

Research Article

Theoretical Study on the Consolidation of Multiple Composite Foundations with Vertical Drains-Impervious Piles considering Temperature Effect and Nonuniform Distribution of Initial Pore Pressure

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For the problem of consolidation of multiple composite foundations of vertical drain-impervious pile, considering the influence of temperature and nonuniform distribution of initial pore pressure, the consolidation control equations based on the assumption of equal strain are established, and the analytical solution method is adopted to give the analytical solution of consolidation of multiple composite foundations of the vertical drain-impervious pile with trapezoidal, rectangular, positive triangular, and inverted triangular distribution of initial pore pressure. The correctness and reasonableness of the solutions are verified by comparing the degenerate solutions of this paper with the existing solutions. The consolidation properties of the multifaceted composite foundation are analyzed according to the solution of this paper. The results show that the initial pore pressure is faster to consolidate in a trapezoidal distribution than in a rectangular distribution, fastest to consolidate in an inverted triangular distribution and slowest to consolidate in a positive triangular distribution. The rate of consolidation of a multiple composite foundation can be significantly accelerated by increasing the temperature, the replacement rate of impervious piles, and the compression modulus. The rate of consolidation of a multiple composite foundation can be significantly increased compared to a single pile type foundation; the size of the permeability coefficient of the vertical drain almost represents the size of the drainage capacity of the composite foundation. The rate of consolidation of a multiple composite foundation is faster as the permeability coefficient of the vertical drain increases.

1. Introduction

With the development of foundation treatment technology, compound foundation treatment method is more applied in engineering, and the compound foundation also changes from single type to multiple type. According to the different permeability, the compound foundation can be divided into drainage pile compound foundation and impervious pile compound foundation. The vertical drain has a strong permeability, and its permeability is much greater than the surrounding soil, and it can also share a certain load, which can reduce the postwork settlement, but the magnitude of improving the bearing capacity of the foundation is small, compared with which the pile strength of the impervious pile

is higher and can significantly improve the bearing capacity of the foundation. Therefore, the joint use of vertical drains and impervious piles to treat soft clay foundations to form combined composite foundations can integrate the advantages of each of them and have higher application value [1]. For this type of multicomposite foundation, the analysis of consolidation theory is the focus of its research. There are more studies on vertical drain in drainage consolidation, and the theory is more mature, and the consolidation theory of multicomposite foundation draws on the research method of the consolidation theory of one-dimensional composite foundation.

In the study of vertical drain consolidation theory, Xie and Zeng [2] derived an analytical solution for radial

consolidation of vertical drain foundations based on the assumption of equal strain. Wang and Jiao [3] discarded the assumption of equal flow around the well and developed a combined radial and vertical seepage calculation model considering both radial and vertical flow within the well and obtained the analytical solution for consolidation in both perforated and unperforated soft clay layers. Chen et al. [4] considered the horizontal permeability coefficient of the smear zone varying along the radial direction, derived the consolidation control equation based on the condition of equal strain assumption, and derived the consolidation solution. Deng et al. [5] derived an analytical solution for radial consolidation of sand well foundations under the variation of well resistance with time, considering that the permeability coefficient of sand wells decreases gradually during consolidation. Nguyen and Kim [6] studied the large deformation consolidation of sand well foundations by considering the variation of sand well permeability coefficient with time with exponential function law. Kim et al. [7] investigated consolidation behavior of soft soil with vertical drains considering well resistance and smear effect under cyclic loadings. Using the variables separation method, a series of analytical solutions were derived to calculate the excess pore water pressure and the average degree of consolidation of the soil subjected to various cyclic loadings, including trapezoidal cyclic loading, rectangular cyclic loading, triangular cyclic loading, and haversine cyclic loading. In addition, in terms of boundary conditions, the actual drainage boundary is usually an incomplete drainage boundary to compensate for the deficiencies of the boundary conditions in the conventional vertical drain consolidation theory under transient constant load [8–10]. The thermal drainage consolidation method is also gradually applied, which is based on the theory of vertical drain consolidation, and the mechanism of action is relatively complex, in which seepage mechanics, geotechnics, and heat transfer interpenetrate and cross each other. Regarding the study of the theoretical problems of thermal drainage consolidation. Wu et al. [11] studied the effect of temperature on the permeability coefficient and gave the relationship equation between permeability coefficient and temperature and obtained the analytical solution of consolidation of single-sided drainage of vertical drain foundation under the influence of temperature effect. Yin et al. [12] derived an analytical solution for consolidation based on the existing theory of consolidation of ideal vertical drain foundations with temperature correction. Consolidation has an important influence on soil strength and building stability [13–15], and vertical drain foundations have also laid a solid theoretical foundation for combined composite foundations with the joint use of multiple pile types in recent years. With the development of composite foundation technology, the combined composite foundation with multipile type joint use has been gradually popularized in engineering, and the research on its consolidation characteristics has also achieved certain results. Chen et al. [16] studied the consolidation of a composite foundation of drained powder-jet piles, with the drainage slab as the center, the dissipation law of the pore pressure is percolated from the surrounding to

the center, assuming that the powder-jet piles are impermeable, considering that the drainage slab has the effect of bearing the load and satisfying the assumption of equal strain, and the consolidation expression of the foundation is obtained using the axisymmetric consolidation theory, but assuming that the compression modulus of the drainage slab as well as the smear zone is the same as the compression modulus of the soil. Liu et al. [17] proposed a combined application method of DJM-PVDs for improving soft clay foundations. Yu et al. [18, 19] simplified the composite foundation with impervious long piles and permeable short piles under combined seepage by considering the volume compression of permeable piles based on the axisymmetric consolidation model and equated the soil within the permeable piles and permeable piles as the interpile soil with larger permeability coefficient and formed a double layer impervious pile composite foundation with different permeability coefficients in the reinforcement zone with impervious long piles, so as to obtain the analytical solution of the composite foundation with permeable piles combined with impervious piles. Lin [20] analyzed the results of the application of CFG pile-gravel pile combination composite foundations in engineering. Ye et al. [21, 22] investigated a combined composite foundation consolidation problem of cemented soil piles combined with plastic drainage boards. The pore water pressure was modeled as percolation from the center to the periphery, the piles were assumed to be permeable piles, the bearing and deformation of the drainage boards were not considered, equivalent vertical and radial consolidation coefficients were established, and the expressions for the consolidation of composite foundations under transient loading were derived. Liu et al. [23] gave a simplified method for calculating the settlement of a composite foundation consolidation used plum blossom pile layout scheme and based on an axisymmetric consolidation model with a combination of powder jetting piles and drainage board. Prakash and Krishnamoorthy [24] investigated the effect of lime and CFG pile composite foundations on the factor of safety of embankments at different time intervals during the consolidation of foundations. Yang et al. [25] derived the controlling equations for the average pore pressure of the soil around a composite foundation of long rigid piles and impervious short piles under ramp loading based on the assumption of equal vertical strain and established the corresponding resolution based on the one-dimensional consolidation theory of double-layered foundations.

Given the available research results, consolidation theory considering temperature is one of the more important research topics in geotechnical engineering today, with a background of applications in energy subsurface engineering, nuclear waste disposal, and thermal foundation treatment, yet it is often overlooked by geotechnical engineers [26]. In practical engineering, the initial pore pressure induced by external loads in the foundation is often non-uniformly distributed along the depth [27–29]. Theoretical derivations have wide application in the analysis and design of many geotechnical engineering problems [30–33]. The study of consolidation theory of multicomposite

foundations is the basis for predicting settlement changes with time. In this paper, the consolidation of multicomposite foundations is considered in the light of the temperature effect and the nonuniform distribution of initial pore pressure. First, the consolidation equation of the vertical drain-impervious pile with trapezoidal distribution of initial pore pressure is derived, and the analytical solution of the consolidation equation is obtained by the method of separation of variables, and the analytical solution of consolidation with rectangular, positive triangular, and inverted triangular distribution of initial pore pressure is also given. The reasonableness of the solution of this paper is verified by comparing the degenerate solution with the existing analytical solution. By compiling the computational procedure of the solution of this paper and plotting the consolidation curves of the relevant parameters, the consolidation law of the multiple composite foundations with vertical drains-impervious piles is investigated.

2. Establishment of Consolidation Control Equation

2.1. Calculation Sketch and Basic Assumptions. Figure 1 shows an axisymmetric simplified model of the vertical drain-impervious pile multiple composite foundation consolidation problem. Multicomposite foundations are mostly arranged in rectangles and triangles. In this paper, both vertical drains and impervious piles are arranged in rectangles, and one unit is taken as the research object, with the vertical drain being the center of the model and the impervious pile located at the outer boundary of the model, where r_w , r_e , and r_n are the radius of the vertical drain, the radius of the drainage influence zone and the radius of the entire foundation, respectively; k_h and k_v are the horizontal and vertical permeability coefficients of the foundation soil, respectively; k_w is the vertical permeability coefficient of the vertical drain; E_p and E_s are the compression modulus of the impervious column and natural soil, respectively; u_w and u_s are the pore pressure within the vertical drain and soil at any point, respectively; H is the thickness of the soft layer; q is the instantaneous applied external load; r and z are the radial and vertical coordinates, respectively, assuming that the drainage conditions of the multicomposite foundation are permeable at the top and impermeable at the bottom.

Figure 2 shows the four cases of initial pore pressure distribution along the depth, where P_T and P_B are the initial pore pressure values at the top and bottom of the soil layer, respectively. When $P_T = P_B = P_0$, as shown in Figure 2(a), the initial pore pressure is evenly distributed. When $P_T = 0$, as shown in Figure 2(b), the initial pore pressure is distributed in a positive triangle along the depth. When $P_B = 0$, as shown in Figure 2(c), the initial pore pressure is distributed in an inverted triangle. When $P_T \neq P_B \neq 0$, as shown in Figure 2(d), the initial pore pressure is distributed in a trapezoid shape.

In the derivation of this paper, the following assumptions are made about the model.

- (1) The equal strain condition holds; that is, there is no lateral deformation in the foundation, and the vertical deformation at any point at the same depth is equal.
- (2) The load is applied instantaneously, and the resulting initial pore pressure is nonuniformly distributed along the depth direction.
- (3) Neglecting the radial seepage in the vertical drain, the water in the soil has both radial and vertical seepage, and the seepage obeys Darcy's law.
- (4) The flow around the well is continuously equal; that is, the flow of water from the soil into the vertical drain at any depth is equal to the increment of water flowing upwards in the well.
- (5) Temperature changes will only affect the permeability of the soil and will not influence other parameters in the soil. The relationship between temperature and permeability coefficient is derived from the literature [34] as follows:

$$k_T = (aT + b)k_R. \quad (1)$$

In the previously mentioned equation, T is a temperature in the soil; R is the room temperature (generally 20°C); k_T and k_R are the coefficients of permeability at T and R , respectively.

2.2. Solving Conditions and Control Equation. The external load is shared by the vertical drain, the soil, and the impervious pile and is obtained from the basic assumption (1) and the vertical equilibrium condition:

$$\pi r_w \bar{\sigma}_w + \pi(r_e^2 - r_w^2) \bar{\sigma}_s + \pi(r_n^2 - r_e^2) \bar{\sigma}_p = \pi r_n^2 q, \quad (2)$$

$$\frac{\bar{\sigma}_w - u_w}{E_w} = \frac{\bar{\sigma}_s - \bar{u}_s}{E_s} = \frac{\bar{\sigma}_p}{E_p} = \varepsilon_v = \varepsilon_z, \quad (3)$$

where E_w is the compressive modulus of the vertical drain; $\bar{\sigma}_w$, $\bar{\sigma}_s$, and $\bar{\sigma}_p$ are the average stresses in the vertical drain, soil, and impervious columns, respectively, in a multicomposite foundation; ε_v and ε_z are the volumetric and vertical strains in this foundation, respectively, which are equal when the foundation is only deformed in the vertical direction.

\bar{u}_s and \bar{u} are the average pore water pressure in the foundation soil at any depth and the average pore water pressure of the whole foundation, respectively; the expressions are

$$\bar{u}_s = \frac{1}{\pi(r_e^2 - r_w^2)} \int_{r_w}^{r_e} 2\pi r u_s dr, \quad (4)$$

$$\bar{u} = \frac{1}{\pi r_n^2} \int_0^{r_e} 2\pi r u_s(r) dr = \frac{N_{ew}^2 - 1}{N_{nw}^2} \bar{u}_s, \quad (5)$$

wherein $N_{ew} = r_e/r_w$, $N_{nw} = r_n/r_w$

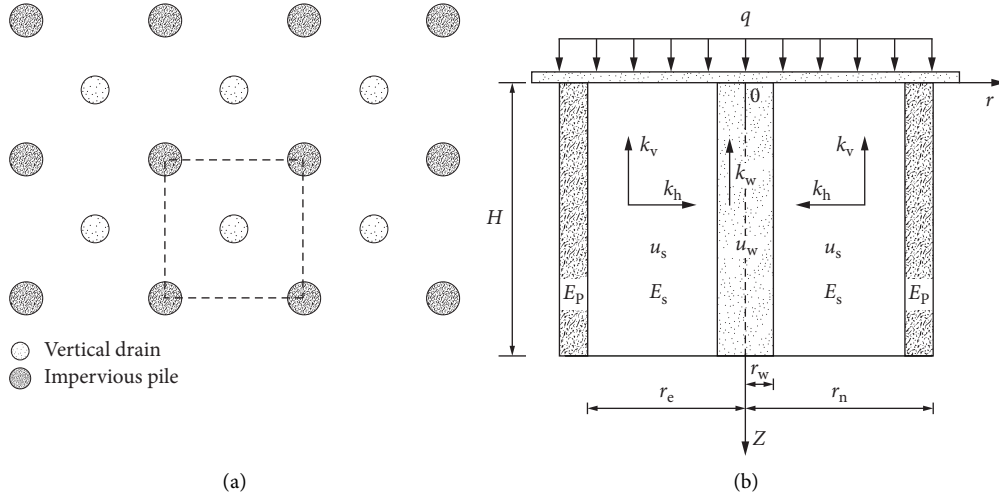


FIGURE 1: Vertical drain-impervious pile multicomposite foundation consolidation model. (a)Layout plan. (b)Cutaway diagram.

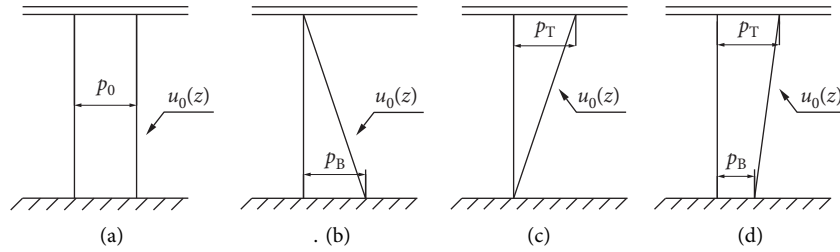


FIGURE 2: Nonuniform distribution of initial excess pore water pressure along depth. (a) Rectangular. (b) Positive triangle. (c) Inverted triangle. (d) Trapezoidal.

From (2), (3), and (5), the partial derivative of the vertical strain ε_z with respect to time is

$$\frac{\partial \varepsilon_z}{\partial t} = \frac{1}{E_{\text{com}}} \frac{\partial \bar{u}}{\partial t}, \quad (6)$$

where E_{com} is the composite modulus of elasticity, and the expression is

$$E_{\text{com}} = \frac{E_w + (N_{\text{ew}}^2 - 1)E_s + (N_{\text{mw}}^2 - N_{\text{ew}}^2)E_p}{N_{\text{mw}}^2}, \quad (7)$$

According to the study of Xie and Zeng [2] and the basic assumption (5), the basic consolidation equation for the soil about the axisymmetric case can be obtained as

$$\frac{k_{hT}}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial u_s}{\partial r} \right] + \frac{k_{vT}}{\gamma_w} \frac{\partial^2 \bar{u}_s}{\partial z^2} = -\frac{\partial \varepsilon_v}{\partial t} \quad r_w \leq r \leq r_e. \quad (8)$$

Since the vertical drain-soil interface of this multiple composite foundation is permeable, and the impervious pile-soil interface is impermeable, the radial boundary condition for this equation as follows:

$$r = r_w: u_s = u_w = \bar{u}_w, \quad (9)$$

$$r = r_e: \frac{\partial u_s}{\partial r} = 0. \quad (10)$$

For this multiple composite foundation with a permeable top surface and an impervious bottom surface, that is, considering one-sided drainage, the vertical boundary conditions are as follows:

$$z = 0: u = 0, \quad (11)$$

$$z = H: \frac{\partial u}{\partial z} = 0 \quad (12)$$

By the basic assumption (2), the initial conditions for this equation are

$$t = 0: \bar{u}(z) = u_0(z) = P_T + (P_B - P_T) \frac{z}{H}, \quad (13)$$

wherein u_0 is the initial pore pressure of the foundation; u is the pore pressure at any point of the foundation.

According to the previously mentioned analysis and assumptions, temperature is uniformly distributed with soil depth, and this paper considers the effect of temperature on both radial and vertical permeability coefficients of the soil, which can be obtained from the basic assumption (5) as follows:

$$k_{hT} = (aT + b)k_h, \quad (14)$$

$$k_{vT} = (aT + b)k_v. \quad (15)$$

From (14) and the basic assumption (4), it follows that

$$\left[2\pi r dz \frac{k_{hT}}{\gamma_w} \frac{\partial u_s}{\partial r} \right]_{r=r_w} = -\pi r_w^2 dz \frac{k_w}{\gamma_w} \frac{\partial^2 \bar{u}_w}{\partial z^2}, \quad (16)$$

where γ_w is the unit weight of water.

Integrating (8) with respect to r and using the boundary condition (10) yields

$$\frac{\partial u_s}{\partial r} = \frac{\gamma_w}{2k_{hT}} \left(\frac{r_e^2}{r} - r \right) \left(\frac{k_{vT}}{\gamma_w} \frac{\partial^2 \bar{u}_s}{\partial z^2} + \frac{\partial \varepsilon_v}{\partial t} \right). \quad (17)$$

According to (4) and (17) and using the boundary condition (9), it follows that

$$\bar{u}_s = \frac{\gamma_w r_e^2 F_a}{2k_{hT}} \left(\frac{k_{vT}}{\gamma_w} \frac{\partial^2 \bar{u}_s}{\partial z^2} + \frac{\partial \varepsilon_v}{\partial t} \right) + \bar{u}_w, \quad (18)$$

where in $F_a = (\ln N_{ew} + (4N_{ew}^4 - 1/4N_{ew}^4) - (3/4))N_{ew}^2 / (N_{ew}^2 - 1)$.

From (17) and (16), the following is obtained:

$$\frac{k_{vT}}{\gamma_w} \frac{\partial^2 \bar{u}_s}{\partial z^2} + \frac{k_w}{\gamma_w (N_{ew}^2 - 1)} \frac{\partial^2 \bar{u}_w}{\partial z^2} = -\frac{\partial \varepsilon_v}{\partial t}. \quad (19)$$

According to (5), (6), (8), and (19), the following can be obtained:

$$N_{mw}^2 \frac{k_{vT}}{\gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{(N_{ew}^2 - 1)}{E_{com}} \frac{\partial \bar{u}}{\partial t} - \frac{k_w}{\gamma_w} \frac{\partial^2 \bar{u}_w}{\partial z^2}. \quad (20)$$

Taking the partial derivative of (18) twice with respect to z and using (5) yields

$$\frac{N_{mw}^2}{N_{ew}^2 - 1} \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial^2 \bar{u}_w}{\partial z^2} + \frac{\gamma_w r_e^2 F_a}{2k_{hT}} \cdot \left(\frac{1}{E_{com}} \frac{\partial^3 \bar{u}}{\partial z^2 \partial t} + \frac{N_{mw}^2}{N_{ew}^2 - 1} \frac{k_{vT}}{\gamma_w} \frac{\partial^4 \bar{u}}{\partial z^4} \right). \quad (21)$$

Equation (24) is a linear differential equation of the first order, and according to the theory of ordinary differential equations, its general solution can be written as

$$T_m(t) = A_m e^{-\beta_m t}. \quad (26)$$

Substituting (26) into (23) gives

$$\bar{u} = \sum_{m=0}^{\infty} A_m \sin\left(\frac{Mz}{H}\right) e^{-\beta_m t}. \quad (27)$$

Substituting the initial condition (13) into (27) and using the trigonometric orthogonality yields

Combining (20) and (21) and introducing (14) and (15) yields the control equation as

$$A \frac{\partial^4 \bar{u}}{\partial z^4} - \frac{B}{(aT + b)} \frac{\partial^3 \bar{u}}{\partial z^2 \partial t} - [C + D(aT + b)] \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{\partial \bar{u}}{\partial t} = 0, \quad (22)$$

where in $A = (N_{ew}^2 - 1)^2 / N_{mw}^2 r_e^2 F_a / 2E_{com} k_w / \gamma_w k_v / k_h$; $B = 1/N_{ew}^2 - 1 r_e^2 F_a / 2k_w / k_h$; $C = N_{mw}^2 / (N_{ew}^2 - 1)^2 E_{com} k_w / \gamma_w$; $D = N_{mw}^2 / N_{ew}^2 - 1 E_{com} k_v / \gamma_w$. where A , B , C , and D are all constants.

3. Solution of Average Pore Pressure and Average Consolidation of Composite Foundation

According to Lu et al. [35], using the separation of variables method, the solution to (22) is assumed to be of the form

$$\bar{u} = \sum_{m=0}^{\infty} T_m(t) \sin\left(\frac{M}{H} z\right), \quad (23)$$

wherein $M = (2m + 1)\pi/2$, $m = 0, 1, 2, \dots$

Substituting (23) into (22), the following can be obtained:

$$T_m'(t) + T_m(t) \cdot \beta_m = 0, \quad (24)$$

where in

$$\beta_m = \frac{N_{mw}^2 / (N_{ew}^2 - 1)^2 r_e^2 F_a k_w k_v / (2k_h) (M/H)^2 + N_{mw}^2 / (N_{ew}^2 - 1) (k_v (aT + b) + k_w / (N_{ew}^2 - 1)) E_{com}}{(H/M)^2 + 1 / (N_{ew}^2 - 1) k_w / k_h r_e^2 F_a / (2(aT + b))} \frac{E_{com}}{\gamma_w}. \quad (25)$$

$$A_m = \frac{\int_0^H u_0(z) \sin(Mz/H) dz}{\int_0^H \sin^2(Mz/H) dz} = \frac{2}{M} \left[p_T + (-1)^m \frac{p_B - p_T}{M} \right]. \quad (28)$$

Substituting (28) into (27) gives

$$\bar{u} = \sum_{m=0}^{\infty} \frac{2}{M} \left[p_T + (-1)^m \frac{p_B - p_T}{M} \right] \sin\left(\frac{Mz}{H}\right) e^{-\beta_m t}. \quad (29)$$

Thus, the total average consolidation of a multiple composite foundations as defined by stress can be derived as

$$U(t) = 1 - \frac{\int_0^H \bar{u} dz}{\int_0^H u_0(z) dz} = 1 - \sum_{m=0}^{\infty} \frac{4}{M^2 (P_B + P_T)} \left[p_T + (-1)^m \frac{P_B - P_T}{M} \right] e^{-\beta_m t}. \quad (30)$$

Equations (29) and (30) are the solutions for multi-composite foundation consolidation with a trapezoidal distribution of initial pore pressure considering the temperature effect. From the distribution of initial pore pressure in Figure 2, further expressions for the mean pore pressure and mean consolidation under rectangular, positive triangular, and inverted triangular distribution of initial pore pressure are given as follows.

Case 1. At $P_T = P_B = P_0$, the initial pore pressure is rectangularly distributed; see Figure 2(a). Degenerate (29) and (30) to the instantaneously loaded initial pore pressure homogeneous vertical drain and impervious column multicomposite foundation consolidation solution; that is,

$$\bar{u} = p_0 \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{Mz}{H}\right) e^{-\beta_m t}, \quad (31)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-\beta_m t}. \quad (32)$$

Case 2. At $P_T = 0$, the initial pore pressure has a positive triangular distribution; see Figure 2(b). According to (29) and (30), the following can be obtained:

$$\bar{u} = p_B \sum_{m=0}^{\infty} (-1)^m \frac{2}{M^2} \sin\left(\frac{Mz}{H}\right) e^{-\beta_m t}, \quad (33)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} (-1)^m \frac{4}{M^3} e^{-\beta_m t}. \quad (34)$$

Case 3. At $P_B = 0$, the initial pore pressure has an inverted triangular distribution; see Figure 2(c). According to (29) and (30), the following can be obtained:

$$\bar{u} = p_T \sum_{m=0}^{\infty} \frac{2}{M} \left[1 - (-1)^m \frac{1}{M} \right] \sin\left(\frac{Mz}{H}\right) e^{-\beta_m t}, \quad (35)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} \frac{4}{M^2} \left[1 - (-1)^m \frac{1}{M} \right] e^{-\beta_m t}. \quad (36)$$

At this point, all the analytical solutions for the consolidation of multiple composite foundations with vertical drain-impervious pile in the temperature effect and the four initial pore pressure distributions have been given in this paper.

4. Degenerations of the Obtained Solution

Referring to the degradation methods in the literature [7, 36], the degradation analysis of the consolidation analytical solutions obtained in this paper is carried out, and the degradation analysis of the solutions can be used to effectively verify the reasonableness of the solutions. The previously mentioned three special cases of pore pressure and consolidation degree are degenerated from (29) and (30), where (31) and (32) are the solutions for consolidation when the initial pore pressure is homogeneous and can be degenerated again, and the steps and methods of degeneration are as follows.

- (1) When temperature effects are not considered, (25) degenerates to

$$\beta_m = \frac{N_{nw}^2 / (N_{ew}^2 - 1)^2 r_e^2 F_a k_w k_v / (2k_h) (M/H)^2 + N_{nw}^2 / (N_{ew}^2 - 1) (k_v + k_w / (N_{ew}^2 - 1))}{(H/M)^2 + 1 / (N_{ew}^2 - 1) k_w / k_h r_e^2 F_a / 2} \frac{E_{com}}{\gamma_w}. \quad (37)$$

Equation (37) agrees with the analytical solution of Li Dongxu [37] for multivariate composite foundation consolidation when temperature effects are not considered.

- (2) If so that $N_{ew} = N_{nw}$, that is, there are no impervious piles in the foundation. Since there are no

impervious piles, the other parameters need to be degraded in the same way, where $E_{com} = E_s$; $N_{ew} = n$. Based on the continued degradation of the previously mentioned equation, (37) can be degraded to

$$\beta_m = \frac{k_w / k_h k_v / k_h r_e^2 F_a / 2 (M/H)^2 1 / (n^2 - 1) + k_v / k_h (1 + 1 / (n^2 - 1) k_w / k_v)}{(H/M)^2 + 1 / (n^2 - 1) k_w / k_h r_e^2 F_a / 2} \frac{E_s k_h}{\gamma_w}. \quad (38)$$

Equation (38) is the same as the consolidation solution of Tang Xiaowu [38] when the smear zone is not considered. At this point, the multifaceted composite foundation consolidation solution degenerates into a single sand well foundation consolidation solution.

- (3) (38) continues to degenerate such that $n \rightarrow \infty$, $k_h \rightarrow \infty$, and then, (38) can be degenerated to

$$\beta_m = c_v \left(\frac{M}{H} \right)^2. \quad (39)$$

Equation (39) is the analytical solution for one-dimensional consolidation of Terzaghi natural foundations, where c_v is the vertical consolidation coefficient of the soil; $c_v = k_v E_s / \gamma_w$.

Through the degradation study of the analytical solution of this paper, it can be seen that the degraded solution of this

paper is completely consistent with the existing analytical solution in different cases, which verifies the correctness of the solution of this paper to a certain extent.

5. Analysis of Consolidation Properties

In this paper, the solution for the consolidation of multivariate composite foundations of vertical drain-impervious column under consideration of temperature effects and nonuniform distribution of initial pore pressure has been derived. According to [39], in order to facilitate calculation and plotting, first, let

$$\beta_m t = \Gamma_m T_h, \quad (40)$$

where T_h is the time factor, $T_h = c_h t / (4r_e^2)$; c_h is the horizontal consolidation coefficient of the soil; $c_h = k_h E_s / \gamma_w$.

From (25) and (40), the following can be obtained:

$$\Gamma_m = \frac{N_{nw}^2 / (N_{ew}^2 - 1)^2 r_e^2 F_a k_w k_v / 2k_h k_h (M/H)^2 + N_{nw}^2 / ((N_{ew}^2 - 1)k_h) (k_v (aT + b) + k_w / (N_{ew}^2 - 1))}{(H/M)^2 + 1 / (N_{ew}^2 - 1) k_w / k_h r_e^2 F_a / (2(aT + b))} 4r_e^2 \frac{E_{com}}{E_s}. \quad (41)$$

The influence of some important parameters on the consolidation properties of multifaceted composite foundations is analyzed by preparing a calculation program for the consolidation solution of this paper and plotting the relevant consolidation curves using a control variable approach, where $a = 0.029$ and $b = 0.428$.

Figure 3 shows the curves of the effect of different temperatures on the consolidation process of the foundation, from which it can be seen that the consolidation rate of the multifaceted composite foundation can be effectively accelerated by the method of increasing the temperature. The temperature is a variable value. The higher the temperature, the faster the rate of foundation consolidation, and with the passage of time and the development of soil consolidation, the effect of temperature on soil consolidation begins to weaken, and the rate of increase of consolidation with the increase of temperature also gradually decreases. The results show that the effect of temperature on soil consolidation decreases with higher temperatures for the same temperature difference.

Figure 4 shows the consolidation curves for different distributions of initial pore pressure. The initial pore pressure distribution conditions have a significant effect on the consolidation of multicomposite foundations, with a trapezoidal distribution consolidating faster than a rectangular distribution, a positive triangular distribution consolidating the slowest, and an inverted triangular distribution consolidating the fastest. The results show that the effect of considering the nonuniform distribution of the initial pore pressure along the depth is not negligible.

Figure 5 shows the effect of the impervious pile replacement rate on the consolidation of the multiple composite foundation. As the radius of the entire unit

foundation continues to decrease, this leads to a decrease in N_{nw} . The smaller the replacement rate of the impervious piles, the slower the rate of consolidation of the whole foundation, because the modulus of elasticity of the impervious piles is much larger than that of the vertical drain and the soil. The smaller the replacement rate of the impervious piles, the smaller the average modulus of elasticity of the whole foundation, the smaller the external load borne by the impervious piles themselves, and the slower the rate of consolidation of the foundation accordingly. When $N_{nw} = 19$, the replacement rate of impervious piles is zero; that is, there are no impervious piles in the composite foundation. It can be seen that the multiple composite foundation can significantly increase the consolidation rate of the foundation compared to a single pile type foundation.

Figure 6 shows the effect of pile-soil compression modulus ratio on the consolidation of a multifaceted composite foundation. As the pile-soil modulus ratio increases, the consolidation rate of the foundation is accelerated. The consolidation rate of the foundation increases with the increase of the compressive modulus of the impervious pile when the compressive modulus of the soil is fixed. This is because the modulus of elasticity of the impervious pile is much greater than the modulus of elasticity of the soil. According to the assumption of equal strain, the impervious pile bears more load, and the overall strain of the composite foundation becomes smaller, and the consolidation coefficient is proportional to the compression modulus, so the consolidation coefficient of the composite foundation becomes larger, and the consolidation rate of the foundation is accelerated. Engineering impervious piles can be CFG piles, cement mixing piles, or even concrete pipe piles with great variation in stiffness, and the increased

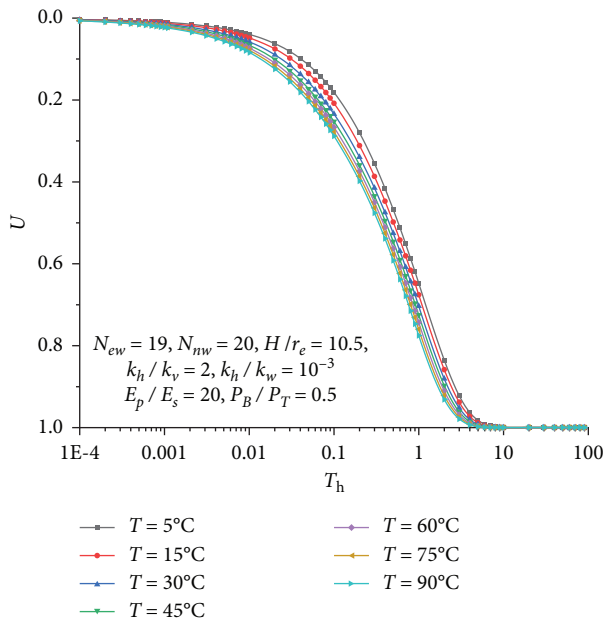


FIGURE 3: Influence of temperature on the consolidation process of foundation.

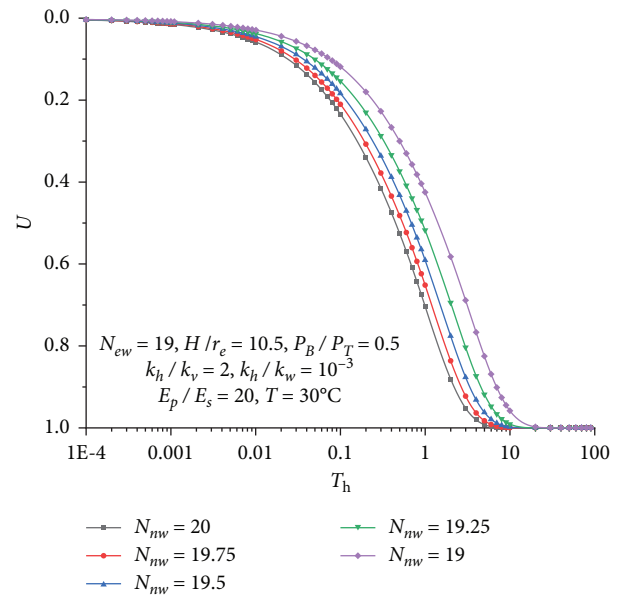


FIGURE 5: Effect of impervious pile replacement rate on foundation consolidation.

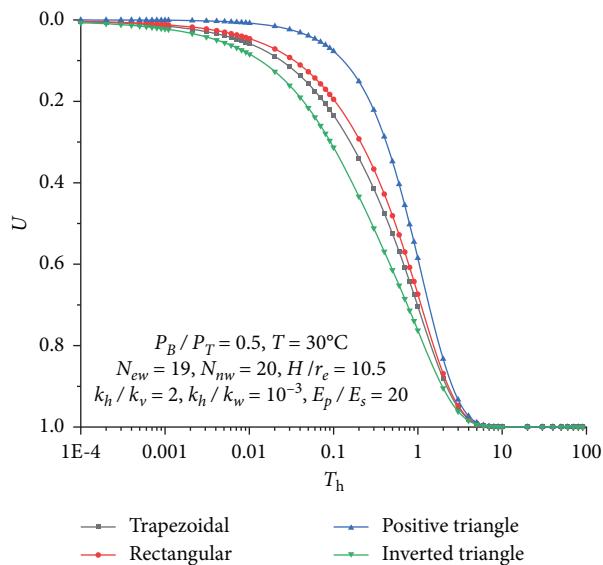


FIGURE 4: Consolidation curves for different distributions of initial pore pressure.

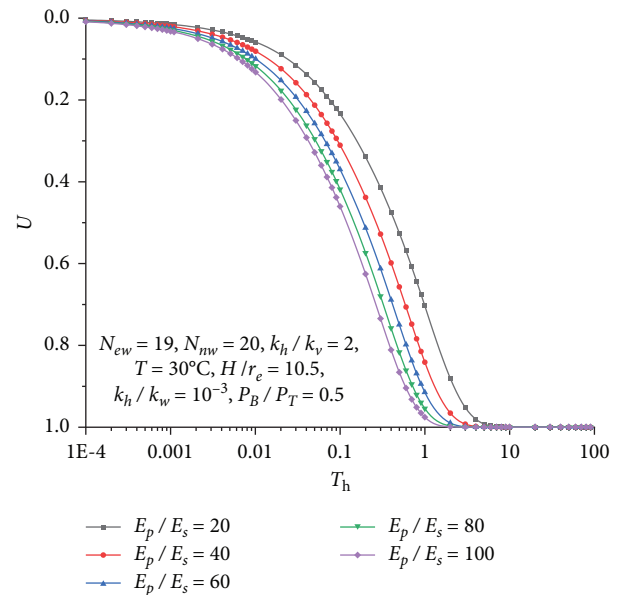


FIGURE 6: Influence of pile-soil compression modulus ratio on foundation consolidation curve.

stiffness of impervious piles has a significant effect on the consolidation rate of multifaceted composite foundations. Therefore, increasing the compression modulus of impervious piles can accelerate the consolidation of the foundation and at the same time increase the bearing capacity of the foundation, which is an effective method to deal with the lack of bearing capacity of the foundation and accelerate the consolidation of the foundation.

Figure 7 gives the consolidation rates of the multifaceted composite foundations for different vertical drain permeability coefficients. It can be seen from the figure that the consolidation rate of the composite foundation accelerates

as the vertical drain permeability coefficient increases (the reduction of the well resistance effect). This is because the multiple composite foundations are mainly consolidated by vertical drain drainage, and the size of the vertical drain permeability coefficient almost represents the size of the drainage capacity of the composite foundations, with the increase of the vertical drain permeability coefficient, the drainage rate is accelerated, and the consolidation rate of the composite foundations is also accelerated accordingly.

Figure 8 shows the comparison between the present solution and the existing solution under transient loading. Under the same parameter conditions, when the smearing

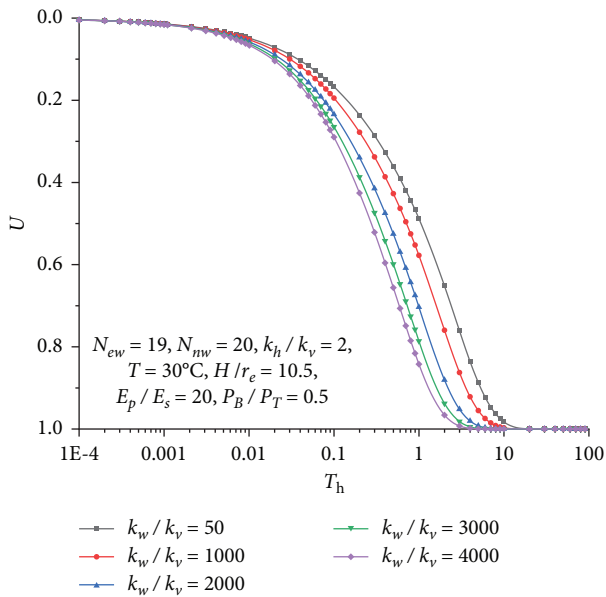


FIGURE 7: Influence of well resistance on consolidation ratio.

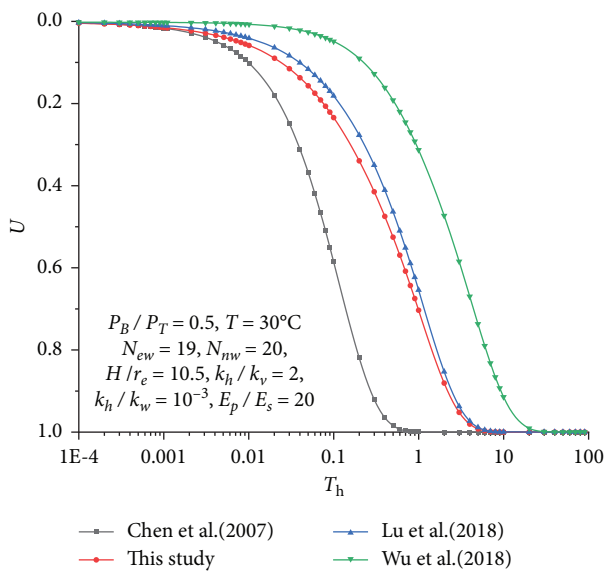


FIGURE 8: Comparison of consolidation of four models.

effect is not considered, the consolidation speed from fast to slow is as follows: Chen et al.'s [16] multicomposite foundation solution, this paper's solution, Lu et al.'s [35] multicomposite foundation solution under the middle B model, and Wu et al.'s [11] sand well foundation solution. The reason for the faster consolidation in the solution of Chen et al. [16] is that the radial and vertical seepage of the drainage body is not considered, and its drainage capacity is considered infinite in the calculation process; that is, the well body has no well resistance effect, while the solution of this paper considers that there is vertical well resistance in the vertical drain, so the consolidation rate is much faster than this paper. The analytical solution for consolidation of model B in Lu et al. [35] does not take into account the temperature

effect and considers that the initial pore pressure is uniformly distributed along the soil, which is equivalent to the case in this paper where the initial pore pressure is uniformly distributed without taking into account the temperature effect, so the consolidation rate is slower than in this paper. The model of Wu et al. [11] without impervious piles and without consideration of the vertical seepage of the soil is the analytical solution for consolidation of a single vertical drain foundation considering temperature effects, so the consolidation rate is the slowest in comparison.

6. Conclusion

In this paper, a model for the consolidation of multiple composite foundations with vertical drain-impervious pile is established, and the analytical solution for consolidation with consideration of temperature effect and nonuniform distribution of initial pore pressure is given. The consolidation characteristics of multifaceted composite foundations are investigated by using the consolidation solutions derived in this paper. The main conclusions are as follows.

- (1) As the temperature increases, it can effectively increase the rate of consolidation of multifaceted composite foundations, and with the passage of time and the development of soil consolidation, the effect of temperature on soil consolidation begins to weaken, and the rate of increase in consolidation with increasing temperature gradually decreases.
- (2) The nonuniform distribution of initial pore pressure along the depth has a significant effect on the consolidation of multifaceted composite foundations, with a trapezoidal distribution of initial pore pressure consolidating faster than a rectangular distribution, an inverted triangular distribution consolidating fastest, and a positive triangular distribution consolidating slowest.
- (3) The rate of consolidation of multicomposite foundations accelerates with the increase in replacement rate and compression modulus of impervious piles, and multicomposite foundations can significantly increase the rate of consolidation of foundations compared to single pile type foundations.
- (4) The magnitude of the vertical drain permeability coefficient almost represents the magnitude of the drainage capacity of the composite foundation, and the greater the vertical drain permeability coefficient, the faster the rate of consolidation of the multiple composite foundation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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