Research Article

Study on Dynamic Characteristics of Circular Tunnel Lining in Fractional Derivative Viscoelastic Radially Inhomogeneous Soil

Zongling Zhang, Ru Li, Qifang Yan, and Linchao Liu

School of Architecture and Civil Engineering, Xinyang Normal University, Xinyang, Henan 464000, China

Correspondence should be addressed to Zongling Zhang; zzl790206@sina.com

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1. Introduction

The mechanical properties of soil are important factors affecting the dynamic response of civil engineering structures. Due to the different engineering environment of the soil, there are great differences in the mechanical properties of the soil under different working conditions and in different regions. Usually, we need to focus on the influence of two properties of soil, one is the influence of the viscosity of soil, and another is the influence of the heterogeneity of soil, including radial heterogeneity and stratification. Under the action of long-term load, the material usually leads to creep or relaxation, that is, it shows viscoelastic properties. In the mechanical analysis of tunnel, pile foundation, and other structures, the influence of viscoelastic properties of soil cannot be ignored. At present, the classical viscoelastic constitutive model is mainly used to describe the viscoelastic properties of soil, but the classical viscoelastic model cannot be in good agreement with the experimental data at the initial stage of creep and relaxation [1]. To accurately fit the experimental data, the high-order differential term often has to be cancelled or the application scope of the constitutive
model is reduced. Because the fractional derivative viscoelastic model can describe the mechanical behavior of soil in a wide frequency range, few experimental parameters are required to determine the model [2]. In recent years, the application of fractional derivative viscoelastic model in the field of geotechnical engineering has been paid more and more attention. Based on viscoelastic theory and theory of fractional calculus, Zhu et al. [3] proposed a fractional Kelvin-Voigt model to describe the time-varying characteristics of foundation under vertical load and gave the analytical solution of foundation settlement. By changing the fractional order of the differential operator and viscosity coefficient, the settlement time relationship can be accurately obtained, and the long-term settlement of foundation can be more accurately predicted. Wang et al. [4] introduced the theory of fractional calculus into the Kelvin-Voigt constitutive model to describe the consolidation characteristics of viscoelastic saturated soil, gave the semi analytical solution of the one-dimensional consolidation equation of the fractional derivative Kelvin-Voigt model for viscoelastic saturated soil under different variable loads, and obtained the analytical solution of effective stress and settlement in Laplace transform domain. Xiao et al. [5] used the three-dimensional fractional derivative viscoelastic model to describe the rheological properties of the soil around the pile and used the correspondence principle and Laplace transform technology to give the long-term settlement of a single pile in the fractional derivative viscoelastic soil under time-dependent load. Li et al. [6] analyzed the triaxial creep and shear test results of deep artificial frozen soil under different confining pressure and temperature, established the fractional derivative constitutive model of deep artificial frozen soil based on Nishihara model, developed the corresponding constitutive finite element program, and simulated the deep artificial freezing curtain of a coal mine, it is verified that the fractional derivative constitutive model can describe the nonattenuation creep deformation characteristics of deep artificial frozen soil under high confining pressure. Engineering construction and in situ stress usually cause the compression, disturbance, and remodeling of the soil near the engineering structure, which will lead to the change of the properties of the soil near the engineering structure, such as shear modulus and density, resulting in the nonuniform distribution of the soil around the structure. At present, considering the influence of soil heterogeneity, the research on the dynamic response of engineering structures is mainly carried out for the vibration of pile foundation [7–12].

With the construction of subway, railway, subsea tunnel, and other projects, the research on the dynamic stability of underground engineering structures is particularly important. The research on the dynamic response of underground engineering structures such as tunnels is of great significance for the seismic design and dynamic monitoring of tunnels, which has attracted extensive attention of scholars. Li [13] obtained the analytical solution of stress-displacement field caused by tunnel excavation in saturated soil in Laplace transform domain and obtained the solution in time domain with the help of inverse Laplace transform numerical technology. The results show that the stress field and displacement field are highly sensitive to the flow conditions imposed on the tunnel wall. Xie et al. [14] gave analytical solutions of stress, displacement, and pore pressure of partially closed circular tunnel lining in viscoelastic saturated soil under progressive loading in Laplace transform domain, the research shows that the local closure of boundary has a significant impact on stress, displacement, and pore pressure, and the radial displacement decreases with the increase of the viscoelastic damping coefficient of soil and the relative stiffness between lining and soil. Lu and Jeng [15] studied the dynamic response of circular tunnel in saturated porous media under moving axisymmetric annular load and obtained the numerical solutions of stress, displacement, and pore pressure. Yang et al. [16] assumed that the tunnel was a beam element, imposed the free field displacement of the horizontal layered viscoelastic foundation on the long tunnel through the viscoelastic foundation model, and analyzed the dynamic response of the long tunnel in the horizontal layered viscoelastic foundation under the action of inclined SH wave by Fourier transform. Yu and Wang [17] established the overall seismic response calculation model of tunnel surface buildings by using the finite element method, and studied the seismic response of dynamic characteristics of a subway tunnel structure under earthquake loading and the interaction characteristics between tunnel structure and ground buildings. Alielahi and Feizi [18] studied the interaction between underground tunnel and surface structure under the action of SV wave by finite element method. It was found that the influence of soil-tunnel structure interaction on surface structure displacement and displacement depends on the characteristics of soil, underground tunnel, number of layers and input motion. Huang et al. [19] considered the slip effect at the interface between lining and ground lining, and obtained the simplified analytical solution of stress and deformation of circular composite lined tunnel under the action of in-plane shear wave, which can be used to approximately calculate the seismic response of composite lined circular tunnel. Lu et al. [20] simplified the blasting P-wave into a triangular pulse, and obtained the distribution functions of dynamic stress concentration coefficient and radial and circumferential dynamic velocity scale coefficient around the circular tunnel based on the Fourier Bessel expansion method. The effects of the rise duration and total duration, Poisson’s ratio on dynamic stress concentration coefficient, and distribution function are discussed. It can be seen that, although many achievements have been made in the current research on the dynamic response and dynamic characteristics of tunnel structure, with the rapid development of tunnel and underground engineering, the engineering environment, and geological environment of tunnel structures are more complex. Due to the influence of excavation disturbance or in situ stress, the heterogeneity of tunnel surrounding rock is common. To ensure
the safety and stability of a tunnel construction and to guide the design and construction of tunnel in inhomogeneous soil, it is necessary to study the dynamic characteristics of a tunnel structure in inhomogeneous soil. Therefore, considering the heterogeneity and viscoelasticity of the soil around the tunnel, based on the fractional derivative viscoelastic model, the dynamic characteristics of the circular tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil are studied in the frequency domain by using the multicycle transfer matrix method and the initial parameter method.

2. Model of Viscoelastic Radially Inhomogeneous Soil-Circular Tunnel Lining

The dynamic characteristics of a wireless long and deep buried circular tunnel in infinite soil shown in Figure 1 will be investigated. There is a simple harmonic load $q = q_0 e^{i \omega t}$ with radial uniform distribution on the inner boundary of the tunnel lining, where $\omega$ is the frequency of the load, $i$ is the imaginary unit, and $q_0$ is the load amplitude. The soil around the tunnel is regarded as viscoelastic medium, and the lining of the tunnel is regarded as elastic solid. The outer radius and lining thickness of the tunnel lining are $R$ and $d$, respectively. It is assumed that the viscoelastic soil in the range from $R$ to $R_0$ around the circular tunnel is disturbed due to construction and other reasons, and the viscoelastic soil outside $R_0$ is not disturbed. The shear modulus of viscoelastic soil in undisturbed area is $\mu_O$. The shear modulus of the soil in the disturbed area is given in reference [21, 22] (as shown in Figure 2). Here, it is assumed that the soil around the tunnel is a viscoelastic medium, and the fractional derivative viscoelastic constitutive relationship is used to describe the relationship between stress and strain of soil [3], that is,

$$\sigma_r - \sigma_\theta = \rho \frac{\partial^2 u_r}{\partial t^2},$$

where $\sigma_r$ and $\sigma_\theta$ are the radial and circumferential stresses of soil, respectively, $u_r$ is the radial displacement of soil, and $\rho$ is the density of soil mass.

3. Dynamic Equations of Fractional Derivative Viscoelastic Soil and the Solution by Multicycle Matrix Transfer Method

According to the theory of elastic dynamics, ignoring the circumferential displacement, the motion equation of soil under polar scale is

$$\frac{\partial^2 \sigma_r}{\partial r^2} + \frac{\sigma_r - \sigma_\theta}{\rho \frac{\partial^2 u_r}{\partial t^2}},$$

where $\lambda$ and $\mu$ are the lame constant of soil, $\lambda = 2\mu/1 - 2\nu$, $\nu$ is Poisson’s ratio of soil mass, $D^\alpha = d^\alpha/dt^\alpha$ is the Riemann Liouville fractional derivative, $\alpha$ is the

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$$(1 + \tau_1 D^\alpha)\sigma_r = (1 + \tau_2 D^\alpha)\left[\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + 2\mu \frac{\partial^2 u_r}{\partial r \partial t}\right],$$

$$(1 + \tau_1 D^\alpha)\sigma_\theta = (1 + \tau_2 D^\alpha)\left[\lambda \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + 2\mu \frac{u_r}{r}\right],$$

where $\tau_1$ and $\tau_2$ are the model parameter of soil fractional derivative viscoelastic model, $\lambda, \mu$ are the lame constant of soil, $\lambda = 2\mu/1 - 2\nu$, $\nu$ is Poisson’s ratio of soil mass, $D^\alpha = d^\alpha/dt^\alpha$ is the Riemann Liouville fractional derivative, $\alpha$ is the

$$\begin{align*}
\mu (r) &= \begin{cases} 
\mu_O f(r), & R < r < R_O, \\
\mu_O, & r \geq R_O,
\end{cases} \\
f(r) &= 1 + \left(1 - \frac{\mu_i}{\mu_O}\right) \left(\frac{R_O - r}{R_S}\right)^m.
\end{align*}$$
order of fractional derivative, and \(0 < \alpha < 1\). Riemann Liouville fractional derivative is defined as \([23]\).

\[
D^\alpha(x(t)) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{d}{dr} \left( x(r) \right)^{-\alpha} dr,
\]

where \(\Gamma(u) = \int_0^{\infty} t^{u-1} e^{-t} dt\) is the gamma function.

From equations (1)–(3), the motion control equation of fractional derivative viscoelastic soil expressed in displacement can be obtained as

\[
\frac{2-2vO}{1-2vO} \rho \left( 1 + \tau_{120}^O D^{\alpha_o} \right) \frac{\partial}{\partial r} \left( \frac{\partial u_{Or}}{\partial r} + \frac{u_{Or}}{r} \right) = \rho \left( 1 + \tau_{120}^O D^{\alpha_o} \right) \frac{\partial^2 u_{Or}}{\partial r^2},
\]

(6)

\[
\frac{2-2v_{Or}}{1-2v_{Or}} \mu_{Or} \left( 1 + \tau_{12}^O D^{\alpha_o} \right) \frac{\partial}{\partial r} \left( \frac{\partial u_{Or}}{\partial r} + \frac{u_{Or}}{r} \right) = \mu_{Or} \left( 1 + \tau_{12}^O D^{\alpha_o} \right) \frac{\partial^2 u_{Or}}{\partial r^2},
\]

(7)

where \(u_{Or}, \rho_{Or}, \nu_{Or}, \text{ and } \mu_{Or}\) are the radial displacement, density, Poisson’s ratio, and shear modulus of viscoelastic soil in the undisturbed area, respectively, \(\tau_{120}^O, \tau_{210}^O \text{ and } \sin \sigma_{Or}\) are the model parameters and the order of the fractional derivative of the fractional derivative viscoelastic model of viscoelastic soil in the undisturbed area, respectively. \(u_{Or}, \rho_{Or}, \nu_{Or}, \text{ and } \mu_{Or}\) are the radial displacement, density, Poisson’s ratio, and shear modulus of viscoelastic soil in the undisturbed area, respectively. \(\tau_{120}^O, \tau_{210}^O \text{ and } \sin \sigma_{Or}\) are the model parameters and the order of the fractional derivative of the fractional derivative viscoelastic model of viscoelastic soil in the undisturbed area.

Because the circular tunnel will vibrate stably under the action of simple harmonic internal pressure, the parameters such as radial displacement and radial stress of soil in the undisturbed area and the irth circle of disturbed area meet \(u_{Or} = \bar{u}_{Or} e^{i\omega t}, \sigma_{Or} = \bar{\sigma}_{Or} e^{i\omega t}, \mu_{Or} = \bar{\mu}_{Or} e^{i\omega t}, \sigma_{ir} = \bar{\sigma}_{ir} e^{i\omega t}, \text{ and } \bar{\sigma}_{ir} / \bar{\mu}_{Or}\) are the amplitudes of radial displacement and radial stress of soil in the undisturbed area and the irth circle of disturbed area, respectively. Then, the parameters are substituted into equations (6) and (7), and let \(U_O = \bar{u}_{Or}/R, U_i = \bar{u}_{Or}/R, \tau = r/R, \bar{\sigma} = \bar{\sigma}_{Or}/\bar{\mu}_{Or}, \nu = \sqrt{\bar{\mu}_{Or}/\bar{\rho}_{Or}}, T_{10} = \tau_{12}^O \rho R, T_{20} = \tau_{21}^O \rho R, \beta_{1i} = \tau_{12}^O / \tau_{10}, \beta_{2i} = \tau_{21}^O / \tau_{20}, \mu = \mu_{Or}/\mu_{Or}, \rho_i = \rho_{Or}/\rho_{Or}, \delta = d/R, \eta_i = (R-d/R) = \tau - \delta. \text{ We have}

\[
\begin{align*}
\frac{\partial^2 U_O}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial U_O}{\partial \tau} - \frac{q_i^2}{\rho_i} U_O &= 0, \\
\frac{\partial^2 U_i}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial U_i}{\partial \tau} - \frac{q_i^2}{\rho_i} U_i &= 0,
\end{align*}
\]

(8)

(9)

in which \(q_i^2 = -(1-2v_{Or}) \nu_i^2 / (2-2v_{Or}) \nu_{Or}^2 + \nu_i^2 = -(1-2v_i) \rho_i \nu_i^2 / (2-2v_i) \nu_{Or}^2, \nu_i = 1 + T_{20}^O (\nu_{Or})^{\alpha_i} + T_{10}^O (\nu_{Or})^{\alpha_i} \text{ and } \nu_i = 1 + \beta_{2i} T_{20}^O (\nu_{Or})^{\alpha_i} + \beta_{1i} T_{10}^O (\nu_{Or})^{\alpha_i}. \text{ By solving Bessel equations (8) and (9), the dimensionless radial displacement of undisturbed fractional derivative viscoelastic soil and irth circle of fractional derivative viscoelastic soil are, respectively, obtained as}

\[
\begin{align*}
U_O(\tau) &= A_i K_i(q_i \tau) + A_i I_i(q_i \tau), \\
U_i(\tau) &= \bar{A}_i K_i(q_i \tau) + \bar{A}_i I_i(q_i \tau),
\end{align*}
\]

(10)

where \(A_i(\cdot) \text{ and } I_i(\cdot)\) are first-order second-class and first-class deformed Bessel functions, respectively, and \(A_i, A_i, A_i, \text{ and } A_i\) are undetermined coefficients.

Since the displacement of the undisturbed fractional derivative viscoelastic soil at infinity is zero and considering the properties of first-order first-class deformation Bessel function, we know \(A_2 = 0\), then, we have

\[
U_O = A_i K_i(q_i \tau).
\]

(11)

Considering the fractional derivative viscoelastic stress-strain relationship (6), the dimensionless radial stress of the undisturbed fractional derivative viscoelastic soil is obtained:

\[
\begin{align*}
\sigma_{Or} &= \frac{2(1-2v_{Or}) K_i(q_i \tau) + (2-2v_{Or}) q_i R K_0(q_i \tau)}{1-2v_{Or}} X_{Or} A_i, \\
\sigma_i &= \frac{2(1-2v_i) K_i(q_i \tau) + (2-2v_i) q_i R K_0(q_i \tau)}{1-2v_i} X_{i} A_i \tau_i, \text{ and }
\end{align*}
\]

(12)

in which \(\sigma_{Or} = \sigma_{Or} / \mu_{Or}. \text{ Similarly, considering the fractional derivative viscoelastic stress-strain relationship (7), it can be obtained that the dimensionless radial stress of irth layer of fractional derivative viscoelastic soil in the disturbed area is}

\[
\begin{align*}
\begin{align*}
\sigma_i &= \frac{2(1-2v_i) K_i(q_i \tau) + (2-2v_i) q_i R K_0(q_i \tau)}{1-2v_i} X_{i} A_i \tau_i, \text{ and }
\end{align*}
\]

(13)
At the junction of the $i$th layer soil and $i-1$th layer soil (i.e., $\tau = \tau_i$), the radial displacement and radial stress of fractional derivative viscoelastic soil are, respectively

$$U_i(\tau_{i-1}) = A_{1i}K_1(q_i\tau_{i-1}) + A_{2i}I_1(q_i\tau_{i-1}),$$

$$\sigma_{ir}(\tau_{i-1}) = \frac{-2(1-2v)K_1(q_i\tau_{i-1}) + (2-2v)q_i\tau_{i-1}K_0(q_i\tau_{i-1})}{(1-2v)\tau_{i-1}} + \frac{(2-2v)q_iI_0(q_i\tau_{i-1}) - 2(1-2v)I_1(q_i\tau_{i-1})}{(1-2v)\tau_{i-1}}$$

where $K_0(\cdot)$ and $I_0(\cdot)$ are zero-order second-class and first-class deformed Bessel functions, respectively. The undetermined coefficients $A_{1i}$ and $A_{2i}$ can be obtained by using the initial parameter method [24] from equations (14) and (15), and then substituted into equations (10) and (13) to obtain the following recurrence relationship:

$$U_i(\tau_i) = \frac{[(2-2v)q_iI_0(q_i\tau_{i-1}) - 2(1-2v)I_1(q_i\tau_{i-1})]K_1(q_i\tau_i)U_i(\tau_{i-1})}{(2-2v)\tau_{i-1}} + \frac{(2-2v)\tau_{i-1}I_1(q_i\tau_i) [K_0(q_i\tau_{i-1})I_1(q_i\tau_{i-1}) + K_1(q_i\tau_{i-1})I_0(q_i\tau_{i-1})]}{(1-2v)\tau_{i-1}} + \frac{(2-2v)\tau_{i-1}I_1(q_i\tau_i) [K_1(q_i\tau_{i-1})I_0(q_i\tau_{i-1}) + K_0(q_i\tau_{i-1})I_1(q_i\tau_{i-1})]}{(1-2v)\tau_{i-1}}$$

$$\sigma_{ir}(\tau_i) = \frac{[2(1-2v)K_1(q_i\tau_i) + (2-2v)q_i\tau_{i-1}K_0(q_i\tau_{i-1})]I_1(q_i\tau_i)U_i(\tau_{i-1})}{(2-2v)\tau_{i-1}} + \frac{[2(1-2v)K_1(q_i\tau_i) + (2-2v)q_i\tau_{i-1}K_0(q_i\tau_{i-1})]I_1(q_i\tau_i)\tau_{i-1}I_1(q_i\tau_i) [K_0(q_i\tau_{i-1})I_1(q_i\tau_{i-1}) + K_1(q_i\tau_{i-1})I_0(q_i\tau_{i-1})]}{(1-2v)\tau_{i-1}} + \frac{[2(1-2v)K_1(q_i\tau_i) + (2-2v)q_i\tau_{i-1}K_0(q_i\tau_{i-1})]I_1(q_i\tau_i)\tau_{i-1}I_1(q_i\tau_i) [K_1(q_i\tau_{i-1})I_0(q_i\tau_{i-1}) + K_0(q_i\tau_{i-1})I_1(q_i\tau_{i-1})]}{(1-2v)\tau_{i-1}}$$

If (16) and (17) are written in matrix form, there are
\[
\begin{bmatrix}
U_i(\bar{r}_i) \\
\sigma_{ir}(\bar{r}_i)
\end{bmatrix} =
\begin{bmatrix}
a_{111} & a_{112} \\
a_{211} & a_{212}
\end{bmatrix}
\begin{bmatrix}
U_{i-1}(\bar{r}) \\
\sigma_{ir}(\bar{r})
\end{bmatrix},
\]  
(18)
in which,

\[
a_{111} = \frac{K_1(q_{\bar{r}})I_0(q_{\bar{r}-1}) + K_0(q_{\bar{r}-1})I_1(q_{\bar{r}})}{K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) + K_1(q_{\bar{r}-1})I_0(q_{\bar{r}-1})}
+ \frac{2(1-2\nu)}{(2-2\nu)q_{\bar{r}-1}[K_1(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) - K_1(q_{\bar{r}-1})I_0(q_{\bar{r}-1})]}
\]

\[
a_{112} = \frac{(1-2\nu)}{(2-2\nu)\chi}\frac{q_{\bar{r}}[K_1(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) - K_1(q_{\bar{r}-1})I_0(q_{\bar{r}-1})]}{K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) + K_0(q_{\bar{r}-1})I_0(q_{\bar{r}-1})}
\]

\[
a_{211} = \frac{2(1-2\nu)\chi\mu_j[K_1(q_{\bar{r}})I_1(q_{\bar{r}-1}) - K_1(q_{\bar{r}-1})I_1(q_{\bar{r}})]}{(1-\nu)q_{\bar{r}}[K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) + K_1(q_{\bar{r}-1})I_0(q_{\bar{r}-1})]}
\]

\[
a_{212} = \frac{2(1-2\nu)\chi\mu_j[K_1(q_{\bar{r}})I_1(q_{\bar{r}-1}) - K_1(q_{\bar{r}-1})I_1(q_{\bar{r}})]}{(1-\nu)q_{\bar{r}}[K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) + K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1})]}
\]

\[
a_{222} = \frac{2(1-2\nu)\chi\mu_j[K_0(q_{\bar{r}})I_1(q_{\bar{r}-1}) - K_0(q_{\bar{r}-1})I_1(q_{\bar{r}})]}{(1-\nu)q_{\bar{r}}[K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1}) + K_0(q_{\bar{r}-1})I_1(q_{\bar{r}-1})]}
\]

Consider the continuity boundary conditions of the radial stress and the radial displacement of the soil at the outer boundary of the 1st circle and the inner boundary of the \(n\)th, we have

\[
U_i(\bar{r}_{i-1}) = U_{i-1}(\bar{r}_i), \sigma_{ir}(\bar{r}_{i-1}) = \sigma_{ir}(\bar{r}_i).
\]
(20)

It can be obtained that the transfer relationship of the radial displacement and the radial stress of the soil at the inner boundary of the first circle and the outer boundary of the \(n\)th circle is

\[
\begin{bmatrix}
U_n(\bar{r}_n) \\
\sigma_{nr}(\bar{r}_n)
\end{bmatrix} =
\begin{bmatrix}
a_{111} & a_{112} \\
a_{211} & a_{212}
\end{bmatrix}
\begin{bmatrix}
U_{1}(\bar{r}_1) \\
\sigma_{1r}(\bar{r}_1)
\end{bmatrix},
\]
(21)

where \(a_{111} \ a_{112} \ a_{211} \ a_{212} \ a_{111} \ a_{122} \ a_{211} \ a_{222} \) is the transfer matrix of radial displacement and radial stress of soil at the inner boundary of first circle and the outer boundary of the \(n\)th circle.

### 4. Solution of Dynamic Response of Circular Tunnel Lining in Fractional Derivative Viscoelastic Radially Inhomogeneous Soil

The lining is regarded as an elastic body, and its dynamic governing equation in polar coordinates is

\[
2 - 2\nu L_t \frac{\partial}{\partial r} \left( \frac{\partial u_{lr}}{\partial r} + \frac{u_{lr}}{r} \right) = \rho L_t \frac{\partial^2 u_{lr}}{\partial t^2},
\]
(22)

where \(u_{lr}, \nu, L_t, \) and \(\rho_t\) are the radial displacement, Poisson’s ratio, shear modulus, and density of lining, respectively.

Similarly, under the action of simple harmonic load, the lining also makes simple harmonic vibration. The radial displacement and stress of the lining meet \(u_{lr} = \bar{u}_{lr} e^{i\omega t}, \) \(\sigma_{lr} = \bar{o}_{lr} e^{i\omega t}\), where \(\bar{u}_{lr}\) and \(\bar{o}_{lr}\) are the radial displacement and radial stress amplitudes of the lining, respectively, which are substituted into equation (22), at the same time, let \(U_l = \bar{u}_{lr}/R, L_t = G_t/\mu_t, L_t = \rho_t/\rho_t\), then equation (22) is dimensionless as
\[
\sigma_{Ul} = - \frac{2(1-2v)q_1 R_0 K_0 q_1 r}{(1-2v)r} + \frac{2(1-2v)q_1 l_0(q_1 r) - 2(1-2v)l_1(q_1 r)}{(1-2v)r} \mu_1 A_{L1} A_{L2},
\]

(25)

where \( q_1^2 = -(1-2v)\bar{c}^2 / (2-2v)\mu_l \). We can obtain the radial displacement of the lining by solving Bessel equation (23)

\[ U_l(r) = A_{L1} K_1(q_1 r) + A_{L2} I_1(q_1 r), \]

(24)

where \( A_{L1} \) and \( A_{L2} \) are undetermined coefficients. According to equation (24), the dimensionless radial stress of the lining is

\[ \sigma_{Ul} = - \frac{2(1-2v)q_1 K_1(q_1 r) + (2-2v)q_1 R_0 K_0(q_1 r)}{(1-2v)r} \mu_1 A_{L1} A_{L2}. \]

(26)

where \( Q = q_0 / \mu_0, \). \( \eta_0 = 1 - \delta. \)

Considering the continuity conditions at the interface between the outer boundary of the lining and the disturbed area viscoelastic soil, and at the interface between the disturbed area viscoelastic soil and the undisturbed area viscoelastic soil, the following continuity boundary conditions can be obtained

\[ U_l(1) = U_1(r_0) |_{r_1} = \sigma_1((r_0)) |_{r_1}, \]

\[ \sigma_l(r_0) = \sigma_1(r_0) |_{r_1} = \eta_0 (R_0) |_{r_1}. \]

(27)

Considering equations (12), (13), (21), and (24)–(27), the following equations can be obtained:

\[ (1-2v)K_1(q_0 R_0) A_1 = a_{11} (1-2v) \left[ A_{L1} K_1(q_1) + A_{L2} I_1(q_1) \right] \\
- a_{12} \left[ 2(1-2v)K_1(q_1) + (2-2v)q_1 K_0(q_1) \right] A_{L1} \\
+ a_{12} \left[ 2(1-2v)q_1 I_0(q_1) - 2(1-2v)l_1(q_1) \right] A_{L2}, \]

(28)

The undetermined coefficients \( A_1, A_{L1}, \) and \( A_{L2} \) can be obtained by solving equations (28)–(30), and then the expressions of the radial displacement and the radial stress of the soil at the contact surface between lining and soil can be obtained as follows:

\[ \varphi = \frac{\sigma_{UL}}{Q} = \frac{\eta_0 (b_{22} b_1 - b_{12} b_2) K_1(q_1) + (b_{11} b_2 - b_{21} b_1) I_1(q_1)}{\mu_1 \left[ (b_{22} b_1 - b_{12} b_2) c_{11} + (b_{11} b_2 - b_{21} b_1) c_{12} \right]}. \]

(29)
5. Numerical Examples and Discussion

The dynamic response characteristics of circular tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil are discussed in the form of numerical examples. In the example, the region of fractional derivative viscoelastic inhomogeneous soil is divided into 10 circles, i.e., $n = 10$. If no description is given, the parameter values in the calculation example are as follows: $m = 2$, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = 10 = 0.5$, $\alpha_0 = 0.5$, $\beta_{11} = \beta_{12} = \beta_{13} = \beta_{14} = \beta_{15} = \beta_{16} = \beta_{17} = \beta_{18} = \beta_{19} = \beta_{10} = 1.0$, $\beta_{21} = \beta_{22} = \beta_{23} = \beta_{24} = \beta_{25} = \beta_{26} = \beta_{27} = \beta_{28} = \beta_{29} = \beta_{20} = 1.0$, $T_{10} = 10$, $T_{20} = 5$, $\nu_0 = 0.33$, $\nu_L = 0.33$, $\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu_5 = \nu_6 = \nu_7 = \nu_8 = \nu_9 = \nu_{10} = 0.33$, $\mu = 0.1$, $\mu_1 = 2.0$, $\mu_2 = 2.5$, $\mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = \mu_9 = \mu_{10} = 1.0$.

5.1. Comparative Analysis of Models. Figures 3 and 4 show the curves of the radial displacement and the radial stress of soil at the contact surface between lining and soil varying with dimensionless frequency obtained by using elastic model, classical viscoelastic model, and fractional derivative viscoelastic model, respectively. No matter what model is used, there are peaks and troughs in the curves of the radial displacement and radial stress varying with frequency. Under the action of harmonic internal pressure, there is resonance in the inhomogeneous soil-circular tunnel lining system. At the same time, it can be seen that the fractional derivative viscoelastic model can degenerate to the classical viscoelastic model and elastic model, which shows the correctness of the analysis method in this paper. When the elastic model is used, the stiffness of the system is relatively large, the radial displacement and radial stress of the soil are the smallest and the fluctuation is relatively stable. When the classical viscoelastic model and fractional derivative viscoelastic model are used, the curves of the radial displacement and radial stress of soil varying with frequency fluctuate more, and the radial displacement and radial stress of the classical viscoelastic model are the largest. Therefore, the influence of viscoelasticity of soil should not be ignored when analyzing the dynamic response of circular tunnel lining in fractional derivative viscoelastic radial inhomogeneous soil.

5.2. The Influence of the Order of the Fractional Derivative. The influence of the order of the fractional derivative on the radially inhomogeneous soil-circle tunnel lining system is shown in Figures 5 and 6. The larger the order of the
fractional derivative, the greater the fluctuation of curves the radial displacement and radial stress of soil varying with the frequency. The fluctuation of curves the radial displacement and radial stress of soil varying with the frequency will be greater when the order of the fractional derivative is larger. The larger the order of the fractional derivative, the larger the peak and valley values of the curves, and the smaller the frequency corresponding to the peak and valley values. We can fit different properties of soil by changing the order of fractional derivative. It can be seen that the application range of fractional derivative viscoelastic model is wider than that of classical viscoelastic model.

5.3. The Influence of the Shear Modulus Ratio of Affected Area. Figures 7 and 8 are the influence of the shear modulus ratio of affected area on radial displacement and radial stress of soil. Obviously, when $\mu = 1.0$, the fractional derivative viscoelastic radial heterogeneous soil can degenerate to the fractional derivative radial homogeneous soil. At low frequency, the influence of heterogeneity of soil around the tunnel on the radial displacement and radial stress of soil is greater. The smaller the shear modulus ratio of affected area, the smaller the rigidity of the system, the larger the radial displacement of soil, and the smaller the radial stress of soil. At high frequency, the influence of shear modulus ratio of
affected area on the radial displacement and radial stress of soil is relatively small, but it has a great influence on the fluctuation of the curves. It can be seen that when studying the dynamic characteristics of tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil, the influence of soil heterogeneity around the tunnel should be considered.

5.4. The Influence of the Thickness of Tunnel Lining. The influence of the thickness of tunnel lining on the radial displacement and radial stress of soil is shown in Figures 9 and 10. At low frequency, the thickness of the tunnel lining has a great influence on the radial displacement of the soil, but it has little influence at high frequency. Moreover, at low frequency, the greater the thickness of tunnel lining, the smaller the radial displacement of soil. The thickness of lining has a great influence on the radial stress of soil. This may be because the lining stiffness will be greater when the thickness of the lining is greater, so the lining will share more internal load and result in the smaller radial displacement and radial stress of the soil.
5.5. The Influence of the Shear Modulus Ratio of Tunnel Lining. Figures 11 and 12 show the influence of the shear modulus ratio of tunnel lining on the radial displacement and radial stress of soil. The shear modulus of tunnel lining has a great influence on the dynamic characteristics of circular tunnel lining in fractional derivative radially inhomogeneous soil, but its influence will decrease with the increase of the shear modulus of lining. It can be seen that when the shear modulus of lining is large, changing the shear modulus of lining has little effect on improving the dynamic characteristics of soil-tunnel lining system.

6. Conclusion

In this paper, considering the viscoelasticity and radial heterogeneity of the soil around the tunnel, the transfer matrix at the outer boundary and inner boundary of the disturbed area is obtained by using the multicycle matrix transfer method. On this basis, the dynamic response of the circular tunnel lining in fractional derivative radially inhomogeneous soil is studied, and the influence of relevant parameters on the dynamic characteristics of the system is numerically analyzed.

The main conclusions are as follows:

(1) Whether it is an elastic model, classical viscoelastic model, or fractional derivative viscoelastic model, there is resonance in a radially inhomogeneous soil-circular tunnel lining system under simple harmonic internal pressure and the fractional derivative viscoelastic model can degenerate to classical viscoelastic model and elastic model.

(2) The curves of the radial displacement and radial stress of the soil varying with frequency obtained by classical viscoelastic model and fractional derivative viscoelastic model fluctuate more than that obtained by an elastic model. When analyzing the dynamic response of circular tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil, the influence of the viscoelasticity of soil should not be ignored.

(3) By changing the order of fractional derivative, the soil with different properties under different working conditions can be fitted. The application range of fractional derivative viscoelastic model is wider than that of classical viscoelastic model.

(4) The influence law of shear modulus ratio of affected area on the dynamic characteristics of circular tunnel lining in radially inhomogeneous soil has a great relationship with the frequency domain. The heterogeneous characteristics of soil around the tunnel have a great influence on the dynamic response of circular tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil. The heterogeneous characteristics of soil around the tunnel should not be ignored.

(5) The thickness of the tunnel lining and shear modulus of the tunnel lining have a great influence on the dynamic characteristics of circular tunnel lining in fractional derivative viscoelastic radially inhomogeneous soil. Increasing the thickness and shear modulus of the tunnel lining can increase the stiffness of the tunnel lining and reduce the radial stress and displacement of soil, but changing the thickness of lining at high frequency has little effect on the radial displacement of soil. When the shear modulus of the lining is large, the influence of the shear modulus of the lining on the radial displacement and radial stress of the soil will be reduced.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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