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Research Article

Calculation and Analysis of Vibration Stress of Guangxi Nanning Subway during Operation

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In this study, based on the Mindlin solution in elastic half-space, the stress calculation formula was determined with consideration of any point in the foundation under the dynamic amplification effect. The train load was simplified as a concentrated force moving in the direction of the train, and the stress of the soil under a single wheel load was analyzed. In the state, σ_z reached a maximum value of 0.56 kPa when the axle was directly above the soil, and the stress decreased until it approached zero as the distance from the soil unit increased. Then, taking the Nanning Metro Line 1 as an example, the load was regarded as the superposition of several single-wheel loads, and the law of the soil stress change at a point directly below the moving line of the train load at different axle loads, speeds, and burial depths was studied. From the analysis of the results, it could be seen that the soil stress under different working conditions was always proportional to the train axle load, speed, and burial depths. The peak stress ratio corresponding to the depths of 9 m, 12 m, and 15 m was 1:2:11, indicating that the closer the load to the soil, the more significantly the stress of the soil element increased. Under multiple wheel loads, the soil stress always exhibited continuous cycle characteristics. The cycle period was related to the time it took for the metro to pass through the point in the soil, and the cycle period was the ratio of the distance between each axle and the vehicle speed.

1. Introduction

With the development of urban underground space, subway rail transit has gradually become the backbone of major cities. At present, there are 37 subway cities in mainland China, and four subway cities are under construction. Nanning Metro Line 1 was the first completed rail transit line in Guangxi Province, and it opened on December 28, 2016. The problem of foundation settlement caused by subway operation has become more and more prominent. The vibration and stress change of soil elements caused by the moving load of a subway have an important impact. At present, there are the Boussinesq solution, the Mindlin solution, the empirical method, and the finite element method for calculating the foundation soil stress under a traffic load. Among them, the Boussinesq solution is suitable for road engineering with a load on a surface [1]. The Mindlin formula can solve for the soil stress at any point in the elastic half-space, so it is suitable for subway engineering at any depth [2]. Ishihara [3] used the Boussinesq solution to simulate ground rail transportation to obtain the stress of a soil unit and the stress path of an inspection point. This method reflected the nature of the rotation of the main stress axis of the soil unit caused by the axle load, but soil element stress of underground traffic could not be considered. Wang et al. [4] studied a series of theoretical problems of the Boussinesq solution and Mindlin solution in depth and in detail. It is believed that the reason for the error of the Boussinesq formula is that the influence of the soil above the load surface on a stress distribution is not considered. The limitation of the Mindlin formula is that when the load surface is not a soil but a foundation, the influence on the stress distribution in the soil needs to be discussed separately. Yang et al. [5] believed that the assumptions of the empirical formula proposed by the American Railway

Engineering Association did not match the actual situation, that the calculation result was relatively high, and that the empirical formula for the calculation of the subgrade stress proposed by the Soviet Union was close to the actual measurement and the difference between the line and the actual when the line was normal or bad. Therefore, it is necessary to find a convenient and practical calculation method.

The instability of the foundation is related to the dynamic effect of a train. There is an error in the pure static solution, and it is difficult to reflect the actual stress of foundation soil. In recent years, underground rail transit has developed rapidly. With the continuous increase of the axle loads and speeds of trains, the dynamic effect has become more intense, so the study of the stress state of the soil inside a foundation under a dynamic load has drawn attention from everyone [6, 7]. Ding et al. [8, 9] analyzed and studied the foundation stress state change caused by subway train operation and the rotation of the main stress axis and the viscoelastic solution of the foundation based on the Mindlin solution in the elastic half-space without considering the effect of dynamic effects on stress. Liu et al. [10] deduced the formula of vertical dynamic stress in subgrade soil under a vehicle dynamic load. They first proposed the concept of the speed coefficient and determined the value through a simulation test. The speed coefficient had nothing to do with the size and weight of the vehicle, and it was about 0.105. The vertical dynamic stress was proportional to the load intensity and vehicle speed. Zhang et al. [11] calculated the dynamic response solution of any point under a train load based on the principle of dynamic reciprocity combined with the generalized Duhamel integral expression. The inverse Floquet transform and the Fourier inverse transform were used. After superposition, the dynamic response of any point under a train load was obtained. The expressions were in the time and frequency domains. Li et al. [12] used the complex variable method, and an analytical solution for the in-situ stress and displacement of a shallow circular tunnel in the elastic half-plane under arbitrary distributed loads in the elastic half-plane was obtained. Gu et al. [13] used the inverse conformal transformation and the Cauchy-Riemann equation. The implicit form of the exact analytical solution based on the complex variable method could be derived from the explicit form of the exact analytical solution. Eason [14] researched the dynamic response of a semi-infinite homogeneous space surface under uniform moving loads. When the velocity of the applied force was lower than the wave velocity, the displacement and stress components of the points below the load action point and the distributed load action area were given. The numerical results of the stress components at the points below the center of the distributed load zone were also given. Grundmann et al. [15] assumed a foundation to be a linear elastic layered half-space. The ordinary differential equation in the vertical direction was obtained through the Fourier transform of the time-space domain, and the wavelet transform was used for error control to analyze the different soils under the motion load of the half-space dynamic response of the layer. Barros et al. [16] proposed a method for calculating the displacement and stress in a layered viscoelastic half-space caused by uniformly moving loads at a surface or a certain depth. This method was based on the wavenumber integral representation of the complete response using the generalization of the displacement and stress fields. The accurate decomposition of the transmission and reflection coefficients took into consideration the effects of layering. Siddharthan et al. [17] studied the dynamic response of layered saturated soil under a moving load. A semi-analytical method with a higher calculation efficiency based on the Biot porous medium formula was proposed for the load under plane strain conditions, which could be used to process complex surface loads such as multiple loads, non-uniform pressure, and other types of time-varying loads. The accuracy of the model was verified through experiments. Alabi [18] gave the parametric research results of the three-dimensional model of a rail traffic vibration load and studied the dynamic response of the moving load speed, train distance, and ground depth to elastic space. The calculation results showed that the displacement was proportional to the moving speed of the load. In all cases, the displacement decreased approximately linearly with the increasing soil depth. Wang et al. [19–21] derived the displacement of the Timoshenko beam in elastic half-space under the action of a moving load and the solution of the ground surface reaction force based on the research of Eason. The steady-state stress caused by the train load in the foundation was obtained when the train speed was less than the Rayleigh wave speed response answer. Hu et al. [22] used the 2.5-dimensional finite element analysis method. The track was simplified to Euler beams, the train load was reduced to single or multiple moving axle loads, the track-embankment-foundation coupling analysis model was established, and the stress of the summarized soil unit body was summarized. The law of rotation of the path and the principal stress axis gave the dynamic response of the track structure and the ground under axial loads moving at various speeds. Chen et al. [23] simplified the vibration effect of rail transit into a uniform load and a single-wheel load, and the stress state changes of the soil element under the two simplified loads were analyzed. The research results showed that the stress path of the soil element was "apple-shaped" in the process of the load from a position close to the soil element to a position far away from the soil element.

Most of the above analyses were related to the dynamic response of a subgrade surface for a high-speed rail or road traffic, or they used pseudo-static solutions to calculate the internal soil stress of the foundation, which were difficult to apply to high-speed underground rail transportation. The greater the train axle load and speed, the violent the train dynamic effect, so the calculation of subway vibration stress considering the dynamic effect is still worthy of study. Based on the above problems, in this study the metro load was simplified as a concentrated force moving in the direction of the metro based on the Mindlin solution of elastic half-space. The stress calculation formula of the dynamic effect analyzed the stress state of the soil at a certain point directly below the moving line of the train under the load of a single axle and multiple sets of axles, and the effects of the vehicle speed, axle load, and buried depth on the stress were studied.

2. Mindlin's analytical solution considering speed

According to Mindelin's research, when the concentrated force P acted on the depth c of the elastic half-space body see Figure 1 , the stress in the soil at any point from the surface depth z in the foundation could be expressed as [24]:

$$\sigma_z = \frac{P}{8\pi(1-\mu)}\left\{\begin{array}{c}\dfrac{(1-2\mu)(z-c)}{R_1^3} - \dfrac{(1-2\mu)(z-c)}{R_2^3} + \dfrac{3(z-c)^3}{R_1^5} + \\ \dfrac{3(3-4\mu)z(z+c)^2 - 3c(z+c)(5z-c)}{R_2^5} + \dfrac{30cz(z+c)^3}{R_2^7}\end{array}\right\} \quad (1)$$

$$R_1 = \sqrt{x^2+y^2+(z-c)^2}$$

$$R_2 = \sqrt{x^2+y^2+(z+c)^2},$$

where σ_z is the stress at any depth directly under the load, z is the depth of a certain point, c is the depth of embedding of the load force, P is the concentration force, μ is Poisson's ratio of the soil layer, and x is the horizontal distance between the load acting point and the calculation point.

The vehicle load was a kind of dynamic load, and it was obviously insufficient to replace its effect on the roadbed with static force only. The stress of the wheel load on the track was related to the lateral bending of the track, the eccentric vertical load, the vehicle speed, and other factors. A dynamic wheel load would result in a dynamic stress value that was higher than the static value. The general method used to determine the wheel load was to empirically express the wheel load as a function of the static wheel load. According to the literature [25], the most comprehensive method for determining the influencing factors is the formula proposed by the International Railway Union Research and Test Office (ORE) based on the measured track results.

The influence coefficient was defined suing three dimensionless velocity coefficients α', β', γ', namely:

$$\phi = 1 + \alpha' + \beta' + \gamma'. \quad (2)$$

Among these coefficients, α' and β' are related to the average value of the impact factor, and γ' is related to the standard deviation of the impact factor. α' depended on the track level, vehicle suspension, and vehicle speed, which was expressed as:

$$\alpha' = 0.04\left(\frac{V}{100}\right)^3. \quad (3)$$

The numerical coefficient of 0.04 mainly depended on the resilience of the vehicle suspension.

According to the calculated value of the SNCF (Societe Nationale des Chemins de Fer Francaise) formula, the value range was 0.13–0.17. According to the observed value, in almost all cases, the measurement coefficient α' on the tangent trajectory was greater than the calculated $\alpha' + \beta'$. Therefore, only α' was regarded as the average value of the impact factor, and β' was ignored.

γ' was related to the vehicle speed, track age, vehicle design, etc. The data showed that γ' increased with the vehicle speed, which could be estimated with the following formula:

$$\gamma' = 0.01 + 0.017\left(\frac{V}{100}\right)^3. \quad (4)$$

By substituting α', γ' into (2), we could obtain:

$$\phi = 1.1 + 0.021\left(\frac{V}{100}\right)^3. \quad (5)$$

In the formula, φ is the dynamic coefficient, and V is the running speed of the train, in units of km/h.

Then, considering the stress of the wheel load on the track under the dynamic effect, the calculation could be simplified as $\sigma_d = \phi \cdot \sigma_z$.

3. Calculation of the vibration stress during the operation of the Nanning subway

3.1. Dynamic stress solution under a single wheel load. The force of the subway was simplified, the wheel load was treated as a concentrated force that ran at a certain speed in the direction of the subway, and the stress state of the foundation was studied based on the Mindlin solution at a certain depth under the load of a single wheel and axle.

Under the load of a single wheel, because the calculated soil element was directly below the load movement line, y = 0. The formula could be simplified to:

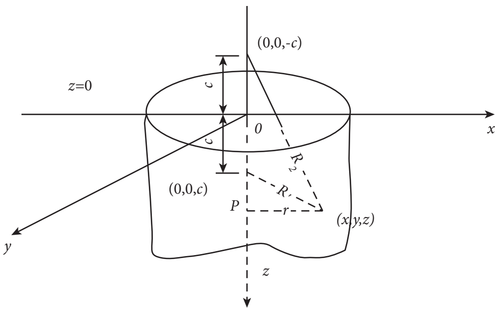

Figure 1: Concentrated force in the soil.

$$\sigma_z = \frac{P}{8\pi(1-\mu)}\left\{\begin{array}{l}\dfrac{(1-2\mu)(z-c)}{\left(x^2+(z-c)^2\right)^{3/2}} - \dfrac{(1-2\mu)(z-c)}{\left(x^2+(z+c)^2\right)^{3/2}} + \dfrac{3(z-c)^3}{\left(x^2+(z-c)^2\right)^{5/2}} + \\ \\ \dfrac{3(3-4\mu)z(z+c)^2-3c(z+c)(5z-c)}{\left(x^2+(z+c)^2\right)^{5/2}} + \dfrac{30cz(z+c)^3}{\left(x^2+(z+c)^2\right)^{7/2}}\end{array}\right\}. \tag{6}$$

The stress of the single-wheel load on the soil element is shown in Figure 2. The foundation soil was homogeneous soil, and the axle load depth was c = 12 m. The calculated soil element was located 4 m directly under the tunnel ($x = 0$, $y = 0$, z = 16 m). The subway ran at v = 40 km/h, and the horizontal distance between the single wheel and the calculated soil element unit was -33 m until the single wheel left the calculated soil element unit by a distance of 33 m. The distance between the load and the calculated soil element simulated the movement of the load.

According to the Mindlin solution at any point in the elastic half-space, the shear stress $\tau_{xy} = 0$ and $\tau_{yz} = 0$ of the soil element were directly under the load. Then only the stress state on the x-z plane was studied. As can be seen in Figure 3, the stress σ_z increased continuously as the wheel load approached the soil unit, and the stress reached a maximum value of 0.56 kPa when it was directly above the soil element. The stress value gradually decreased toward 0 when it was away from the soil unit, and its graph was symmetrical about the z-axis. The stress σ_x graph was symmetrical about the z-axis, and the stress reached a maximum value of 0.093 kPa at a distance of approximately 5 m from the soil unit. The shear stress τ_{xz} gradually increased as the train approached the soil unit, it reached a maximum value of 0.16 kPa at a distance of approximately 2 m from the soil unit, and then it was reduced to 0. The graph was antisymmetric about the z-axis. It could be seen that when the load was close to the soil element, the stress changed greatly, and σ_z was much larger than τ_{xz} and σ_x, and when the load was far from the soil unit, the stress value was small.

Based on the above research, the stress of the soil at different burial depths was calculated. Figures 4 and 5 show the calculation of the stress change of the soil unit when the burial depths were 9 m, 12 m, and 15 m. It can be seen from the figures that when the load was far away, the stress value of the soil unit was very small. Within the horizontal distance range of -2 m to 2 m from the soil unit, the stress value changed significantly, and it gradually increased as it approached the soil unit. The maximum values of σ_z and τ_{xz} were obtained at the horizontal distances of 0 m and ±1 m from the soil unit, respectively. When the buried depth was 9 m, σ_{zmax} = 0.20 kPa and τ_{xzmax} = 0.05 kPa. When the buried depth was 12 m, σ_{zmax} = 0.57 kPa and τ_{xzmax} = 0.15 kPa. At a buried depth of 15 m, σ_{zmax} = 8.74 kPa and τ_{xzmax} = 2.49 kPa. It could be seen that the soil stress was proportional to the buried depth, the greater the depth of the tunnel, the greater the stress value, and the more rapid the stress change. Therefore, the horizontal and vertical loads were smaller, the distance from the soil was smaller, and the impact on the stress state of the soil was greater.

3.2. Dynamic stress solution under multi-wheel load. The Nanning Metro Line 1 subway train used a 6B formation, which meant that the metro was arranged with six type B cars and the type B cars had a length of 19 m, a fixed distance of 12.6 m, a fixed wheelbase of 2.2 m, an axle weight of 14 t, and a maximum operating speed of 80 km/h. The train model is shown in Figure 6.

The train load acted on the foundation soil through the wheels, and the stress of the soil unit was calculated as the superposition of the stress generated by each wheel load at the point of action. The train had four pairs of wheels in each car, and the total stress value was the stress accumulation of 24 pairs of wheel pairs at the calculated point. The stress calculation formula was:

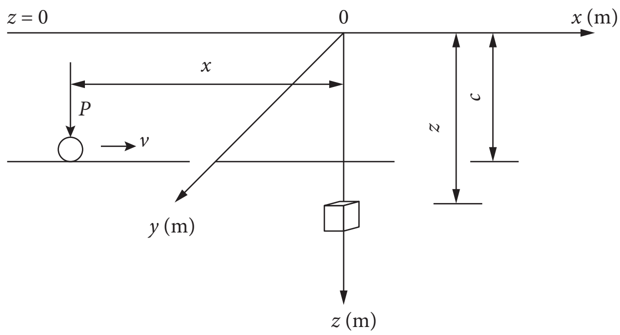

FIGURE 2: Stress state of the soil element under a single wheel load.

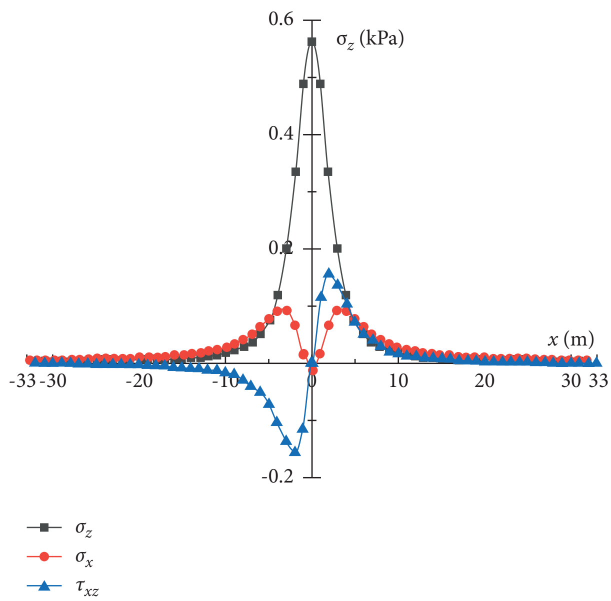

FIGURE 3: Changes of the soil stress state under a single wheel load.

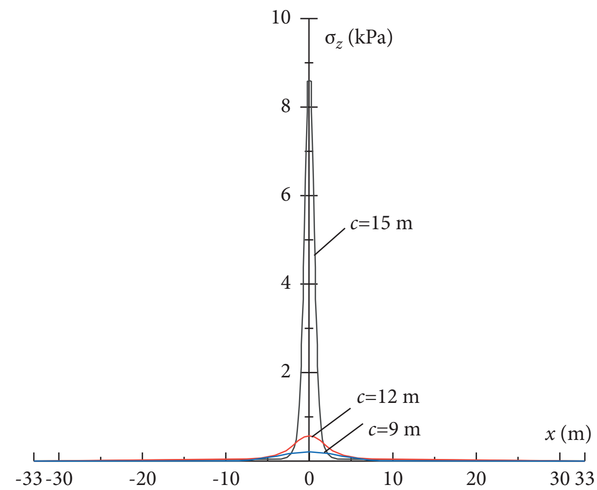

FIGURE 4: σ_z at different burial depths.

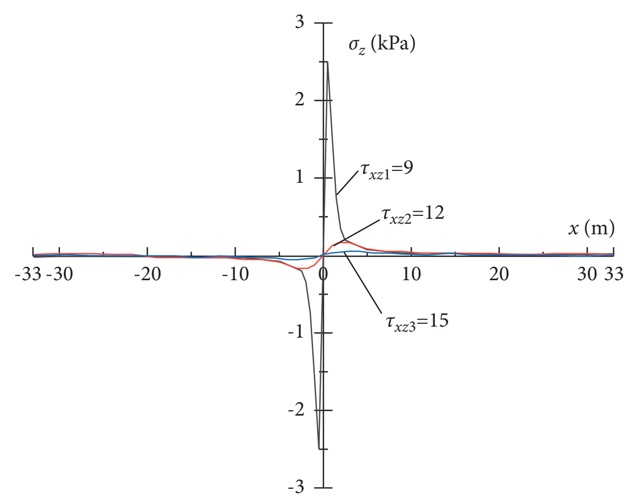

FIGURE 5: τ_{xz} at different burial depths.

$$\sigma_z = \sum \sigma_{z_i}$$

$$= \sum \frac{P}{8\pi(1-\mu)} \left\{ \begin{array}{c} \dfrac{(1-2\mu)(z-c)}{\left(x_i^2+(z-c)^2\right)^{3/2}} - \dfrac{(1-2\mu)(z-c)}{\left(x_i^2+(z+c)^2\right)^{3/2}} \\ + \dfrac{3(z-c)^3}{\left(x_i^2+(z-c)^2\right)^{5/2}} + \dfrac{3(3-4\mu)z(z+c)^2 - 3c(z+c)(5z-c)}{\left(x_i^2+(z+c)^2\right)^{5/2}} \\ + \dfrac{30cz(z+c)^3}{\left(x_i^2+(z+c)^2\right)^{7/2}} \end{array} \right\}. \quad (7)$$

3.2.1. Different loads. The subway train load depth was taken as $c = 12$ m, and it was calculated that the soil unit was located 4 m directly under the tunnel ($x = 0$, $y = 0$, $z = 16$ m). When the train ran at a speed of $v = 80$ km/h and when the axle weights were 11 t, 14 t, and 16 t, the stress state of the soil changed. The train ran at $v = 80$ km/h, and the head distance was calculated from the soil unit as -33 m until the car left the soil unit for a distance of 33 m, that is, for t =-4 s to $t = 4$ s. The stress state change of the soil element was calculated, and the load movement was simulated with the distance change between the load and the calculated soil element. The calculation result is shown in Figure 7.

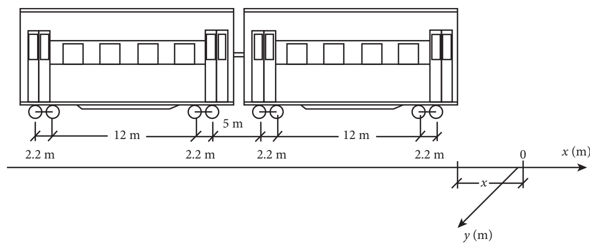

Figure 6: Schematic diagram of the vehicle model.

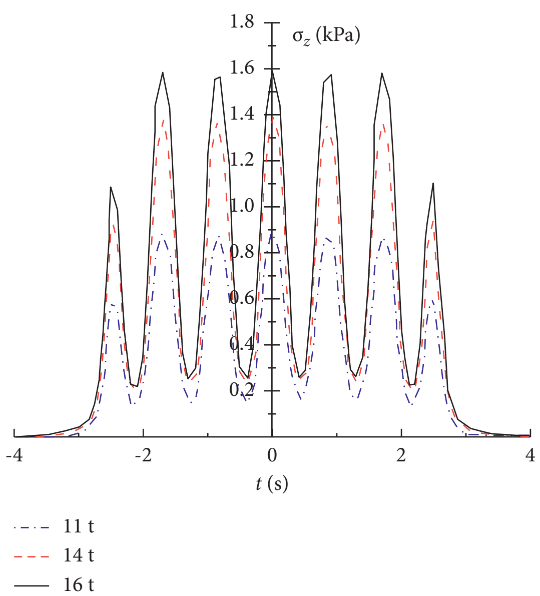

Figure 7: σ_z (o)f soil with different axes.

3.2.2. Different buried depth. The depths of the load were taken as $c = 9$ m, 12 m, and 15 m. The calculated soil unit was located 4 m directly under the tunnel ($x = 0$, $y = 0$, $z = 16$ m). The axle load of the train was 14 t. When running at a speed of $v = 80$ km/h, the change of the soil stress state was studied.

3.2.3. Different speeds. The axle load of the train was 14 t, and the buried depth was 12 m. The initial distance between the train and the soil unit was -33 m, when the train ran through the soil unit at speeds of 40 km/h, 80 km/h, and 120 km/h. At different speeds, the times for the vehicles passing through the soil unit at different speeds were 16 s, 8 s, and 4 s. The stress state of the calculated unit is shown in Figure 9.

It can be seen in Figures 7 and 9 that under different operating conditions, the stress of the soil unit of the metro load always had a continuous cycle characteristic, and the dynamic stress value of each cycle period was not completely equal. The stress state change at the calculation point in the process of moving away from the soil unit was symmetrical; that is, where the horizontal distance from the soil body was equal, the stress value was almost equal.

The cycle period was related to the length of the metro and the speed; that is, the faster the metro passed through the soil unit, the shorter of the cycle period. The rear wheel of each car and the front wheel of the next car were regarded as a group. Each group of the wheel axle caused a group of stress cycles. The first car front wheel and the last car rear wheel caused a group of cycles, for a total of seven cycles. The number of cycles was related to the number of wheels and axles. The peak stress in each cycle that appeared in the wheel shaft reached directly above the soil element. The closer the wheel shaft to the soil element, the greater the influence on the stress of the soil element. Therefore, the stress was related to the burial depth of the tunnel. In addition, the stress was positively related to the magnitude of the load applied to the soil. The load was related to the axle weight and speed.

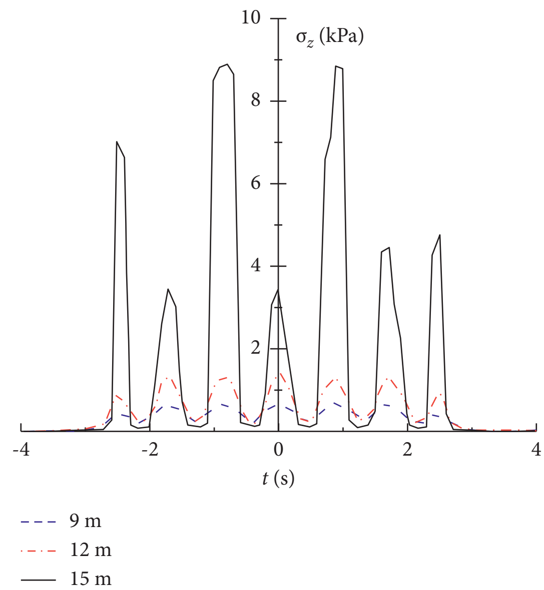

Figure 8: σ_z of soils with different depths.

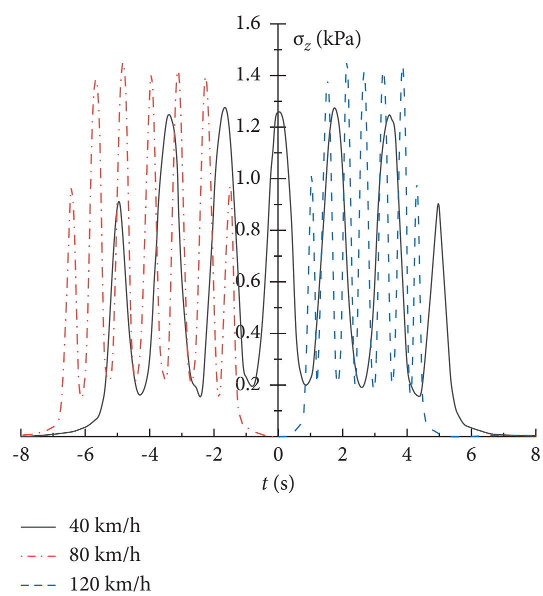

Figure 9: σ_z of soil at different speeds.

It can be seen in Figure 7 that when the axle loads of the train were 11 t, 14 t, and 16 t, the corresponding peak stresses of the soil were 0.90 kPa, 1.40 kPa, and 1.60 kPa, respectively, and the stress increased with the increase of the axle load. The stress that was exerted by the lower wheel on the track was proportional to the train speed. As shown in Figure 8, when the train speeds were 40 km/h, 80 km/h, and 120 km/h, the corresponding peak stresses of the calculated soil unit were 1.26 kPa, 1.43 kPa, and 1.60 kPa.

It can be seen in Figures 4 and 9 that the effects of different tunnel burial depths on the stresses of the soil elements under multi-wheel loads and the dynamic responses under single wheel loads showed a proportional relationship between the stress and the burial depth. Additionally, the soil stress at the depth of 15 m was significantly greater than the soil stresses at the depths of 12 m and 9 m.

4. Conclusions

In this study, based on the Mindlin solution, the change of the stress state of the foundation under a single wheel load and multiple wheel loads under different conditions was calculated. The law of the soil stress at a certain point directly below the moving line of the train load was as follows.

1. Under the load of a single wheel, the stress σ_z reached a maximum value of 0.56 kPa when the axle was directly above the soil element, and the stress decreased until it approached zero as the distance from the soil unit increased. The stress value and the burial depth had a positive correlation, and the σ_{zmax} values corresponding to 9 m, 12 m, and 15 m were 0.20 kPa, 0.57 kPa, and 8.74 kPa, respectively. Therefore, the greater the burial depth, the greater the stress value, and the more significant the stress change.

2. When a multi-wheel load was applied, the stresses of the soil passing through the soil unit under different working conditions all exhibited continuous cycle characteristics. The number of cycles was related to the number of wheels, and the cycle period corresponded to the time it took for the wheel loads to pass through the soil element, which was related to the length of the metro and the speed of the vehicle.

3. Under the load of multiple wheels, the soil stress peaked when each train axle was directly above the calculated soil element. When the metro axle weights were 11 t, 14 t, and 16 t, the corresponding peak soil stresses were 0.90 kPa, 1.40 kPa, and 1.60 kPa. When the train speeds were 40 km/h, 80 km/h, and 120 km/h, the corresponding peak stress values of the calculated soil unit were 1.26 kPa, 1.40 kPa, and 1.48 kPa, respectively. The corresponding peaks of the soil stress at the burial depths of 9 m, 12 m, and 15 m were 0.72 kPa, 1.40 kPa, and 8.9 kPa, respectively. It could be seen that the magnitude of the stress was directly proportional to the axle load, speed, and burial depth of the tunnel.

In this paper, the calculated and analyzed metro axle weights are 11 t, 14 t, and 16 t, and the metro speeds are 40 km/h, 80 km/h, and 120 km/h. The vibration stress under other metro axle weights and speeds needs to be further studied.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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