \varphi_{m1})$. 

Figure 1: Schematic diagrams of the failure mode of a rock slope containing structural planes and karst caves (Figure 1 is reproduced from Jin Xu et al. 2021). (a) Single structural plane sliding failure. (b) Curved face-plane sliding failure. (c) Polyline sliding failure between multiple structural planes.

Figure 2: Analysis and calculation model of the upper bound method for rock slopes containing structural planes and karst caves.
\[ W_q = q \cdot L \cdot v_1 \cdot \cos \varphi_m, \]
\[ W_p = F_p \cdot v_1 \cdot T, \]
\[ W_{OEF} = \frac{1}{2} \cdot r_0^2 \cdot v_1 \cdot \frac{T^3 \cdot \left(3 \tan \varphi_m \cdot \cos(\varphi_m + \theta_1) + \sin(\varphi_m + \theta_1) - 4 \sin \varphi_m \right)}{1 + 9 \tan^2 \varphi_m}, \]

If \( E_{\text{total}} = W_{\text{total}} \).

Then, the general equation can be written as
\[ AL^2 + BL + Cr_0^2 + Dr_0 + Er_0L + F = 0, \quad (9) \]

\[
B = \gamma \cos \varphi_m \cdot \frac{h}{\cos \beta} - c_m \cdot \cos \varphi_m \cdot \tan \beta + q \cos \varphi_m, \\
C = \frac{1}{2} T^3 \cdot \left[ \frac{1}{1 + 9 \tan^2 \varphi_m} \right] + \frac{1}{2} T \cdot \cos \varphi_{m2} \cdot \tan(90° - \varphi_m - \theta_1), \\
D = \frac{c_m(T^2 - 1)}{\tan \varphi_m} - c_{m2} \cdot T^2 \cdot \cos \varphi_{m2} \cdot \tan(90° - \varphi_m - \theta_1), \\
E = -
\gamma \cos \varphi_m \cdot \frac{T}{\cos(90° - \varphi_m - \theta_1)}, \\
F = -c_m \cdot \cos \varphi_m \cdot \frac{h}{\cos \beta} \cdot F_p \cdot T - \cos \varphi_m \cdot \sin \varphi_m, \\
q = \frac{(N - L)^2 \cdot (\sin^2 \beta) \cdot (1/2) \gamma + (N - L)^2 \tan \beta \cdot \cos \beta \cdot (1/2) \gamma \tan \varphi_{m2} - c_{m2} \cdot (N - L) \tan \beta \cdot \gamma}{L}, \\
N = \left[ H_c - \frac{h}{\sin(90° - \beta)} \right] \cdot \tan(90° - \beta). 
\]

\( L \) is a function of \( r_0 \). In order to find the position in the cave between the two structural planes where penetration occurs, it is necessary to determine the minimum weight generated by the overlying rock mass, that is, the minimum value of \( L \), defined as \( dL/dr_0 = 0 \).

\[ 2Cr_0 + D + EL = 0, \quad (11) \]

\[ L = \frac{-2Cr_0 + D}{E}. \]

\[ L = f(\gamma, c_m, \varphi_m, \varphi_{m1}, \beta, r_0, c_{m2}, \varphi_{m2}, H_c). \quad (12) \]

When the retaining force \( F_p = 0 \), the safety factor can be expressed as
\[ K = f(\gamma, c_m, \varphi_m, \beta, L, c_{m2}, \varphi_{m2}, H_c), \quad (13) \]

\[ \text{where} \; c_{m1} \; \text{and} \; c_{m2} \; \text{are the cohesion and internal friction angle of the rock mass, respectively, and} \; \varphi_{m1} \; \text{and} \; \varphi_{m2} \; \text{are the cohesion and internal friction angle of the structural plane, respectively.} \; H_c \; \text{is the depth of the underlying structural plane from the slope’s surface.} \]

When the rock mass slides along the fracture surface (defined as \( k \leq 0 \)), the lateral earth pressure \( F_p \) in formula (9) can be expressed as
\[ F_p = f(\gamma, c_{m1}, \varphi_{m1}, \beta, L, c_{m2}, \varphi_{m2}, H_c, k). \quad (14) \]

The above equations can be programmed into MATLAB to constrain several parameters to find the optimal solution to the equation.

It should be noted that if the location of the cave is placed on the top surface of the foundation pit, the above equations can be used for the case of the slope sliding along a single
structural plane in Figure 1(a). If the above equations do not take into account the shift in rock mass ratio of the virtual action surface, then they can be used when the slope is damaged by sliding from a curved surface to a plane in Figure 1(b). Applying the equations sequentially as a combination of multiple structural planes and karst caves, the equations can be used in the case of multiple lines of sliding failure between multiple structural planes occurs in the slope.

3.2. Comparison of the Safety Factor Results Obtained Using the Theoretical Method and Simulations. To verify the accuracy of the theoretical equations, karst cave penetration failure along parallel structural planes in a foundation pit slope was used as a test case. A calculation was carried out using MATLAB to obtain the safety factor change curve, which was then compared with the numerical simulation calculation results.

The comparison and verification were carried out by substituting the numerical simulation parameters ($c_1 = 300$ kPa, $\varphi_1 = 30^\circ$, $c_2 = 144$ kPa, $\varphi_2 = 30^\circ$, $\gamma = 23$ kN/m$^3$, and $H_{c2} = 25$ m) into equation (13), and by setting the dip angle of the structural plane to $60^\circ$ and $70^\circ$ (the point labels of the karst cave model are shown in Figure 4). The curves of the relationship between the safety factor and the position of the karst cave’s centroid are shown in Figure 4. At a dip angle of $60^\circ$, the load of the overlying rock mass at point 3 is $43.55$ kN/m, $L = 9.63$ m. The factor of safety $k$ is 1.24 for the theoretical calculation and 1.3 for the numerical simulation. The result of the theoretical calculation is slightly smaller than that of the numerical simulation in Figure 4(a). The result of the theoretical calculation tends to be more conservative.

The same calculation was carried out at point 1 for a dip angle of $70^\circ$. As shown in Figure 4(b), the result of the theoretical calculation is slightly larger than the numerical simulation result, with a maximum difference in the safety factor of 0.08. The calculation results are in general agreement with the simulation results.

3.3. Analysis of Support Design Based on the Upper Bound Method. To better guide the design of engineering support, the safety evaluation equations for a foundation pit need to reflect the retaining forces required by the foundation pits for conditions where the factor of safety is lower than the specification. Figure 5 shows the relationship between the dip angle of the structural plane and the retaining force (the negative sign indicates that a retaining force is required), where the parameters are $c_1 = 300$ kPa, $\varphi_1 = 30^\circ$, $c_2 = 144$ kPa, $\varphi_2 = 30^\circ$, $\gamma = 23$ kN/m$^3$, $h = 2.25$ m, and $k = 2.0$. As the dip angle of the structural plane changes from $60^\circ$ to $80^\circ$, the retaining forces required by the foundation pit change from large to small and then to large. For a constant length $L$, the retaining force required is maximum when the dip angle of the structure plane reaches $80^\circ$. In addition, the greater the distance from the cave to the slope of the foundation pit, that is, the closer the cave to the ground surface in the same structural plane, the greater the retaining force required. Similarly, in Figure 6, the curves of the retaining force versus the strength reduction factor also indicate that the higher the reduction factor, the greater the additional retaining force required, which is consistent with the actual conditions.

Table 1 lists the common methods used to calculate the lateral earth pressure on a pile. Since a vertical slope is used, and the top layer of the slope is not overloaded, the calculation result of Coulomb’s earth pressure (without considering the friction angle between the earth and the back of the wall) is the same as the calculation result of Rankine’s earth pressure. The parameters used in the calculation are as follows: rock weight of $23$ kN/m$^3$, rock mass friction angle of $45^\circ$, cohesion of $150$ kPa, Poisson’s ratio of 0.17, additional load on the top of the slope of 0, angle between the fracture plane and the horizontal plane of $60^\circ$, internal friction angle of the outward sloping structural plane of $15^\circ$, and the surface cohesion of 0 kPa at the structural surface of the outer slope. The method of calculating the earth pressure for sliding along outwardly sloping rigid structural planes is specific to rock slope engineering. The calculation result that considers the structural plane is closer to the actual range of damage. The calculation result obtained using our proposed upper bound method is smaller than that obtained using the method for an outward sloping rigid structural plane. This shows that when the operating conditions of karst caves are not considered, the calculation method along the outward sloping structural plane provides a slightly more conservative result. The reason for this is that the upper bound analysis is used to satisfy the equilibrium of the virtual powers based on static and dynamic conditions. The calculation result is the minimum remaining sliding force of the rock mass that satisfies the equilibrium equation, which is more stringent than the traditional static equilibrium equation. Therefore, the calculated result for the slip along the outward sloping rigid structural plane satisfies the calculation equations of the upper bound method.
Rankine and Coulomb's earth pressure equations are generally used for loose structures. When the retaining wall is short, the calculations are conservative, while when the retaining wall is high, the calculations tend to be aggressive. The calculation result of the wedge analysis is closer to that of the upper bound analysis, but the disadvantage of this method is that it depends on the empirical value of the internal friction angle of the soil-rock interface.

4. Method of Calculating Pile Deformation Control

The pile displacement calculation method based on numerical analysis is a method for calculating the forces on retaining piles based on the upper bound method. In the analysis, the rock model only restricts the portion of the pile embedded in the rock. The range of the forces on the piles is determined by the actual excavation depth. The incremental rock-soil lateral pressure generated by the excavation can be...
obtained from the above calculations based on the upper boundary analysis.

4.1. Numerical Method of Calculating the Pile Displacement.
The pile deformation calculations based on the finite element method for a pole system can be divided into two methods. The first method uses the total load method and does not take into account the construction process of the foundation pit. The second method adopts the incremental method to consider each phase of the pit construction. The calculations for both methods depend on the forces on the pile and the restraint forces of the soil around the pile that is below the excavation face. A rectangular parallelepiped rock mass of $120 \times 120 \times 100$ m was modelled in the calculations using 3DEC distinct-element calculation software to simulate the actual stress state of the foundation pit by applying loads to the rock mass to replace the calculation of the earth reaction force in the embedded portion (Figures 7 and 8). The interaction between the pile and the rock mass is simulated by setting up two sets of nonlinear coupling springs in the normal and tangential directions, obeying the Mohr–Coulomb theory. The cohesive force and angle of internal friction on the interface are set at 0.8 times as high as the adjacent rock formation, and the normal and shear stiffnesses of the interface are set at 10 times the stiffness of the rock mass [31].

When the incremental method is used to consider the excavation process of a foundation pit, the rectangular parallelepiped model was divided into vertical layers, and each layer represented the thickness of each excavation until the designed excavation depth of the foundation pit was reached. In addition, a retaining pile was set at the center point of the model, with the top of the pile being flush with the top surface of the model. An example is shown in Figure 8. A retaining pile with a diameter of 1 m and a length

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Depth (m)</th>
<th>Total active earth force (kN)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rankine’s active earth pressure</td>
<td>0</td>
<td>0</td>
<td>Coefficient of cohesion is ignored; Fracture angle is $45^\circ + \phi/2$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>197.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>789.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1775.8</td>
<td></td>
</tr>
<tr>
<td>Coulomb’s active earth pressure</td>
<td>0</td>
<td>0</td>
<td>Friction angle between the rock mass and the back of the wall is</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>966.4</td>
<td>$0.33 \phi$; Fracture angle is $45^\circ + \phi/2$</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1573.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1814.9</td>
<td></td>
</tr>
<tr>
<td>Wedge analysis</td>
<td>0</td>
<td>0</td>
<td>Friction angle between the earth fill and the rock interface is</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>499.3</td>
<td>$18^\circ$; Fracture angle is $60^\circ$</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1997.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4493.6</td>
<td></td>
</tr>
<tr>
<td>Outward sloping rigid structural plane</td>
<td>0</td>
<td>0</td>
<td>Fracture angle is $60^\circ$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>542.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2171.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4886.4</td>
<td></td>
</tr>
<tr>
<td>Upper bound analysis</td>
<td>0</td>
<td>0</td>
<td>No cave. Fracture angle is $60^\circ$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>314</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1834.7</td>
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</tr>
<tr>
<td></td>
<td>30</td>
<td>4351.2</td>
<td></td>
</tr>
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</table>
of 34 m was installed with a total design excavation depth of 30 m, and each excavation step had a depth of 3 m. There were 10 excavation steps. A concrete support was installed at the top of the pile. When the excavation reached a depth of 15 m, a steel support was set at 10 m, and a prestress of 200 kN was applied. When the excavation reached 20 m, an anchor cable was installed at a depth of 18 m. The final deformation of the pile can be calculated using the incremental method.

4.2. Calculation of Pile Deformation Based on Numerical Analysis. Based on the above calculation principles, Table 2 shows the incremental earth pressure per 3 m layer for a 30 m excavation depth using Rankine's earth pressure calculation equation.

Substitution of the values in Table 2 into the model shown in Figure 8 yields the results shown in Figure 9. Prior to the excavation of the pit to a depth of 9 m, the displacement of the pile was very small, and the portion of the retaining pile below the excavation face was barely deformed. After the installation of steel supports, as the excavation depth increased, the upper half of the pile tended to deform toward the outside of the foundation pit, while the lower half of the pile appears to convexly deform toward the inside of the foundation pit, with the maximum displacement point exhibiting a downward trend as the excavation depth of the pit increased. When the excavation depth reached 30 m, the maximum displacement point of the deformation curve occurred at about two-thirds of the pile length.

Figure 10 shows the pile displacement curves calculated using the traditional finite element method for a pole system. In the calculation, the rock mass model below the unexcavated surface was not defined, and the active earth pressure was used on the outside of the foundation pit, and the earth reaction force on the embedded portion is used for the inner side of the foundation [32]. It can be seen from Figure 10 that the calculated soil reaction forces produced excessive pile deformation prior to the installation of the steel supports, with the maximum deformation occurring at an excavation depth of 3 m, which is far from consistent with the actual situation. However, when the excavation of the foundation pit reached a depth of 21 m, the trend of the deformation was consistent with the calculation conditions and the rock mass constraints. In addition, when the excavation reached the bottom of the foundation pit (30 m), the maximum displacements reached by the two were basically the same, and the maximum displacement occurred at about two-thirds of the pile length.

5. Method of Support Design for Rock Slopes Containing Karst Caves

The rock slope stability analysis based on the upper bound analysis principle can predict the range of the slip surfaces. Provided that the approximate location of the cave's centroid can be determined via geophysical field testing methods, equation (12) and MATLAB can be used to calculate the minimum overburden load distance for cave penetration failure $L_{\text{min}}$. When the horizontal distance between the actual position of the cave and the slope of the foundation pit is $L < L_{\text{min}}$, penetration failure will not occur. Then, the position of the karst cave in equation (13) is placed on the top surface of the foundation pit, which becomes a safety assessment of the overall slip along the structural plane. By using this method to assess each karst cave where penetration can occur, a final extent of sliding fracture of the rock mass can be calculated. Figure 11 presents a flowchart of this process.

Substituting the calculated safety factor $k$ and rock mass parameters into equation (14), the resultant earth pressure within the range of the slip surface can be obtained. If the excavation process of the foundation pit is considered, and a virtual structural plane is set at the corresponding depth of each excavation layer, the increment of the earth pressure generated by each excavation layer can be calculated. By combining this with a numerical simulation based calculation method for the force and the deformation of the retaining pile, the retaining effectiveness of the pile can be evaluated.

6. Verification of an Engineering Case: The Foundation Pit of the Nansanhuanslu Station

The Nansanhuanslu Station of the Xuzhou Metro Line 3 is located at the intersection of the Nanshan Road and the Beijing Road. The main foundation pit of the station was constructed using the open-cut and bottom-up construction method. Retaining piles and horizontal internal supports (anchor cables) were used as the retaining structures. The excavation depth of the foundation pit was about 30 m, and the stratigraphic structure is dominated by limestone (Table 3).

The section we studied is located on the west side of the foundation pit and belongs to the slope of the outward sloping structural plane (at a dip angle of 70°). Bored piles of Ø1000 mm × 1500 mm were used for this section. The first support is made of concrete, and the second and third supports are made of steel. An inclinometer tube was installed on the west side of the deepest intermediate mileage section. The monitoring positions are presented in Figure 12. The inclinometer tube was damaged during the construction of the W2 pile and is therefore not discussed here.

Detection data from the seismic tomography survey (provided by Professor Song Lei’s research group) show the caves (Figure 13). Karst cave 1 exhibits a longitudinal extension of the foundation pit, which is a massive cave with a length of 15 m.

In the calculations, a 5 m deep layer was used to simulate the construction. The first concrete support was constructed at a depth of −5 m. When the excavation reached −15 m, the first steel support was installed at −10 m. When the excavation went to −25 m, the second steel support was installed at −20 m, giving a final excavation depth of −30 m.

In the theoretical calculations, Cave 1 (Figure 13) was transformed into a plane model, that is, without regard to the longitudinal shape and length. Cave 2 was not analyzed due
The parameters presented in Table 3 and the overlying rocklayerofthekarstcavewereconvertedintoanequivalentpressureof122kN/m. By substituting the measured distance from the center of the cave to the vertical excavation boundary of the foundation pit (5.8 m) and the safety factor \( k \) of 1.2 into equation (14), the remaining sliding force at an excavation depth of 15 m was calculated to be 185.64 kN, which is greater than zero. When the excavation depth is 25 m, the calculated remaining sliding force is 865.77 kN, which is also greater than zero. Therefore, the upper and lower structural planes of the cave satisfy the conditions of overall sliding.

Substituting the actual horizontal distance between the cave and the excavation boundary of the foundation pit \( (L = 4.3 \text{ m}) \) into equation (12) yields \( L_{\text{min}} = 10.22 \text{ m} \), which is greater than the actual horizontal distance \( L \), thus indicating that cave penetration failure may occur. Based on the above analysis results, the eventual slope of the foundation pit will suffer overall sliding failure. Figure 14 shows the slope displacement failure heat map calculated using the discrete element method. Karst cave penetration has occurred, but there is also relative sliding of the upper rock mass of the karst cave, which is consistent with the theoretical analysis result.

Figures 15(a)-15(d) show a comparison of the theoretical value and the measured value for each excavation stage. Figure 15(a) shows that the displacement curve sliding along the 70° structural plane is similar to the deformation of piles \( W_1 \) and \( W_3 \) on the measured-slope curves above the excavation face. In Figures 15(b) and 15(c), the displacement curve sliding along the 60° structural plane reflects the deformation characteristics of piles \( W_1 \) and \( W_3 \) on the measured-slope curves above the excavation face. The parts below the excavation face deviate from the theoretical calculation results due to the rock mass structural planes and measurement errors but basically reflect the deformation characteristics of the part above the excavation face.
When the pit was excavated to 30 m, the sliding displacement curve along the 60° structural plane was consistent with the deformation of \( W_4 \) in Figure 15(d). This is because the displacement of retaining pile \( W_4 \) occurs at the inflection point of the edge of the foundation pit, and the early excavation is strongly influenced by the spatial effect.

In addition, it can be seen that, for a slope with a 70° dip angle, the slip area is dominated by the surface of the high dip structure until the excavation reaches 15 m, while after
Figure 13: Locations of the karst caves in the west side of the foundation pit.

Figure 14: Displacement diagram of karst cave penetration failure.

Figure 15: Continued.
the excavation reaches 15 m, the slip surface defined by $45^\circ + \varphi$ is dominated.

7. Conclusions

In this paper, calculation equations for stability analysis of slopes based on the upper limit analysis and improved methods for calculating the deformation of retaining piles based on numerical analysis are proposed, respectively, for rock slopes developed with a combination of structural planes and caves, and the major conclusions can be summarized as follows:

1. Calculation equations for stability analysis of slopes based on the upper limit analysis can calculate the stability of sliding slopes caused by caves in three different failure modes separately, so that the range of possible sliding slopes in the foundation pit can be predicted, and the analysis covers a wider range.

2. Calculation equations for the remaining sliding force of a foundation pit under ultimate conditions are derived from rock slope stability analysis, which enables a safety factor to be set according to the importance of the project and the calculation of the minimum supporting force required to be provided by the supporting structure.

3. The results of the remaining sliding force calculation equations proposed in this paper are slightly smaller compared to the traditional earth pressure calculation equations, and the conditions satisfied are more stringent, which can be used as a reference for the value of the structure design.

The improved method of calculating the deformation of the retaining pile based on numerical analysis avoids the process of calculating the lateral Earth pressure in the embedded portion of the pile compared with the traditional elastic fulcrum method, and the results of the pile deformation calculation are more in line with reality.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request. Yansen Wang TB18220020B0@cumt.edu.cn.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Conceptualization, formal analysis, data curation, and writing were performed by J. X; supervision and funding acquisition were done by Y. W. The above authors have read and agreed to the published version of the manuscript.

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